Binning scheme for γ measurement in $B^{\pm} \to (K^{+}K^{-}\pi^{+}\pi^{-})_{D}K^{\pm}$ decays

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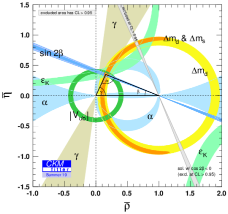


Outline

1. Current progress
2. Unbinned fit
3. Binned fit and pull studies
4. Attempts at better binning scheme
   - Rectangular binning scheme
   - Amplitude model binning
   - Amplitude model binning optimized for interference
5. Summary

Current progress

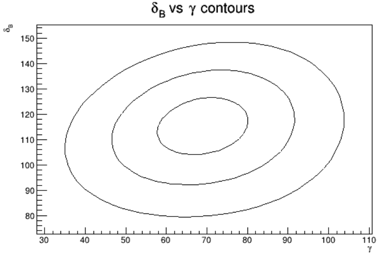
- Events generated in AmpGen using amplitude model arXiv:1811.08304
  - Assumed event yield: 2000 (1000 B^+, 1000 B^-)
  - Assumed parameters in toy model: $\gamma = 75°$, $\delta_B = 130°$, $r_B = 0.1$
- Unbinned fit using SimFit in AmpGen
- Binned fit and pull studies
- Develop suitable binning scheme

Unbinned fit with amplitude model

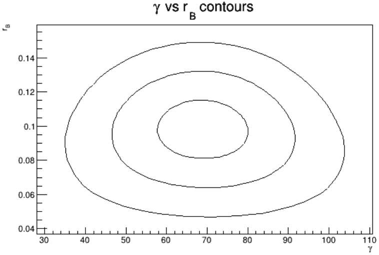
$$\mathcal{A}(B^- \to (K^+K^-\pi^+\pi^-)_D K^-) = \mathcal{A}_B \mathcal{A}(D^0 \to K^+K^-\pi^+\pi^-) + \mathcal{A}_B \mathcal{A}(\bar{D}^0 \to K^+K^-\pi^+\pi^-) r_B e^{i(\delta_B - \gamma)}$$

- $\mathcal{A}(D \to K^+K^-\pi^+\pi^-)$ obtained from amplitude model
- Fit with γ, δ_B and r_B as free parameters
- Results from unbinned fit of 2×10^3 events:
  - $\gamma = (69 \pm 11)^\circ$
  - $\delta_B = (115 \pm 11)^\circ$
  - $r_B = 0.098 \pm 0.017$
- Pulls of γ, δ_B and r_B all have mean 0 and std 1 (see backup slides)

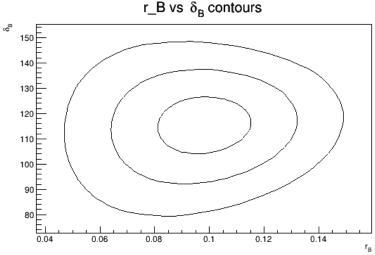
Unbinned fit of 2×10^3 events with amplitude model



(a) γ vs δ_B



(b) γ vs r_B



(c) r_B vs δ_B

Binned fit of $D \to K^+K^-\pi^+\pi^-$

$$\mathcal{A}(B^- \to (K^+K^-\pi^+\pi^-)_D K^-) = \mathcal{A}_B \mathcal{A}(D^0 \to K^+K^-\pi^+\pi^-) + \mathcal{A}_B \mathcal{A}(\bar{D}^0 \to K^+K^-\pi^+\pi^-) r_B e^{i(\delta_B - \gamma)}$$

Event yield in bin i

$$N_i^- = h_{B^-}\left(K_i + (x_-^2 + y_-^2)\bar{K}_i + 2\sqrt{K_i\bar{K}_i}(x_- c_i + y_- s_i)\right)$$
$$N_i^+ = h_{B^+}\left(\bar{K}_i + (x_+^2 + y_+^2)K_i + 2\sqrt{K_i\bar{K}_i}(x_+ c_i - y_+ s_i)\right)$$

CP-violating observables

$$x_\pm = r_B \cos(\delta_B \pm \gamma), \quad y_\pm = r_B \sin(\delta_B \pm \gamma)$$

Pull studies

- Used an arbitrary and naive binning scheme with 4 bins
- $x_\pm$ pulls show asymmetric tails for 2×10^3 events
- Pulls for γ, δ_B, r_B are rubbish

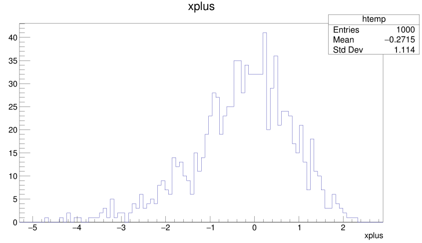
Naive binning scheme

Split phase space along the boundaries $E_{K^+} = E_{K_-}$ and $E_{\pi^+} = E_{\pi^-}$
Bin 1: $E_{K^+} > E_{K^-}, \quad E_{\pi^+} > E_{\pi^-}, \ldots$

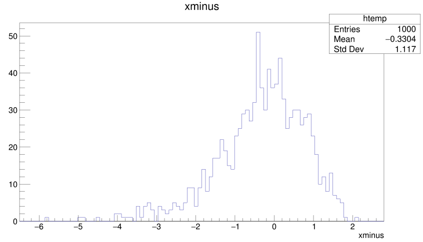
D decay hadronic parameters

$$c_i = \frac{\int_i \mathrm{d}\Phi\, |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)| \cos(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi\, |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}}, \quad K_i = \frac{\int_i \mathrm{d}\Phi\, |\mathcal{A}(D^0)|^2}{\sum_j \int_j \mathrm{d}\Phi\, |\mathcal{A}(D^0)|^2}$$

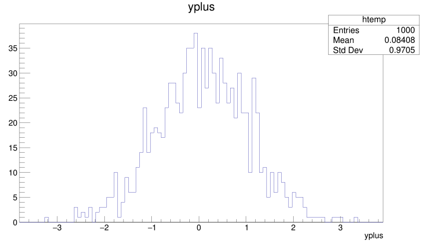
Naive binning pull study



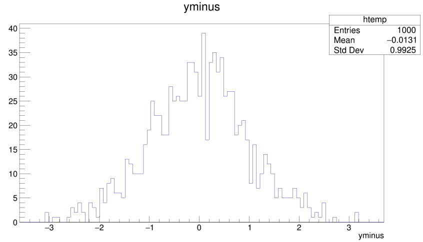
(a) x_+ pull



(b) x_- pull



(c) y_+ pull



(d) y_- pull

Rectangular binning scheme

- Inspired by arXiv:1709.03467
- 4-body phase space is 5-dimensional
- Convenient to choose rectangular coordinates

Phase space parameterisation

$$
\begin{aligned}
x_1 &= m(K^+\pi^+) + \alpha \\
x_2 &= m(K^-\pi^-) + \alpha, \quad \alpha = \min\big(m(K^+\pi^+), m(K^-\pi^-)\big) - m_\pi - m_K \\
x_3 &= \cos(\theta_+), \quad \text{(Helicity angles)} \\
x_4 &= \cos(\theta_-) \\
x_5 &= \phi
\end{aligned}
$$

- Study phase space in terms of these coordinates

Rectangular (x_3, x_4) plane

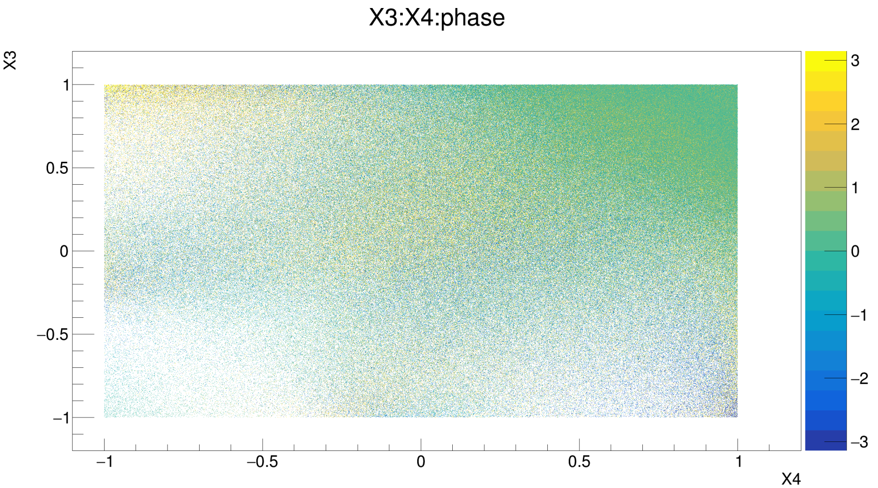


Figure 3: Strong phase differences in the (x_3, x_4) plane

Rectangular (x_3, x_4) plane

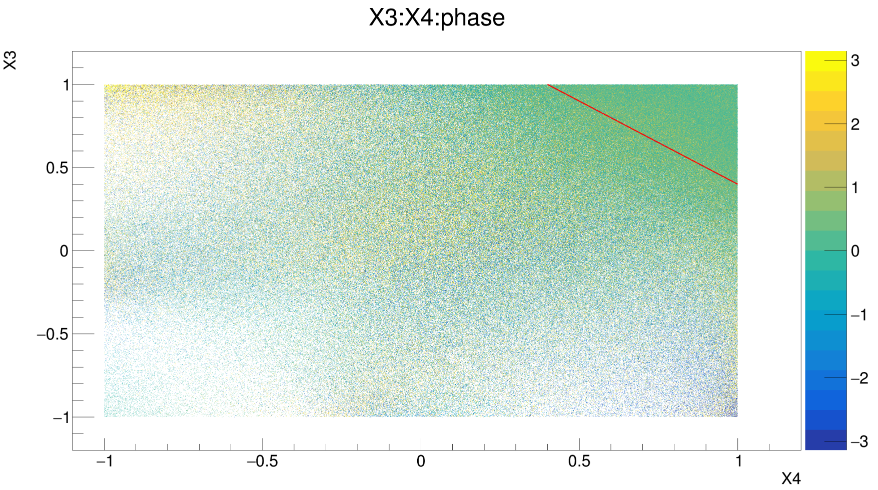


Figure 4: Strong phase differences in the (x_3, x_4) plane

Rectangular (x_3, x_4) plane

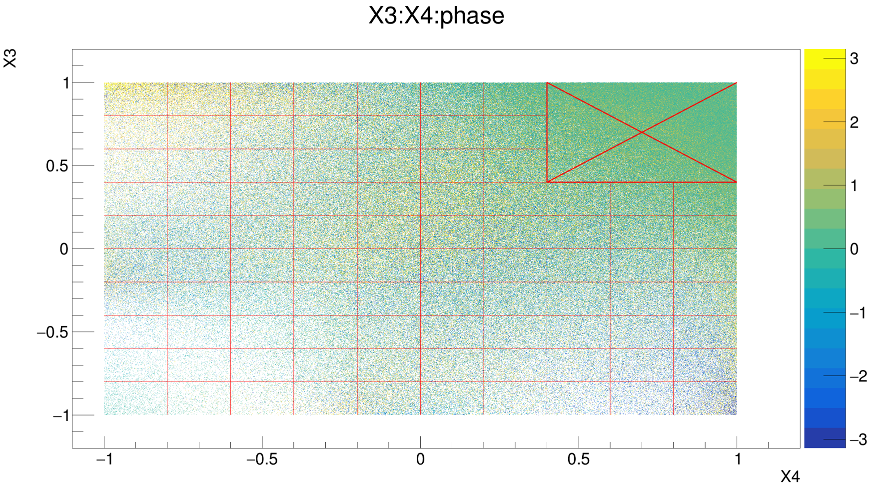
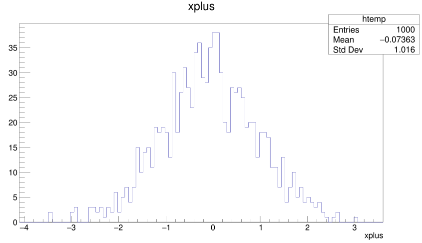


Figure 5: Strong phase differences in the (x_3, x_4) plane

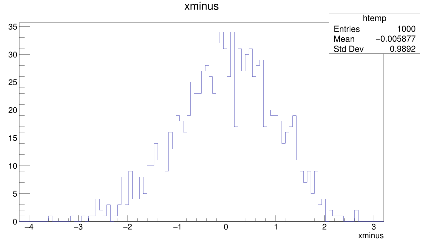
Description of rectangular scheme

- Define upper right corner as bin 0 and 1
- Divide the (x_3, x_4) plane into regions
- Make a $100 \times 100 \times 100$ grid in (x_1, x_2, x_5) volume
- For each gridpoint in (x_1, x_2, x_5), average strong phases over (x_3, x_4)
- Sort each gridpoint into one of 6 bins according to strong phase
- $6 + 2 = 8$ bins

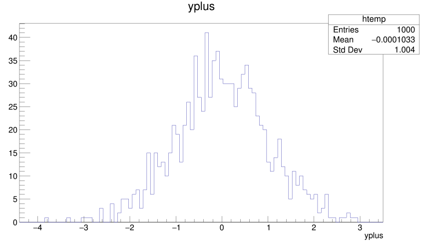
Rectangular binning pull study



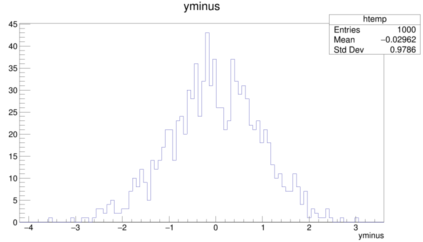
(a) x_+ pull



(b) x_- pull

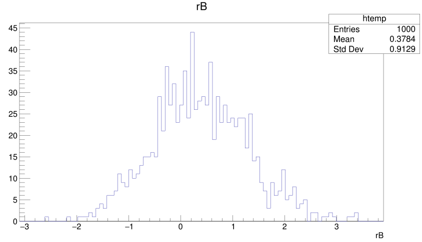


(c) y_+ pull

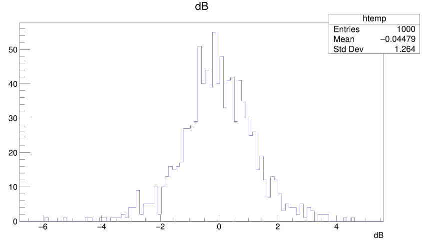


(d) y_- pull

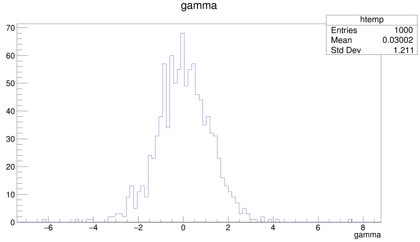
Rectangular binning pull study



(a) r_B pull



(b) δ_B pull



(c) γ pull

Fitted γ values

γ precision of 21°

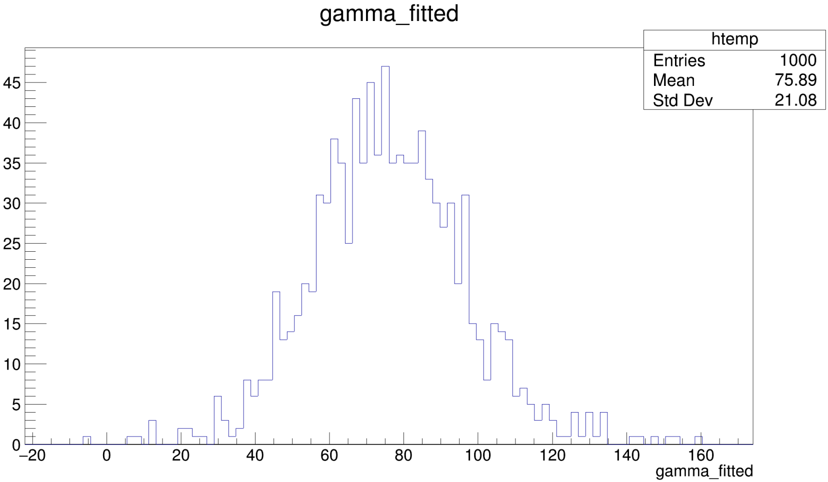


Figure 8: Histogram of fitted γ values

Amplitude model binning

- Calculate strong phase difference of each event
- Divide into evenly spaced bins according to their strong phase difference
- 8 bins

Amplitude model binning

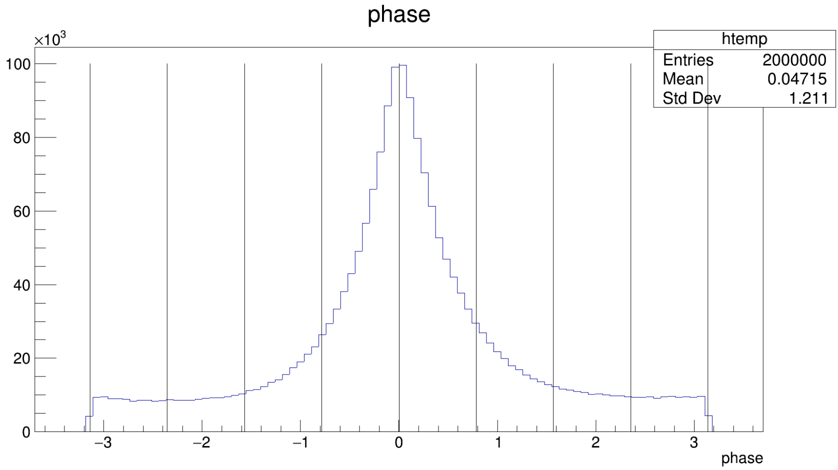
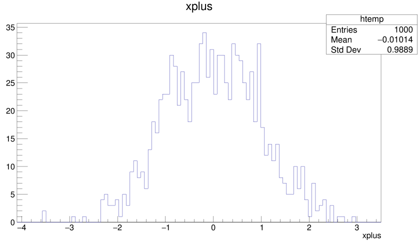
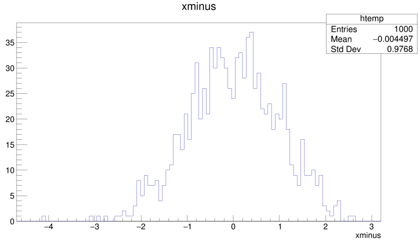


Figure 9: Histogram of strong phases, with black lines indicating bins

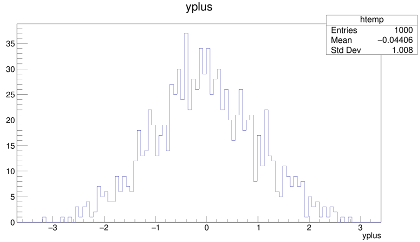
Amplitude model binning pull study



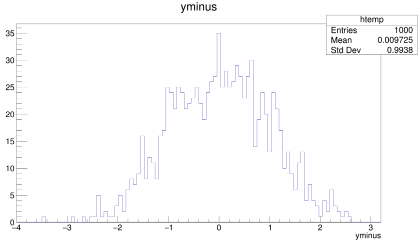
(a) x_+ pull



(b) x_- pull

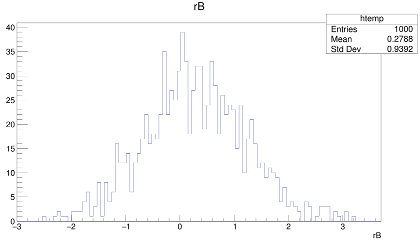


(c) y_+ pull

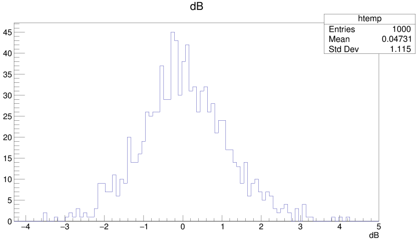


(d) y_- pull

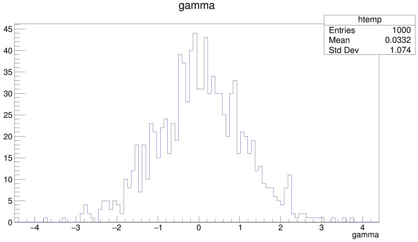
Amplitude model binning pull study



(a) r_B pull



(b) δ_B pull



(c) γ pull

Fitted γ values

γ precision of 15.4°

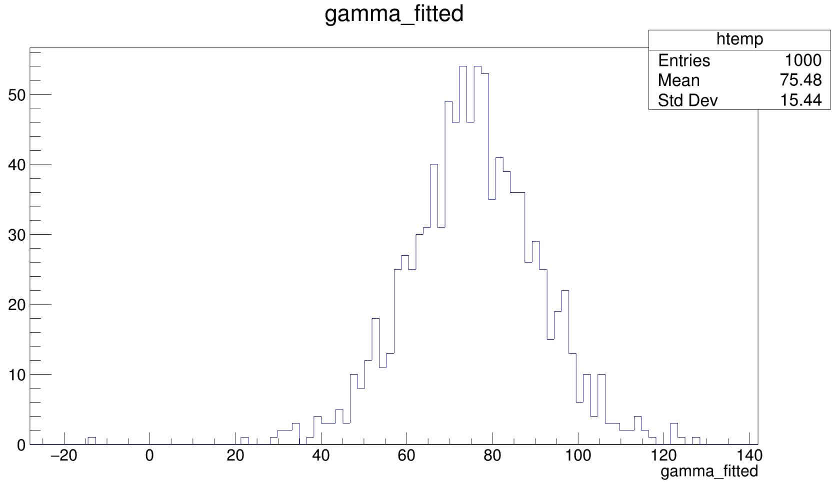


Figure 12: Histogram of fitted γ values

Maximize interference terms

Event yield in bin i

$$N_i^- = h_{B^-}\left(K_i + \left(x_-^2 + y_-^2\right)\bar{K}_i + 2\sqrt{K_i\bar{K}_i}\left(x_- c_i + y_- s_i\right)\right)$$
$$N_i^+ = h_{B^+}\left(\bar{K}_i + \left(x_+^2 + y_+^2\right)K_i + 2\sqrt{K_i\bar{K}_i}\left(x_+ c_i - y_+ s_i\right)\right)$$

- Difference of interference terms $\propto \sin(\delta_D + \gamma)$
- Choose bins such that $\sin(\delta_D + \gamma)$ always has the same sign in each bin
- 8 bins

Optimized amplitude model binning

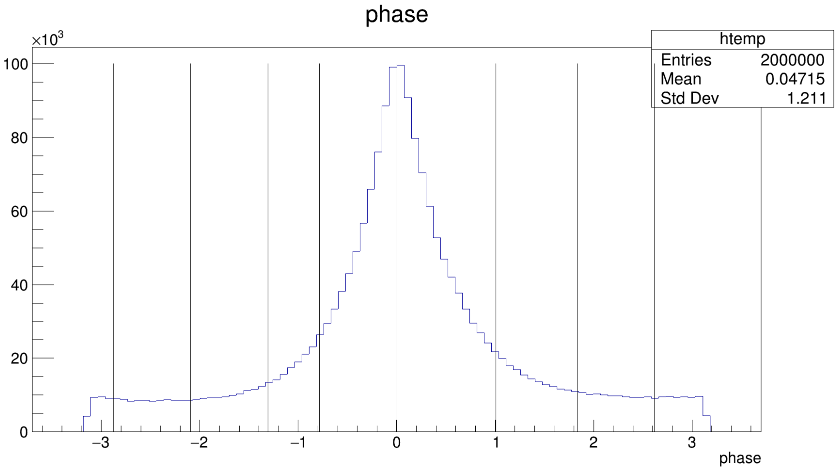
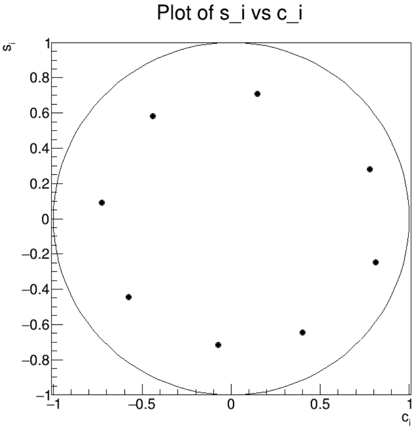
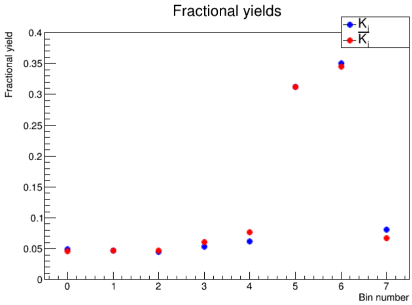


Figure 13: Histogram of strong phases, with black lines indicating bins

Optimized amplitude model binning D hadronic parameters

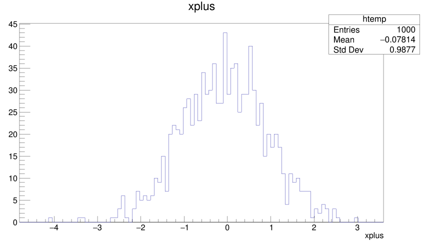


(a) c_i vs s_i

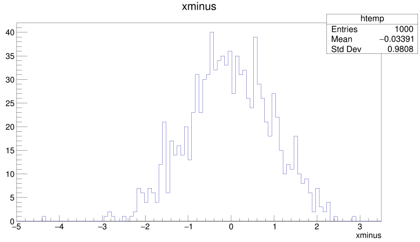


(b) K_i and $\bar{K}_i$

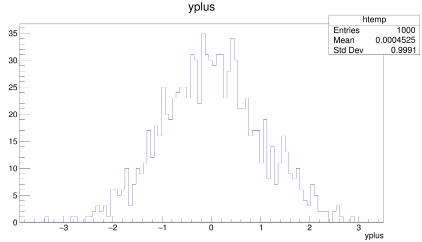
Optimized amplitude model binning pull study



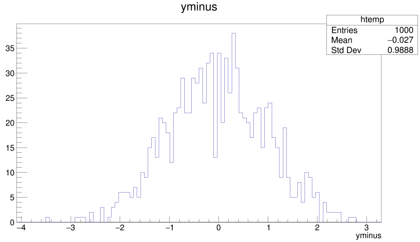
(a) x_+ pull



(b) x_- pull

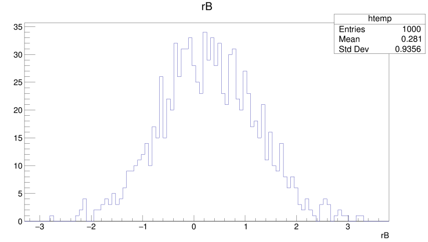


(c) y_+ pull

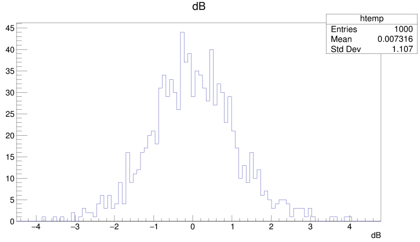


(d) y_- pull

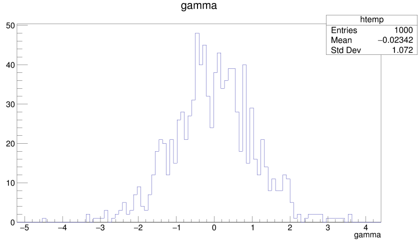
Optimized amplitude model binning pull study



(a) r_B pull



(b) δ_B pull



(c) γ pull

Fitted γ values

γ precision of 15.1° Can reach 14.8° with > 30 bins

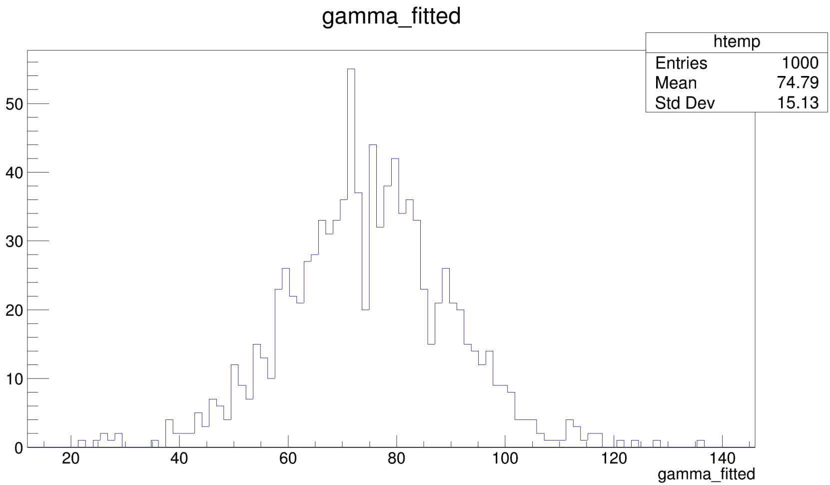


Figure 17: Histogram of fitted γ values

Summary and next steps

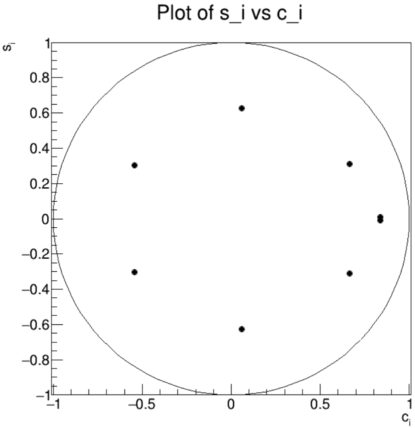
Summary:

- Unbinned fit: 11° precision with 2000 events
- Binned fit: 15.1° with 8 bins
- Can reach 14.8° with > 30 bins

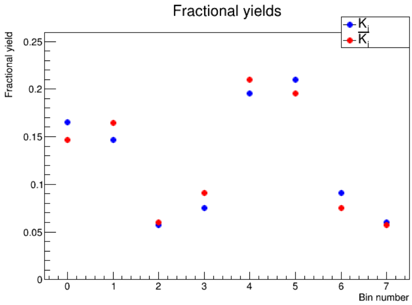
Any suggestions for improving the binning scheme? Why does the binned fit precision not approach 11°?

Backup slides

Rectangular D hadronic parameters

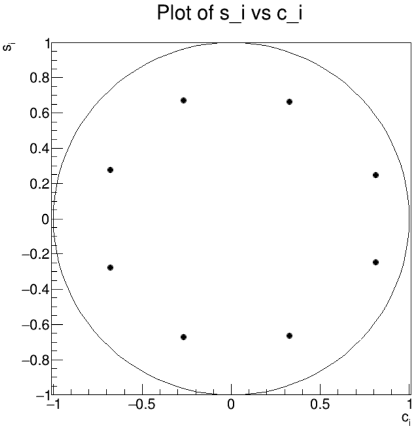


(a) c_i vs s_i

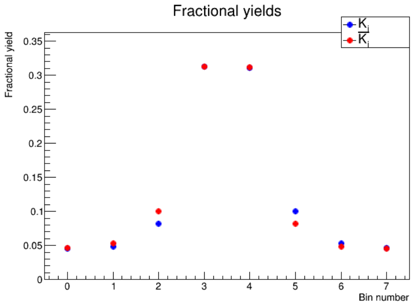


(b) K_i and $\bar{K}_i$

Amplitude D hadronic parameters

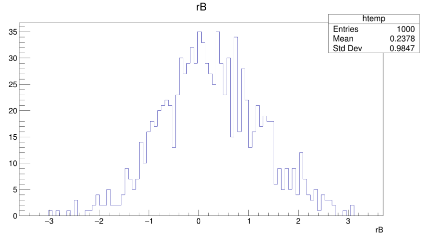


(a) c_i vs s_i

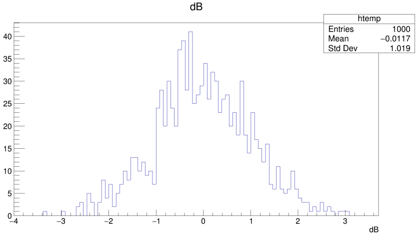


(b) K_i and $\bar{K}_i$

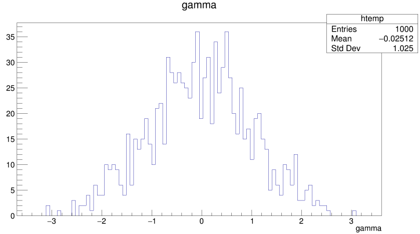
Pull study of unbinned fit



(a) r_B pull



(b) δ_B pull



(c) γ pull

Fitted γ values

γ precision of 10.6°

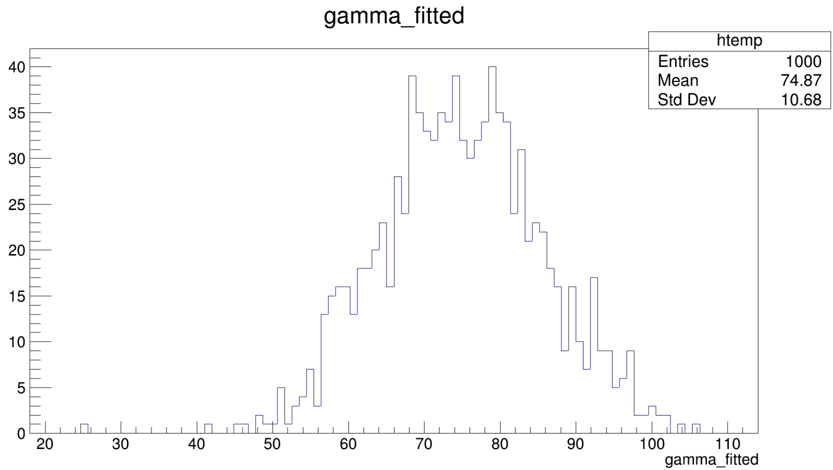


Figure 21: Histogram of fitted γ values