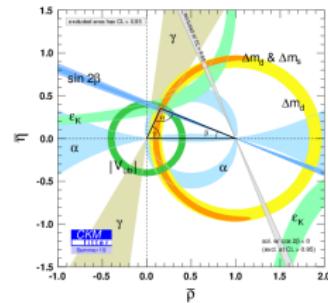


# Binning scheme for $\gamma$ measurement in $B^\pm \rightarrow (K^+ K^- \pi^+ \pi^-)_D K^\pm$ decays

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# Outline

- 1 Current progress
- 2 Unbinned fit
- 3 Binned fit and pull studies
- 4 Attempts at better binning scheme
  - Rectangular binning scheme
  - Amplitude model binning
  - Amplitude model binning optimized for interference
- 5 Summary

# Current progress

- Events generated in AmpGen using amplitude model  
[arXiv:1811.08304](https://arxiv.org/abs/1811.08304)
  - Assumed event yield: 2000 (1000  $B^+$ , 1000  $B^-$ )
  - Assumed parameters in toy model:  $\gamma = 75^\circ$ ,  $\delta_B = 130^\circ$ ,  $r_B = 0.1$
- Unbinned fit using SimFit in AmpGen
- Binned fit and pull studies
- Develop suitable binning scheme

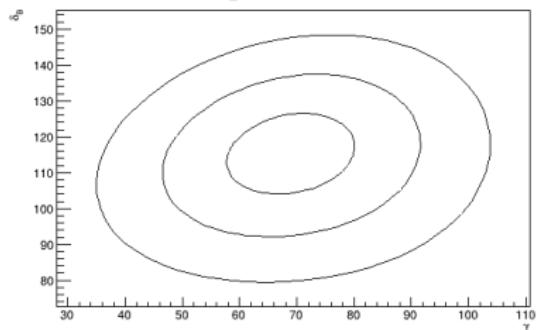
# Unbinned fit with amplitude model

$$\begin{aligned}\mathcal{A}(B^- \rightarrow (K^+ K^- \pi^+ \pi^-)_D K^-) = & \mathcal{A}_B \mathcal{A}(D^0 \rightarrow K^+ K^- \pi^+ \pi^-) \\ & + \mathcal{A}_B \mathcal{A}(\bar{D}^0 \rightarrow K^+ K^- \pi^+ \pi^-) r_B e^{i(\delta_B - \gamma)}\end{aligned}$$

- $\mathcal{A}(D \rightarrow K^+ K^- \pi^+ \pi^-)$  obtained from amplitude model
- Fit with  $\gamma$ ,  $\delta_B$  and  $r_B$  as free parameters
- Results from unbinned fit of  $2 \times 10^3$  events:
  - $\gamma = (69 \pm 11)^\circ$
  - $\delta_B = (115 \pm 11)^\circ$
  - $r_B = 0.098 \pm 0.017$
- Pulls of  $\gamma$ ,  $\delta_B$  and  $r_B$  all have mean 0 and std 1 (see backup slides)

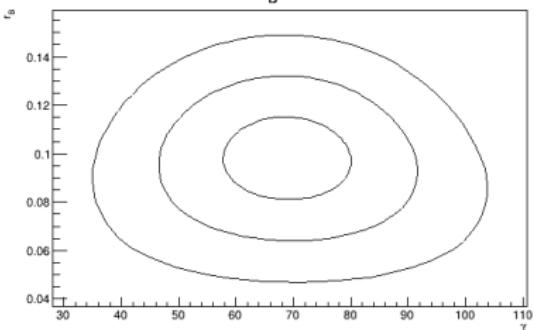
# Unbinned fit of $2 \times 10^3$ events with amplitude model

$\delta_B$  vs  $\gamma$  contours



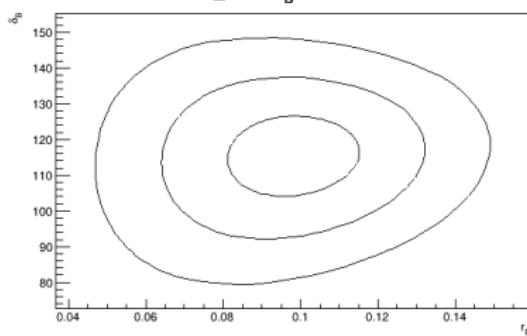
(a)  $\gamma$  vs  $\delta_B$

$\gamma$  vs  $r_B$  contours



(b)  $\gamma$  vs  $r_B$

$r_B$  vs  $\delta_B$  contours



(c)  $r_B$  vs  $\delta_B$

## Binned fit of $D \rightarrow K^+ K^- \pi^+ \pi^-$

$$\begin{aligned}\mathcal{A}(B^- \rightarrow (K^+ K^- \pi^+ \pi^-)_D K^-) = & \mathcal{A}_B \mathcal{A}(D^0 \rightarrow K^+ K^- \pi^+ \pi^-) \\ & + \mathcal{A}_B \mathcal{A}(\bar{D}^0 \rightarrow K^+ K^- \pi^+ \pi^-) r_B e^{i(\delta_B - \gamma)}\end{aligned}$$

Event yield in bin  $i$

$$\begin{aligned}N_i^- &= h_{B^-} \left( K_i + (x_-^2 + y_-^2) \bar{K}_i + 2\sqrt{K_i \bar{K}_i} (x_- c_i + y_- s_i) \right) \\ N_i^+ &= h_{B^+} \left( \bar{K}_i + (x_+^2 + y_+^2) K_i + 2\sqrt{K_i \bar{K}_i} (x_+ c_i - y_+ s_i) \right)\end{aligned}$$

CP-violating observables

$$x_\pm = r_B \cos(\delta_B \pm \gamma), \quad y_\pm = r_B \sin(\delta_B \pm \gamma)$$

# Pull studies

- Used an arbitrary and naive binning scheme with 4 bins
- $x_{\pm}$  pulls show asymmetric tails for  $2 \times 10^3$  events
- Pulls for  $\gamma$ ,  $\delta_B$ ,  $r_B$  are rubbish

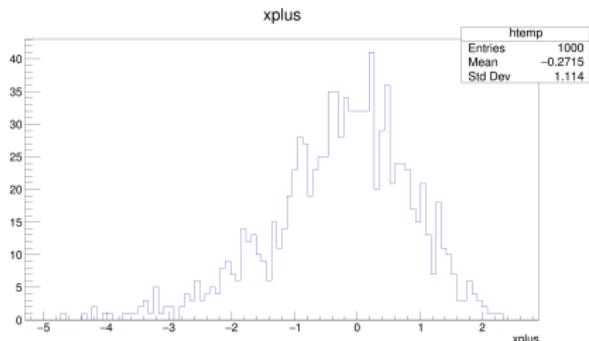
## Naive binning scheme

Split phase space along the boundaries  $E_{K^+} = E_{K^-}$  and  $E_{\pi^+} = E_{\pi^-}$   
Bin 1:  $E_{K^+} > E_{K^-}$ ,  $E_{\pi^+} > E_{\pi^-}$ , ...

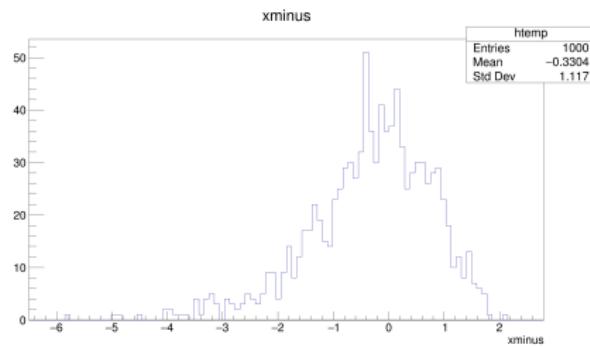
## $D$ decay hadronic parameters

$$c_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D}^0)| \cos(\delta_D)}{\sqrt{\int_i d\Phi |\mathcal{A}(D^0)|^2 \int_i d\Phi |\mathcal{A}(\bar{D}^0)|^2}}, \quad K_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j d\Phi |\mathcal{A}(D^0)|^2}$$

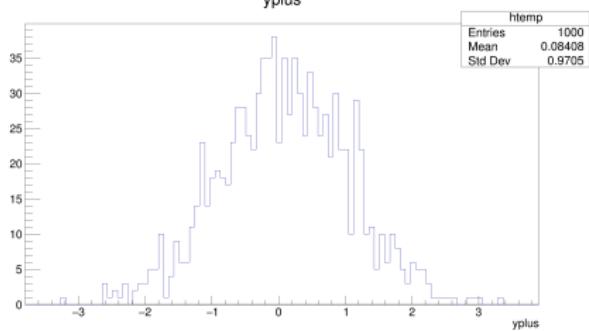
# Pull study with $2 \times 10^3$ events



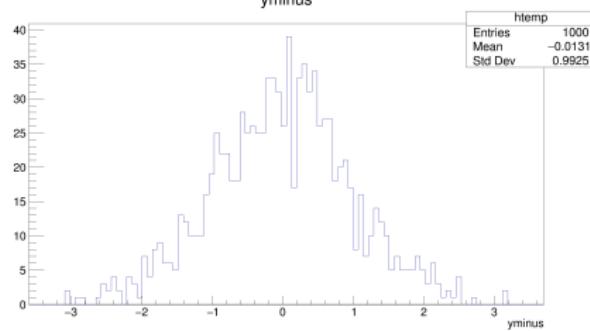
(a)  $x_{+}$  pull



(b)  $x_{-}$  pull



(c)  $y_{+}$  pull



(d)  $y_{-}$  pull

# Rectangular binning scheme

- Inspired by [arXiv:1709.03467](#)
- 4-body phase space is 5-dimensional
- Convenient to choose rectangular coordinates

## Phase space parameterisation

$$x_1 = m(K^+ \pi^+) + \alpha$$

$$x_2 = m(K^- \pi^-) + \alpha, \quad \alpha = \min(m(K^+ \pi^+), m(K^- \pi^-)) - m_\pi - m_K$$

$$x_3 = \cos(\theta_+), \quad (\text{Helicity angles})$$

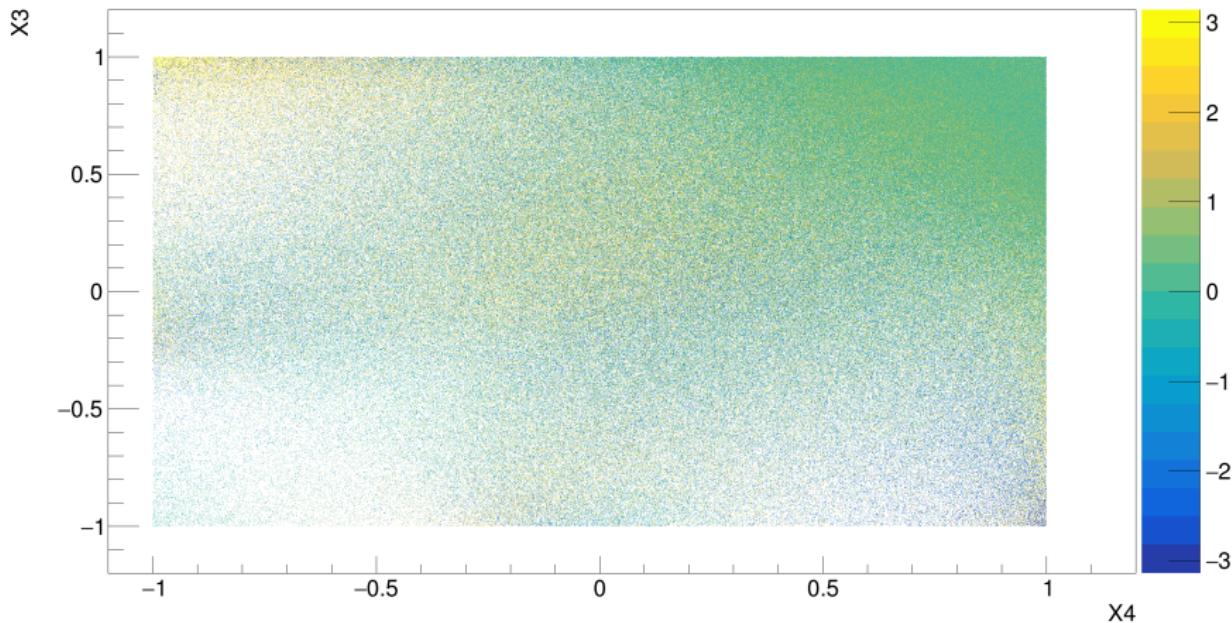
$$x_4 = \cos(\theta_-)$$

$$x_5 = \phi$$

- Study phase space in terms of these coordinates

# Rectangular binning scheme

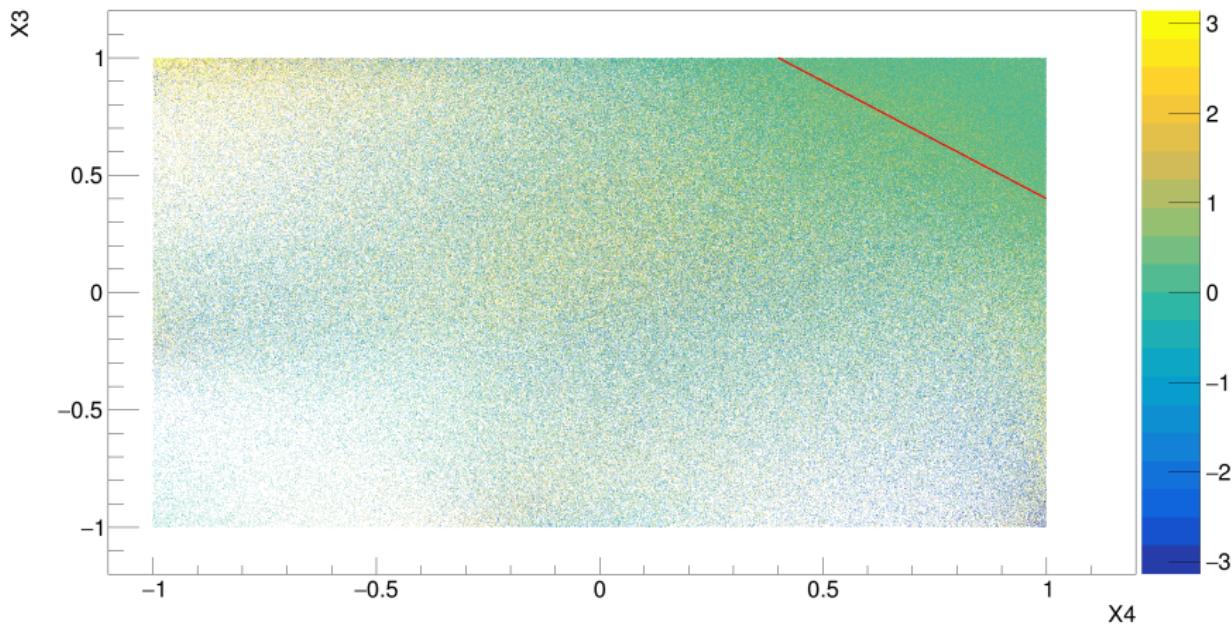
X3:X4:phase



**Figure 3:** Strong phase differences in the  $(x_3, x_4)$  plane

# Rectangular binning scheme

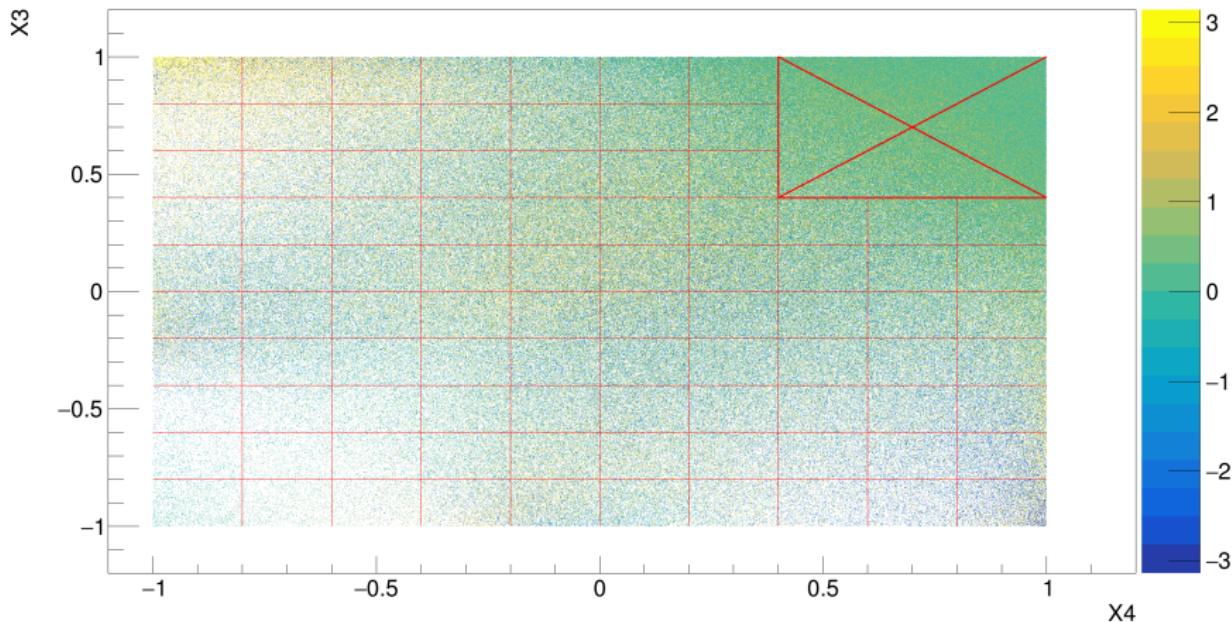
X3:X4:phase



**Figure 4:** Strong phase differences in the  $(x_3, x_4)$  plane

# Rectangular binning scheme

X3:X4:phase

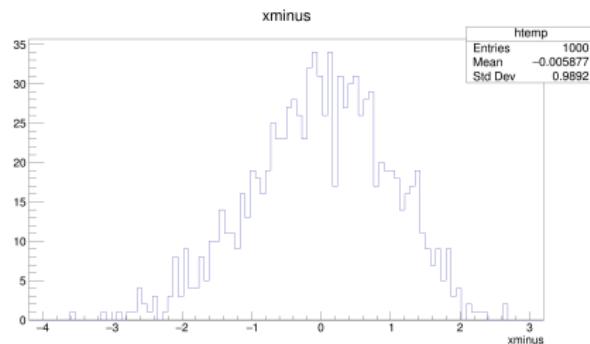
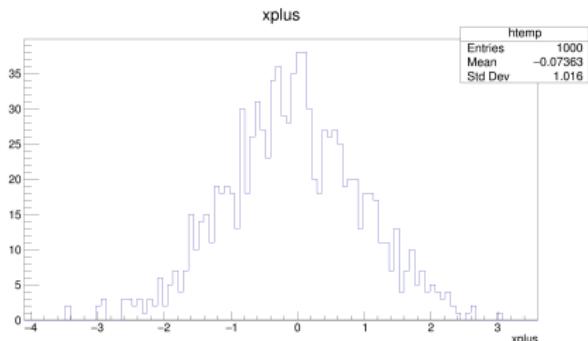


**Figure 5:** Strong phase differences in the  $(x_3, x_4)$  plane

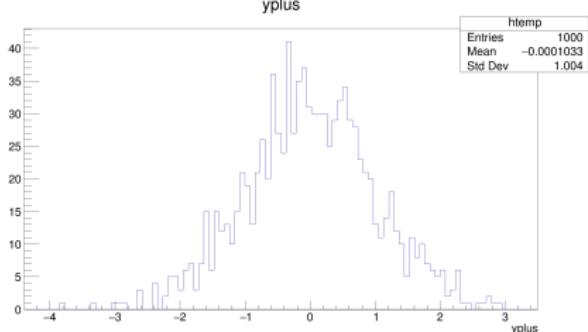
# Description of rectangular scheme

- Define upper right corner as bin 0 and 1
- Divide the  $(x_3, x_4)$  plane into regions
- Make a  $100 \times 100 \times 100$  grid in  $(x_1, x_2, x_5)$  volume
- For each gridpoint in  $(x_1, x_2, x_5)$ , average strong phases over  $(x_3, x_4)$
- Sort each gridpoint into one of 6 bins according to strong phase
- $6 + 2 = 8$  bins

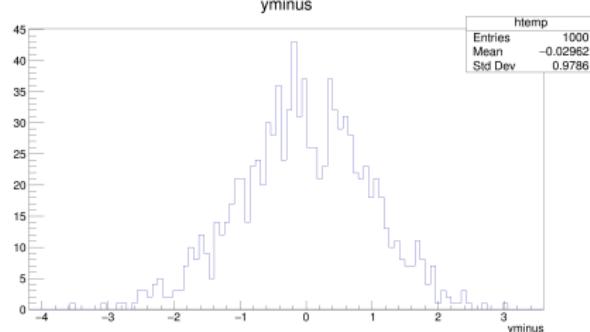
# Pull study with $2 \times 10^3$ events



(a)  $x_{+}$  pull



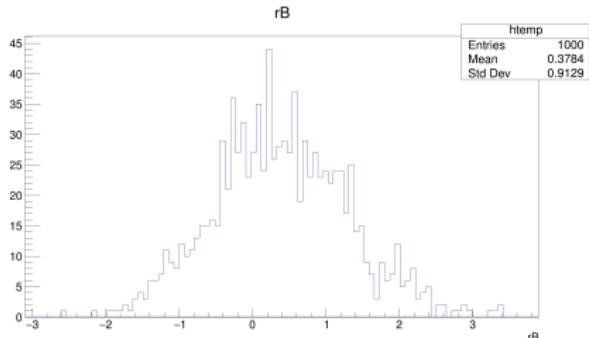
(b)  $x_{-}$  pull



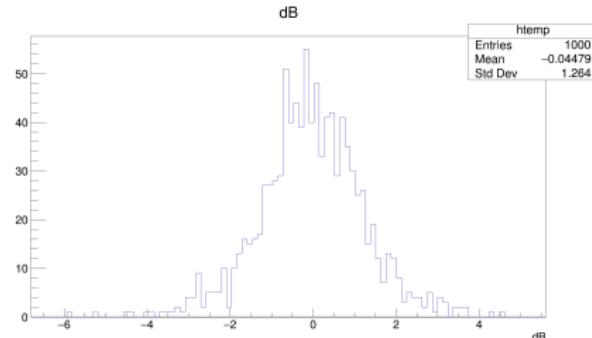
(c)  $y_{+}$  pull

(d)  $y_{-}$  pull

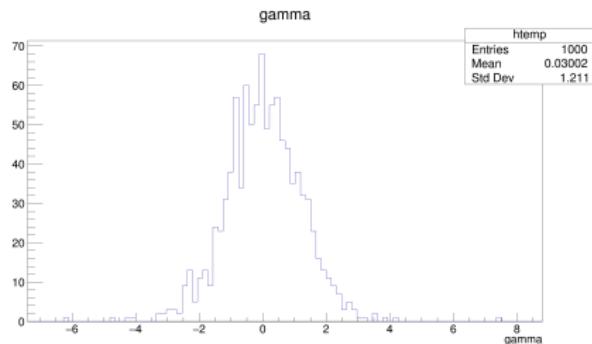
# Pull study with $2 \times 10^3$ events



(a)  $r_B$  pull



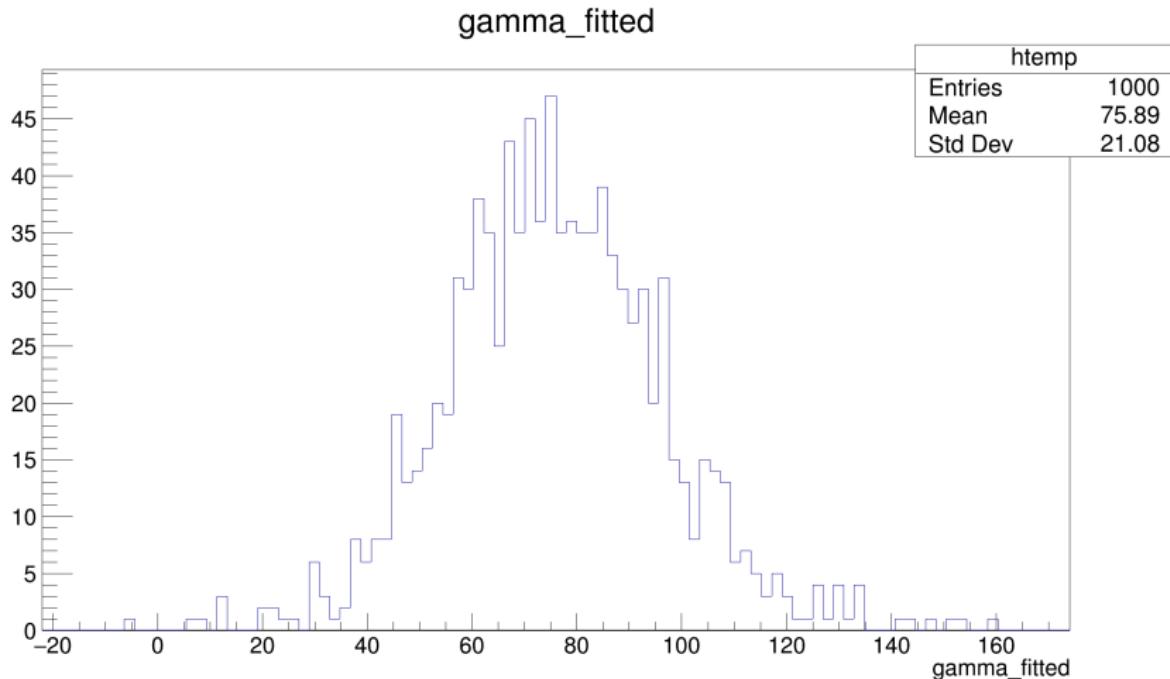
(b)  $\delta_B$  pull



(c)  $\gamma$  pull

# Fitted $\gamma$ values

$\gamma$  precision of  $21^\circ$

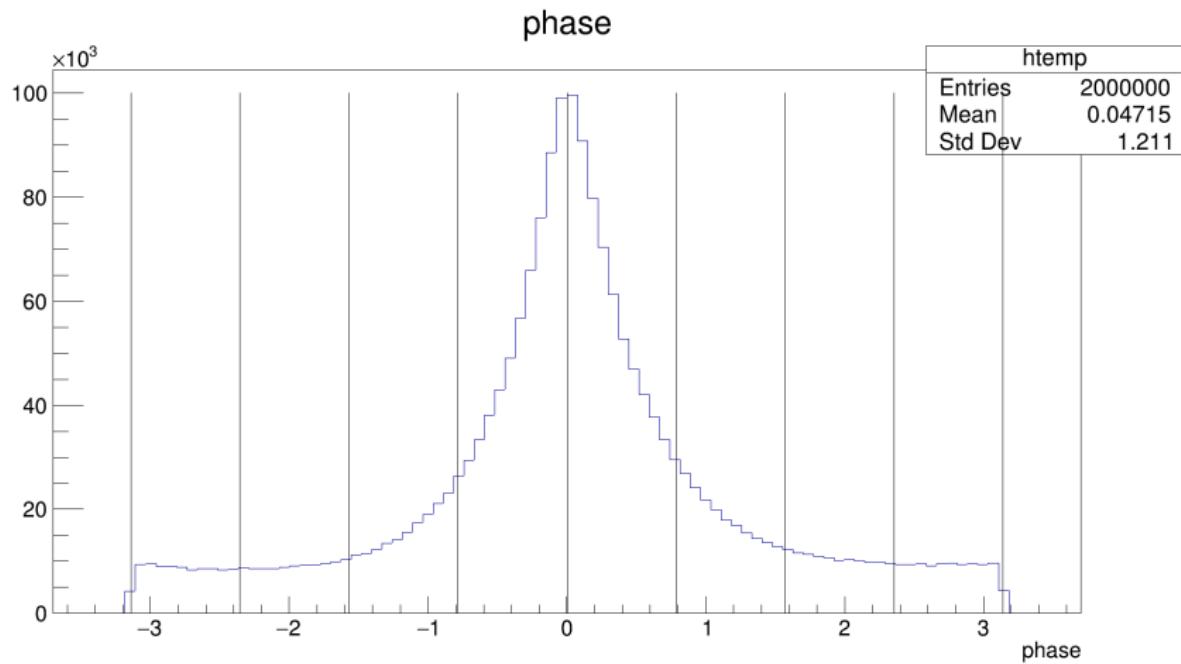


**Figure 8:** Histogram of fitted  $\gamma$  values

# Amplitude model binning

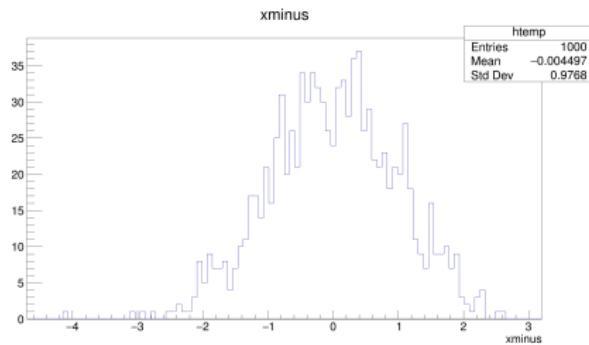
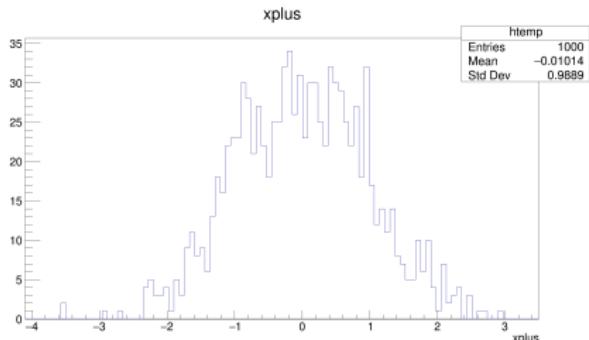
- Calculate strong phase difference of each event
- Divide into evenly spaced bins according to their strong phase difference
- 8 bins

# Amplitude model binning

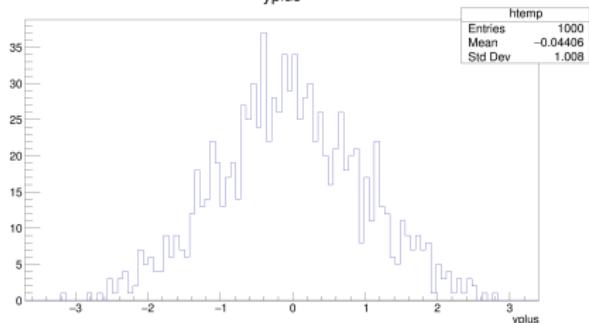


**Figure 9:** Histogram of strong phases, with black lines indicating bins

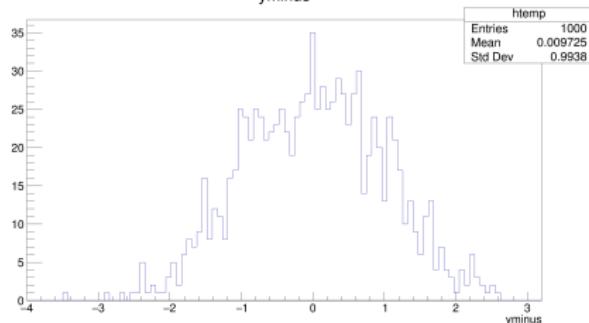
# Pull study with $2 \times 10^3$ events



(a)  $x_{+}$  pull  
yplus



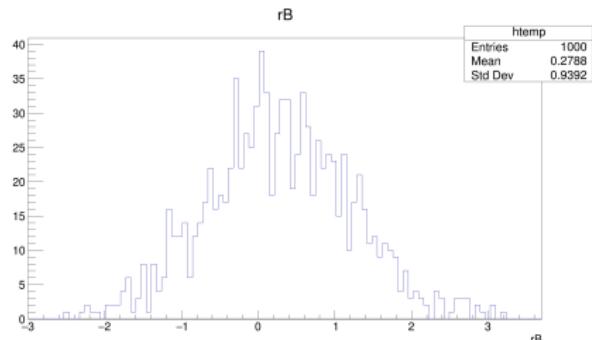
(b)  $x_{-}$  pull  
yminus



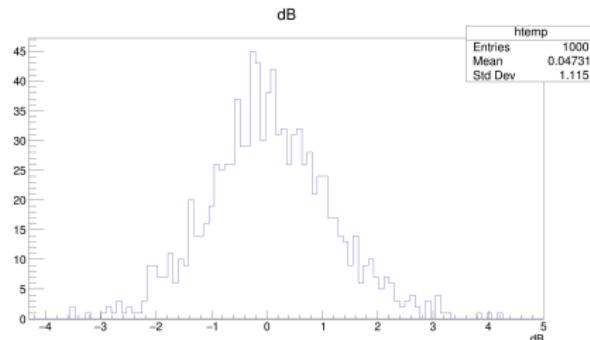
(c)  $y_{+}$  pull

(d)  $y_{-}$  pull

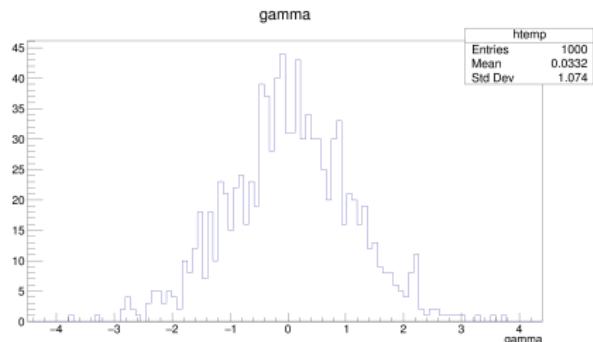
# Pull study with $2 \times 10^3$ events



(a)  $r_B$  pull



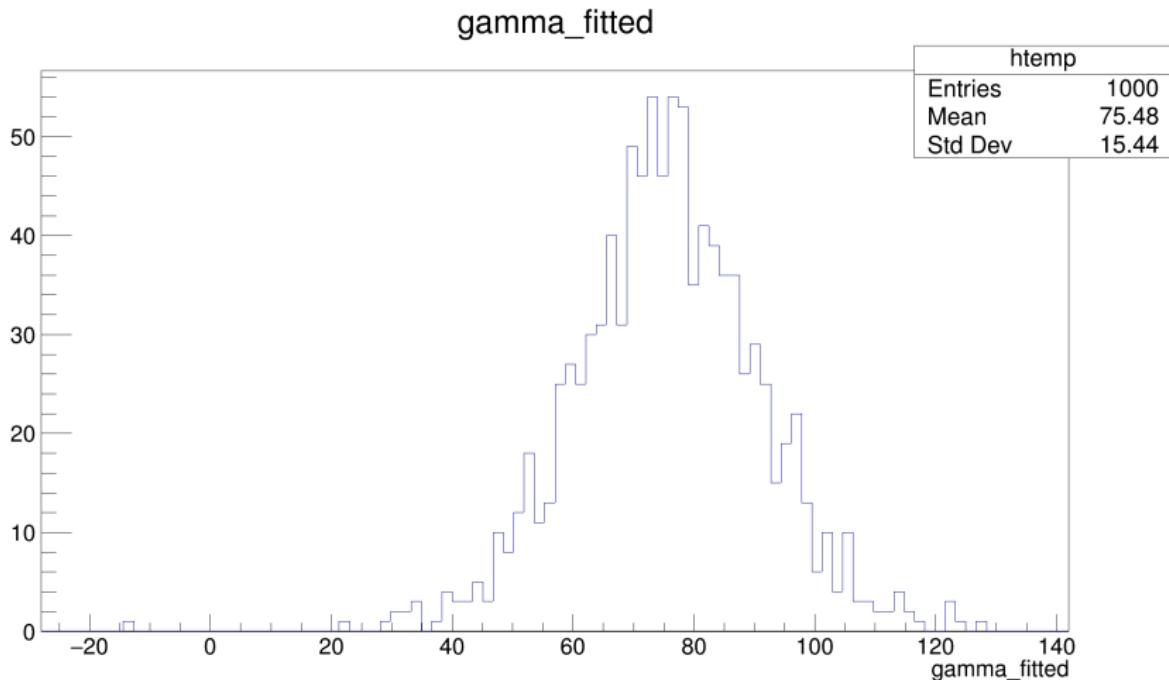
(b)  $\delta_B$  pull



(c)  $\gamma$  pull

# Fitted $\gamma$ values

$\gamma$  precision of  $15.4^\circ$



**Figure 12:** Histogram of fitted  $\gamma$  values

# Maximize interference terms

Event yield in bin  $i$

$$N_i^- = h_{B^-} \left( K_i + (x_-^2 + y_-^2) \bar{K}_i + 2\sqrt{K_i \bar{K}_i} (x_- c_i + y_- s_i) \right)$$

$$N_i^+ = h_{B^+} \left( \bar{K}_i + (x_+^2 + y_+^2) K_i + 2\sqrt{K_i \bar{K}_i} (x_+ c_i - y_+ s_i) \right)$$

- Difference of interference terms  $\propto \sin(\delta_D + \gamma)$
- Choose bins such that  $\sin(\delta_D + \gamma)$  always has the same sign in each bin
- 8 bins
- Can reach  $14.8^\circ$  with  $> 30$  bins

## Summary and next steps

Summary:

- Unbinned fit:  $11^\circ$  precision with 2000 events
- Binned fit:  $15.3^\circ$  with 8 bins
- Can reach  $14.8^\circ$  with  $> 30$  bins

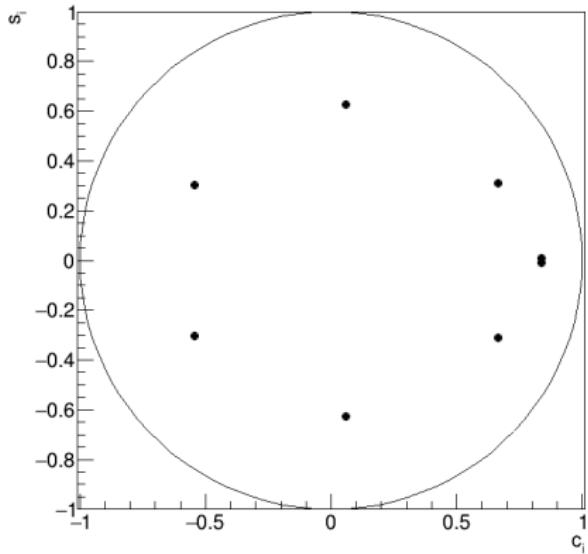
Any suggestions for improving the binning scheme? Why does the binned fit precision not approach  $11^\circ$ ?

# Backup slides

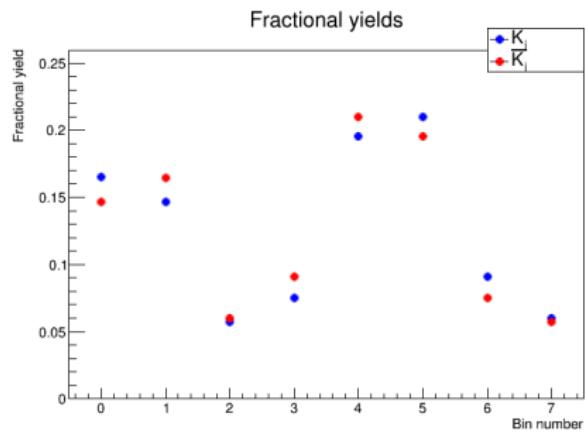
Backup slides

# Rectangular $D$ hadronic parameters

Plot of  $s_i$  vs  $c_i$



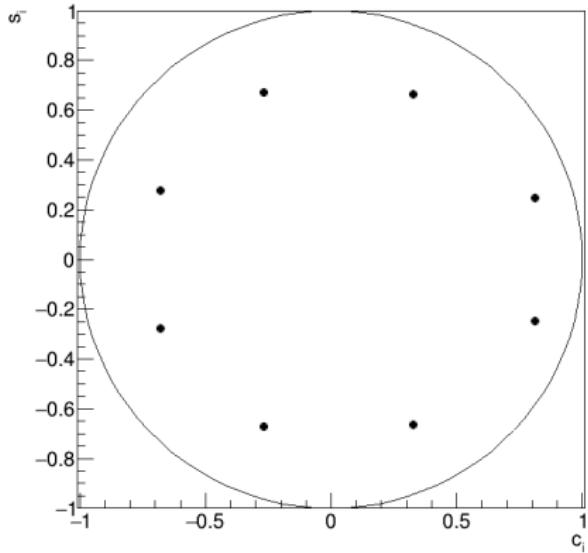
(a)  $c_i$  vs  $s_i$



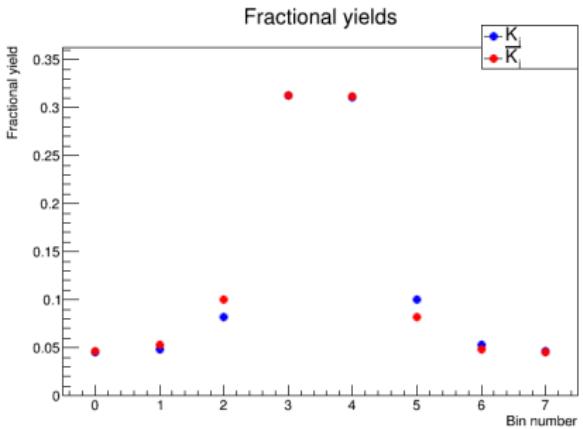
(b)  $K_i$  and  $\bar{K}_i$

# Amplitude $D$ hadronic parameters

Plot of  $s_i$  vs  $c_i$

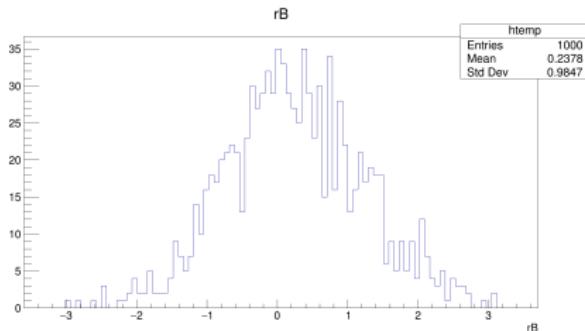


(a)  $c_i$  vs  $s_i$

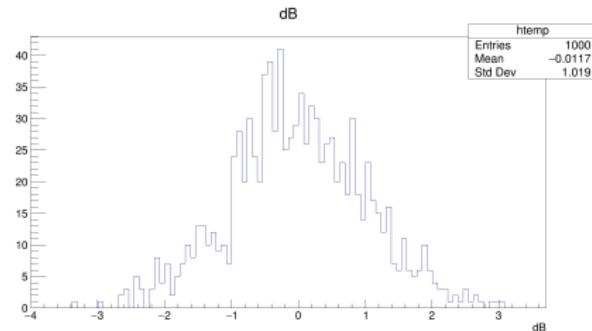


(b)  $K_i$  and  $\bar{K}_i$

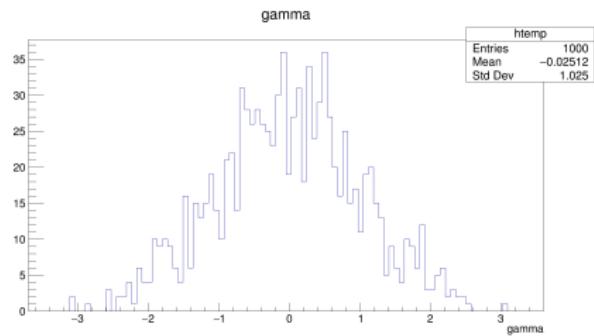
# Pull study of unbinned fit



(a)  $r_B$  pull



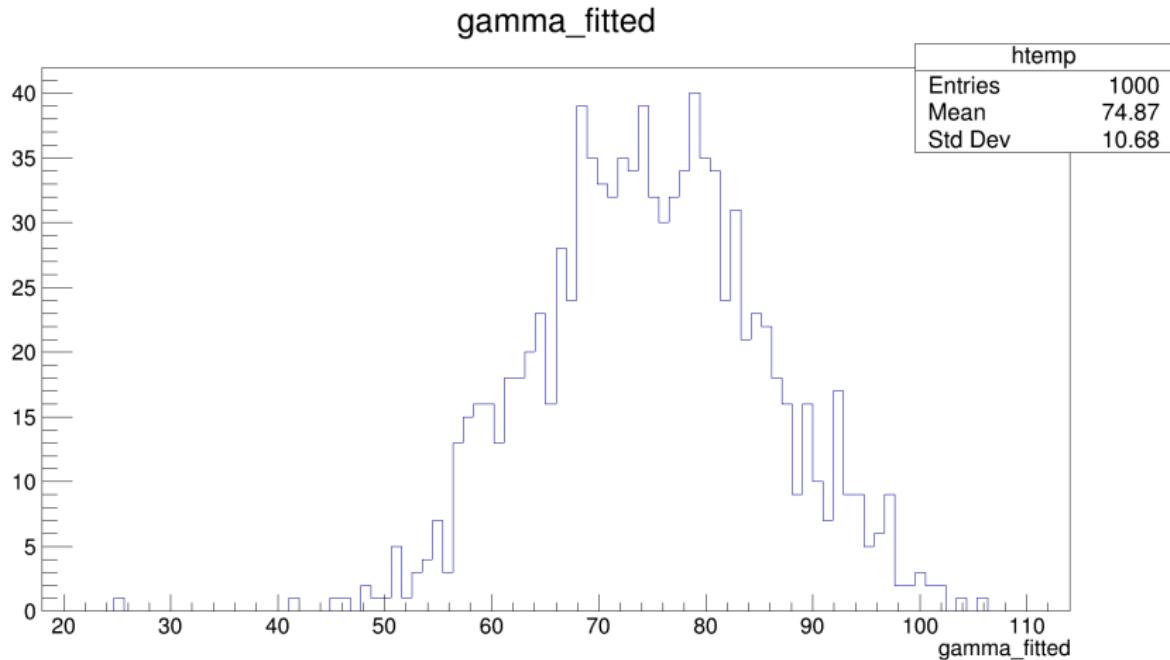
(b)  $\delta_B$  pull



(c)  $\gamma$  pull

# Fitted $\gamma$ values

$\gamma$  precision of  $10.6^\circ$



**Figure 16:** Histogram of fitted  $\gamma$  values