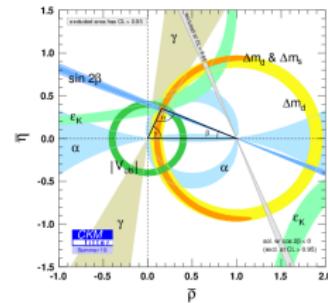


Binning scheme for γ measurement in $B^\pm \rightarrow (K^+ K^- \pi^+ \pi^-)_D K^\pm$ decays

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Oxford LHCb

17th December 2020



Outline

- 1 Current progress
- 2 Unbinned fit
- 3 Binned fit and pull studies
- 4 Attempts at better binning scheme
 - Rectangular binning scheme
 - Amplitude model binning
 - Amplitude model binning optimized for interference
- 5 Summary

Current progress

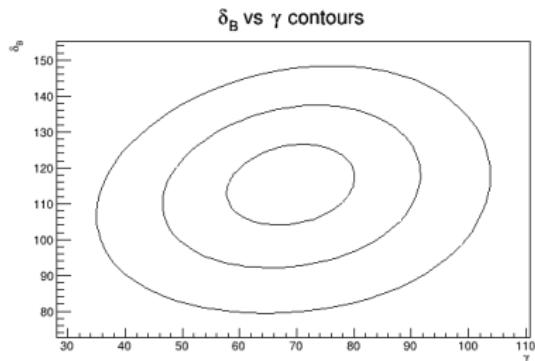
- Events generated in AmpGen using amplitude model
[arXiv:1811.08304](https://arxiv.org/abs/1811.08304)
 - Assumed event yield: 2000 (1000 B^+ , 1000 B^-)
 - Assumed parameters in toy model: $\gamma = 75^\circ$, $\delta_B = 130^\circ$, $r_B = 0.1$
- Unbinned fit using SimFit in AmpGen
- Binned fit and pull studies
- Develop suitable binning scheme

Unbinned fit with amplitude model

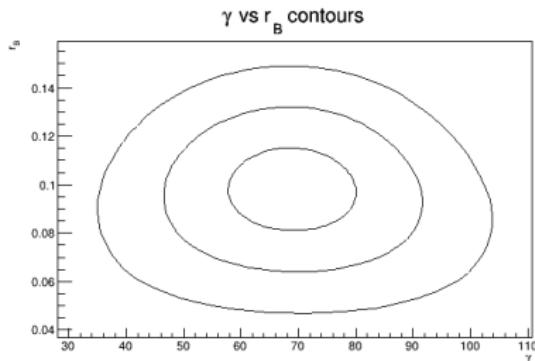
$$\begin{aligned}\mathcal{A}(B^- \rightarrow (K^+ K^- \pi^+ \pi^-)_D K^-) = & \mathcal{A}_B \mathcal{A}(D^0 \rightarrow K^+ K^- \pi^+ \pi^-) \\ & + \mathcal{A}_B \mathcal{A}(\bar{D}^0 \rightarrow K^+ K^- \pi^+ \pi^-) r_B e^{i(\delta_B - \gamma)}\end{aligned}$$

- $\mathcal{A}(D \rightarrow K^+ K^- \pi^+ \pi^-)$ obtained from amplitude model
- Fit with γ , δ_B and r_B as free parameters
- Results from unbinned fit of 2×10^3 events:
 - $\gamma = (69 \pm 11)^\circ$
 - $\delta_B = (115 \pm 11)^\circ$
 - $r_B = 0.098 \pm 0.017$
- Pulls of γ , δ_B and r_B all have mean 0 and std 1 (see backup slides)

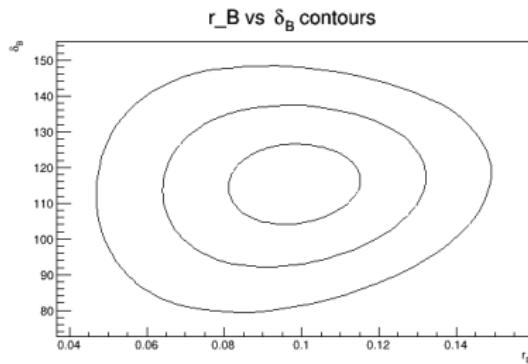
Unbinned fit of 2×10^3 events with amplitude model



(a) γ vs δ_B



(b) γ vs r_B



(c) r_B vs δ_B

Binned fit of $D \rightarrow K^+ K^- \pi^+ \pi^-$

$$\begin{aligned}\mathcal{A}(B^- \rightarrow (K^+ K^- \pi^+ \pi^-)_D K^-) = & \mathcal{A}_B \mathcal{A}(D^0 \rightarrow K^+ K^- \pi^+ \pi^-) \\ & + \mathcal{A}_B \mathcal{A}(\bar{D}^0 \rightarrow K^+ K^- \pi^+ \pi^-) r_B e^{i(\delta_B - \gamma)}\end{aligned}$$

Event yield in bin i

$$\begin{aligned}N_i^- &= h_{B^-} \left(K_i + (x_-^2 + y_-^2) \bar{K}_i + 2\sqrt{K_i \bar{K}_i} (x_- c_i + y_- s_i) \right) \\ N_i^+ &= h_{B^+} \left(\bar{K}_i + (x_+^2 + y_+^2) K_i + 2\sqrt{K_i \bar{K}_i} (x_+ c_i - y_+ s_i) \right)\end{aligned}$$

CP-violating observables

$$x_\pm = r_B \cos(\delta_B \pm \gamma), \quad y_\pm = r_B \sin(\delta_B \pm \gamma)$$

Pull studies

- Used an arbitrary and naive binning scheme with 4 bins
- x_{\pm} pulls show asymmetric tails for 2×10^3 events
- Pulls for γ , δ_B , r_B are rubbish

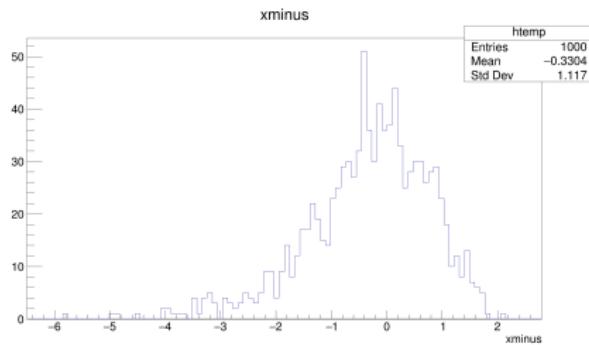
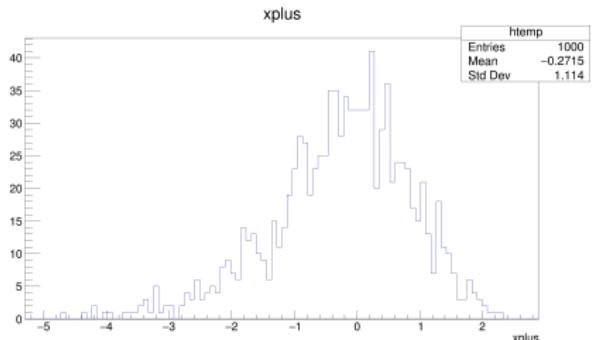
Naive binning scheme

Split phase space along the boundaries $E_{K^+} = E_{K^-}$ and $E_{\pi^+} = E_{\pi^-}$
Bin 1: $E_{K^+} > E_{K^-}$, $E_{\pi^+} > E_{\pi^-}$, ...

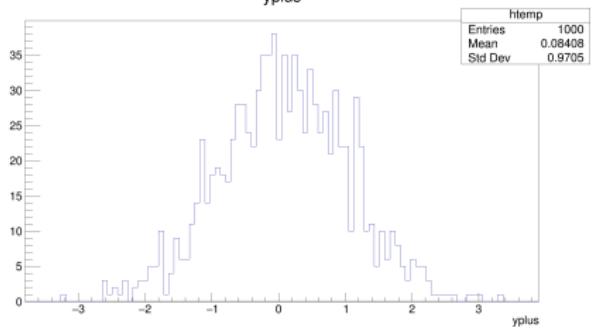
D decay hadronic parameters

$$c_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D}^0)| \cos(\delta_D)}{\sqrt{\int_i d\Phi |\mathcal{A}(D^0)|^2 \int_i d\Phi |\mathcal{A}(\bar{D}^0)|^2}}, \quad K_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j d\Phi |\mathcal{A}(D^0)|^2}$$

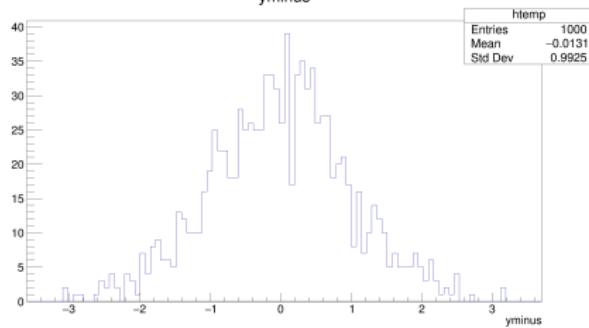
Naive binning pull study



(a) x_{+} pull



(b) x_{-} pull



(c) y_{+} pull

(d) y_{-} pull

Rectangular binning scheme

- Inspired by [arXiv:1709.03467](#)
- 4-body phase space is 5-dimensional
- Convenient to choose rectangular coordinates

Phase space parameterisation

$$x_1 = m(K^+ \pi^+) + \alpha$$

$$x_2 = m(K^- \pi^-) + \alpha, \quad \alpha = \min(m(K^+ \pi^+), m(K^- \pi^-)) - m_\pi - m_K$$

$$x_3 = \cos(\theta_+), \quad (\text{Helicity angles})$$

$$x_4 = \cos(\theta_-)$$

$$x_5 = \phi$$

- Study phase space in terms of these coordinates

Rectangular (x_3, x_4) plane

X3:X4:phase

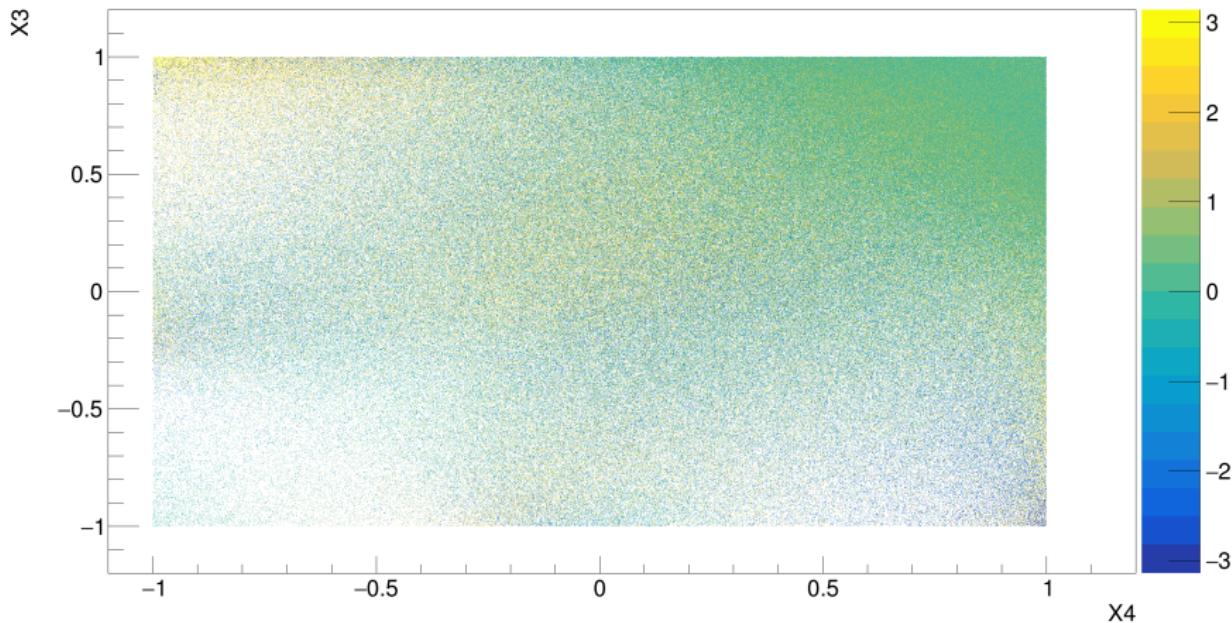


Figure 3: Strong phase differences in the (x_3, x_4) plane

Rectangular (x_3, x_4) plane

X3:X4:phase

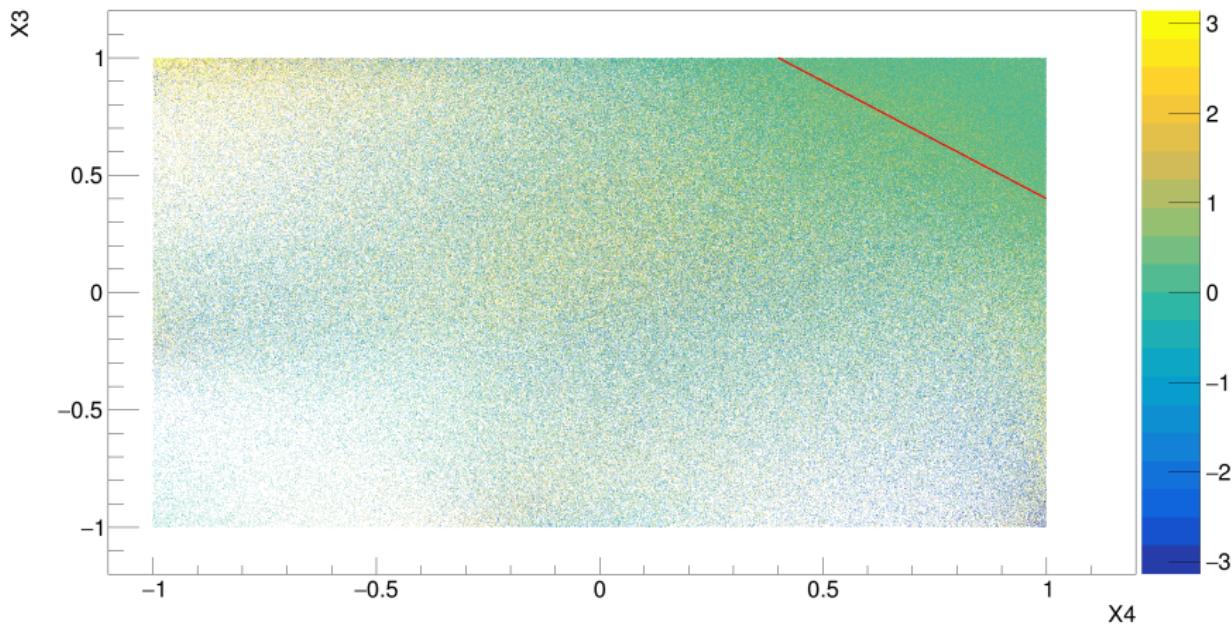


Figure 4: Strong phase differences in the (x_3, x_4) plane

Rectangular (x_3, x_4) plane

X3:X4:phase

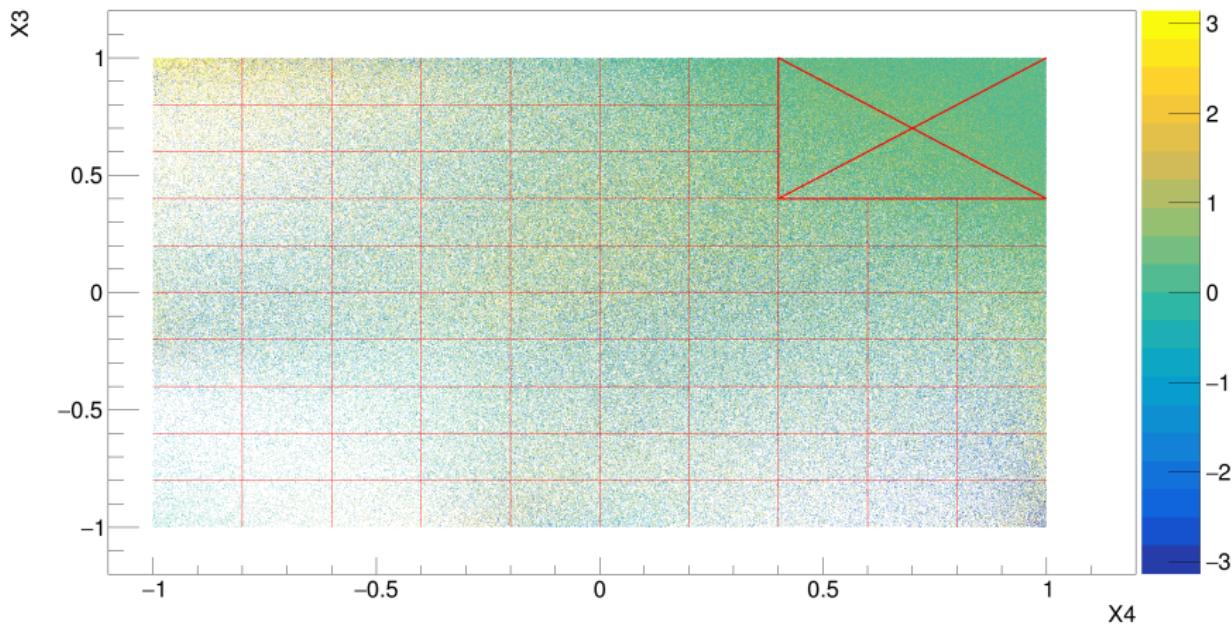
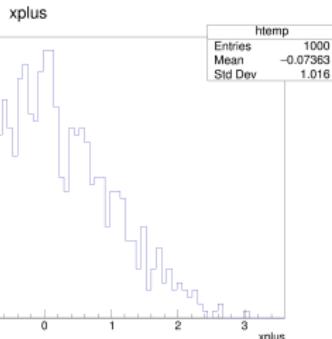


Figure 5: Strong phase differences in the (x_3, x_4) plane

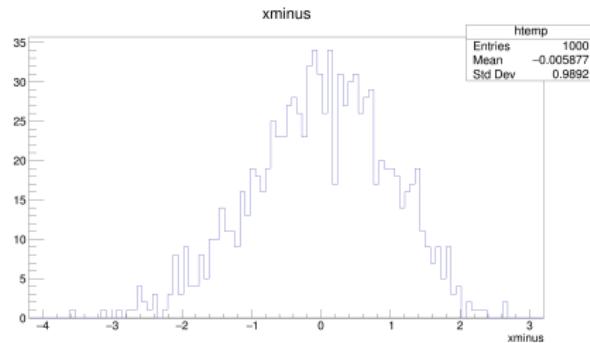
Description of rectangular scheme

- Define upper right corner as bin 0 and 1
- Divide the (x_3, x_4) plane into regions
- Make a $100 \times 100 \times 100$ grid in (x_1, x_2, x_5) volume
- For each gridpoint in (x_1, x_2, x_5) , average strong phases over (x_3, x_4)
- Sort each gridpoint into one of 6 bins according to strong phase
- $6 + 2 = 8$ bins

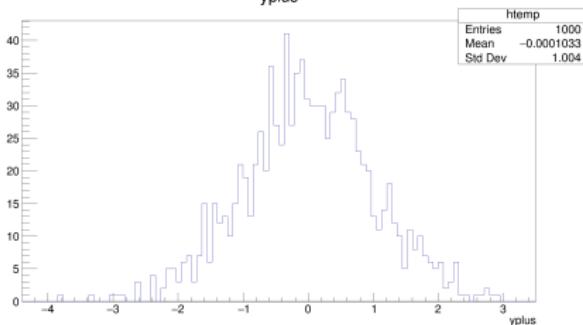
Rectangular binning pull study



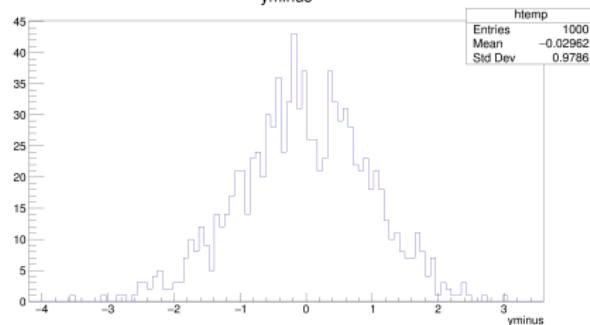
(a) x_{+} pull



(b) x_{-} pull

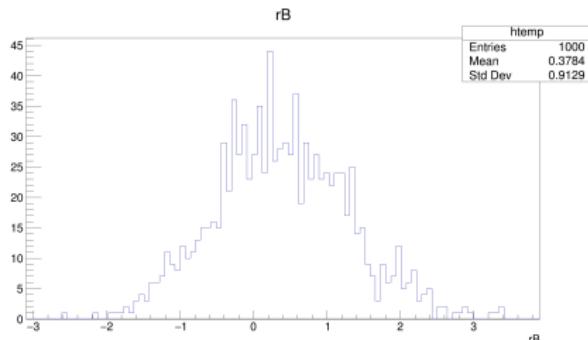


(c) y_{+} pull

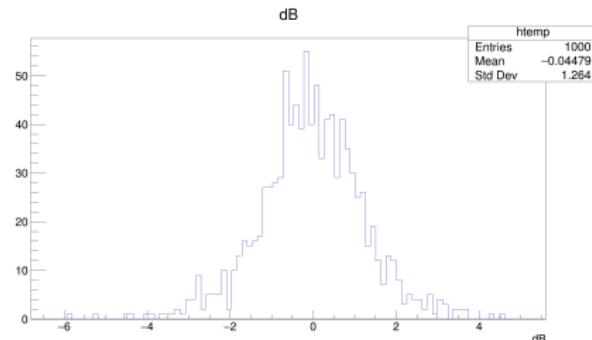


(d) y_{-} pull

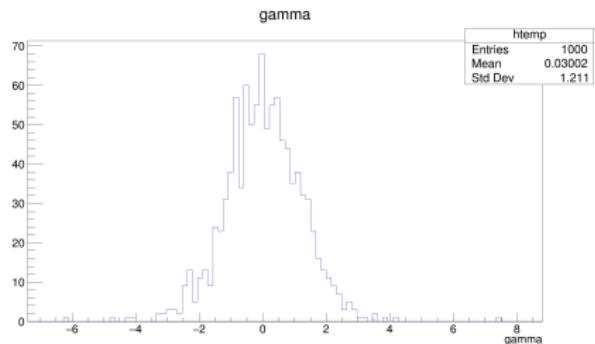
Rectangular binning pull study



(a) r_B pull



(b) δ_B pull



(c) γ pull

Fitted γ values

γ precision of 21°

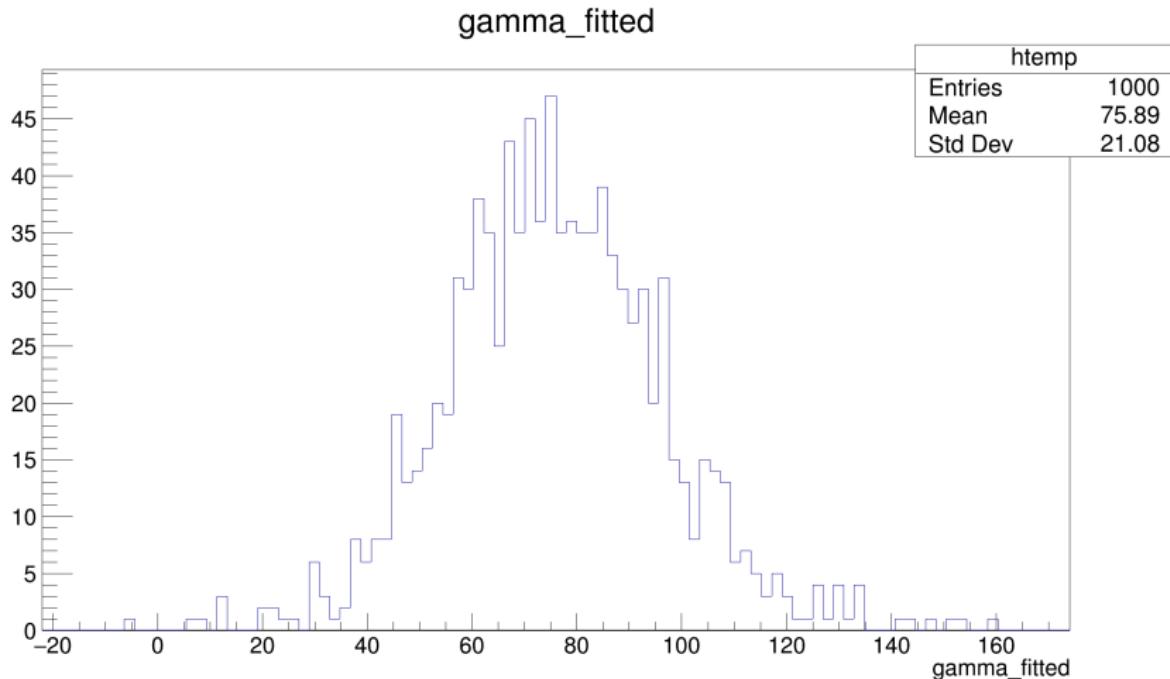


Figure 8: Histogram of fitted γ values

Amplitude model binning

- Calculate strong phase difference of each event
- Divide into evenly spaced bins according to their strong phase difference
- 8 bins

Amplitude model binning

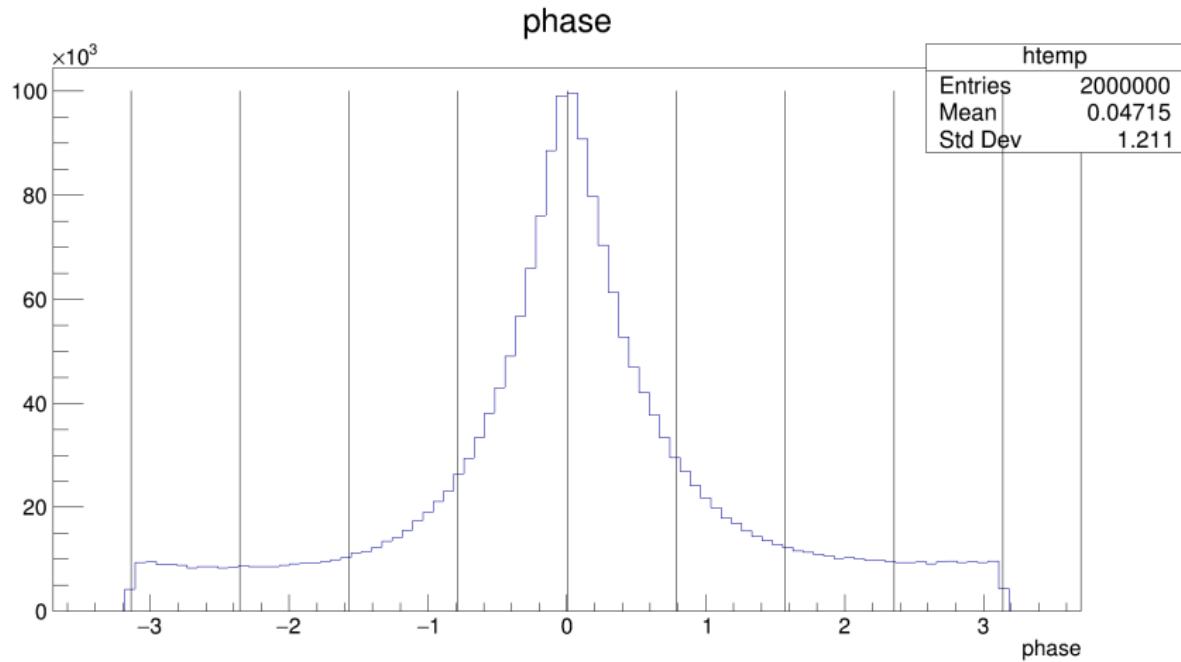
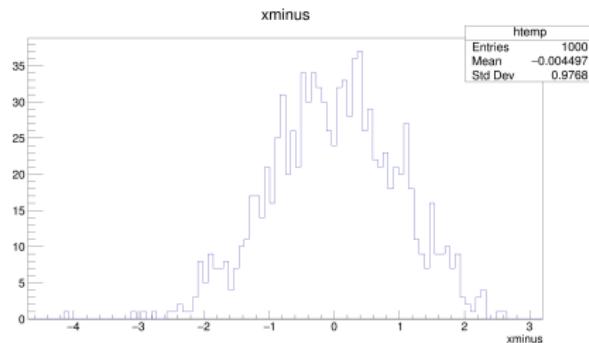
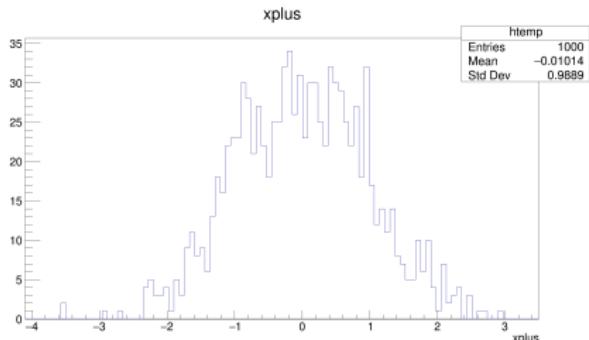
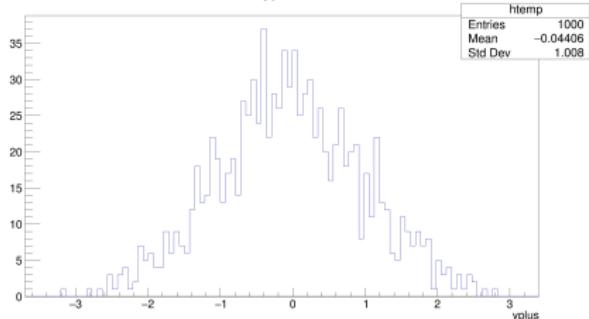


Figure 9: Histogram of strong phases, with black lines indicating bins

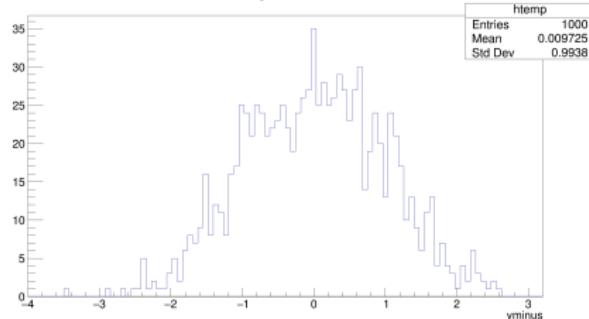
Amplitude model binning pull study



(a) x_{+} pull
yplus



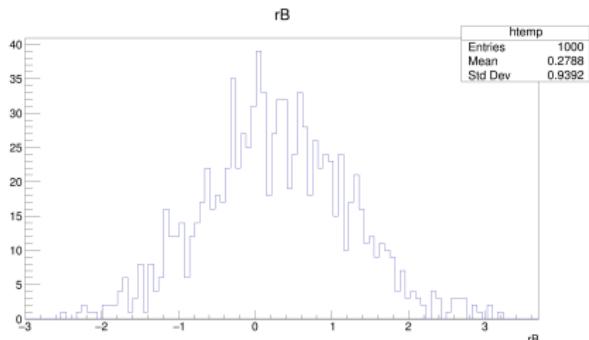
(b) x_{-} pull
yminus



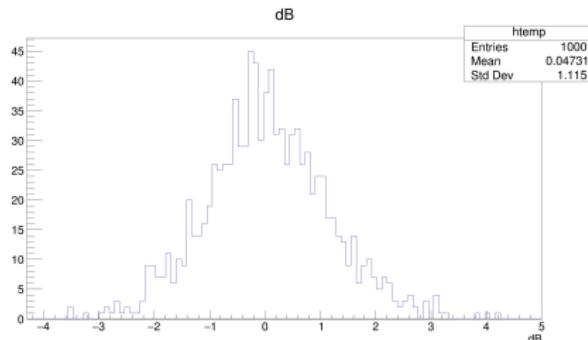
(c) y_{+} pull

(d) y_{-} pull

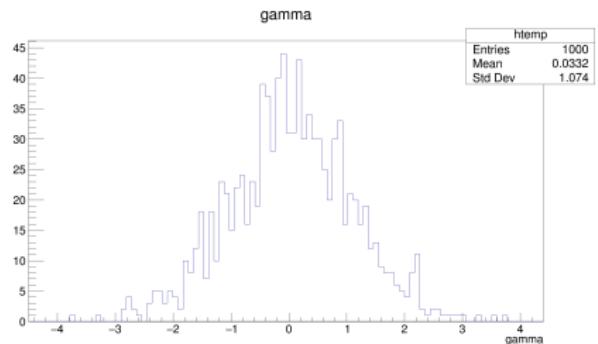
Amplitude model binning pull study



(a) r_B pull



(b) δ_B pull



(c) γ pull

Fitted γ values

γ precision of 15.4°

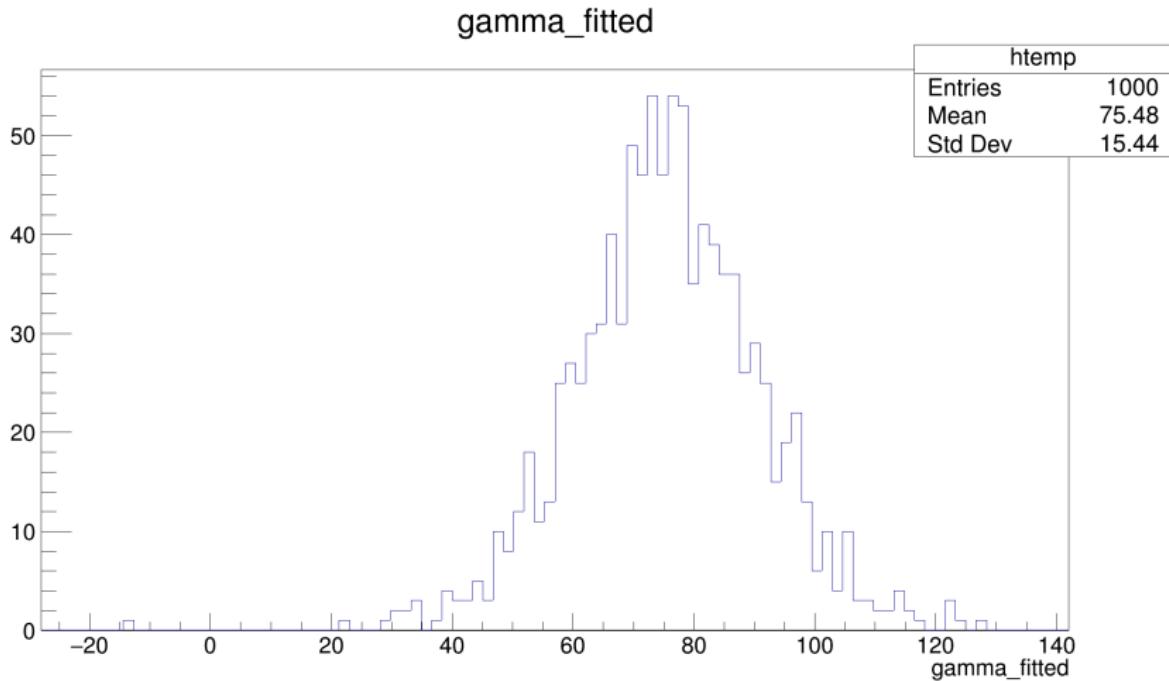


Figure 12: Histogram of fitted γ values

Maximize interference terms

Event yield in bin i

$$N_i^- = h_{B^-} \left(K_i + (x_-^2 + y_-^2) \bar{K}_i + 2\sqrt{K_i \bar{K}_i} (x_- c_i + y_- s_i) \right)$$

$$N_i^+ = h_{B^+} \left(\bar{K}_i + (x_+^2 + y_+^2) K_i + 2\sqrt{K_i \bar{K}_i} (x_+ c_i - y_+ s_i) \right)$$

- Difference of interference terms $\propto \sin(\delta_D + \gamma)$
- Choose bins such that $\sin(\delta_D + \gamma)$ always has the same sign in each bin
- 8 bins

Optimized amplitude model binning

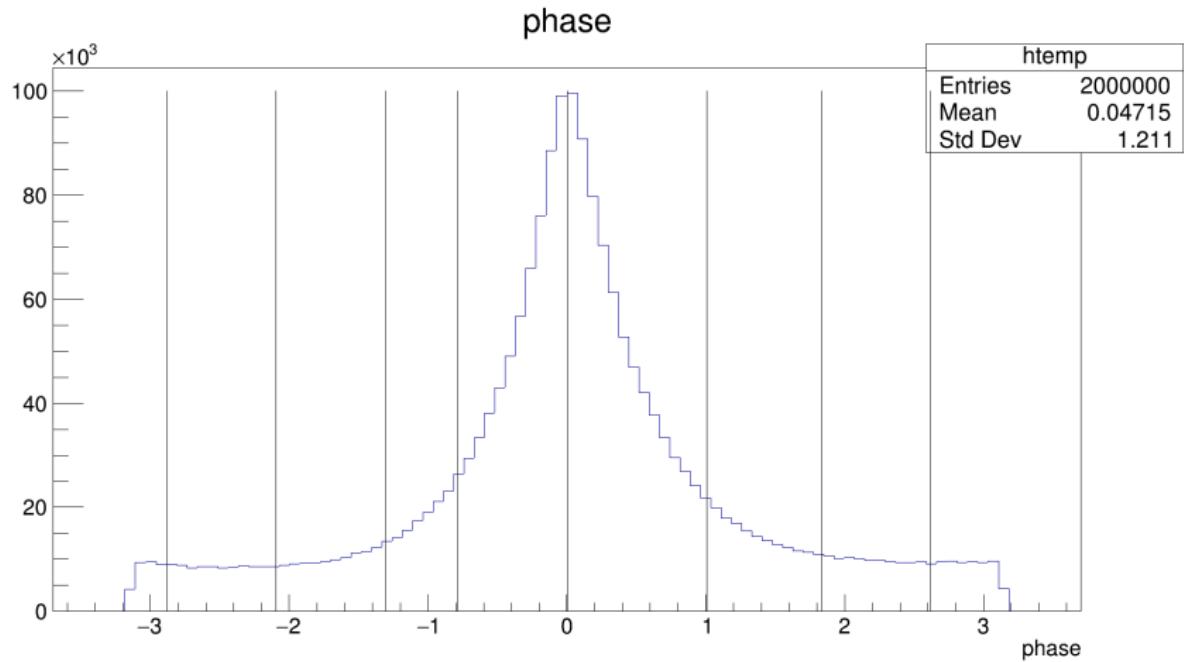
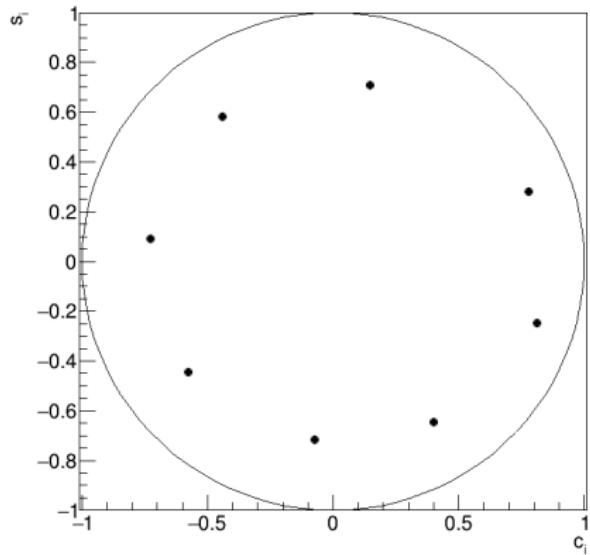


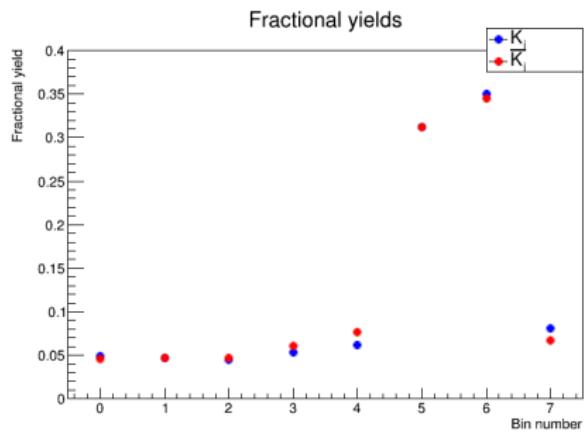
Figure 13: Histogram of strong phases, with black lines indicating bins

Optimized amplitude model binning D hadronic parameters

Plot of s_i vs c_i

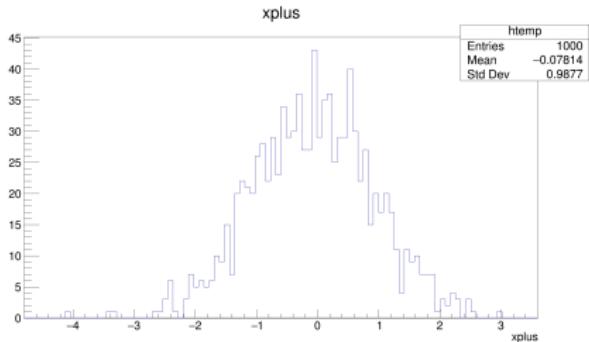


(a) c_i vs s_i

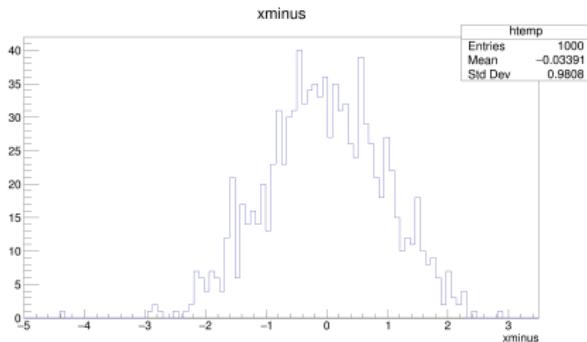


(b) K_i and \bar{K}_i

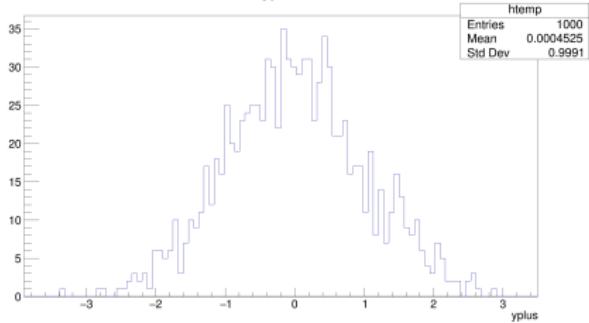
Optimized amplitude model binning pull study



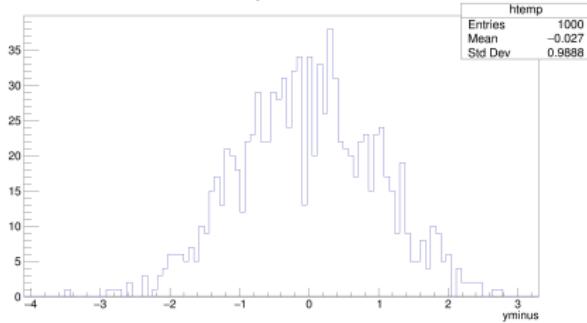
(a) x_{+} pull
yplus



(b) x_{-} pull
yminus

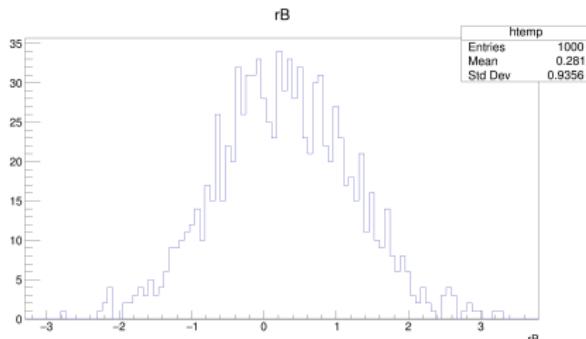


(c) y_{+} pull

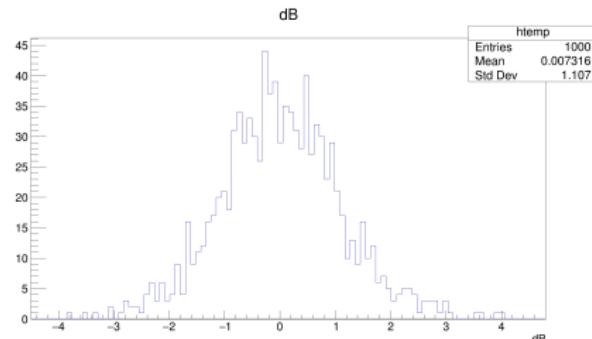


(d) y_{-} pull

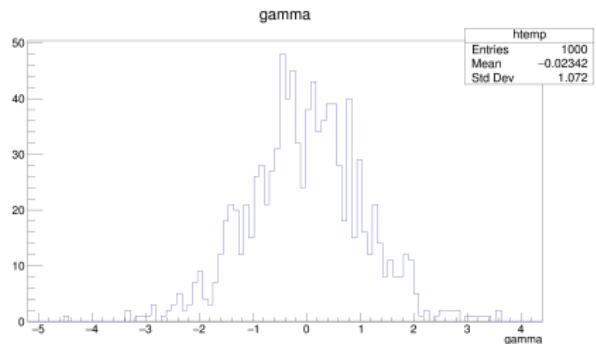
Optimized amplitude model binning pull study



(a) r_B pull



(b) δ_B pull



(c) γ pull

Fitted γ values

γ precision of 15.1°

Can reach 14.8° with > 30 bins

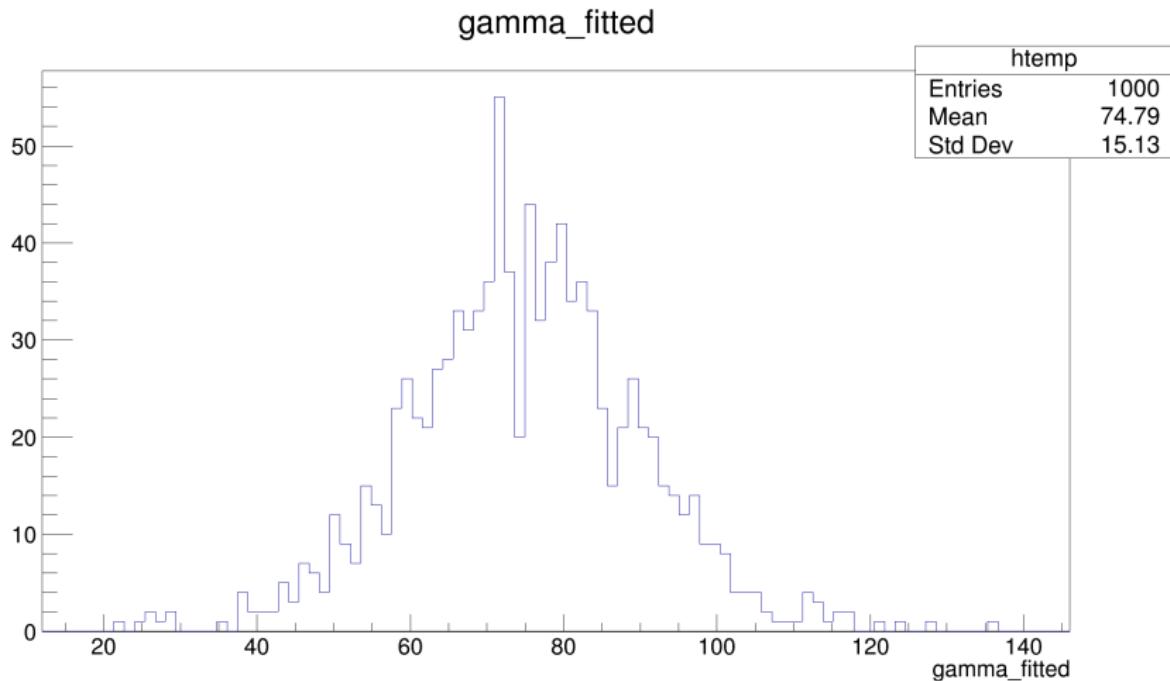


Figure 17: Histogram of fitted γ values

Summary and next steps

Summary:

- Unbinned fit: 11° precision with 2000 events
- Binned fit: 15.1° with 8 bins
- Can reach 14.8° with > 30 bins

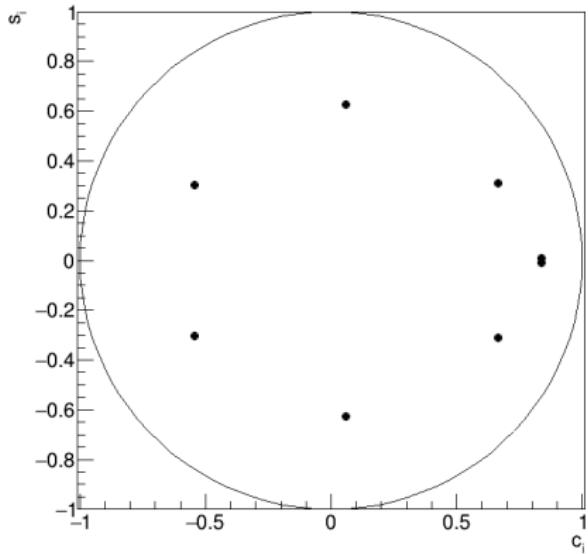
Any suggestions for improving the binning scheme? Why does the binned fit precision not approach 11° ?

Backup slides

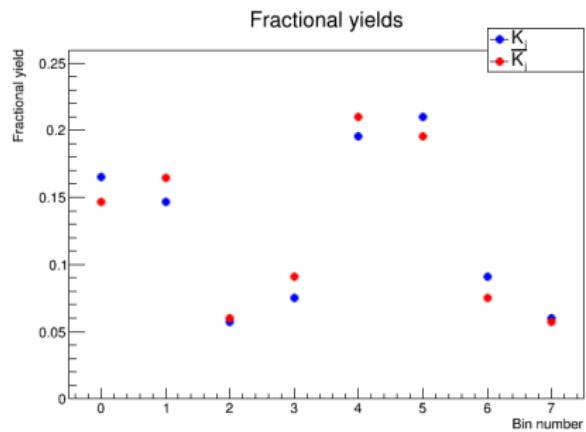
Backup slides

Rectangular D hadronic parameters

Plot of s_i vs c_i



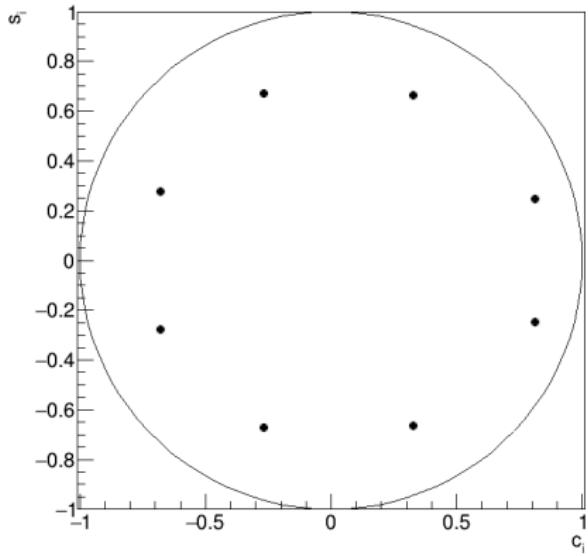
(a) c_i vs s_i



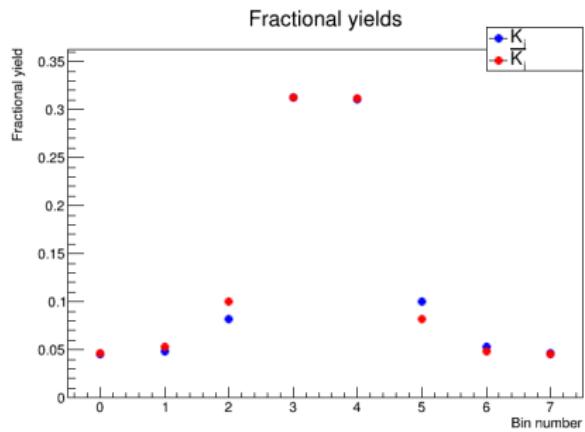
(b) K_i and \bar{K}_i

Amplitude D hadronic parameters

Plot of s_i vs c_i

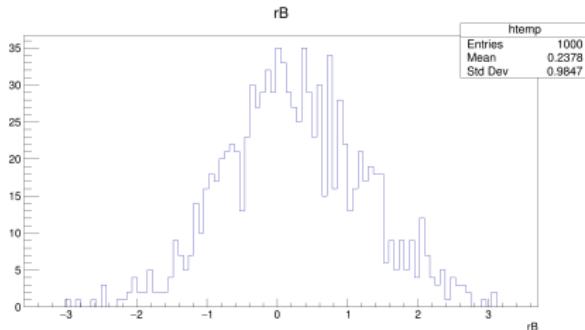


(a) c_i vs s_i

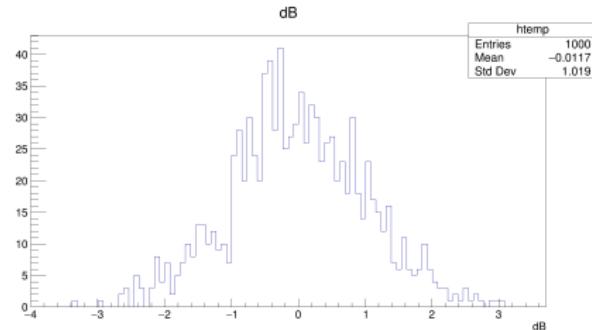


(b) K_i and \bar{K}_i

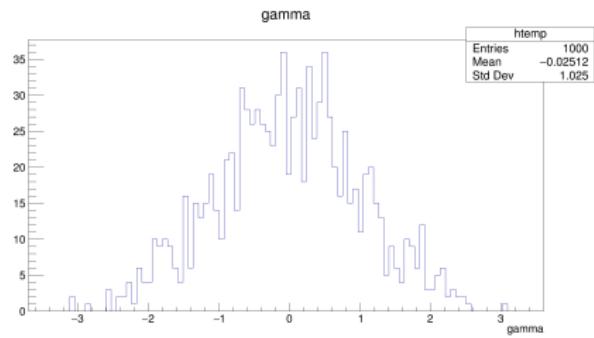
Pull study of unbinned fit



(a) r_B pull



(b) δ_B pull



(c) γ pull

Fitted γ values

γ precision of 10.6°

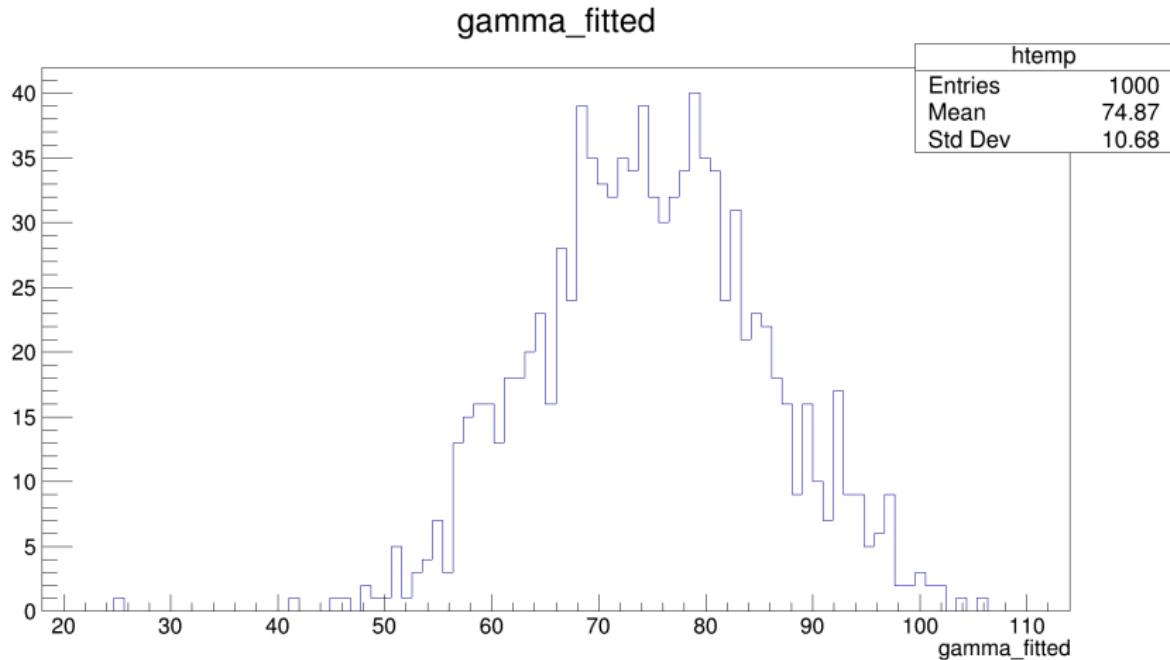


Figure 21: Histogram of fitted γ values