

Determination of the CKM angle γ in $B^\pm \rightarrow D(\rightarrow K^+K^-\pi^+\pi^-)h^\pm$ decays

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Abstract

A model-independent measurement of the CKM angle γ is performed in $B^\pm \rightarrow Dh^\pm$ ($h = K, \pi$) decays, where $D \rightarrow K^+K^-\pi^+\pi^-$, using the LHCb Run 1+2 dataset. An optimal phase space binning scheme is developed. Analysis of external strong phase inputs of $D \rightarrow K^+K^-\pi^+\pi^-$ from BESIII is performed in parallel.

1 Introduction

In the Standard Model, CP-violation is studied by measuring the lengths and angles of the Unitary Triangle of the CKM matrix [1]. The angle $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$, in particular, is the only angle accessible at tree level, with negligible theoretical uncertainties. Thus, a precise determination of γ is a good Standard Model benchmark.

γ is sensitive to $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$ interference, such as in $B^\pm \rightarrow DK^\pm$ decays. Fig. 1 shows the $B^- \rightarrow D^0K^-$ mode on the left and the colour suppressed $B^- \rightarrow \bar{D}^0K^-$ channel on the right. They interfere when D^0 and \bar{D}^0 decay to a common state.

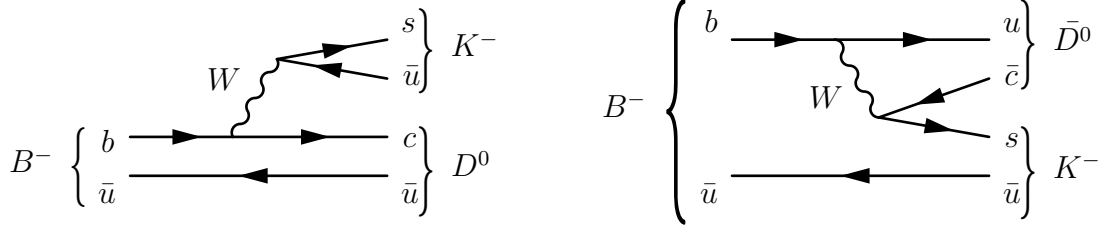


Figure 1: Feynman diagrams of $B^- \rightarrow DK^-$ decays at tree level.

A wide range of subsequent D decays has been studied, where $\gamma = (68.7^{+5.2}_{-5.1})^\circ$ is the single most precise measurement using $D \rightarrow K_S^0 h^+ h^-$ ($h = \pi, K$) [2]. In the following analysis, the decay $D \rightarrow K^+K^-\pi^+\pi^-$ is considered. From similar channels, one expects 2000 candidates from the LHCb Run 1+2 dataset. The challenge with $D \rightarrow K^+K^-\pi^+\pi^-$ is the 5D phase space, where the D^0 and \bar{D}^0 strong phases vary non-trivially. An amplitude model may be used to predict the strong phases, at the cost of a model uncertainty.

In this analysis, a model-independent approach is chosen. Strong phases are independently measured, in phase space bins, at the BESIII charm factory. The current BESIII 2010-11 dataset is insufficient, but significantly more data is expected from 2022. An amplitude model for $D \rightarrow K^+K^-\pi^+\pi^-$, which is referred to as the LHCb model [3], will be used to study strong phase variations and develop an effective binning scheme. A poor binning scheme may decrease the statistical sensitivity, but will not bias the result. Thus, with a model-independent approach there is no modelling systematic uncertainty.

2 Formalism

2.1 γ sensitivity through B^\pm decays

The amplitude of $B^\pm \rightarrow DK^\pm$ is a coherent sum of the diagrams in Fig. 1,

$$\mathcal{A}(B^- \rightarrow DK^-) = \mathcal{A}_D(\Phi) + r_B^{DK} e^{i(\delta_B^{DK} - \gamma)} \mathcal{A}_{\bar{D}}(\Phi), \quad (1)$$

$$\mathcal{A}(B^+ \rightarrow DK^+) = \mathcal{A}_{\bar{D}}(\Phi) + r_B^{DK} e^{i(\delta_B^{DK} + \gamma)} \mathcal{A}_D(\Phi). \quad (2)$$

r_B is the relative magnitude of the diagrams and $A_{D,\bar{D}}$ are D decay amplitudes. Under CP, the B^\pm decay strong phase δ_B^{DK} is invariant while the weak phase γ swaps sign.

The $B^\pm \rightarrow DK^\pm$ decay is considered in $2 \times N$ phase space bins, labelled $i = -N, \dots, N$, excluding zero. Bin i is related to $-i$ by a CP transformation. When integrating the square of Eqs. (1)-(2) over phase space Φ , the $B^\mp \rightarrow DK^\mp$ yield in bin $\pm i$ are

$$N_i^- = h_{B^-} \left[F_i + \left((x_-^{DK})^2 + (y_-^{DK})^2 \right) \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_-^{DK} c_i + y_-^{DK} s_i) \right], \quad (3)$$

$$N_{-i}^+ = h_{B^+} \left[F_i + \left((x_+^{DK})^2 + (y_+^{DK})^2 \right) \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_+^{DK} c_i + y_+^{DK} s_i) \right], \quad (4)$$

$$\begin{pmatrix} c_i \\ s_i \end{pmatrix} = \frac{\int_i d\Phi |\mathcal{A}_D| |\mathcal{A}_{\bar{D}}| \begin{pmatrix} \cos(\Delta\delta_D) \\ \sin(\Delta\delta_D) \end{pmatrix}}{\sqrt{\int_i d\Phi |\mathcal{A}_D|^2 \int_i d\Phi |\mathcal{A}_{\bar{D}}|^2}}, \quad F_i = \frac{\int_i d\Phi |\mathcal{A}_D|^2}{\sum_i \int_i d\Phi |\mathcal{A}_D|^2}, \quad \bar{F}_i = \frac{\int_i d\Phi |\mathcal{A}_{\bar{D}}|^2}{\sum_i \int_i d\Phi |\mathcal{A}_{\bar{D}}|^2}. \quad (5)$$

h_{B^\pm} are a normalization constants. c_i (s_i) is the cosine (sine) of the strong phase difference $\Delta\delta_D$ between the D^0 and \bar{D}^0 decays, amplitude-averaged over bin i . F_i is the fractional yield of $B^- \rightarrow D^0 K^-$ in bin i . Assuming CP conservation in D decays, $\bar{F}_i = F_{-i}$. Furthermore, the CP observables in Eqs. (1)-(2) are

$$x_\pm^{DK} = r_B^{DK} \cos(\delta_B^{DK} \pm \gamma), \quad y_\pm^{DK} = r_B^{DK} \sin(\delta_B^{DK} \pm \gamma). \quad (6)$$

From the binned yields of $B^\pm \rightarrow DK^\pm$, one can do a Maximum Likelihood (ML) fit of Eqs. (3)-(4), with external inputs of c_i and s_i from BESIII, to obtain the CP observables x_\pm^{DK} and y_\pm^{DK} . These are interpreted in terms of γ , δ_B^{DK} and r_B^{DK} .

To improve the fit stability and constrain F_i , which are free parameters, the decay mode $B^\pm \rightarrow D\pi^\pm$ is included fit. This adds another set of Eqs. (1)-(2) with $K \rightarrow \pi$. The common topology means $B^\pm \rightarrow DK^\pm$ and $D\pi^\pm$ will share the same F_i , but $B^\pm \rightarrow D\pi^\pm$ has smaller CP-violation because $r_B^{D\pi} \ll r_B^{DK}$. The fit stability is improved by introducing

$$x_\xi^{D\pi} = \text{Re}(\xi^{D\pi}), \quad y_\xi^{D\pi} = \text{Im}(\xi^{D\pi}), \quad \xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} e^{i(\delta_B^{D\pi} - \delta_B^{DK})}. \quad (7)$$

Therefore, the six CP observables in the ML fit are x_\pm^{DK} , y_\pm^{DK} , $x_\xi^{D\pi}$ and $y_\xi^{D\pi}$.

2.2 Strong phase from quantum correlations

In $\psi(3770) \rightarrow D^0 \bar{D}^0$ decays, $D^0 \bar{D}^0$ is a correlated antisymmetric state, since $\psi(3770)$ has charge conjugation $\mathcal{C} = -1$. Thus, c_i and s_i can be measured using a double tag method.

The number of events where only one D meson is reconstructed as $D \rightarrow f$ is the single tag yield of f . If both D mesons are reconstructed in the signal mode $D \rightarrow K^+ K^- \pi^+ \pi^-$ and tag mode $D \rightarrow f$, one obtains the double tag yield.

c_i is measured in events where the tag mode is a CP eigenstate or a mode with known CP-even fraction F_+ . The yield of CP tagged $D \rightarrow K^+ K^- \pi^+ \pi^-$ events in bin i is

$$M_i^\pm = \frac{S_\pm}{2S_f} \left[K_i - 2c_i(2F_+ - 1)\sqrt{K_i K_{-i}} + K_{-i} \right]. \quad (8)$$

K_i are the double tag yields, where the tag is a flavour tag, such as $D^0 \rightarrow K^- \pi^+$. S_\pm and S_f are single tag yields of the CP and flavour tag modes used, respectively. For CP even (odd) modes, $F_+ = 1$ (0). Table 1 shows the tag modes in this analysis. To obtain s_i , the tag mode phase space must also be binned, labelled by j . The analogous expression is

$$M_{ij} = \frac{N_{D\bar{D}}}{2S_f S'_f} \left[K_i K'_{-j} + 2\sqrt{K_i K'_{-j} K_{-i} K'_j} (c'_i c_j + s'_i s_j) + K_{-i} K'_j \right]. \quad (9)$$

$N_{D\bar{D}}$ is the total number of $D^0 \bar{D}^0$ pairs. If the tag is $K_S^0 \pi^+ \pi^-$, the $D \rightarrow K_S^0 \pi^+ \pi^-$ strong phases c'_i and s'_i are known [4]. If the tag is $K^+ K^- \pi^+ \pi^-$, then $c'_i = c_i$ and $s'_i = s_i$.

Table 1: List of tag modes in the double tag analysis. Flavour mode conjugates are implied.

Flavour	CP even	CP odd	Self conjugate
$K^- \pi^+, K^- \pi^+ \pi^0,$ $K^- \pi^+ \pi^- \pi^+, K^- e^+ \nu_e$	$K^+ K^-, \pi^+ \pi^-, \pi^+ \pi^- \pi^0,$ $K_S^0 \pi^0 \pi^0, K_L^0 \pi^0, K_L^0 \eta, K_L^0 \omega$	$K_S^0 \pi^0, K_S^0 \phi,$ $K_S^0 \eta, K_S^0 \eta', K_S^0 \omega$	$K_S^0 \pi^+ \pi^-,$ $K^+ K^- \pi^+ \pi^-$

3 LHCb and BESIII detectors

The LHCb [5] is a single arm forward spectrometer designed to study beauty and charm hadrons in pp collisions. The components important for this analysis are the tracking system and the Ring Imaging Cherenkov counters (RICH1 and RICH2). The tracking system includes the Vertex Locator (VELO), which are silicon strip modules close to the interaction point, providing high precision tracking and identification of displaced secondary vertices. A dipole magnet and the tracking stations measure charged particle momenta. The RICH detectors provides identification of kaons and pions.

BESIII [6] is a general purpose solenoidal detector. The parts relevant to this analysis are the Helium gas drift chamber for measuring the momenta and dE/dx of charged particles, a time of flight system for particle ID and an electromagnetic calorimeter to measure shower energies. $e^+ e^-$ collisions at the $\psi(3770)$ resonance is used for this analysis.

4 Binning scheme

In Ref. [2], the 2D phase space, visualized on a Dalitz plot, was separated into bins of similar strong phase to avoid diluting the amplitude-averaged c_i and s_i . Cabbibo favoured and suppressed resonances were assigned bin numbers with opposite sign to enhance the interference terms in Eqs. (3)-(4), and thus enhance the sensitivity to CP observables.

A binning scheme for the four-body decay $D \rightarrow K^+ K^- \pi^+ \pi^-$, where phase space is 5D, cannot easily be visualized. Instead, the LHCb model, implemented in AmpGen [7], is used to calculate the amplitude $\mathcal{A}(D)$ from the D daughter momenta. Defining $\mathcal{A}(D^0)/\mathcal{A}(\bar{D}^0) \equiv r_D e^{i\Delta\delta_D}$, where $\Delta\delta_D$ and r_D are the strong phase difference and relative magnitude, a simple, but naive, binning choice is uniformly spaced bins along $\Delta\delta_D$.

Under CP, $\Delta\delta_D \rightarrow -\Delta\delta_D$ and $\ln(r_D) \rightarrow -\ln(r_D)$. For $\ln(r_D) > 0$ (< 0), the \bar{D}^0 decay is suppressed (favoured), relative to the D^0 decay. Therefore, the interference terms in Eqs. (3)-(4) may be enhanced if bins with $i > 0$ and $i < 0$ are separated along $\ln(r_D) = 0$.

To optimize the binning, the bin boundaries are moved symmetrically around $\Delta\delta_D = 0$ to maximize Q , defined as the x_{\pm} and y_{\pm} sensitivity in a binned fit divided by that of an unbinned fit. It can be shown, with N_i^{\pm} from Eqs. (3)-(4), that

$$Q^2 = \frac{1}{2}(Q_+^2 + Q_-^2), \quad Q_{\pm}^2 = 1 - \sum_i \frac{F_i F_{-i} (1 - c_i^2 - s_i^2)}{N_i^{\pm}}. \quad (10)$$

To assess the binning scheme efficiency, 1000 toy experiments, each with 2000 B^{\pm} candidates, were generated with the LHCb model in AmpGen, using $\gamma = 75^\circ$, $\delta_B = 130^\circ$ and $r_B = 0.1$. An unbinned ML fit was performed to establish a benchmark of $\Delta\gamma = 11^\circ$.

With 2×8 bins, Fig. 2a shows the optimal binning scheme, with $Q = 0.90$, indicating that 10% sensitivity is lost due to binning. Using the binning in Fig. 2a, a ML fit was performed on each toy experiment using Eqs. (3)-(4) to extract x_{\pm}^{DK} and y_{\pm}^{DK} . c_i , s_i and F_i were obtained from Monte Carlo integration of Eqs. (5) with $\mathcal{A}(D)$ from the LHCb model. Finally, γ , δ_B and r_B were obtained from x_{\pm}^{DK} and y_{\pm}^{DK} using Eqs. (6).

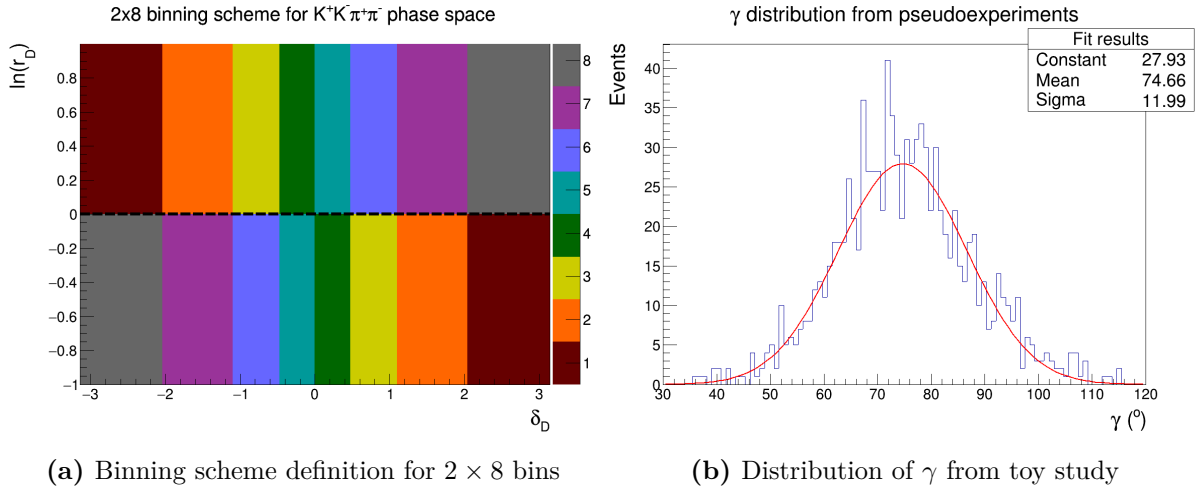


Figure 2

Fig. 2b shows the γ distribution in the toy experiments. The binned γ precision is $\Delta\gamma = (12.0 \pm 0.4)^\circ$, which is consistent with the unbinned benchmark and $Q = 0.90$. The pull distributions of x_{\pm}^{DK} , y_{\pm}^{DK} , γ , δ_B and r_B have mean and standard deviations consistent with zero and one, respectively, except for r_B , which has a small bias.

5 B^{\pm} candidate selection

The $B^{\pm} \rightarrow (K^+K^-\pi^+\pi^-)_D h^{\pm}$ candidates are reconstructed using standard candidate requirements from Ref. [2]. The $K^+K^-\pi^+\pi^-$ invariant mass must be within 25 MeV of the D^0 mass. The tracks are refitted with the D invariant mass constrained and its momentum pointing towards B^{\pm} . A cut on χ^2 from this fit removes the D mass sidebands.

$B^{\pm} \rightarrow K^+K^-\pi^+\pi^-h^{\pm}$ candidates form a charmless background in the B^{\pm} mass spectrum, but does not peak in the D mass spectrum. To estimate the contamination,

the χ^2 cut was removed to preserve the D mass sidebands. Inside the mass region $m(K^+K^-\pi^+\pi^-) \in [1770, 1820]\text{MeV}$, a fit to the B^\pm mass spectrum gives a yield of 2605 ± 57 . A flight significance cut, defined as the flight distance divided by its error, reduces this yield to 110 ± 19 . No charmless background was found for $B^\pm \rightarrow D\pi^\pm$.

A significant mis-ID background is $B^\pm \rightarrow Dh^\pm$, $D \rightarrow K^\pm\pi^\mp\pi^\pm\pi^\mp$, where a pion is identified as a kaon. A Monte Carlo (MC) simulation sample indicated that 7.2% of B^\pm candidates inside the signal region are $D \rightarrow K^\pm\pi^\mp\pi^\pm\pi^\mp$. A tighter particle identification requirement reduces this to 1.8% while keeping 93% of $D \rightarrow K^+K^-\pi^+\pi^-$ candidates.

For combinatorial background, a Boosted Decision Tree (BDT) was trained using MC samples as signal and the region $m(Dh^\pm) \in [5800, 7000]\text{MeV}$ in data as background. 99.4% of the combinatorial background was removed and 93.0% of the signal remained.

6 B^\pm global mass fit and binned CP fit

A global ML fit of the B^\pm mass spectrum, shown in Fig. 3, was performed to determine global yields and shape parameters, with shape parameterizations from Ref. [2]. The final analysis requires minor adjustments. Currently, only Run 2 data has been processed.

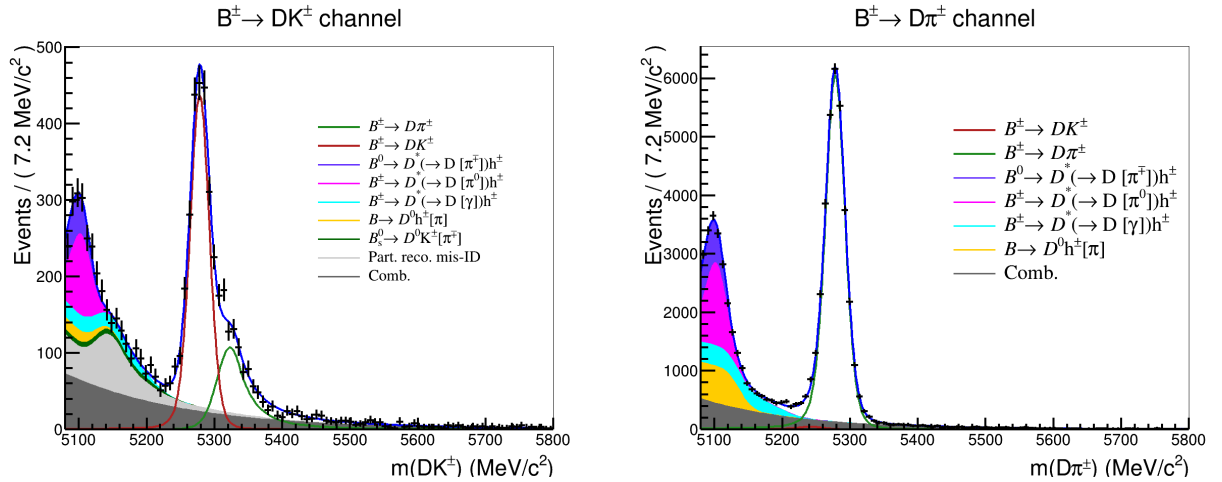


Figure 3: Global mass fit of $B^\pm \rightarrow DK^\pm$ (left) and $B^\pm \rightarrow D\pi^\pm$ (right).

The combinatorial background is parameterized by an exponential function. The signal is a sum of a Gaussian $f_G(m|m_B, \sigma)$ and a modified Gaussian,

$$f_{\text{MG}}(m|m_B, \sigma, \alpha_L, \alpha_R, \beta) \propto \begin{cases} \exp\left(\frac{-\Delta m^2(1+\beta\Delta m^2)}{2\sigma^2+\alpha_L\Delta m^2}\right), & \Delta m = m - m_B < 0 \\ \exp\left(\frac{-\Delta m^2(1+\beta\Delta m^2)}{2\sigma^2+\alpha_R\Delta m^2}\right), & \Delta m = m - m_B > 0, \end{cases} \quad (11)$$

which accounts for the radiated tail and wider resolution. To the left of the B^\pm mass peak are partially reconstructed B^\pm , B^0 and B_s decays where a π^\pm or γ is missed, indicated by $[\pi^\pm/\gamma]$ in the legend. In the $B^\pm \rightarrow DK^\pm$ channel, there is a mis-ID component of $B^\pm \rightarrow D\pi^\pm$, and a partially reconstructed mis-ID component, where π^\pm is misidentified as K^\pm . The yields of $B^\pm \rightarrow DK^\pm$ and $D\pi^\pm$ candidates are 2290 ± 59 and 33113 ± 211 .

After the global fit, B^\pm candidates are split by charge and bins, using the binning in Fig. 2a. A ML fit is performed, with shape parameters from the global fit fixed, but the

yields are floated. The fitted CP observables are shown in Fig. 4. Geometrically, the angle between (x_+^{DK}, y_+^{DK}) and (x_-^{DK}, y_-^{DK}) is 2γ , but the exact value is blinded.

To check the fit robustness, 1000 toy datasets are generated, for both global and CP fits, using the fitted parameters. Each toy is run through the fitting procedure. All CP observables had pulls consistent with zero mean and unit standard deviation.

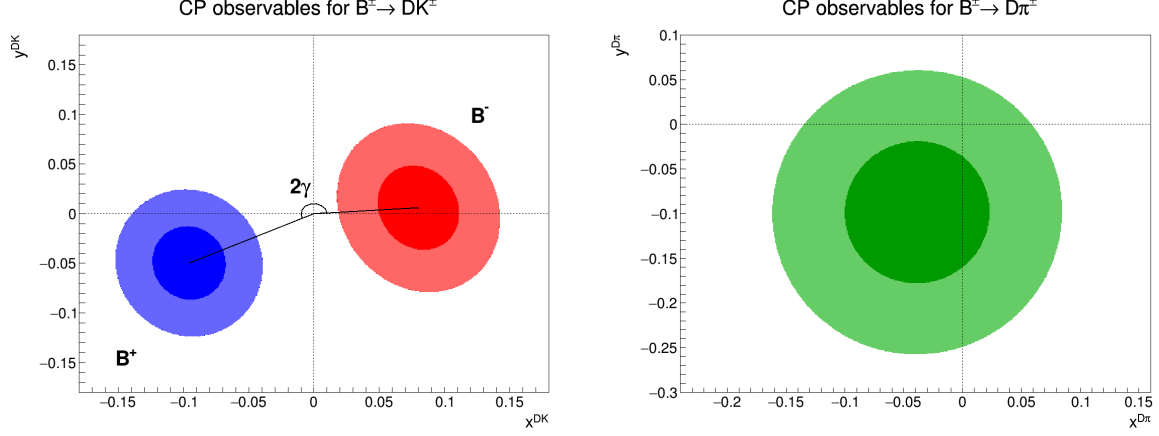


Figure 4: Confidence levels at 68.2% and 95.5% of $(x_{\pm}^{DK}, y_{\pm}^{DK})$ (left) and $(x_{\xi}^{D\pi}, y_{\xi}^{D\pi})$ (right).

7 External strong phase input from BESIII

Event reconstruction at BESIII follows Ref. [4]. The single tag yield is obtained from a ML fit to the beam constrained D mass $M_{BC} = (E_{\text{beam}}^2 - |\sum_i \mathbf{p}_i|^2)^{1/2}$, shown in Fig. 5a for $D \rightarrow K^+K^-\pi^+\pi^-$. Peaking backgrounds are accounted for using inclusive MC. The main component, in green, is $D \rightarrow K_S^0 K^+ K^-$. The signal shape is a MC sample convolved with a Gaussian. An Argus shape parameterizes the combinatorial background.

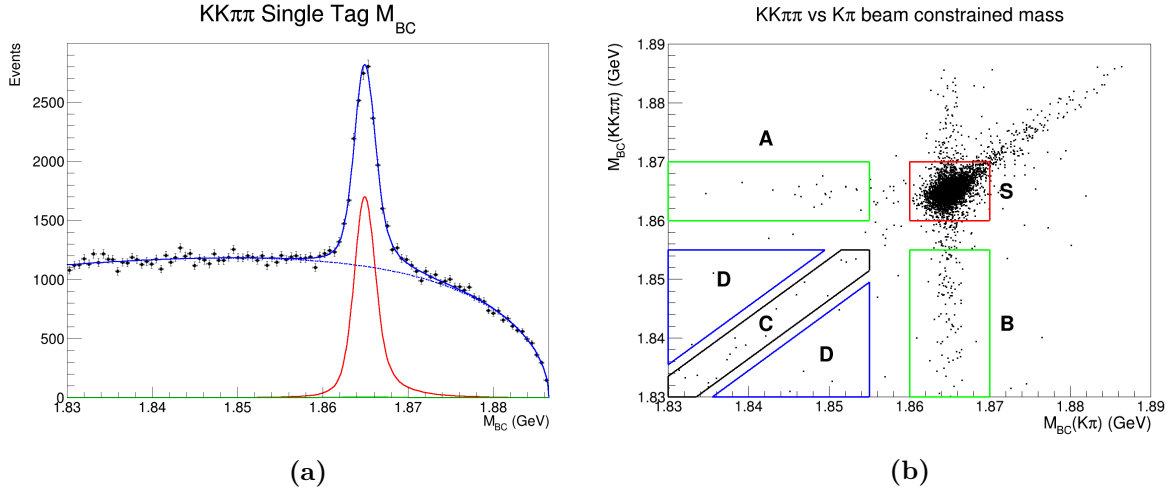


Figure 5: Fit of $K^+K^-\pi^+\pi^-$ single tag M_{BC} distribution (left) and background subtraction technique of $K^+K^-\pi^+\pi^-$ vs $K^{\mp}\pi^{\pm}$ double tag (right).

For double tag yields, a sideband subtraction technique is used. Fig. 5b shows $D \rightarrow K^+K^-\pi^+\pi^-$ versus $D \rightarrow K^{\pm}\pi^{\mp}$ events, where the M_{BC} for each D meson is plotted in a 2D scatter plot. Region S is the signal region, while A and B contain only

one real D meson. In region C , daughters have been swapped between the D mesons. Region D contains non-charm background. The background is estimated using

$$B = P + \frac{a_S}{a_D} Y_D + \sum_{i=A,B,C} \frac{a_S}{a_i} \left(Y_i - \frac{a_i}{a_D} Y_D \right),$$

where P is the peaking background, and Y_i and a_i are the yield and area of region i . The single and double tag yields are used in a fit of Eqs. (8)-(9) to obtain c_i and s_i .

Fig. 6 shows, on the left, $D \rightarrow K^\pm \pi^\mp$ double tag yields with the LHCb model prediction, using 2×4 bins. On the right are the double tag yields with CP odd and even tags $K_S^0 \pi^0$ and $K^+ K^-$, normalized by $K^\pm \pi^\mp$ yields. The agreement is satisfactory, but the yields are not corrected for efficiencies yet, so perfect agreement is not expected.

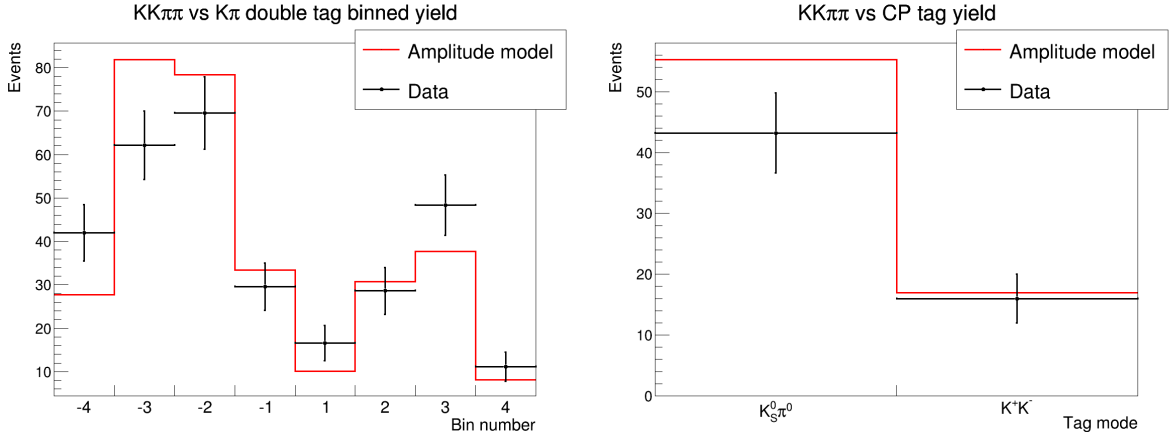


Figure 6: Double tag yield of $K^\pm \pi^\mp$ (left), and $K^+ K^-$ and $K_S^0 \pi^0$ (right).

8 Conclusion and future work

The current results are consistent with prior expectations in both LHCb and BESIII. The next steps in the LHCb analysis is optimization of cuts and adjusting shape parameters, in addition to a systematics study. For BESIII, the yields of all tag modes must be finalized. Finally, the analyses will be combined into a first γ determination in this channel.

References

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9 DPhil thesis plan

Discuss DPhil thesis plan with Guy first!