

Model-independent measurement of the CKM angle γ with $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$ at LHCb and BESIII

Martin Tat

University of Oxford

IOP Joint APP and HEPP Conference

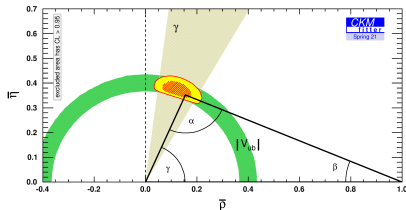
3rd-5th April 2023



Introduction to γ and CP violation

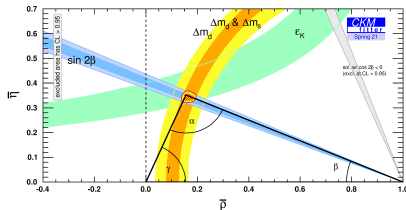
Introduction to γ and CP violation

- CPV in SM is described by the Unitary Triangle, with angles α , β , γ
- The angle $\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$ is very important:
 - 1 Negligible theoretical uncertainties: Ideal SM benchmark
 - 2 Accessible at tree level: Indirectly probe New Physics that enter loops
 - 3 Compare with a global CKM fit: Is the Unitary Triangle a triangle?



(a) Tree level: $\gamma = (72.1^{+5.4}_{-5.7})^\circ$

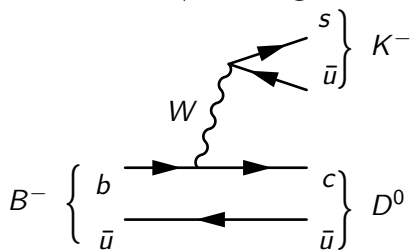
CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005)



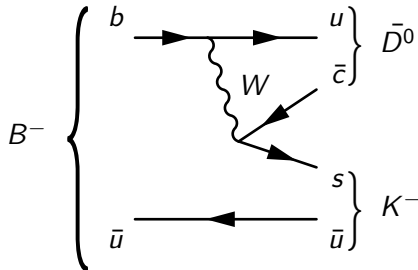
(b) Loop level: $\gamma = (65.5^{+1.1}_{-2.7})^\circ$

Sensitivity through interference

Measure γ through interference effects in $B^\pm \rightarrow DK^\pm$



Favoured $B^- \rightarrow D^0 K^-$



Suppressed $B^- \rightarrow \bar{D}^0 K^-$

- Superposition of D^0 and \bar{D}^0
 - Consider D^0/\bar{D}^0 decays to the same final state, such as $D \rightarrow K^+ K^-$
- $b \rightarrow u\bar{c}s$ and $b \rightarrow c\bar{u}s$ interference \rightarrow Sensitivity to γ

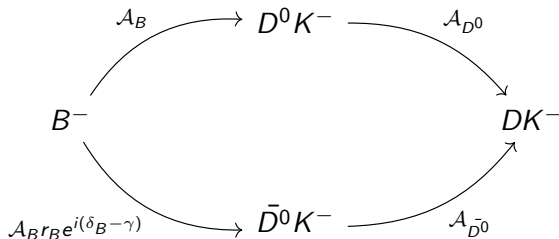
$$\mathcal{A}(B^-) = \mathcal{A}_B \left(\mathcal{A}_{D^0} + r_B e^{i(\delta_B - \gamma)} \mathcal{A}_{\bar{D}^0} \right)$$

$$\mathcal{A}(B^+) = \mathcal{A}_B \left(\mathcal{A}_{\bar{D}^0} + r_B e^{i(\delta_B + \gamma)} \mathcal{A}_{D^0} \right)$$

Sensitivity through interference

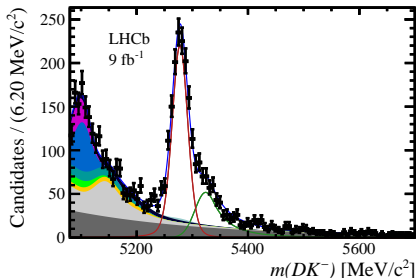
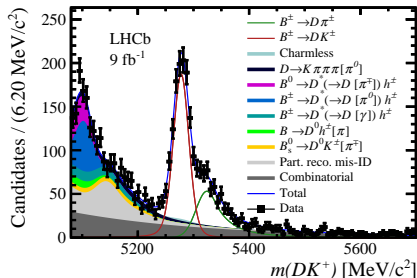
This talk: Focus on multi-body D decays, where interference effects vary across phase space

- First, consider a phase-space integrated analysis
- Compare yields of B^+ and B^- and determine the asymmetry
- Interference effects are diluted by c_1 when integrated over phase space



$$|\mathcal{A}(B^-)|^2 \propto 1 + r_B^2 + 2r_B c_1 \cos(\delta_B - \gamma)$$

First look at $B^\pm \rightarrow [K^+K^-\pi^+\pi^-]_D K^\pm$



arXiv:2301.10328

Measuring the $B^\pm \rightarrow DK^\pm$ asymmetry \mathcal{A} provides useful constraints on γ , but with some caveats:

- ① Interference effects are diluted by a factor $c_1 = 0.46 \pm 0.08$
 - [Phys. Rev. D **107**, 032009](#)
- ② Four-fold degeneracy:
 - $(\gamma, \delta_B) \rightarrow (\delta_B, \gamma)$
 - $(\gamma, \delta_B) \rightarrow (\pi - \gamma, \pi - \delta_B)$

Phase-space binned analysis of $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$

Phase-space binned analysis of $B^\pm \rightarrow [K^+K^-\pi^+\pi^-]_D K^\pm$

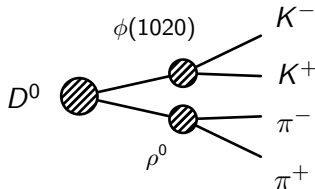
Our aim: First binned model independent measurement of γ in $B^\pm \rightarrow [K^+K^-\pi^+\pi^-]_D K^\pm$

- Need external information about the D^0 - \bar{D}^0 strong-phase difference δ_D across a 5-dimensional phase space to extract γ
- Model dependent measurement first proposed in 2007
 - [Phys. Lett. B647 \(2007\) 400](#) by J. Rademacker and G. Wilkinson
 - FOCUS amplitude model: 14° precision with 1000 candidates

$$\mathcal{A}(\Phi) = \sum a_i B_i(\Phi)$$

a_i = Complex coefficients

$B_i(\Phi)$ = Lineshapes (Breit-Wigner, etc)



Phase-space binned analysis of $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$

- It is better to measure δ_D directly at a charm factory, such as BESIII

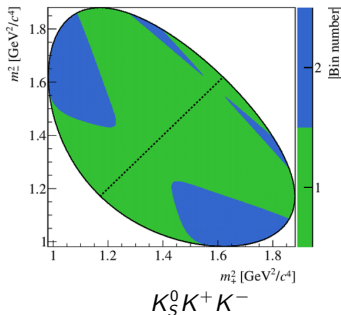
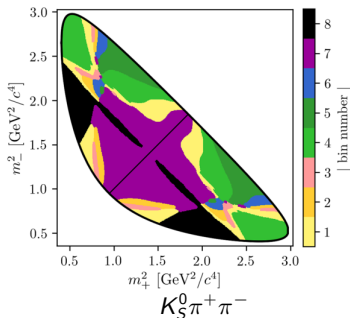
Amplitude-average cosine of δ_D in bin i

$$c_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D}^0)| \cos(\delta_D)}{\sqrt{\int_i d\Phi |\mathcal{A}(D^0)|^2 \int_i d\Phi |\mathcal{A}(\bar{D}^0)|^2}}$$

- State of the art amplitude analysis by LHCb: [JHEP 02 \(2019\) 126](#)
 - Identify regions with expected large asymmetries
 - Place regions with similar asymmetries together in bins
 - Final measurement will be model independent

Phase-space binned analysis of $B^\pm \rightarrow [K^+K^-\pi^+\pi^-]_D K^\pm$

- Analogous to the decays $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ and $K_S^0 K^+ K^-$, where the binning scheme may be visualised on a Dalitz plot
- Single most precise measurement: $\gamma = (68.7^{+5.2}_{-5.1})^\circ$
 - JHEP 02 (2021) 0169



- Measurements of c_i and s_i were performed at:
 - CLEO: Phys. Rev. **D82** (2010) 112006
 - BESIII: Phys. Rev. **D101** (2020) 112002

Binning scheme

A binning scheme must satisfy the following:

- Minimal dilution of strong phases when integrating over bins
- Enhance interference between $B^\pm \rightarrow D^0 K^\pm$ and $B^\pm \rightarrow \bar{D}^0 K^\pm$

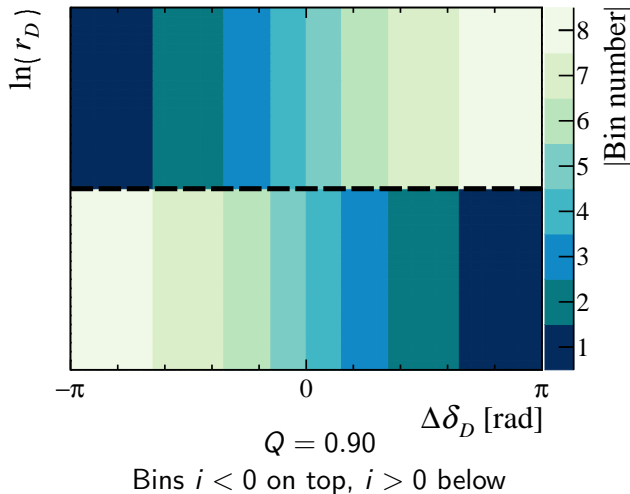
How to bin a 5-dimensional phase space?

- 1 For each B^\pm candidate, use the amplitude model to calculate

$$\frac{\mathcal{A}(D^0)}{\mathcal{A}(\bar{D}^0)} = r_D e^{i\delta_D}$$

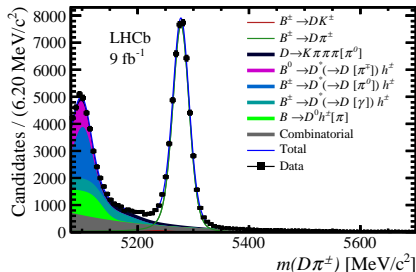
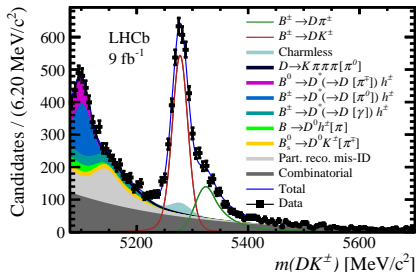
- 2 Split δ_D into uniformly spaced bins
- 3 Use the symmetry line $r_D = 1$ to separate bin $+i$ from $-i$
- 4 Optimise the binning scheme by adjusting the bin boundaries in δ_D

Binning scheme



Mass fits, CP fit and γ

Global invariant mass fit: Fit everything to understand mass shapes and background yields

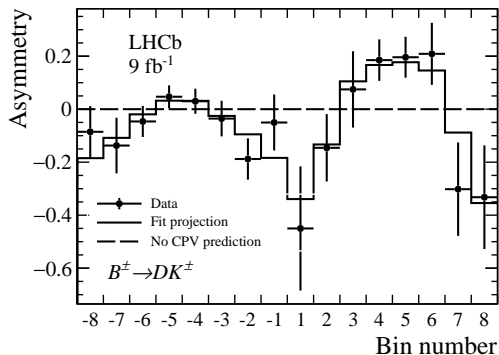


arXiv:2301.10328

Signal yield:

$$B^\pm \rightarrow DK^\pm : 3026 \pm 38$$

$$B^\pm \rightarrow D\pi^\pm : 44\,349 \pm 218$$



arXiv:2301.10328

- Clear bin asymmetries are seen, and the non-trivial distribution is driven by the change in strong phases across phase space
- While the interpretation of γ require charm inputs, the observed bin asymmetries are model independent

CP fit: Simultaneously fit bin yields and determine physics parameters, using model predictions of c_i and s_i :

$$\gamma = (116^{+12}_{-14})^\circ$$

[arXiv:2301.10328](#)

However, the latest γ and charm combination result is:

$$\gamma = (63.8^{+3.5}_{-3.7})^\circ$$

Results are compatible at 3σ , but could there be some other issue?

CP fit: Simultaneously fit bin yields and determine physics parameters, using model predictions of c_i and s_i :

$$\gamma = (116^{+12}_{-14})^\circ$$

[arXiv:2301.10328](https://arxiv.org/abs/2301.10328)

However, the latest γ and charm combination result is:

$$\gamma = (63.8^{+3.5}_{-3.7})^\circ$$

Do we trust the model predicted c_i and s_i , or their uncertainties?

There are several reasons why amplitude models cannot be trusted

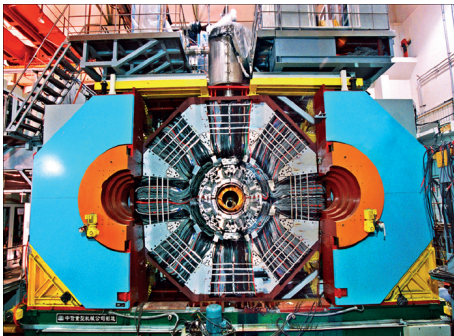
- ① Amplitude models are just models, which may not reflect reality
- ② In fact, the model is fitted to data that knows nothing about δ_D
- ③ It is impossible to assign an objective error to a model!

We wish to do a model independent measurement
Let's go and measure c_i and s_i at BESIII!

Strong phase analysis of $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ at BESIII

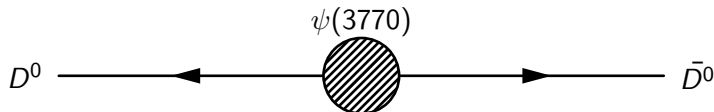
Strong phase analysis of $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ at BESIII

- BESIII: Beijing Spectrometer III, a detector at the Beijing Electron-Positron Collider II, located at IHEP
- e^+e^- collider at the $\psi(3770) \rightarrow D^0 \bar{D}^0$ threshold
 - 2010-2011: 3 fb^{-1}
 - 2021-2022: 5 fb^{-1}
 - 2023-: 5 fb^{-1}
 - Expect 20 fb^{-1} in total by end of 2024

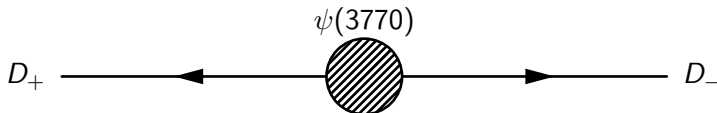


Strong phase analysis of $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ at BESIII

- Double-tag analysis: Reconstruct signal ($KK\pi\pi$) and tag mode
- $D^0 \bar{D}^0$ pair is quantum correlated



- Equivalently, we can consider $D_+ D_-$
 - $D_{\pm} = \frac{1}{\sqrt{2}}(D^0 \pm \bar{D}^0)$ are CP eigenstates



The DD pair is quantum correlated, spooky action at a distance!

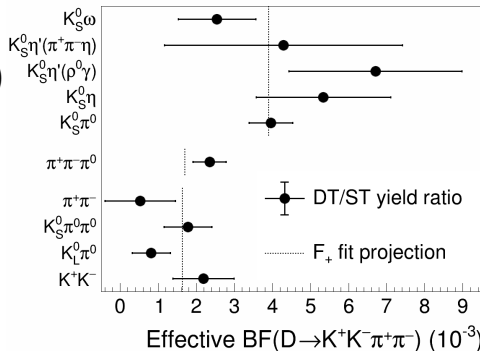
Strong phase analysis of $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ at BESIII

Quantum correlation: The CP content of the tag can modify the effective branching fraction

$$\frac{N^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi)(1 \pm c_1)$$

c_1 is the cosine of the strong phase, averaged over the whole phase space

Result: $c_1 = 0.46 \pm 0.08$



Phys. Rev. D **107**, 032009

Strong phase analysis of $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ at BESIII

Our next task is to change the phase-space integrated analysis,

$$\frac{N^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi) \quad (\text{flavour tag})$$

$$\frac{N^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi)(1 \pm c_1) \quad (\text{CP tag})$$

into a binned phase space analysis:

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi)F_i \quad (\text{flavour tag})$$

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi)(F_i + \bar{F}_i \pm 2\sqrt{F_i \bar{F}_i} c_i) \quad (\text{CP tag})$$

- 1 F_i : Measure using flavour tags
- 2 c_i : Determine from asymmetry of CP even and odd tags
- 3 s_i : Analogous to c_i , but requires binning of tag mode

Strong phase analysis of $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ at BESIII

Our next task is to change the phase-space integrated analysis,

$$\frac{N^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi) \quad (f')$$

$$\frac{N^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi) \quad (\text{CP tag})$$

into a binned phase analysis:

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK) \quad (\text{flavour tag})$$

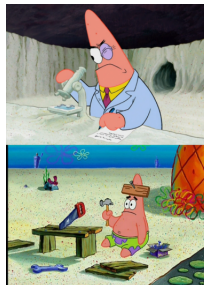
$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0) (F_i + \bar{F}_i \pm 2\sqrt{F_i \bar{F}_i} c_i) \quad (\text{CP tag})$$

- ① F_i : Measure using 1. our tags
- ② c_i : Determine from asymmetry of CP even and odd tags
- ③ s_i : Analogous to c_i , but requires binning of tag mode

Work in progress!

Summary and conclusion

- 1 I have presented the first model-independent measurement of γ using $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$
- 2 The optimised binning scheme, developed with an amplitude model, successfully identified regions with large, local CP asymmetries
- 3 Model predictions of δ_D must not be trusted for γ extraction
- 4 LHCb analysis has 3σ tension with world average
- 5 External inputs from charm factories, such as BESIII, are crucial to eliminate biases due to modelling



Making binning scheme with amplitude model

Predicting strong phases with amplitude model

Thanks for your attention!

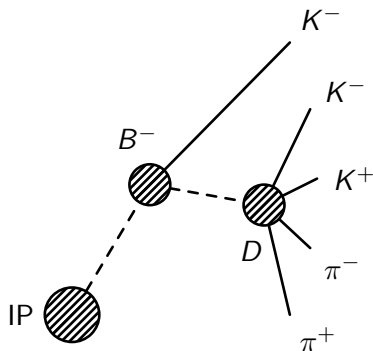
Backup slides

Event selection

Decay topology

Look for:

- 1 5 charged tracks
- 2 Displaced B vertex
- 3 1 bachelor track with good PID information
- 4 Displaced D vertex with invariant mass within 25 MeV of the D^0 mass



Offline selection has 3 stages

Initial cuts:

- 1 Invariant D and B mass cuts
- 2 Momentum and RICH requirements

Boosted Decision Tree (BDT)

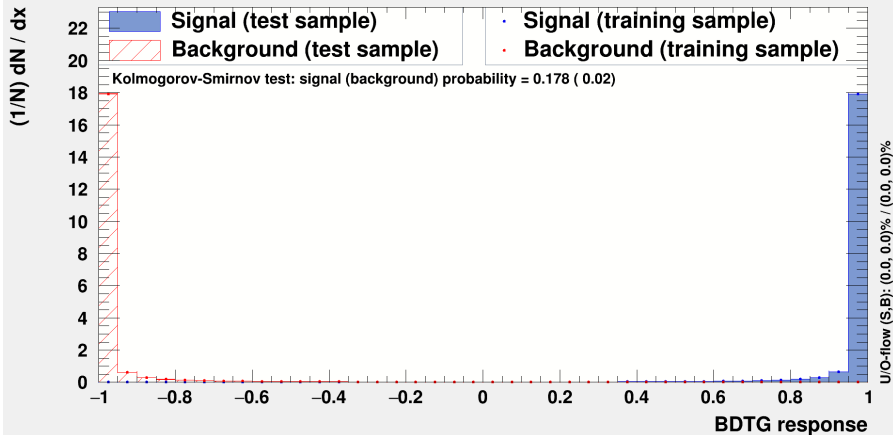
- Signal sample: Simulation samples
- Background sample: Upper B mass sideband
- 28 variables describing kinematics, impact parameters, vertex quality

Final selection

- 1 D Flight distance
- 2 Particle Identification of bachelor
- 3 K_S^0 veto

Event selection

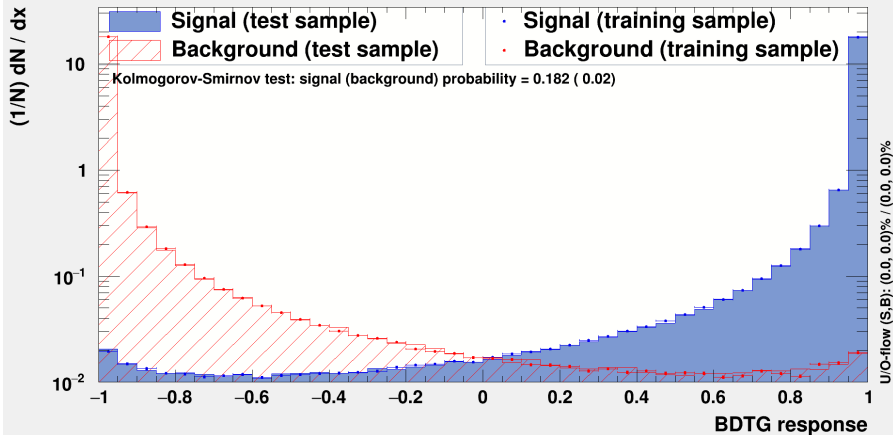
TMVA overtraining check for classifier: BDTG



BDT is highly efficient at rejecting combinatorial background

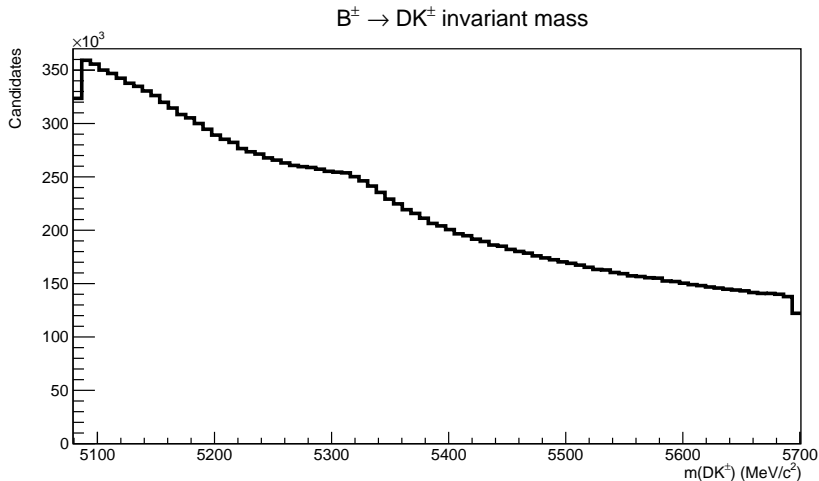
Event selection

TMVA overtraining check for classifier: BDTG



Very important, combinatorial background is large in multi-body decays

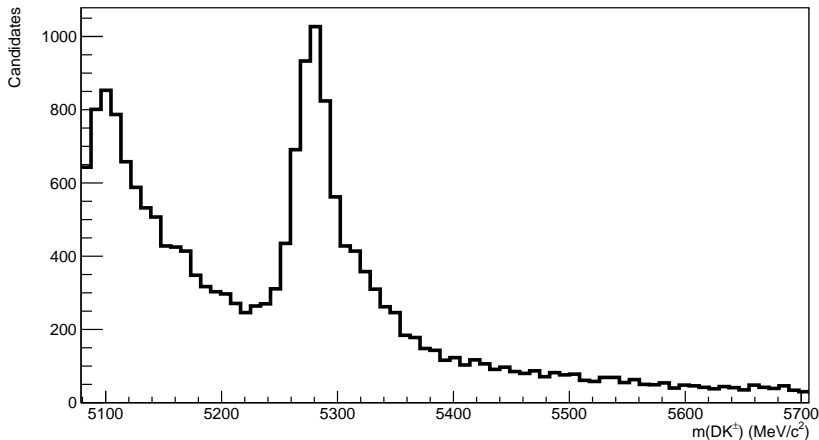
Event selection



The invariant B mass, after online selection, show no visible signal...

Event selection

$B^\pm \rightarrow DK^\pm$ invariant mass



... but the BDT does a great job cleaning this up!