Model-independent measurement of γ in $B^\pm \to [h^+h^-\pi^+\pi^-]_D h^\pm$ at LHCb and BESIII

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Introduction

Last presentation since starting my PhD journey 1295 days ago!

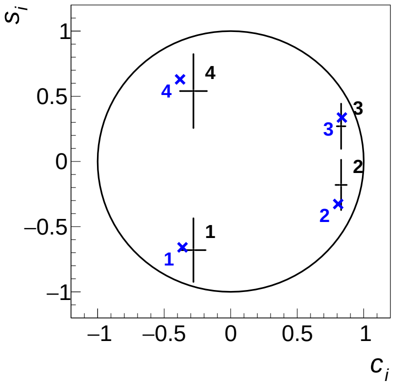
1. October 2020: Sensitivity studies with $B^\pm \to [K^+K^-\pi^+\pi^-]_D K^\pm$
2. April 2021: First data fit reveals large tension in γ

3. January 2023: Model-dependent γ publication with 3 σ tension
4. March 2024: BESIII c_i/s_i analysis approved by review committee
5. April 2024: Presented $B^\pm \to [h^+h^-\pi^+\pi^-]_D h^\pm$ to B2OC

Recap of BESIII $D^0 \to K^+K^-\pi^+\pi^-$ strong-phase results

With additional BESIII data ($16\,\text{fb}^{-1}$), c_i/s_i agree perfectly with model
Analysis approved by review committee, paper currently in review

$$c_1 = -\,0.28 \pm 0.09 \pm 0.01$$
$$s_1 = -\,0.68 \pm 0.24 \pm 0.04$$
$$c_2 = +\,0.83 \pm 0.04 \pm 0.01$$
$$s_2 = -\,0.18 \pm 0.19 \pm 0.03$$
$$c_3 = +\,0.83 \pm 0.03 \pm 0.01$$
$$s_3 = +\,0.27 \pm 0.17 \pm 0.03$$
$$c_4 = -\,0.28 \pm 0.10 \pm 0.01$$
$$s_4 = +\,0.54 \pm 0.28 \pm 0.04$$



Measured values (black) are consistent and close to LHCb model predictions (blue), so central value of γ is not expected to change much

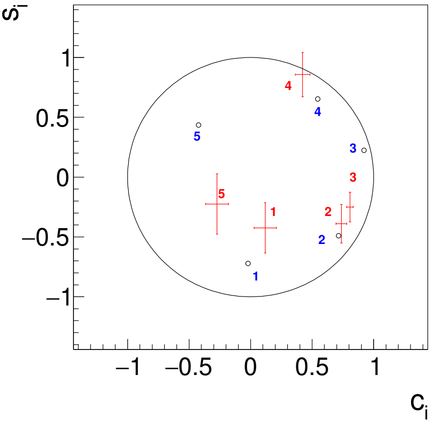
BESIII preliminary $D^0 \to \pi^+\pi^-\pi^+\pi^-$ strong-phase results

- Binned strong-phase analysis of $D^0 \to \pi^+\pi^-\pi^+\pi^-$ uses the 2×5 "optimal" binning scheme with 3 fb^{-1} $\psi(3770)$
- Earlier CLEO-c analysis with 0.8 fb^{-1} JHEP **01** (2018) 144
- New BESIII analysis uses a new binning scheme optimised with a BESIII amplitude model arXiv:2312.02524
  - Amplitude model constructed from a larger data set
  - In principle more sensitive
- Two binning schemes are available:
  - We use the more sensitive "optimal" binning with $Q = 0.85$
  - The other "equal δ" binning has $Q = 0.80$
- Analysis also approved by review committee, currently in paper review

BESIII preliminary $D^0 \to \pi^+\pi^-\pi^+\pi^-$ strong-phase results

Small differences between model prediction and measurement, but data points are generally close to the unit circle

$$c_1 = +0.12 \pm 0.09 \pm 0.02$$
$$s_1 = -0.42 \pm 0.21 \pm 0.04$$
$$c_2 = +0.74 \pm 0.04 \pm 0.02$$
$$s_2 = -0.39 \pm 0.16 \pm 0.06$$
$$s_3 = -0.25 \pm 0.12 \pm 0.03$$
$$c_3 = +0.81 \pm 0.03 \pm 0.01$$
$$c_4 = +0.42 \pm 0.06 \pm 0.02$$
$$s_4 = +0.86 \pm 0.19 \pm 0.07$$
$$c_5 = -0.27 \pm 0.09 \pm 0.03$$
$$s_5 = -0.22 \pm 0.25 \pm 0.08$$



Plan: Publish measurement of γ using both $K^+K^-\pi^+\pi^-$ and $\pi^+\pi^-\pi^+\pi^-$

The BPGGSZ method

Event yield in bin i

$$N_i^- = h_{B^-}\left(F_i + (x_-^2 + y_-^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_- c_i + y_- s_i)\right)$$
$$N_{-i}^+ = h_{B^+}\left(F_i + (x_+^2 + y_+^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_+ c_i + y_+ s_i)\right)$$

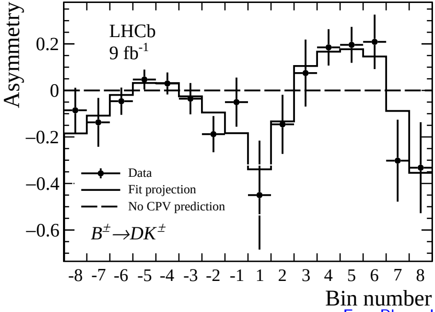
- CP observables:
  - $x_\pm^{DK} = r_B^{DK}\cos\left(\delta_B^{DK} \pm \gamma\right), \quad y_\pm^{DK} = r_B^{DK}\sin\left(\delta_B^{DK} \pm \gamma\right)$
  - $x_\xi^{D\pi} = \mathrm{Re}(\xi^{D\pi}),\ y_\xi^{D\pi} = \mathrm{Im}(\xi^{D\pi}) \qquad \left(\xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} e^{i(\delta_B^{D\pi} - \delta_B^{DK})}\right)$
- Fractional bin yield:
  - $F_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}$
  - Floated in the fit, mostly constrained by $B^\pm \to D\pi^\pm$
- Amplitude averaged strong phases:

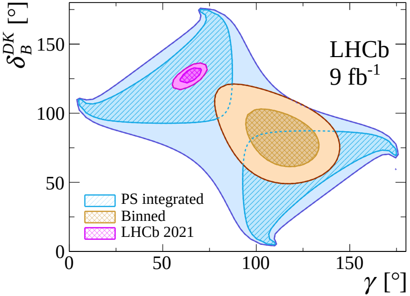
$$c_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\cos(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}} \qquad s_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\sin(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}}$$

Model-dependent measurement with $D \to K^+K^-\pi^+\pi^-$

From the phase-space binned asymmetries, we obtain:

$$\gamma = (116^{+12}_{-14})^\circ$$



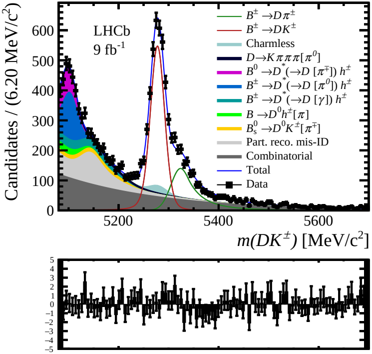


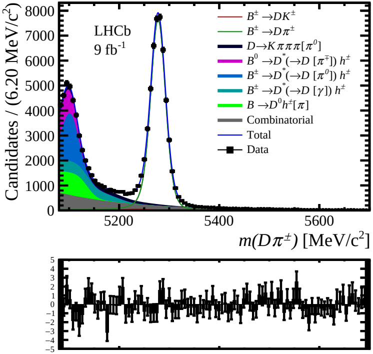
Eur. Phys. J. C **83**, 547 (2023)

How will this evolve with model-independent BESIII inputs? Will the 3σ tension reduce?

Global fit

Global fit of $K^+K^-\pi^+\pi^-$ remains as in model-dependent publication:





- $B^\pm \to [K^+K^-\pi^+\pi^-]_D h^\pm$ signal yield:
  - $B^\pm \to DK^\pm$: 3051 ± 38
  - $B^\pm \to D\pi^\pm$: 44356 ± 218

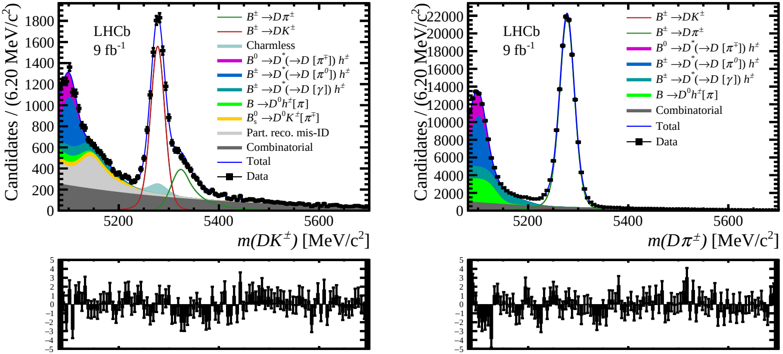
Global fit

How do we include the $\pi^+\pi^-\pi^+\pi^-$ mode?

- We have already studied $B^\pm \to [\pi^+\pi^-\pi^+\pi^-]_D h^\pm$ for phase-space integrated measurement
  1. Different D daughter PID cuts in stripping
  2. No $D \to K\pi\pi\pi\pi^0$ background
  3. Charmless background recalculated using the sideband
  4. Use same BDT
  5. No additional peaking backgrounds
- Sort candidates into phase-space bins using BESIII binning scheme
- Can fit separately or simultaneously with $K^+K^-\pi^+\pi^-$

Global fit

Global fit of $\pi^+\pi^-\pi^+\pi^-$ has a good fit quality:



- $B^\pm \to [\pi^+\pi^-\pi^+\pi^-]_D h^\pm$ signal yield:
  - $B^\pm \to DK^\pm$: 8745 ± 105
  - $B^\pm \to D\pi^\pm$: 126314 ± 385

CP fit

After global fit, perform a "CP fit" to study CP violation:

- Split candidates by:
  1. B^+ and B^- charges
  2. $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$ decays
  3. D phase-space bins
- Combinatorial and low-mass backgrounds are floating in each category
- Parameterise signal yields in terms of $x_\pm^{DK}$, $y_\pm^{DK}$, $x_\xi^{D\pi}$, $y_\xi^{D\pi}$
- $2N - 1$ floating F_i parameters
- c_i and s_i are Gaussian constrained

CP fit categories

Summary of free parameters in the CP fit:

$K^+K^-\pi^+\pi^-$

$2 \times 2 \times 2 \times 4 = 32$ categories

- 6 CP observables
- 7 F_i parameters
- 8 c_i and s_i parameters
- 32 combinatorial yields
- 32 low mass yields
- 4 global normalisations
- Total: 89 parameters

$\pi^+\pi^-\pi^+\pi^-$

$2 \times 2 \times 2 \times 5 = 40$ categories

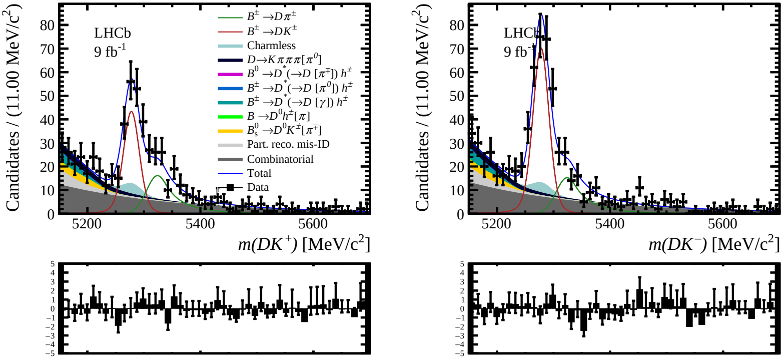
- 6 CP observables
- 9 F_i parameters
- 10 c_i and s_i parameters
- 40 combinatorial yields
- 40 low mass yields
- 4 global normalisations
- Total: 109 parameters

In a combined fit where CP observables are shared, there are

$89 + 109 - 6 = 192$ parameters

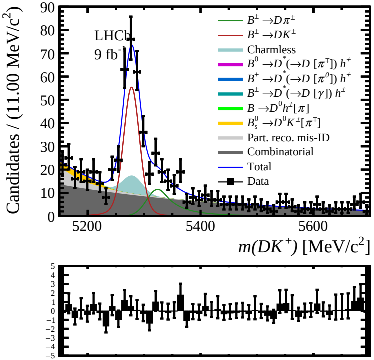
CP fit bin asymmetry

Example of bin asymmetry in $D \to K^+K^-\pi^+\pi^-$ bin -3:



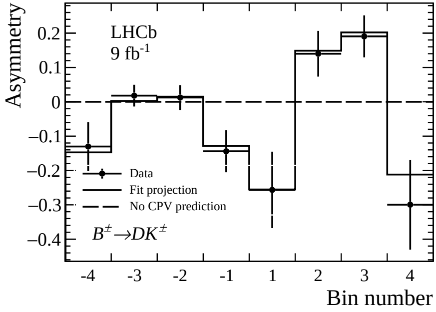
CP fit bin asymmetry

Example of bin asymmetry in $D \to \pi^+\pi^-\pi^+\pi^-$ bin $+5$:

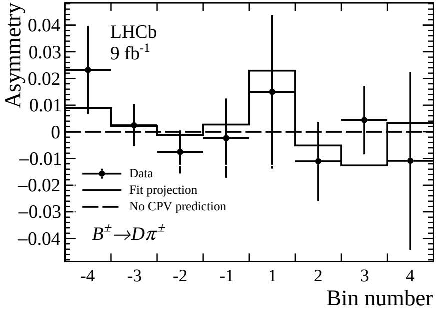


Bin asymmetries

$B^{\pm} \to [K^+K^-\pi^+\pi^-]_D h^{\pm}$ bin asymmetries



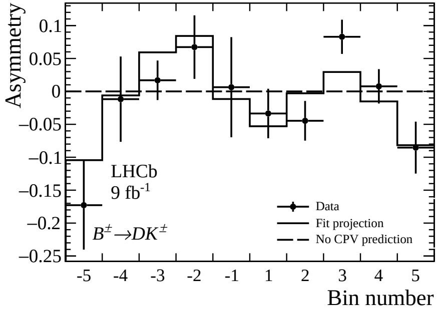
$B^{\pm} \to DK^{\pm}$



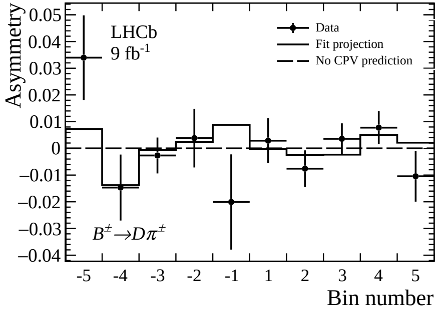
$B^{\pm} \to D\pi^{\pm}$

Bin asymmetries

$B^{\pm} \to [\pi^+\pi^-\pi^+\pi^-]_D h^{\pm}$ bin asymmetries



$B^{\pm} \to DK^{\pm}$



$B^{\pm} \to D\pi^{\pm}$

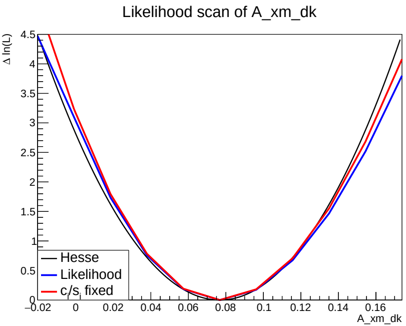
Strong-phase parameters in CP fit

Why are c_i and s_i Gaussian constrained?

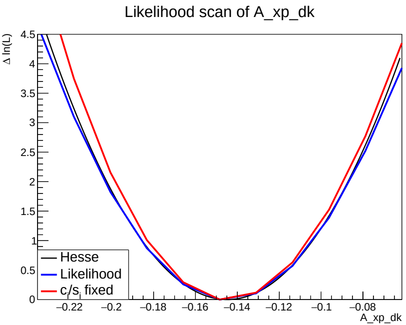
- Previous BPGGSZ analyses have kept c_i and s_i fixed
  1. c_i and s_i uncertainties are added as a systematic through smearing
  2. Convenient for calculating correlations between different analyses
  3. Appropriate when c_i and s_i uncertainties are small
- In four-body analyses, uncertainties on γ from c_i and s_i are almost the same size as the statistical uncertainty
- Large s_i uncertainties introduces non-Gaussian uncertainties on $y_\pm$
- γ moves significantly when fixing s_i instead of constraining them
- These effects are largest for $K^+K^-\pi^+\pi^-$, but are also seen in $\pi^+\pi^-\pi^+\pi^-$ and in the combined fit

Likelihood scan of CP observables

$x_{\pm}^{DK}$ agree well between likelihood scan and Hesse approximation



x_{-}^{DK}

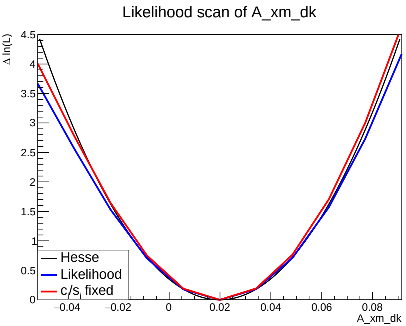


x_{+}^{DK}

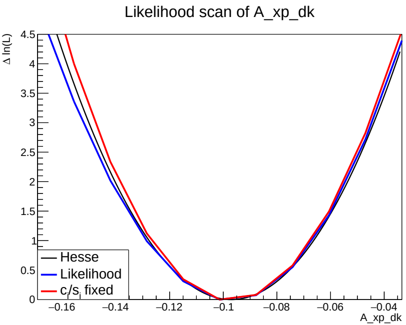
$$D^0 \to K^+K^-\pi^+\pi^-$$

Likelihood scan of CP observables

$x_{\pm}^{DK}$ agree well between likelihood scan and Hesse approximation



x_{-}^{DK}

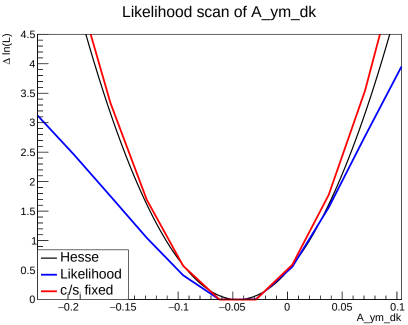


x_{+}^{DK}

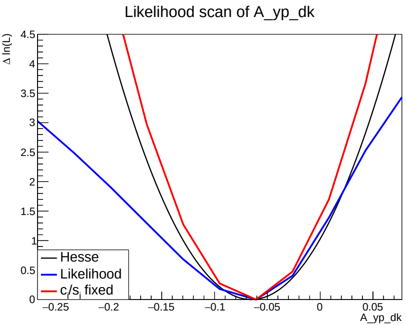
$$D^0 \to \pi^+\pi^-\pi^+\pi^-$$

Likelihood scan of CP observables

$y_{\pm}^{DK}$ diverges from Hesse approximation outside 1σ



y_{-}^{DK}

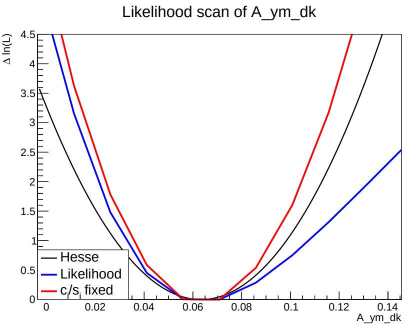


y_{+}^{DK}

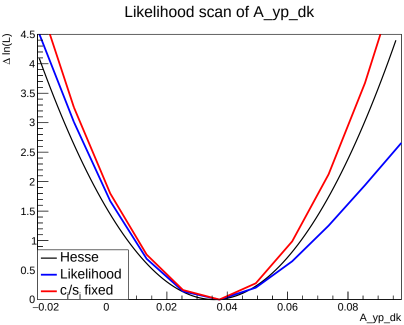
$$D^0 \to K^+K^-\pi^+\pi^-$$

Likelihood scan of CP observables

$y_{\pm}^{DK}$ diverges from Hesse approximation outside 1σ



y_{-}^{DK}



y_{+}^{DK}

$$D^0 \to \pi^+\pi^-\pi^+\pi^-$$

Likelihood scan of CP observables

What do the likelihood scans tell us?

- Uncertainties from c_i and s_i are significant, which justifies Gaussian constraining c_i and s_i
- Non-Gaussian uncertainties means GammaCombo cannot be used
- New strategy:
  1. Produce a likelihood function from CP fit
  2. Interpret CP observables in terms of γ, etc
  3. Must profile all nuisance parameters (F_i, c_i, s_i, backgrounds yields, normalisation constants)
  4. Provide direct measurements of γ, δ_B and r_B without GammaCombo

Summary of LHCb internal systematic uncertainties

Internal LHCb systematic uncertainties from model-dependent analysis of $K^+K^-\pi^+\pi^-$:

Source	x_-^{DK}	y_-^{DK}	x_+^{DK}	y_+^{DK}	$x_\xi^{D\pi}$	$y_\xi^{D\pi}$
Statistical	2.87	3.40	2.51	3.05	4.24	5.17
Mass shape	0.02	0.02	0.03	0.06	0.02	0.04
Bin-dependent mass shape	0.11	0.05	0.10	0.19	0.68	0.16
PID efficiency	0.02	0.02	0.03	0.06	0.02	0.04
Low-mass background model	0.02	0.02	0.03	0.04	0.02	0.02
Charmless background	0.14	0.15	0.12	0.14	0.01	0.02
CP violation in low-mass background	0.01	0.10	0.08	0.12	0.07	0.26
Semi-leptonic *b*-hadron decays	0.05	0.27	0.06	0.01	0.07	0.19
Semi-leptonic charm decays	0.02	0.07	0.03	0.15	0.06	0.24
$D \to K^-\pi^+\pi^-\pi^+$ background	0.11	0.05	0.07	0.04	0.09	0.05
$\Lambda_b \to pD\pi^-$ background	0.01	0.25	0.14	0.04	0.06	0.34
$D \to K^-\pi^+\pi^-\pi^+\pi^0$ background	0.30	0.05	0.19	0.07	0.05	0.01
Fit bias	0.06	0.05	0.13	0.02	0.06	0.13
Total LHCb systematic	0.37	0.43	0.34	0.32	0.70	0.57

Give systematic uncertainties in terms of CP observables (not γ) since these are more Gaussian and better behaved

Summary of LHCb internal systematic uncertainties

Internal LHCb systematics for $\pi^+\pi^-\pi^+\pi^-$ have not been calculated yet, but the plan is to run the same procedure from $K^+K^-\pi^+\pi^-$ on this mode during review, with minor simplifications:

1. No $K\pi\pi\pi$ background
2. No $K\pi\pi\pi\pi^0$ background
3. No D semileptonic background

Reminder: LHCb internal systematic uncertainties are expected to be an order of magnitude smaller than statistical uncertainties!

Interpretation strategy

From CP fit, we have a (negative log) likelihood function with nuisance parameters n_k:

$$\mathcal{L}(x_-^{DK}, y_-^{DK}, x_+^{DK}, y_+^{DK}, x_\xi^{D\pi}, y_\xi^{D\pi}, \{n_k\})$$

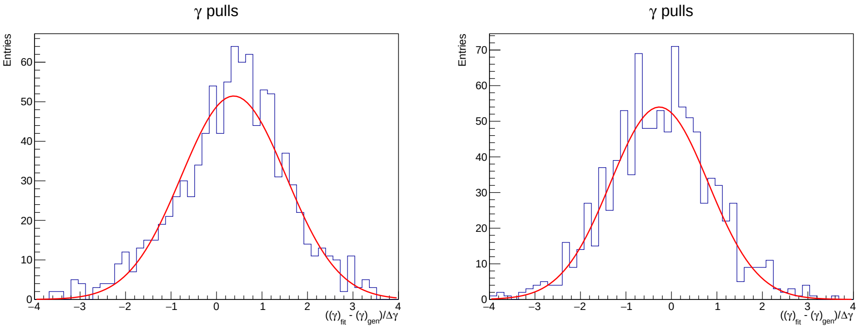
Express in terms of physics parameters:

$$\mathcal{L}(\gamma, \delta_B^{DK}, r_B^{DK}, \delta_B^{D\pi}, r_B^{D\pi}, \{n_k\})$$

In this step, also add a Gaussian smearing term on CP observables to account for internal LHCb systematics

Interpretation toys

We can perform toy studies on the interpretation fit, but we do not expect these to behave very Gaussian...



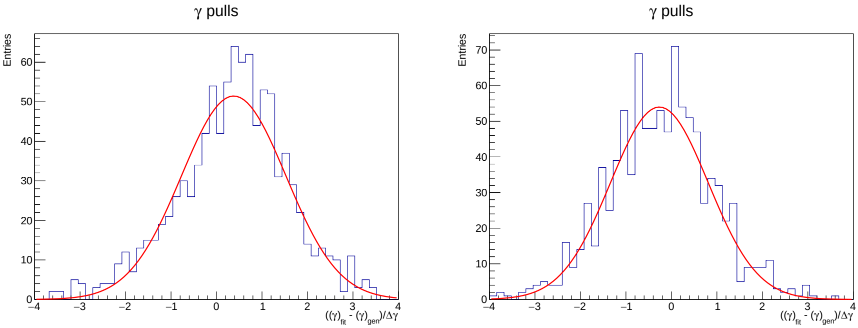
$K^+K^-\pi^+\pi^-$ $\quad$ γ pull distributions $\quad$ $\pi^+\pi^-\pi^+\pi^-$

Indeed, small but significant biases are observed!

Use pull distributions to correct central values of physics parameters

Interpretation toys

We can perform toy studies on the interpretation fit, but we do <u>not</u> expect these to behave very Gaussian...



$K^+K^-\pi^+\pi^-$ $\qquad$ $\pi^+\pi^-\pi^+\pi^-$

γ pull distributions

The absolute bias corrections are:
$K^+K^-\pi^+\pi^-$: $+5.6^\circ$, $\pi^+\pi^-\pi^+\pi-$: -3.0°, combined: -3.0°

Interpretation results

Results from interpretation of $K^+K^-\pi^+\pi^-$, after correcting for biases in central values (not uncertainties):

Model independent

$$
\begin{aligned}
\gamma =& (117 \pm 15)^\circ \\
\delta_B^{DK} =& (83 \pm 12)^\circ \\
r_B^{DK} =& (12.1 \pm 2.6) \times 10^{-2} \\
\delta_B^{D\pi} =& (295 \pm 74)^\circ \\
r_B^{D\pi} =& (0 \pm 5) \times 10^{-3}
\end{aligned}
$$

Model dependent

$$
\begin{aligned}
\gamma =& (116^{+12}_{-14})^\circ \\
\delta_B^{DK} =& (81^{+14}_{-13})^\circ \\
r_B^{DK} =& (11.0 \pm 2.0) \times 10^{-2} \\
\delta_B^{D\pi} =& (298^{+62}_{-118})^\circ \\
r_B^{D\pi} =& (4^{+5}_{-4}) \times 10^{-3}
\end{aligned}
$$

LHCb systematics not included yet, but central value of γ remains high...
... it seems that the large tension with the LHCb global result $\gamma = (63.8^{+3.5}_{-3.7})^\circ$ remains

Interpretation results

Results from interpretation of $h^+h^-\pi^+\pi^-$, after correcting for biases in central values (not uncertainties):

$K^+K^-\pi^+\pi^-$

$$\begin{aligned}\gamma &= (117 \pm 15)^\circ \\ \delta_B^{DK} &= (83 \pm 12)^\circ \\ r_B^{DK} &= (12.1 \pm 2.6) \times 10^{-2} \\ \delta_B^{D\pi} &= (295 \pm 74)^\circ \\ r_B^{D\pi} &= (0 \pm 5) \times 10^{-3}\end{aligned}$$

$\pi^+\pi^-\pi^+\pi^-$

$$\begin{aligned}\gamma &= (45 \pm 11)^\circ \\ \delta_B^{DK} &= (115 \pm 11)^\circ \\ r_B^{DK} &= (8.2 \pm 1.9) \times 10^{-2} \\ \delta_B^{D\pi} &= (204 \pm 42)^\circ \\ r_B^{D\pi} &= (4 \pm 5) \times 10^{-3}\end{aligned}$$

$\pi^+\pi^-\pi^+\pi^-$ is in much better agreement with LHCb global result, but there is a tension with $K^+K^-\pi^+\pi^-$...

Interpretation results

Results from interpretation of $h^+h^-\pi^+\pi^-$, after correcting for biases in central values (not uncertainties):

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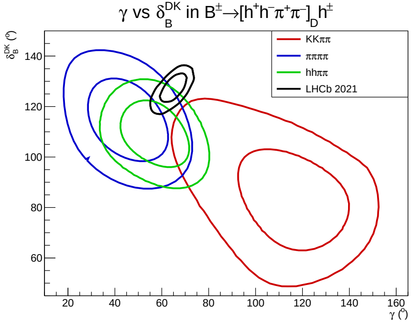
$\pi^+\pi^-\pi^+\pi^-$ is in much better agreement with LHCb global result, but there is a tension with $K^+K^-\pi^+\pi^-$...

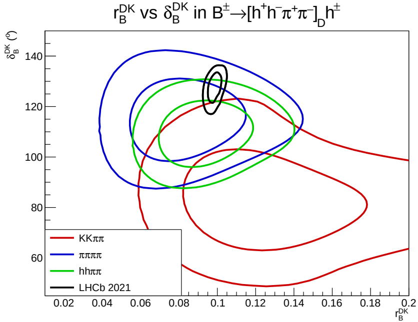
...but how Gaussian are these uncertainties?

Likelihood scan of interpretation fit

In fact, a likelihood scan shows that $D \to K^+K^-\pi^+\pi^-$ and $D \to \pi^+\pi^-\pi^+\pi^-$ agree within 2σ (no LHCb systematics yet)



γ vs δ_B^{DK}

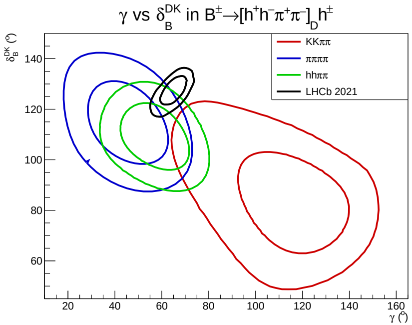


r_B^{DK} vs δ_B^{DK}

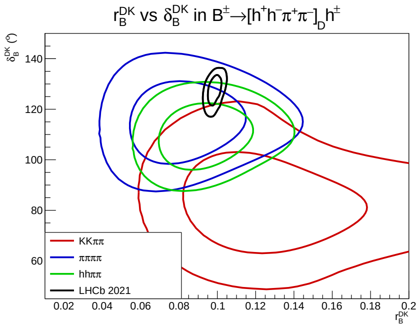
When all biases, correlations and non-Gaussian uncertainties are accounted for, the tension with the LHCb average has reduced significantly

Likelihood scan of interpretation fit

In fact, a likelihood scan shows that $D \to K^+K^-\pi^+\pi^-$ and $D \to \pi^+\pi^-\pi^+\pi^-$ agree within 2σ (no LHCb systematics yet)



γ vs δ_B^{DK}



r_B^{DK} vs δ_B^{DK}

However, with all the non-Gaussian behaviour, are we sure these contours cover 68% and 95% ?

Plugin/Feldman-Cousins method

Feldman-Cousins method, or Plugin, is a “brute-force” approach to assigning a confidence interval

At each scan point of γ, perform these fits to data:

1. Fit with all parameters floating, and save the log-likelihood χ^2
2. Fit with γ fixed to scan point, and save χ^2_{fix}
3. Calculate $\Delta\chi^2_{\mathrm{data}} = \chi^2_{\mathrm{fix}} - \chi^2$

We expect $\Delta\chi^2_{\mathrm{data}}$ to become large as we move away from best-fit value, but without direct knowledge of underlying PDF, we cannot determine any confidence intervals from this

Plugin/Feldman-Cousins method

Feldman-Cousins method, or Plugin, is a "brute-force" approach to assigning a confidence interval

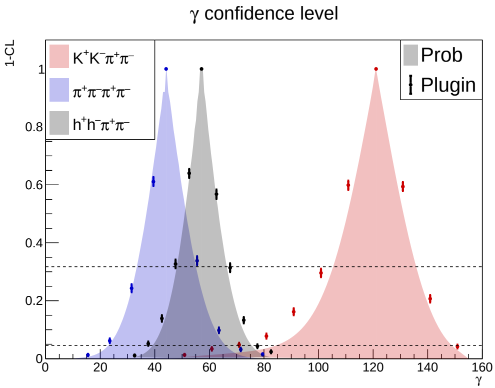
At each scan point of γ, perform these fits to toy:

1. Fix γ to scan point and generate 1000 toys
2. Perform fits to each toy, with γ both floating and fixed
3. Calculate $\Delta\chi^2_{\mathrm{toy}}$

At each scan point, the fraction of toys with $\Delta\chi^2_{\mathrm{toy}} > \Delta\chi^2_{\mathrm{data}}$ is equal to $1 - \mathrm{CL}$, and the exact 68% confidence interval can then be obtained using an interpolation between points

Plugin/Feldman-Cousins method

LHCb average within 2σ of $D \to K^+K^-\pi^+\pi^-$ Plugin result
Combined fit shows good agreement between Plugin and Prob scans



Combined fit result: $\gamma = (57 \pm 9)^\circ$
Third most precise single measurement of γ in $B^\pm$ decays

Summary and next steps

In summary:

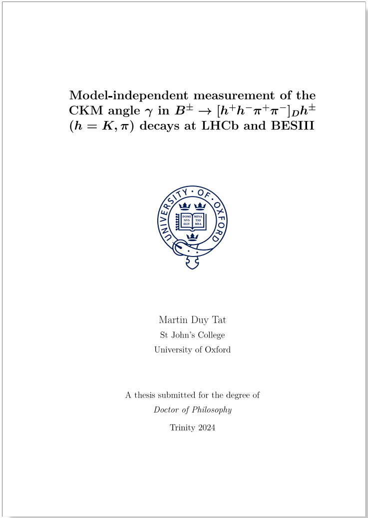
1. BESIII results approved, paper in review
2. Model-independent measurement of γ with $B^{\pm} \to [h^{+}h^{-}\pi^{+}\pi^{-}]_{D}h^{\pm}$ presented to B2OC, and ANA note circulated to B2OC conveners
3. 3σ tension in $D \to K^{+}K^{-}\pi^{+}\pi^{-}$ has reduced to less than 2σ due to:
    1. Non-Gaussian uncertainties in $y_{\pm}^{DK}$ originating from s_i uncertainties
    2. Large anti-correlation between γ and δ_{B}^{DK}
4. Main takeaway: Important to meausure γ model independently!

Summary and next steps

My thesis is slowly taking shape...

- ❏ 1. Introduction
- ✓ 2. Theoretical background
- ❏ 3. Optimised binning scheme for $D^0 \to K^+K^-\pi^+\pi^-$
- ❏ 4. The BESIII detector
- ❏ 5. Selection of single- and double-tag events
- ❏ 6. Measurement of strong-phase differences
- ❏ 7. The LHCb detector
- ❏ 8. Selection of $B^\pm \to [h^+h^-\pi^+\pi^-]_D h^\pm$ events
- ❏ 9. Model-independent measurement of γ
- ❏ 10. Summary and outlook

... I will do my best to stop procrastinating!



Model-independent measurement of the CKM angle γ in $B^\pm \to [h^+h^-\pi^+\pi^-]_D h^\pm$ ($h = K, \pi$) decays at LHCb and BESIII

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St John's College
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A thesis submitted for the degree of
Doctor of Philosophy
Trinity 2024

Summary and next steps

Next steps:

- Aim to finish thesis by the end of June
- Unfortunately I failed to obtain useful TORCH results...
- ...but it was a useful experience with testbeam and RICH work!
- Future plans: Start new postdoc position in Heidelberg in September

Thanks for your attention and thanks for all your support during my PhD!