

Measuring γ in $B^\pm \rightarrow (K^+ K^- \pi^+ \pi^-)_D K^\pm$ decays

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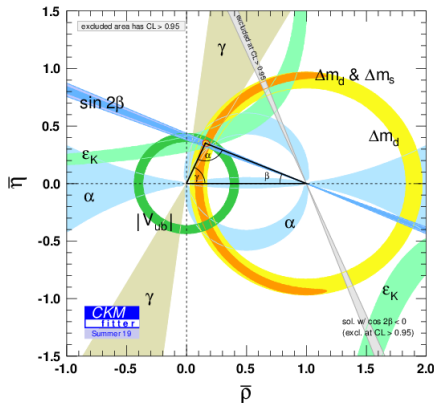
Background

- Supervisors:
 - Prof Guy Wilkinson (γ measurement)
 - Prof Neville Harnew (TORCH)
- 4-year MPhys in Oxford
 - Performance of monolithic CMOS sensors
 - Prof Daniela Bortoletto
- CERN Summer Student 2019
 - Beam loss reduction in TT20 transfer line
 - Dr Yann Dutheil, Dr Matthew Fraser
- Oxford Summer Student 2018
 - Study of PDF uncertainties in W-boson mass measurement
 - Prof Chris Hays
- RAL Summer Student 2018
 - Bending magnets in accelerator simulations
 - Dr Chris Rogers

γ and the unitary triangle

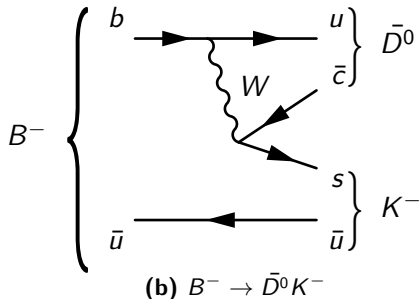
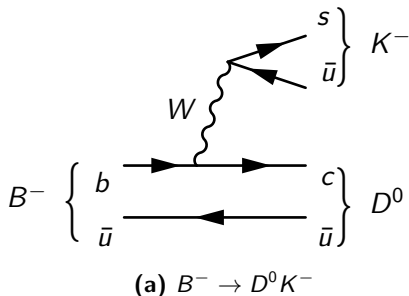
Unitarity of CKM matrix: $V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$

Define $\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$



CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005)

$b \rightarrow u$ and $b \rightarrow c$ interference



Similar diagrams for $B^+ \rightarrow DK^+$, with $\gamma \rightarrow -\gamma \Rightarrow$

Interference when D^0 and \bar{D}^0 decay into a common final state

Single most precise measurement: $\gamma = (68.7^{+5.2}_{-5.1})^\circ$ [arXiv:2010.08483](https://arxiv.org/abs/2010.08483)

$B^\pm \rightarrow DK^\pm$, $B^\pm \rightarrow D\pi^\pm$, $D \rightarrow K_S^0 h^+ h^-$

The $D \rightarrow K^+ K^- \pi^+ \pi^-$ decay

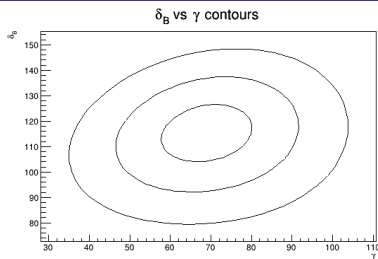
- Estimated 2000 events from LHCb Run 1 and 2
- First proposed by G. Wilkinson and J. Rademacker
[arXiv:hep-ph/0611272](#)
 - Amplitude analysis: Isobar model
 - Estimated γ precision: 14° with 1000 events
- CLEO amplitude analysis [arXiv:1201.5716](#)
 - Estimated γ precision: 11° with 2000 events
- LHCb amplitude analysis [arXiv:1811.08304](#)
 - AmpGen for generating events and unbinned fit
- Other 4-body decay studies:
 - Binned analysis of $K_S^0 \pi^+ \pi^- \pi^0$ by Belle [arXiv:1908.09499](#)
 - Inclusive analysis of $h^\pm \pi^\mp \pi^+ \pi^-$ [arXiv:1906.08297](#), [arXiv:1709.05855](#), [arXiv:1603.08993](#) by LHCb

Unbinned fit with amplitude model

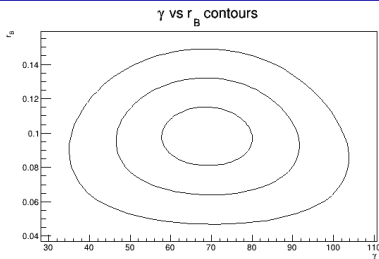
$$\mathcal{A}(B^- \rightarrow (K^+ K^- \pi^+ \pi^-)_D K^-) = \mathcal{A}_B \mathcal{A}(D^0 \rightarrow K^+ K^- \pi^+ \pi^-) \\ + \mathcal{A}_B \mathcal{A}(\bar{D}^0 \rightarrow K^+ K^- \pi^+ \pi^-) r_B e^{i(\delta_B - \gamma)}$$

- $\mathcal{A}(D \rightarrow K^+ K^- \pi^+ \pi^-)$ obtained from amplitude model
- Fit with γ , δ_B and r_B as free parameters
- Initial values:
 - $\gamma = 75^\circ$
 - $\delta_B = 130^\circ$
 - $r_B = 0.1$
- Results from unbinned fit of 2×10^3 events:
 - $\gamma = (69 \pm 11)^\circ$
 - $\delta_B = (115 \pm 11)^\circ$
 - $r_B = 0.098 \pm 0.017$
- Results from unbinned fit of 2×10^4 events:
 - $\gamma = (77.1 \pm 3.5)^\circ$
 - $\delta_B = (129 \pm 4)^\circ$
 - $r_B = 0.093 \pm 0.006$

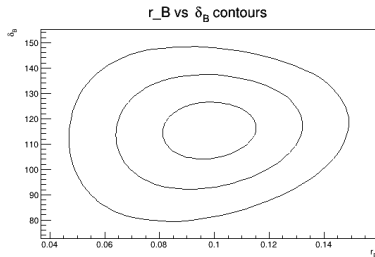
Unbinned fit of 2×10^3 events with amplitude model



(a) γ vs δ_B

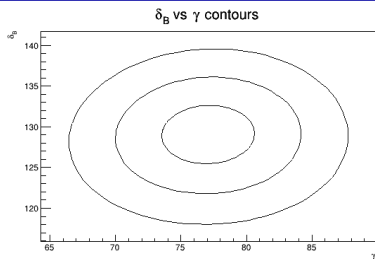


(b) γ vs r_B

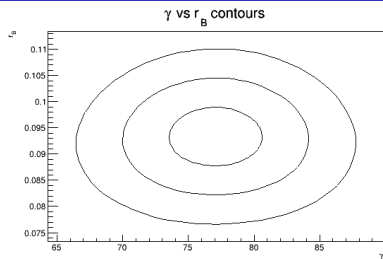


(c) r_B vs δ_B

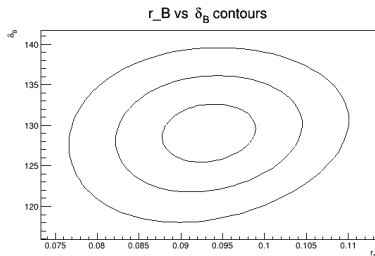
Unbinned fit of 2×10^4 events with amplitude model



(a) γ vs δ_B



(b) γ vs r_B



(c) r_B vs δ_B

Binned fit of $D \rightarrow K^+ K^- \pi^+ \pi^-$

$$\begin{aligned}\mathcal{A}(B^- \rightarrow (K^+ K^- \pi^+ \pi^-)_D K^-) &= \mathcal{A}_B \mathcal{A}(D^0 \rightarrow K^+ K^- \pi^+ \pi^-) \\ &\quad + \mathcal{A}_B \mathcal{A}(\bar{D}^0 \rightarrow K^+ K^- \pi^+ \pi^-) r_B e^{i(\delta_B - \gamma)}\end{aligned}$$

Event yield in bin i

$$\begin{aligned}N_i^- &= h_{B^-} \left(K_i + (x_-^2 + y_-^2) \bar{K}_i + 2\sqrt{K_i \bar{K}_i} (x_- c_i + y_- s_i) \right) \\ N_i^+ &= h_{B^+} \left(\bar{K}_i + (x_+^2 + y_+^2) K_i + 2\sqrt{K_i \bar{K}_i} (x_+ c_i - y_+ s_i) \right)\end{aligned}$$

CP-violating observables

$$x_{\pm} = r_B \cos(\delta_B \pm \gamma), \quad y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

- c_i, s_i : Amplitude-averaged strong phase difference of D decay
- Measure c_i and s_i at BES III detector
- Can measure K_i and \bar{K}_i at both LHCb and BES III
- Need to divide phase space into bins

Pull studies

- Pull studies shows that this works
- Used an arbitrary and naive binning scheme with 4 bins
- x_{\pm} pulls show asymmetric tails for 2×10^3 events
- x_{\pm} and y_{\pm} are all satisfactory for 2×10^4 events

Naive binning scheme

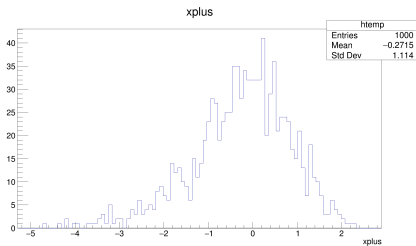
Split phase space along the boundaries $E_{K^+} = E_{K^-}$ and $E_{\pi^+} = E_{\pi^-}$

Bin 1: $E_{K^+} > E_{K^-}$, $E_{\pi^+} > E_{\pi^-}$, ...

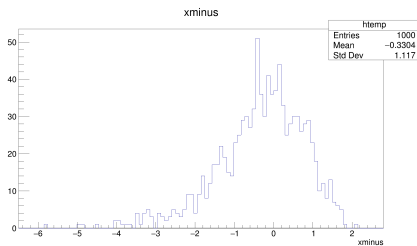
D decay hadronic parameters

$$c_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D}^0)| \cos(\delta_D)}{\sqrt{\int_i d\Phi |\mathcal{A}(D^0)|^2 \int_i d\Phi |\mathcal{A}(D^0)|^2}}, \quad K_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j d\Phi |\mathcal{A}(D^0)|^2}$$

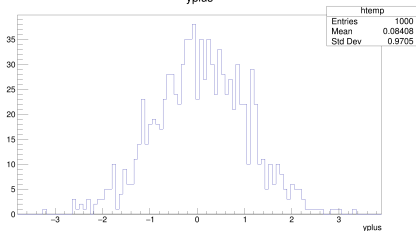
Pull study with 2×10^3 events



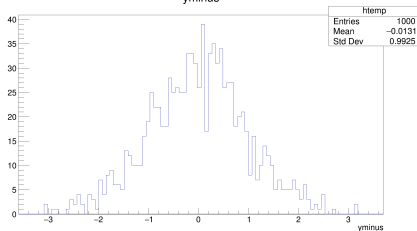
(a) x_+ pull



(b) x_- pull

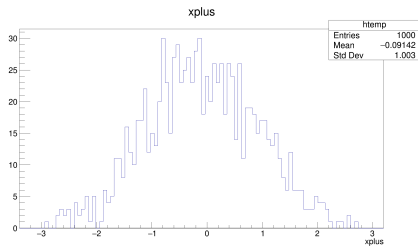


(c) y_+ pull

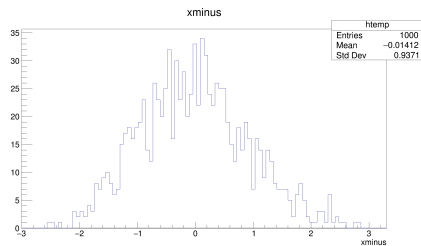


(d) y_- pull

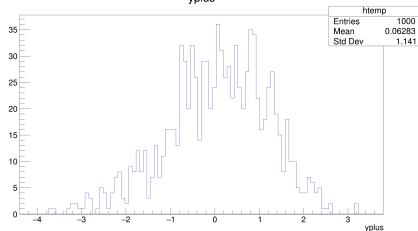
Pull study with 2×10^4 events



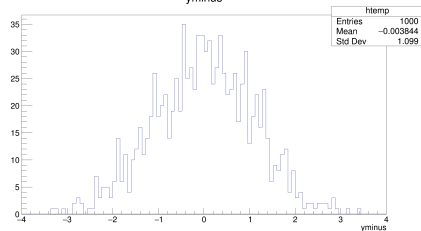
(a) x_+ pull



(b) x_- pull



(c) y_+ pull



(d) y_- pull

Need a more sophisticated binning scheme!

- 4-body phase space is 5-dimensional
- Convenient to choose rectangular coordinates

Phase space parameterisation

$$x_1 = m(K^+\pi^+) + \alpha$$

$$x_2 = m(K^-\pi^-) + \alpha, \quad \alpha = \min(m(K^+\pi^+), m(K^-\pi^-))$$

$$x_3 = \cos(\theta_+), \quad (\text{Helicity angles})$$

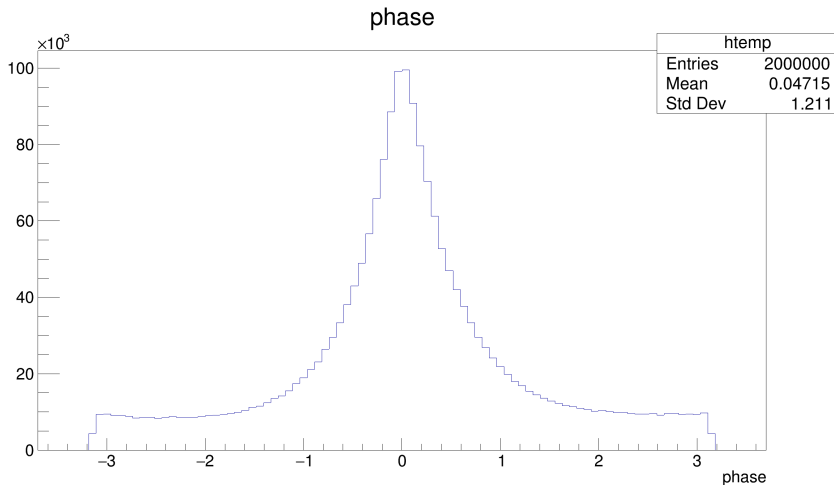
$$x_4 = \cos(\theta_-)$$

$$x_5 = \phi$$

- Define binning scheme in terms of these coordinates
- Ideally have a constant strong phase difference in each bin
- Determine binning scheme based on amplitude model, but final fit is model independent

Histogram of strong phase difference

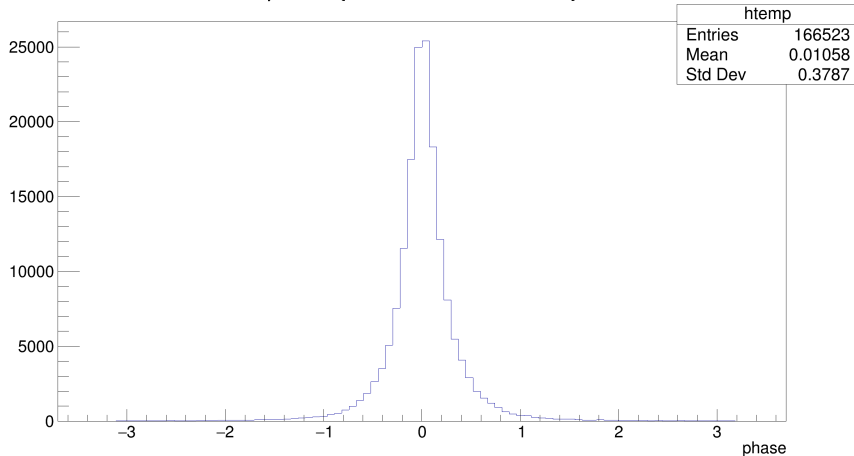
Strong phase difference from amplitude model:



Histogram of strong phase difference

Place cuts $x_3 > 0.8, x_4 > 0.8$:

phase {X3 > 0.8 && X4 > 0.8}



Summary and next steps

Summary:

- Model independent determination of γ from $B^\pm \rightarrow (K^+ K^- \pi^+ \pi^-)_D K^\pm$ decay
- External input from BES III
- Precision from unbinned fit with 2×10^3 events: $\sigma(\gamma) \approx 11^\circ$
- Binned fit in 5 dimensions, need to understand phase space

Next steps:

- Finish mapping out phase space
- Decide on a suitable binning scheme
- Look at data from BES III
- Look at LHCb data