

# Determination of the CKM angle $\gamma$ in $B^\pm \rightarrow (K^+ K^- \pi^+ \pi^-)_D h^\pm$ decays

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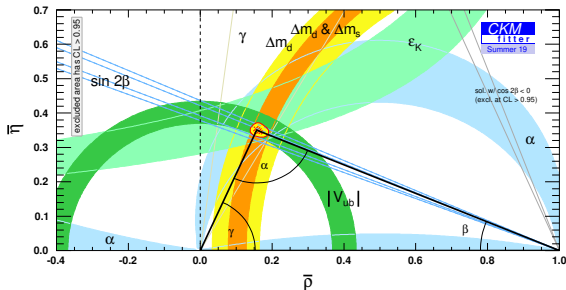
- 1 Introduction to binned  $\gamma$  analysis
- 2 Backgrounds
- 3 Systematic uncertainty
- 4 Summary and conclusion

# $\gamma$ and the unitary triangle

- Unitarity of CKM matrix:  $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \Rightarrow$

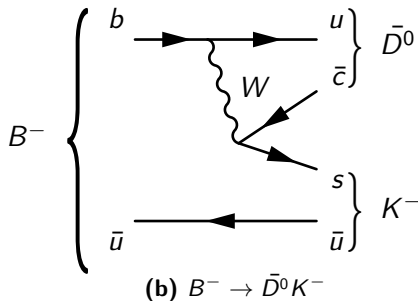
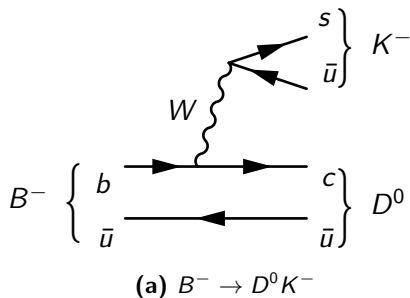
$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

- Only CKM angle accessible at tree level  $\Rightarrow$ 
  - Negligible theoretical uncertainties
  - Ideal Standard Model benchmark
  - Compare with indirect measurements



CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005)

# Sensitivity through interference



- Superposition of  $D^0$  and  $\bar{D}^0$
- $b \rightarrow u\bar{c}s$  and  $b \rightarrow c\bar{u}s$  interference  $\rightarrow$  Sensitivity to  $\gamma$

$$\begin{aligned}\mathcal{A}(B^-) &= \mathcal{A}(D^0) + r_B e^{i(\delta_B - \gamma)} \mathcal{A}(\bar{D}^0) \\ \mathcal{A}(B^+) &= \mathcal{A}(\bar{D}^0) + r_B e^{i(\delta_B + \gamma)} \mathcal{A}(D^0)\end{aligned}$$

# The BPGGSZ method

- $B^\pm \rightarrow Dh^\pm$  amplitude:

$$\begin{aligned}\mathcal{A}(B^-) &= \mathcal{A}(D^0) + r_B e^{i(\delta_B - \gamma)} \mathcal{A}(\bar{D}^0) \\ \mathcal{A}(B^+) &= \mathcal{A}(\bar{D}^0) + r_B e^{i(\delta_B + \gamma)} \mathcal{A}(D^0)\end{aligned}$$

- $\mathcal{A}(D^0)$  and  $\mathcal{A}(\bar{D}^0)$  depend on  $D$  phase space
- Strong-phase difference of  $D^0$  and  $\bar{D}^0$  decays inaccessible at LHCb
- Model-independent measurement: Integrate over bins of phase space

## Event yield in bin $i$

$$\begin{aligned}N_i^- &= h_{B^-} \left( F_i + (x_-^2 + y_-^2) \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_- c_i + y_- s_i) \right) \\ N_{-i}^+ &= h_{B^+} \left( F_i + (x_+^2 + y_+^2) \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_+ c_i + y_+ s_i) \right)\end{aligned}$$

# The BPGGSZ method

## Event yield in bin $i$

$$N_i^- = h_{B^-} (F_i + (x_-^2 + y_-^2) \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_- c_i + y_- s_i))$$

$$N_{-i}^+ = h_{B^+} (F_i + (x_+^2 + y_+^2) \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_+ c_i + y_+ s_i))$$

- CP observables:

- $x_{\pm}^{DK} = r_B^{DK} \cos(\delta_B^{DK} \pm \gamma), \quad y_{\pm}^{DK} = r_B^{DK} \sin(\delta_B^{DK} \pm \gamma)$
- $x_{\xi}^{D\pi} = \text{Re}(\xi^{D\pi}), \quad y_{\xi}^{D\pi} = \text{Im}(\xi^{D\pi}) \quad \left( \xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} e^{i(\delta_B^{D\pi} - \delta_B^{DK})} \right)$

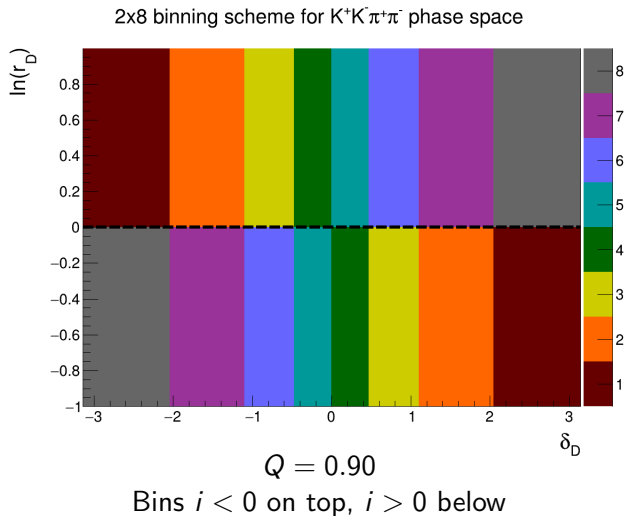
- Fractional bin yield:

- $F_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j d\Phi |\mathcal{A}(D^0)|^2}$
- Floated in the fit, mostly constrained by  $B^{\pm} \rightarrow D\pi^{\pm}$

- Amplitude averaged strong phases from BESIII:

$$c_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D}^0)| \cos(\delta_D)}{\sqrt{\int_i d\Phi |\mathcal{A}(D^0)|^2 \int_i d\Phi |\mathcal{A}(\bar{D}^0)|^2}}, \quad s_i = \frac{\int_i d\Phi |\mathcal{A}(D^0)| |\mathcal{A}(\bar{D}^0)| \sin(\delta_D)}{\sqrt{\int_i d\Phi |\mathcal{A}(D^0)|^2 \int_i d\Phi |\mathcal{A}(\bar{D}^0)|^2}}$$

# Binning scheme



## Backgrounds

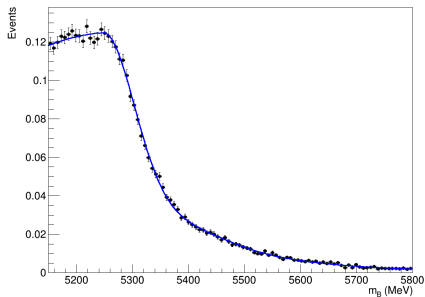


# $D$ semileptonic backgrounds

- $B^\pm \rightarrow Dh^\pm$ ,  $D \rightarrow K(X)l\nu$ ,  $K(X) \rightarrow K\pi\pi$ 
  - $K_1(1270)$
  - $K_1(1400)$
  - $K^*(1410)$
  - $K^*(1680)$
  - $K_2^*(1430)$
- Single mis-ID:  $K\mu\pi\pi \rightarrow KK\pi\pi$
- Double mis-ID:  $K\pi\pi\mu \rightarrow KK\pi\pi$
- Generate Rapidsim samples, reweight with PIDCalib2

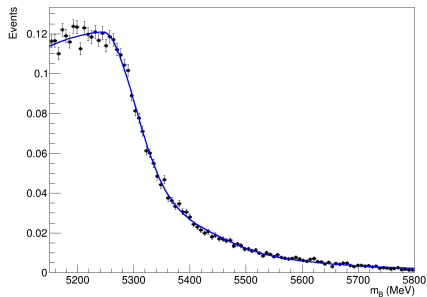
# D semileptonic backgrounds

B mass of SL  $D \rightarrow K(X)\mu\nu$  backgrounds,  $B \rightarrow DK$



(a)  $B \rightarrow DK$

B mass of SL  $D \rightarrow K(X)\mu\nu$  backgrounds,  $B \rightarrow D\pi$

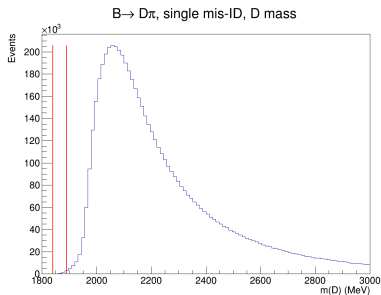


(b)  $B \rightarrow D\pi$

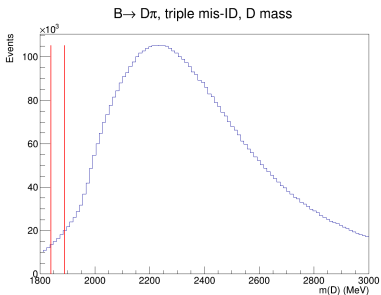
Conclusion: Negligible impact, include in systematics

# $D \rightarrow K\pi\pi\pi$ mis-ID background

- $B^\pm \rightarrow Dh^\pm$ ,  $D \rightarrow K\pi\pi\pi$
- Single mis-ID:  $K\pi\pi\pi \rightarrow KK\pi\pi$
- Triple mis-ID:  $\pi\pi K\pi \rightarrow KK\pi\pi$
- Use LHCb MC generated with AmpGen, reweight with PIDCalib2



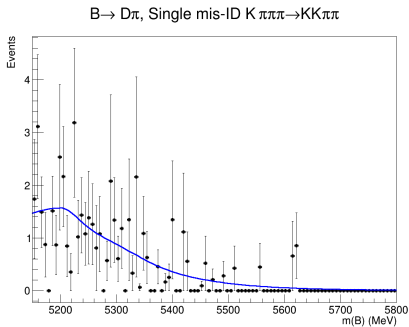
(a) Single mis-ID



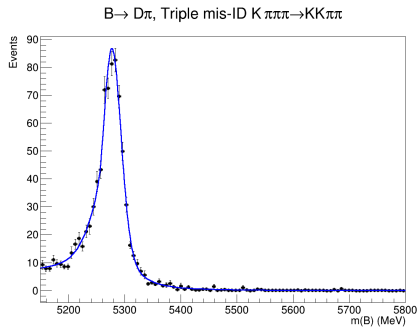
(b) Triple mis-ID

**Figure 3:**  $D$  invariant mass

# $D \rightarrow K\pi\pi\pi$ mis-ID background



(a) Single mis-ID



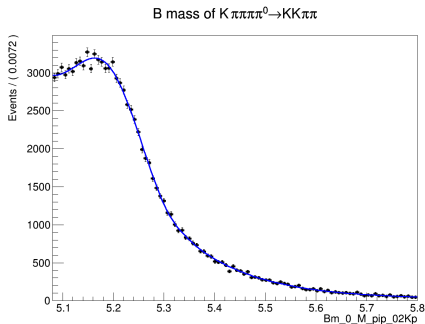
(b) Triple mis-ID

**Figure 4:**  $B$  invariant mass

Conclusion: Negligible impact, include in systematics

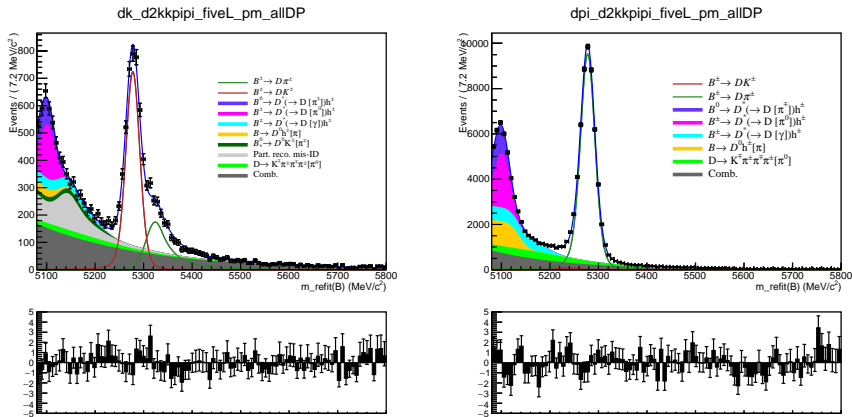
# $D \rightarrow K\pi\pi\pi\pi^0$ mis-ID background

- $B^\pm \rightarrow Dh^\pm$ ,  $D \rightarrow K\pi\pi\pi[\pi^0]$
- $\pi^0$  not reconstructed  $\rightarrow$  Lower  $D$  mass
- Single mis-ID:  $K\pi\pi\pi \rightarrow KK\pi\pi \rightarrow$  Higher  $D$  mass
- Generate RapidSim samples, reweight with PIDCalib2



Conclusion: Fix shape from RapidSim, allow yield to float

## Global fit



**Figure 5:**  $B^\pm \rightarrow DK^\pm$  channel (left) and  $B^\pm \rightarrow D\pi^\pm$  channel (right)

- $B^\pm \rightarrow DK^\pm$  yield:  $3543 \pm 75$
- $B^\pm \rightarrow D\pi^\pm$  yield:  $47\,503 \pm 260$

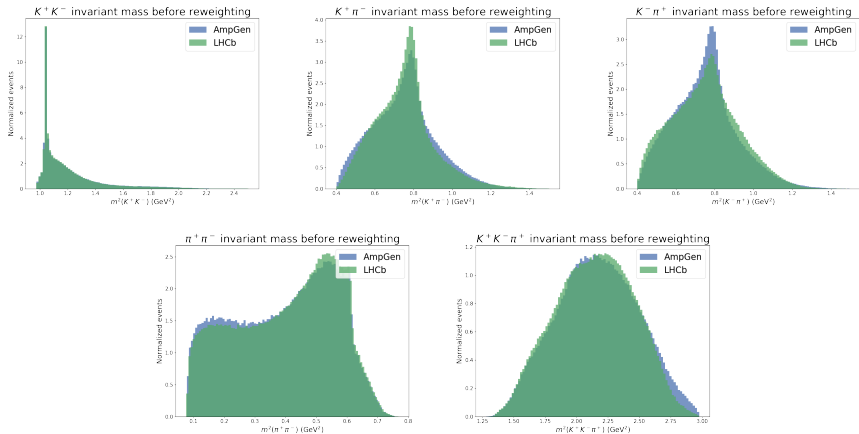
## Systematic uncertainties

Different strategies for evaluating systematic uncertainties:

- Generate toy datasets with systematics, fit with default model and take the bias as a systematic:
  - Small backgrounds ( $D \rightarrow K(X)l\nu_l$ ,  $D \rightarrow K\pi\pi\pi$ ,  $B \rightarrow Dl\nu_l$ ,  $\Lambda_b$ )
  - Bin dependent mass shape
  - Low mass physics effects
- Do multiple fits to data while smearing parameters:
  - $c_i$  and  $s_i$
  - Mass shape
  - Fixed yield fractions
  - PID efficiency
- Fit bias: Take bias toys as systematic uncertainty

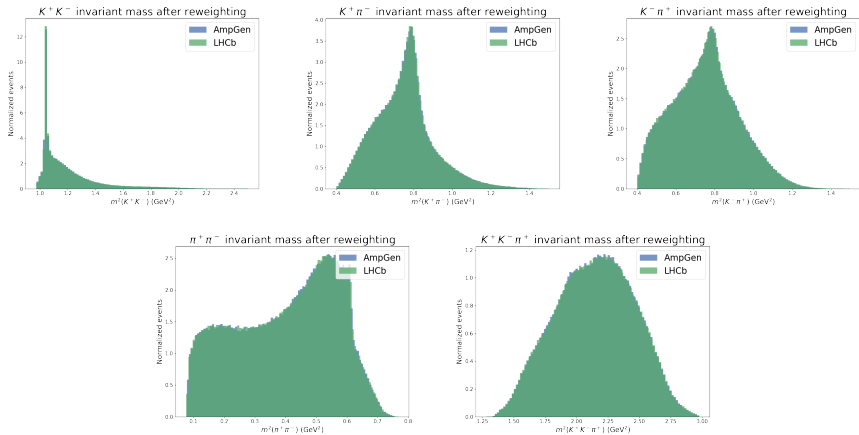


# Efficiency correction of $c_i$ and $s_i$



Need to reweight events to account for efficiency differences between AmpGen samples and LHCb MC

# Efficiency correction of $c_i$ and $s_i$



After reweighting, use weights to recalculate  $c_i$  and  $s_i$

Conclusion: Efficiency correction of  $c_i$  and  $s_i$  is an order of magnitude smaller than their uncertainties, no systematic uncertainty considered

# Summary of all systematic uncertainties

Source	$x_-^{DK}$	$y_-^{DK}$	$x_+^{DK}$	$y_+^{DK}$	$x_\xi^{D\pi}$	$y_\xi^{D\pi}$
Statistical	2.73	3.23	2.38	2.90	4.30	5.27
$c_i, s_i$	0.66	1.55	0.32	1.31	1.73	1.03
$B^\pm \rightarrow D\mu\nu$ background	0.04	0.03	0.02	0.15	0.30	0.10
$D \rightarrow K(X)l\nu_l$ background	0.15	0.05	0.11	0.03	0.35	0.25
$D \rightarrow K\pi\pi\pi$ background	0.17	0.03	0.04	0.01	0.46	0.18
$\Lambda_b$ background	0.09	0.11	0.00	0.18	0.16	0.21
Bin dependent mass shape	0.21	0.05	0.17	0.01	0.37	0.11
Fit bias	0.19	0.03	0.16	0.04	0.30	0.16
Fixed yield fractions	0.02	0.03	0.02	0.02	0.01	0.01
Low mass physics effects	0.05	0.09	0.05	0.18	0.41	0.48
Mass shape	0.03	0.03	0.02	0.02	0.04	0.01
PID Efficiency	0.03	0.03	0.02	0.02	0.04	0.01
Total LHCb systematic	0.39	0.17	0.27	0.30	0.92	0.65
Total systematic	0.77	1.55	0.41	1.34	1.96	1.22

## Summary and conclusion

# Summary of CP observables

- Measured CP observables:

$$x_-^{DK} = (x.x \pm 2.7 \pm 0.4 \pm 0.7) \times 10^{-2},$$

$$y_-^{DK} = (x.x \pm 3.2 \pm 0.2 \pm 1.6) \times 10^{-2},$$

$$x_+^{DK} = (x.x \pm 2.4 \pm 0.3 \pm 0.3) \times 10^{-2},$$

$$y_+^{DK} = (x.x \pm 2.9 \pm 0.3 \pm 1.3) \times 10^{-2},$$

$$x_\xi^{D\pi} = (x.x \pm 4.3 \pm 0.9 \pm 1.7) \times 10^{-2},$$

$$y_\xi^{D\pi} = (x.x \pm 5.3 \pm 0.6 \pm 1.0) \times 10^{-2},$$

- Note: Currently using  $c_i$  and  $s_i$  from the LHCb model
- Publication strategy: Publish current results together with binned yields  $\rightarrow$  Redo fit to obtain model-independent CP observables once  $c_i$  and  $s_i$  from BESIII are available

- Interpret in terms of physics parameters:

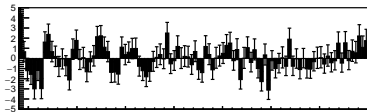
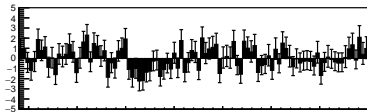
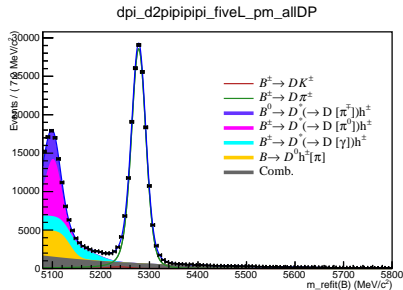
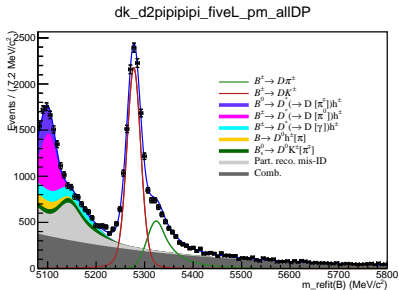
$$\begin{aligned}\gamma &= (x.x_{-15}^{+14})^\circ, \\ \delta_B^{DK} &= (x.x_{-14}^{+15})^\circ, \\ r_B^{DK} &= x.x_{-0.018}^{+0.019}, \\ \delta_B^{D\pi} &= (x.x_{-63}^{+117})^\circ, \\ r_B^{D\pi} &= x.x_{-0.0024}^{+0.0052}.\end{aligned}$$

- The mode  $B^\pm \rightarrow Dh^\pm$ ,  $D \rightarrow \pi^+\pi^-\pi^+\pi^-$  very similar
- Run this through same selection (including BDT)
- Can measure GLW CP observables as additional constraints on  $\gamma$ :

$$A_h = \frac{\Gamma(B^- \rightarrow Dh^-) - \Gamma(B^+ \rightarrow Dh^+)}{\Gamma(B^- \rightarrow Dh^-) + \Gamma(B^+ \rightarrow Dh^+)},$$
$$R_{\text{CP}} = \frac{R(4\pi)}{R(K3\pi)},$$
$$R = \frac{\Gamma(B \rightarrow DK)}{\Gamma(B \rightarrow D\pi)}.$$

- $B^\pm \rightarrow Dh^\pm$ ,  $D \rightarrow K\pi\pi\pi$  yields provided by Tim Evans

# Global fit of $B^\pm \rightarrow Dh^\pm$ , $D \rightarrow \pi^+\pi^-\pi^+\pi^-$



$$\frac{\text{Yield}(\pi\pi\pi\pi)}{\text{Yield}(KK\pi\pi)} = \frac{161\,900 \pm 455}{54\,039 \pm 256} = 2.996 \pm 0.017$$

Compare with PDG:  $3.06 \pm 0.16$



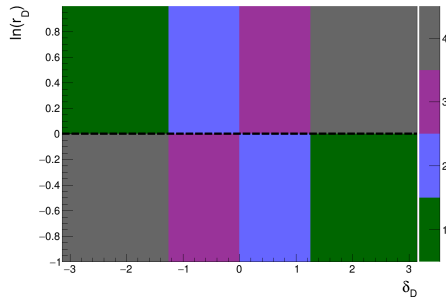
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Thank you!

Backup

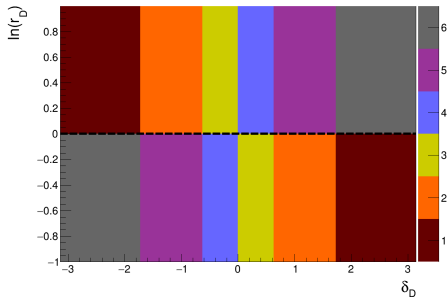
# Binning scheme

2x4 binning scheme for  $K^+K^-\pi^+\pi^-$  phase space

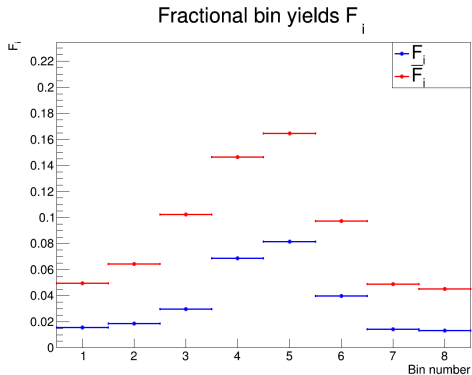
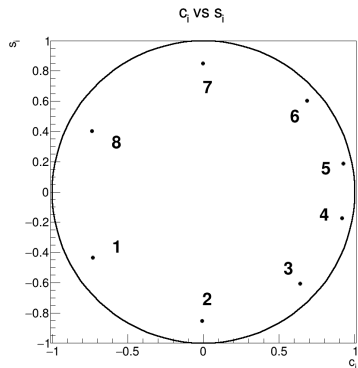


(a)  $Q = 0.85$

2x6 binning scheme for  $K^+K^-\pi^+\pi^-$  phase space

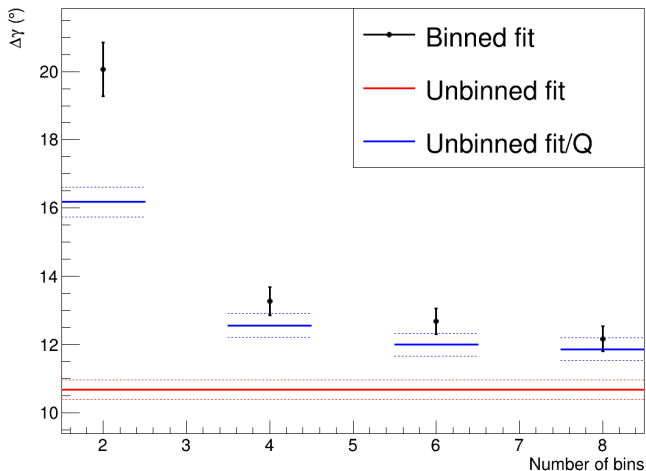


(b)  $Q = 0.89$



# Comparison of binned fit precision with unbinned fit

$\gamma$  precision vs number of bins



# Trigger requirements

Run 1 trigger requirements	(Bu_LOGlobal_TIS or Bu_LOHadronDecision_TOS) and (Bu_Hlt1TrackAllL0Decision_TOS) and (Bu_Hlt2Topo2BodyBBDTDecision_TOS or Bu_Hlt2Topo3BodyBBDTDecision_TOS or Bu_Hlt2Topo4BodyBBDTDecision_TOS or Bu_Hlt2IncPhiDecision_TOS)
Run 2 trigger requirements	(Bu_LOGlobal_TIS or Bu_LOHadronDecision_TOS) and (Bu_Hlt1TrackMVADecision_TOS or Bu_Hlt1TwoTrackMVADecision_TOS) and (Bu_Hlt2Topo2BodyDecision_TOS or Bu_Hlt2Topo3BodyDecision_TOS or Bu_Hlt2Topo4BodyDecision_TOS or Bu_Hlt2IncPhiDecision_TOS)

## Rectangular cuts before BDT

Number	Variable description	Cut
1	DTF converged	True
2	Bachelor momentum	$< 100\text{GeV}$
3	Bachelor has RICH	True
4	$D$ invariant mass	$[1839.84, 1889.84]\text{MeV}$
5	$B^\pm$ invariant mass	$[5080, 5800]\text{MeV}$
6	$K^\pm$ daughter PID	$> -10$
7	$\pi^\pm$ daughter PID	$< 20$

## Rectangular cuts after BDT

Number	Variable description	Cut
8	$K^\pm$ bachelor PID	$> 4$
9	$\pi^\pm$ bachelor PID	$< 4$
10	Bachelor is muon	False
11	$z$ flight significance	$> 2$
12	$K^\pm$ PID	$> 0$
13	$K_S^0$ mass veto	$[477, 507]\text{MeV}$



# BDT training variables

Name	Rank (%)	Description
$\log(D0\_RHO\_BPV)$	7.7	$D$ radial distance to beamline
$\log(Bu\_FDCHI2\_OWNPV)$	6.3	$B^\pm$ flight distance $\chi^2$
$\log(Bu\_RHO\_BPV)$	6.1	$B^\pm$ radial distance to beamline
$\log(Bach\_PT)$	6.1	Bachelor transverse momentum
$Bu\_PTASY\_1.5$	5.3	$B^\pm$ asymmetry parameter
$\log(1-D0\_DIRA\_BPV)$	5.0	Angle between PV and $D$
$\log(Bu\_IPCHI2\_OWNPV)$	4.8	$B^\pm$ impact parameter $\chi^2$
$\log(1-Bu\_DIRA\_BPV)$	4.7	Angle between PV and $B^\pm$
$\log(h[1,2]\_PT)$	4.4	$K^\pm$ transverse momentum
$Bu\_MAXDOCA$	4.4	$B^\pm$ distance of closest approach
$\log(Bach\_IPCHI2\_OWNPV)$	4.1	Bachelor impact parameter $\chi^2$

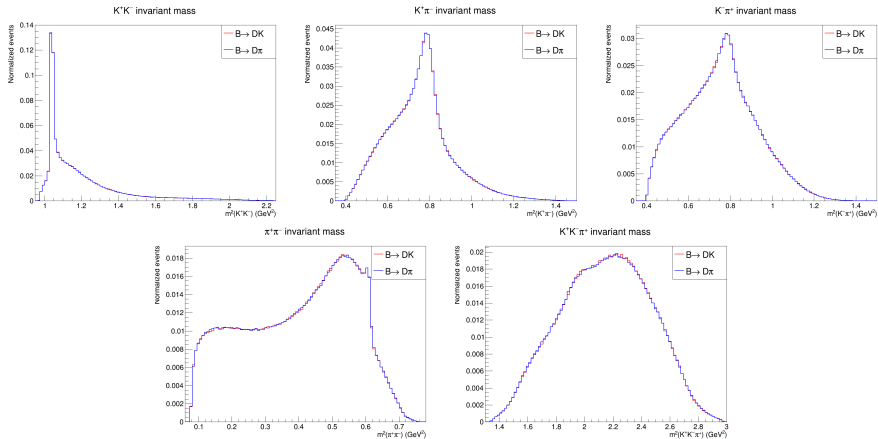
# BDT training particles

Name	Rank (%)	Description
$\log(\text{Bu\_constD0PV\_D0\_P})$	3.7	$D$ momentum from DTF
$\log(\text{D0\_VTXCHI2D0F})$	3.3	$D0$ vertex fit $\chi^2$
$\log(\text{h}[3,4]_{\text{-IPCHI2\_OWNPV}})$	3.3	$\pi^\pm$ impact parameter $\chi^2$
$\log(\text{D0\_IPCHI2\_OWNPV})$	3.2	$D$ impact parameter $\chi^2$
$\log(\text{h}[3,4]_{\text{-PT}})$	3.2	$\pi^\pm$ transverse momentum
$\log(\text{Bu\_PT})$	2.8	$B^\pm$ transverse momentum
$\log(\text{h}[1,2]_{\text{-P}})$	2.8	$K^\pm$ momentum
$\log(\text{Bach\_P})$	2.7	Bachelor momentum
$\log(\text{Bu\_constD0PV\_P})$	2.6	$B^\pm$ momentum from DTF
$\log(\text{h}[1,2]_{\text{-IPCHI2\_OWNPV}})$	2.5	$K^\pm$ impact parameter $\chi^2$
$\text{D0\_MAXDOCA}$	2.5	$D$ distance of closest approach
$\log(\text{Bu\_VTXCHI2D0F})$	2.0	$B^\pm$ vertex fit $\chi^2$
$\log(\text{h}[3,4]_{\text{-P}})$	1.9	$\pi^\pm$ momentum

Efficiency related systematics:

- Difference in  $B^\pm \rightarrow DK^\pm$  and  $B^\pm \rightarrow D\pi^\pm$  phase space acceptance
- Efficiency correction of  $c_i$  and  $s_i$

# Efficiency differences between $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$



Conclusion: More or less identical phase space acceptance, no systematic uncertainty considered