Determination of the CKM angle γ in $B^{\pm} \to (K^{+}K^{-}\pi^{+}\pi^{-})_{D}h^{\pm}$ decays

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Outline

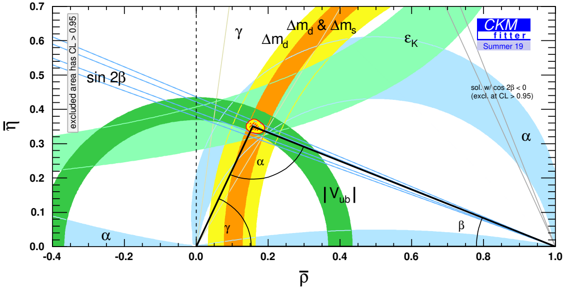
1. Introduction to the CKM angle γ
2. Binned γ analysis of the $D \to K^+K^-\pi^+\pi^-$ mode
3. Backgrounds
4. Systematic uncertainty
5. Summary and conclusion

γ and the unitary triangle

- Unitarity of CKM matrix: $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \implies$

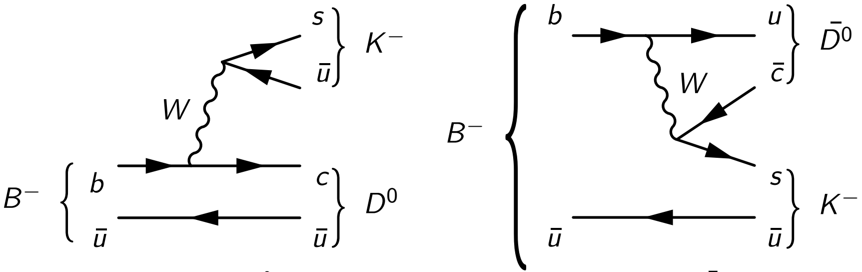
$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

- Only CKM angle accessible at tree level $\implies$
  - Negligible theoretical uncertainties
  - Ideal Standard Model benchmark
  - Compare with indirect measurements



CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005)

Sensitivity through interference



(a) $B^- \to D^0 K^-$ **(b)** $B^- \to \bar{D}^0 K^-$

- Superposition of D^0 and $\bar{D}^0$
- $b \to u\bar{c}s$ and $b \to c\bar{u}s$ interference $\to$ Sensitivity to γ

$$\mathcal{A}(B^-) = \mathcal{A}(D^0) + r_B e^{i(\delta_B - \gamma)} \mathcal{A}(\bar{D}^0)$$
$$\mathcal{A}(B^+) = \mathcal{A}(\bar{D}^0) + r_B e^{i(\delta_B + \gamma)} \mathcal{A}(D^0)$$

Binned γ analysis of the $D \to K^+K^-\pi^+\pi^-$ mode

The BPGGSZ method

- $B^\pm \to Dh^\pm$ amplitude:

$$\mathcal{A}(B^-) = \mathcal{A}(D^0) + r_B e^{i(\delta_B - \gamma)} \mathcal{A}(\bar{D}^0)$$
$$\mathcal{A}(B^+) = \mathcal{A}(\bar{D}^0) + r_B e^{i(\delta_B + \gamma)} \mathcal{A}(D^0)$$

- $\mathcal{A}(D^0)$ and $\mathcal{A}(\bar{D}^0)$ depend on D phase space
- Strong-phase difference of D^0 and $\bar{D}^0$ decays inaccessible at LHCb
- Model-independent measurement: Integrate over bins of phase space

Event yield in bin i

$$N_i^- = h_{B^-}\left(F_i + (x_-^2 + y_-^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_- c_i + y_- s_i)\right)$$
$$N_{-i}^+ = h_{B^+}\left(F_i + (x_+^2 + y_+^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_+ c_i + y_+ s_i)\right)$$

The BPGGSZ method

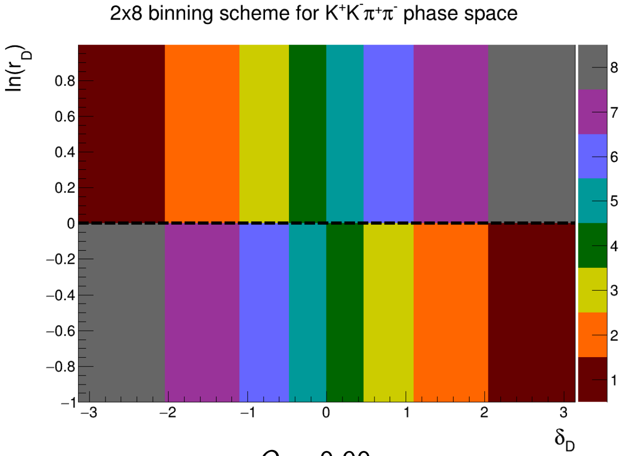
Event yield in bin i

$$N_i^- = h_{B^-}\left(F_i + (x_-^2 + y_-^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_- c_i + y_- s_i)\right)$$
$$N_{-i}^+ = h_{B^+}\left(F_i + (x_+^2 + y_+^2)\bar{F}_i + 2\sqrt{F_i\bar{F}_i}(x_+ c_i + y_+ s_i)\right)$$

- CP observables:
  - $x_\pm^{DK} = r_B^{DK}\cos\left(\delta_B^{DK} \pm \gamma\right), \quad y_\pm^{DK} = r_B^{DK}\sin\left(\delta_B^{DK} \pm \gamma\right)$
  - $x_\xi^{D\pi} = \mathrm{Re}(\xi^{D\pi}),\ y_\xi^{D\pi} = \mathrm{Im}(\xi^{D\pi}) \qquad \left(\xi^{D\pi} = \frac{r_B^{D\pi}}{r_B^{DK}} e^{i(\delta_B^{D\pi} - \delta_B^{DK})}\right)$
- Fractional bin yield:
  - $F_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}{\sum_j \int_j \mathrm{d}\Phi |\mathcal{A}(D^0)|^2}$
  - Floated in the fit, mostly constrained by $B^\pm \to D\pi^\pm$
- Amplitude averaged strong phases from BESIII:

$$c_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\cos(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}} \qquad s_i = \frac{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)|\sin(\delta_D)}{\sqrt{\int_i \mathrm{d}\Phi |\mathcal{A}(D^0)|^2 \int_i \mathrm{d}\Phi \left|\mathcal{A}(\bar{D}^0)\right|^2}}$$

Binning scheme



$$Q = 0.90$$

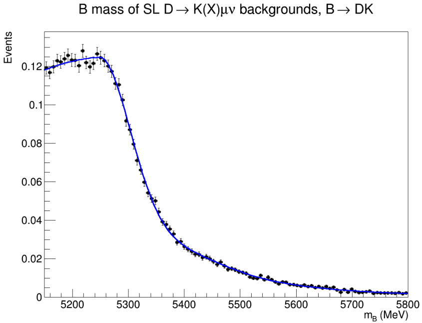
Bins $i < 0$ on top, $i > 0$ below

Backgrounds

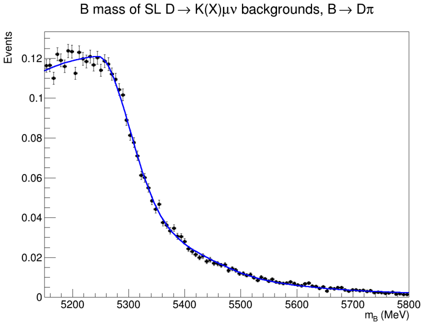
D semileptonic backgrounds

- $B^{\pm} \to Dh^{\pm}$, $D \to K(X)l\nu$, $K(X) \to K\pi\pi$
  - $K_1(1270)$
  - $K_1(1400)$
  - $K^*(1410)$
  - $K^*(1680)$
  - $K_2^*(1430)$
- Single mis-ID: $K\mu\pi\pi \to KK\pi\pi$
- Double mis-ID: $K\pi\pi\mu \to KK\pi\pi$
- Generate Rapidsim samples, reweight with PIDCalib2

D semileptonic backgrounds



(a) $B \to DK$

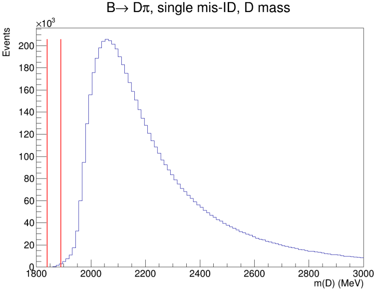


(b) $B \to D\pi$

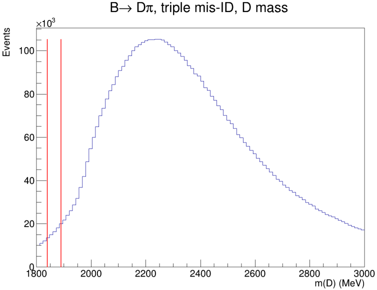
Conclusion: Negligible impact, include in systematics

$D \to K\pi\pi\pi$ mis-ID background

- $B^{\pm} \to Dh^{\pm}$, $D \to K\pi\pi\pi$
- Single mis-ID: $K\pi\pi\pi \to KK\pi\pi$
- Triple mis-ID: $\pi\pi K\pi \to KK\pi\pi$
- Use LHCb MC generated with AmpGen, reweight with PIDCalib2



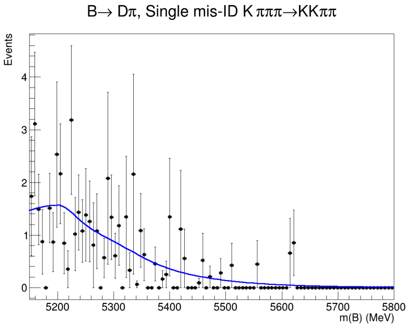
(a) Single mis-ID



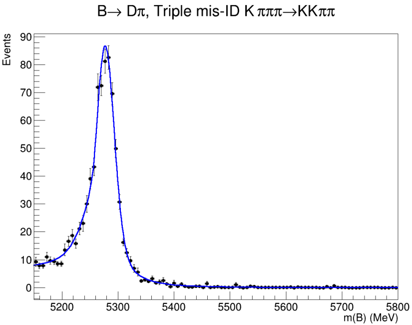
(b) Triple mis-ID

Figure 3: D invariant mass

$D \to K\pi\pi\pi$ mis-ID background



(a) Single mis-ID



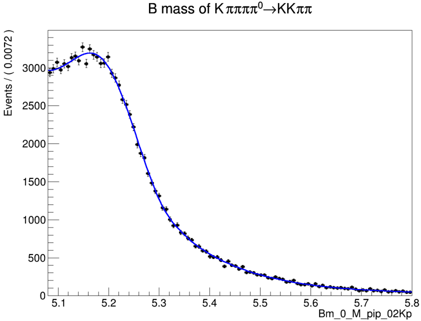
(b) Triple mis-ID

Figure 4: B invariant mass

Conclusion: Negligible impact, include in systematics

$D \to K\pi\pi\pi^0$ mis-ID background

- $B^{\pm} \to Dh^{\pm}$, $D \to K\pi\pi\pi[\pi^0]$
- π^0 not reconstructed $\to$ Lower D mass
- Single mis-ID: $K\pi\pi\pi \to KK\pi\pi \to$ Higher D mass
- Generate RapidSim samples, reweight with PIDCalib2



Conclusion: Fix shape from RapidSim, allow yield to float

Global fit

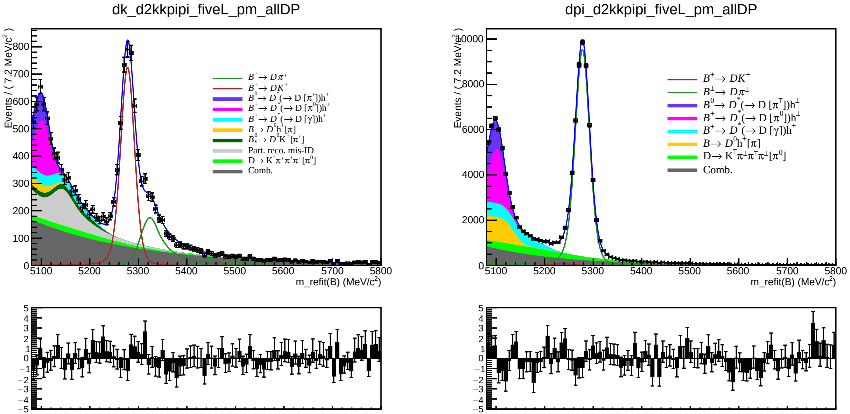


Figure 5: $B^{\pm} \to DK^{\pm}$ channel (left) and $B^{\pm} \to D\pi^{\pm}$ channel (right)

- $B^{\pm} \to DK^{\pm}$ yield: 3543 ± 75
- $B^{\pm} \to D\pi^{\pm}$ yield: $47\,503 \pm 260$

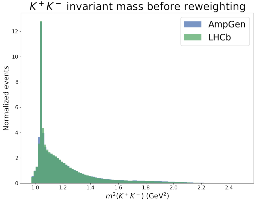
Systematic uncertainties

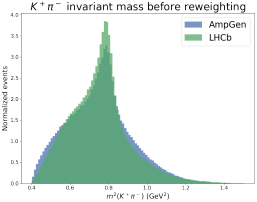
Systematic uncertainties

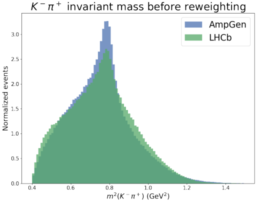
Different strategies for evaluating systematic uncertainties:

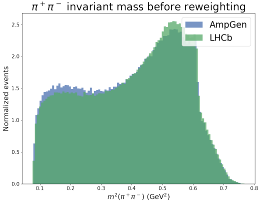
- Generate toy datasets with systematics, fit with default model and take the bias as a systematic:
  - Small backgrounds ($D \to K(X) l \nu_l$, $D \to K\pi\pi\pi$, $B \to D l \nu_l$, Λ_b)
  - Bin dependent mass shape
  - Low mass physics effects
- Do multiple fits to data while smearing parameters:
  - c_i and s_i
  - Mass shape
  - Fixed yield fractions
  - PID efficiency
- Fit bias: Take bias toys as systematic uncertainty

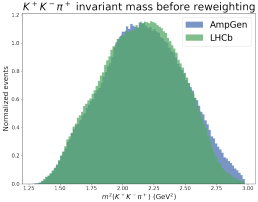
Efficiency correction of c_i and s_i





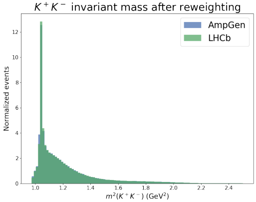


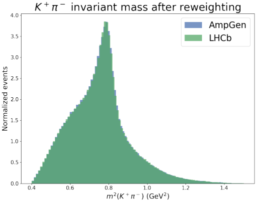


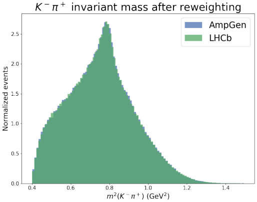


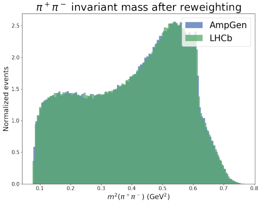
Need to reweight events to account for efficiency differences between AmpGen samples and LHCb MC

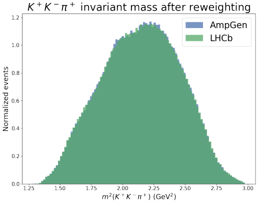
Efficiency correction of c_i and s_i











After reweighing, use weights to recalculate c_i and s_i

Conclusion: Efficiency correction of c_i and s_i is an order of magnitude smaller than their uncertainties, no systematic uncertainty considered

Summary of all systematic uncertainties

Source	x_-^{DK}	y_-^{DK}	x_+^{DK}	y_+^{DK}	$x_\xi^{D\pi}$	$y_\xi^{D\pi}$
Statistical	2.73	3.23	2.38	2.90	4.30	5.27
c_i, s_i	0.66	1.55	0.32	1.31	1.73	1.03
$B^\pm \to D\mu\nu$ background	0.04	0.03	0.02	0.15	0.30	0.10
$D \to K(X)l\nu_l$ background	0.15	0.05	0.11	0.03	0.35	0.25
$D \to K\pi\pi\pi$ background	0.17	0.03	0.04	0.01	0.46	0.18
Λ_b background	0.09	0.11	0.00	0.18	0.16	0.21
Bin dependent mass shape	0.21	0.05	0.17	0.01	0.37	0.11
Fit bias	0.19	0.03	0.16	0.04	0.30	0.16
Fixed yield fractions	0.02	0.03	0.02	0.02	0.01	0.01
Low mass physics effects	0.05	0.09	0.05	0.18	0.41	0.48
Mass shape	0.03	0.03	0.02	0.02	0.04	0.01
PID Efficiency	0.03	0.03	0.02	0.02	0.04	0.01
Total LHCb systematic	0.39	0.17	0.27	0.30	0.92	0.65
Total systematic	0.77	1.55	0.41	1.34	1.96	1.22

Summary and conclusion

Summary of CP observables

- Measured CP observables:

$$
\begin{aligned}
x_-^{DK} &= (x.x \pm 2.7 \pm 0.4 \pm 0.7) \times 10^{-2}, \\
y_-^{DK} &= (x.x \pm 3.2 \pm 0.2 \pm 1.6) \times 10^{-2}, \\
x_+^{DK} &= (x.x \pm 2.4 \pm 0.3 \pm 0.3) \times 10^{-2}, \\
y_+^{DK} &= (x.x \pm 2.9 \pm 0.3 \pm 1.3) \times 10^{-2}, \\
x_\xi^{D\pi} &= (x.x \pm 4.3 \pm 0.9 \pm 1.7) \times 10^{-2}, \\
y_\xi^{D\pi} &= (x.x \pm 5.3 \pm 0.6 \pm 1.0) \times 10^{-2},
\end{aligned}
$$

- Note: Currently using c_i and s_i from the LHCb model
- Publication strategy: Publish current results together with binned yields $\rightarrow$ Redo fit to obtain model-independent CP observables once c_i and s_i from BESIII are available

Interpretation in terms of γ

- Interpret in terms of physics parameters:

$$\begin{aligned}
\gamma &= (x.x^{+14}_{-15})^\circ, \\
\delta_B^{DK} &= (x.x^{+15}_{-14})^\circ, \\
r_B^{DK} &= x.x^{+0.019}_{-0.018}, \\
\delta_B^{D\pi} &= (x.x^{+117}_{-63})^\circ, \\
r_B^{D\pi} &= x.x^{+0.0052}_{-0.0024}.
\end{aligned}$$

Bonus measurement

- The mode $B^\pm \to Dh^\pm$, $D \to \pi^+\pi^-\pi^+\pi^-$ very similar
- Run this through same selection (including BDT)
- Can measure GLW CP observables as additional constraints on γ:

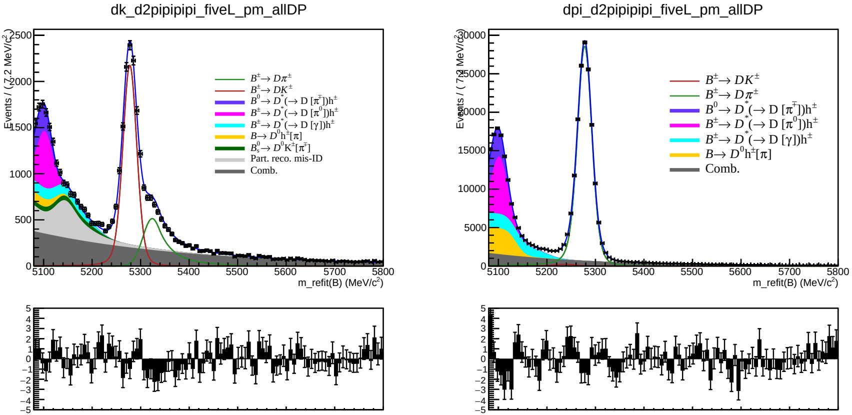
$$A_h = \frac{\Gamma(B^- \to Dh^-) - \Gamma(B^+ \to Dh^+)}{\Gamma(B^- \to Dh^-) + \Gamma(B^+ \to Dh^+)},$$

$$R_{\mathrm{CP}} = \frac{R(4\pi)}{R(K3\pi)},$$

$$R = \frac{\Gamma(B \to DK)}{\Gamma(B \to D\pi)}.$$

- $B^\pm \to Dh^\pm$, $D \to K\pi\pi\pi$ yields provided by Tim Evans

Global fit of $B^\pm \to Dh^\pm$, $D \to \pi^+\pi^-\pi^+\pi^-$



$$\frac{\mathrm{Yield}(\pi\pi\pi\pi)}{\mathrm{Yield}(KK\pi\pi)} = \frac{161\,900 \pm 455}{54\,039 \pm 256} = 2.996 \pm 0.017$$

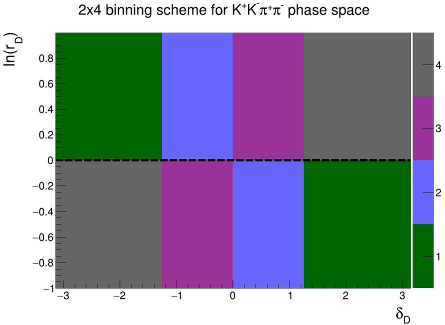
Compare with PDG: 3.06 ± 0.16

Thank you!

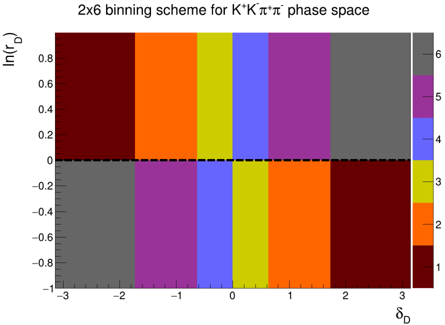
Thank you!

Backup

Binning scheme

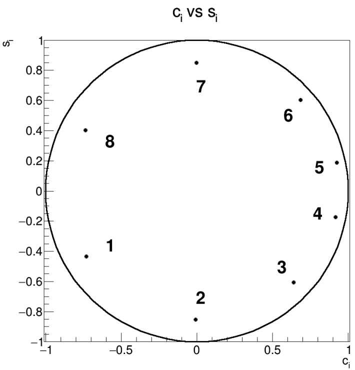


(a) $Q = 0.85$

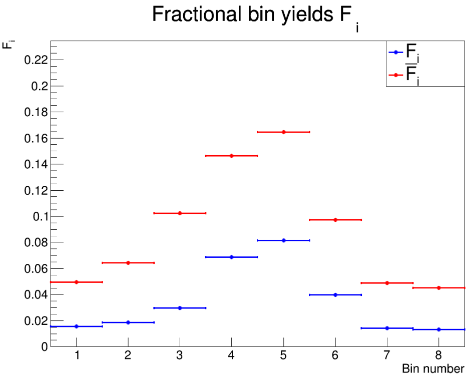


(b) $Q = 0.89$

c_i, s_i and F_i

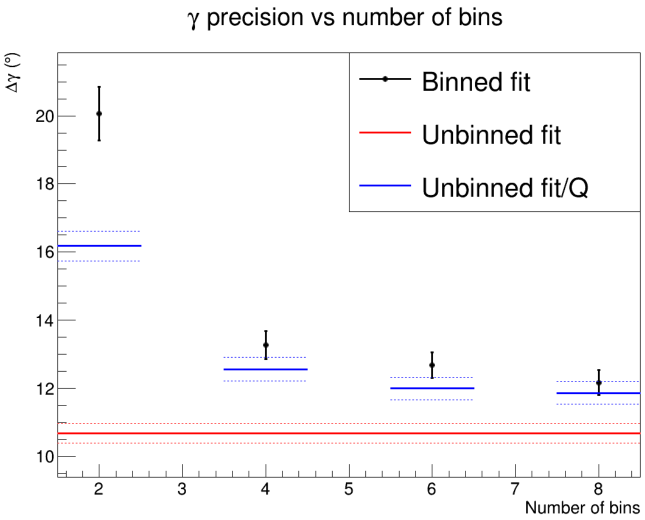


c_i vs s_i
s_i
c_i
1
2
3
4
5
6
7
8



Fractional bin yields F_i
F_i
$\overline{F}_i$
Bin number

Comparison of binned fit precision with unbinned fit



Trigger requirements

Run 1 trigger requirements	(Bu_L0Global_TIS or Bu_L0HadronDecision_TOS) and (Bu_Hlt1TrackAllL0Decision_TOS) and (Bu_Hlt2Topo2BodyBBDTDecision_TOS or Bu_Hlt2Topo3BodyBBDTDecision_TOS or Bu_Hlt2Topo4BodyBBDTDecision_TOS or Bu_Hlt2IncPhiDecision_TOS)
Run 2 trigger requirements	(Bu_L0Global_TIS or Bu_L0HadronDecision_TOS) and (Bu_Hlt1TrackMVADecision_TOS or Bu_Hlt1TwoTrackMVADecision_TOS) and (Bu_Hlt2Topo2BodyDecision_TOS or Bu_Hlt2Topo3BodyDecision_TOS or Bu_Hlt2Topo4BodyDecision_TOS or Bu_Hlt2IncPhiDecision_TOS)

Initial cuts

Rectangular cuts before BDT

Number	Variable description	Cut
1	DTF converged	True
2	Bachelor momentum	$< 100\text{GeV}$
3	Bachelor has RICH	True
4	D invariant mass	$[1839.84, 1889.84]\text{MeV}$
5	$B^{\pm}$ invariant mass	$[5080, 5800]\text{MeV}$
6	$K^{\pm}$ daughter PID	> -10
7	$\pi^{\pm}$ daughter PID	< 20

Final cuts

Rectangular cuts after BDT

Number	Variable description	Cut
8	$K^{\pm}$ bachelor PID	> 4
9	$\pi^{\pm}$ bachelor PID	< 4
10	Bachelor is muon	False
11	z flight significance	> 2
12	$K^{\pm}$ PID	> 0
13	K_S^0 mass veto	$[477, 507]$MeV

BDT training variables

Name	Rank (%)	Description
log(D0_RHO_BPV)	7.7	D radial distance to beamline
log(Bu_FDCHI2_OWNPV)	6.3	$B^{\pm}$ flight distance χ^2
log(Bu_RHO_BPV)	6.1	$B^{\pm}$ radial distance to beamline
log(Bach_PT)	6.1	Bachelor transverse momentum
Bu_PTASY_1.5	5.3	$B^{\pm}$ asymmetry parameter
log(1-D0_DIRA_BPV)	5.0	Angle between PV and D
log(Bu_IPCHI2_OWNPV)	4.8	$B^{\pm}$ impact parameter χ^2
log(1-Bu_DIRA_BPV)	4.7	Angle between PV and $B^{\pm}$
log(h[1,2]_PT)	4.4	$K^{\pm}$ transverse momentum
Bu_MAXDOCA	4.4	$B^{\pm}$ distance of closest approach
log(Bach_IPCHI2_OWNPV)	4.1	Bachelor impact parameter χ^2

BDT training particles

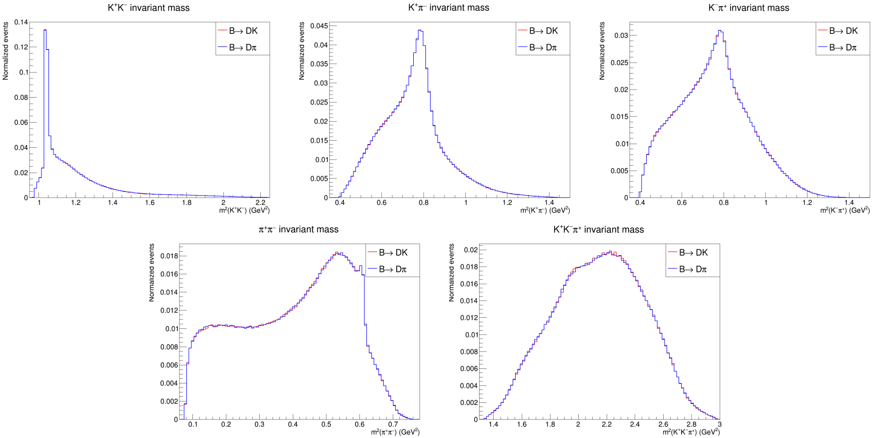
Name	Rank (%)	Description
`log(Bu_constD0PV_D0_P)`	3.7	D momentum from DTF
`log(D0_VTXCHI2DOF)`	3.3	$D0$ vertex fit χ^2
`log(h[3,4]_IPCHI2_OWNPV)`	3.3	$\pi^\pm$ impact parameter χ^2
`log(D0_IPCHI2_OWNPV)`	3.2	D impact parameter χ^2
`log(h[3,4]_PT)`	3.2	$\pi^\pm$ transverse momentum
`log(Bu_PT)`	2.8	$B^\pm$ transverse momentum
`log(h[1,2]_P)`	2.8	$K^\pm$ momentum
`log(Bach_P)`	2.7	Bachelor momentum
`log(Bu_constD0PV_P)`	2.6	$B^\pm$ momentum from DTF
`log(h[1,2]_IPCHI2_OWNPV)`	2.5	$K^\pm$ impact parameter χ^2
`D0_MAXDOCA`	2.5	D distance of closest approach
`log(Bu_VTXCHI2DOF)`	2.0	$B^\pm$ vertex fit χ^2
`log(h[3,4]_P)`	1.9	$\pi^\pm$ momentum

Efficiency related systematics

Efficiency related systematics:

- Difference in $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$ phase space acceptance
- Efficiency correction of c_i and s_i

Efficiency differences between $B^\pm \to DK^\pm$ and $B^\pm \to D\pi^\pm$



Conclusion: More or less identical phase space acceptance, no systematic uncertainty considered