$D \to K^+K^-\pi^+\pi^-$ strong phase analysis at BESIII

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Summary of BESIII analysis progress

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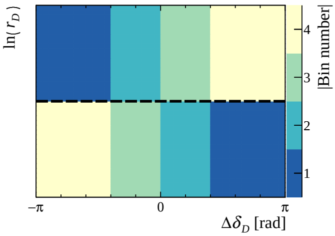
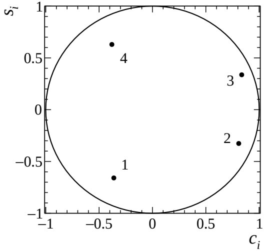
Analysis of $D^0 \to K^+K^-\pi^+\pi^-$

- Study D^0-$\bar{D}^0$ strong phase difference in bins of the 5D phase space
- Measurement of amplitude averaged strong phases c_i and s_i
- c_i and s_i are important inputs to:
  - Measurement of γ using BPGGSZ method
  - Charm mixing and CPV studies





Progress since last group meeting presentation:

1. Selection has been finalised
2. New $\psi(3770)$ data has been added ($3\,\text{fb}^{-1} \to 8\,\text{fb}^{-1}$)
3. All single and double tag yields have been (re)fitted
4. New strong phase fit has been developed
5. Toy studies with new fit
6. Preliminary result of c_i and s_i is ready

Brief summary of formalism

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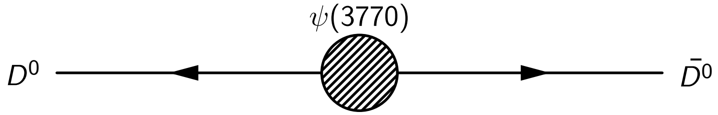
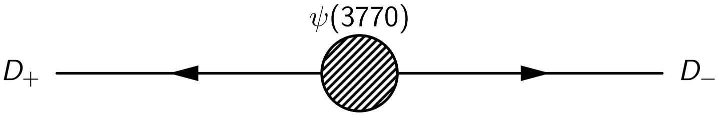
- $\psi(3770) \to D^0 \bar{D}^0$ decay conserves $\mathcal{C} = -1$



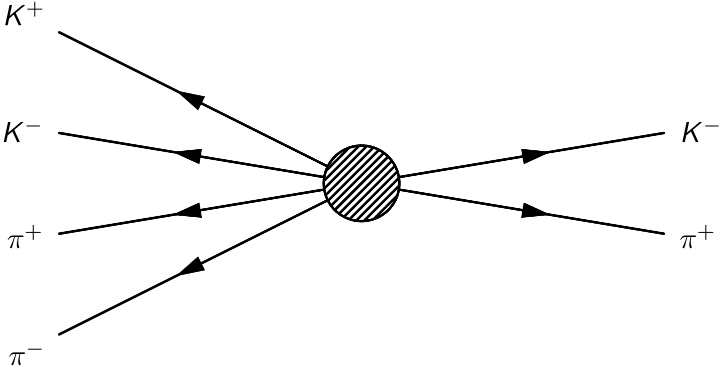
- But since they are quantum correlated, we must consider their CP eigenstates $D_\pm = (|D^0\rangle \pm |\bar{D}^0\rangle)/\sqrt{2}$
- Total wavefunction is $|D^0\rangle|\bar{D}^0\rangle - |\bar{D}^0\rangle|D^0\rangle = |D_+\rangle|D_-\rangle + |D_-\rangle|D_+\rangle$



The two D mesons do <u>not</u> communicate, but the $D \to KK\pi\pi$ decay is perfectly correlated with the tagged D

Brief summary of formalism

- Tag mode can be a flavour tag
  - $K\pi$, $K\pi\pi^0$, $K\pi\pi\pi$, $Ke\nu$

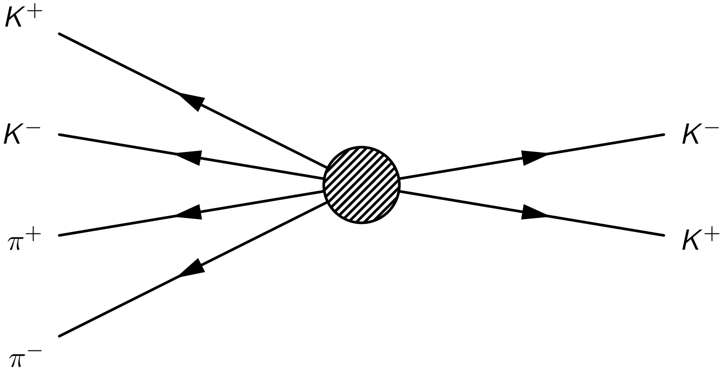


Use flavour tags to measure fraction of $D^0 \to KK\pi\pi$ decays in bin i:

$$\frac{N_i^{\mathrm{DT}}}{N^{\mathrm{ST}}} = \mathcal{B}(KK\pi\pi) \times \left(K_{-i} + r_D^2 K_i - 2r_D R\sqrt{K_i K_{-i}}(c_i \cos(\delta_D) + s_i \sin(\delta_D))\right)$$

Recap of BESIII analysis

- Tag mode can be a CP even tag
  - KK, $\pi\pi$, $\pi\pi\pi^0$, $K_S\pi^0\pi^0$, $K_L\pi^0$, $K_L\omega$

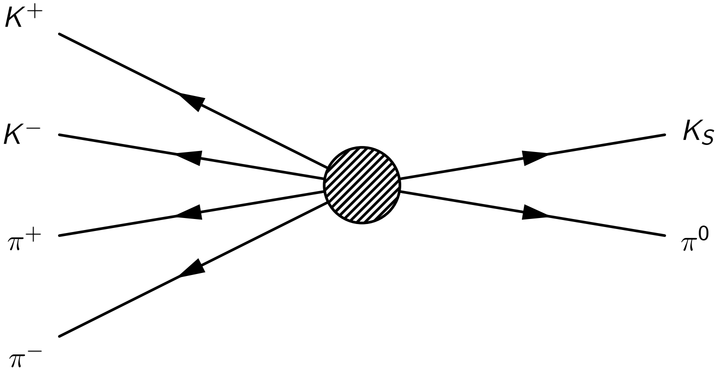


$D \to K^+K^-$, which is CP even, forces $D \to K^+K^-\pi^+\pi^-$ to be CP odd:

$$\frac{N_i^{\mathrm{DT}}}{N^{\mathrm{ST}}} = \mathcal{B}(KK\pi\pi) \times \left(K_{-i} + K_i - 2\sqrt{K_iK_{-i}}c_i\right)$$

Recap of BESIII analysis

- Tag mode can be a CP odd tag
  - $K_S\pi^0$, $K_S\omega$, $K_S\eta$, $K_S\eta'$, $K_L\pi^0\pi^0$

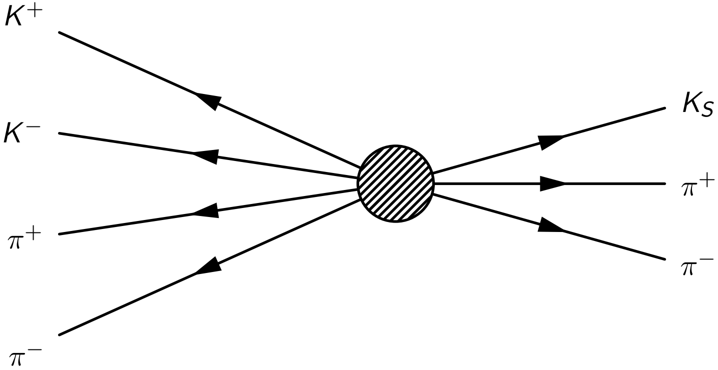


$D \to K_S^0\pi^0$, which is CP odd, forces $D \to K^+K^-\pi^+\pi^-$ to be CP even:

$$\frac{N_i^{\mathrm{DT}}}{N^{\mathrm{ST}}} = \mathcal{B}(KK\pi\pi) \times \left(K_{-i} + K_i + 2\sqrt{K_iK_{-i}}c_i\right)$$

Recap of BESIII analysis

- Tag mode can be a multi-body tag
  - $K_S\pi^+\pi^-$, $K_L\pi^+\pi^-$

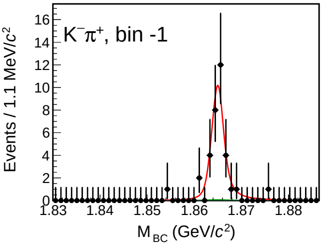


Multi-body tags, such as $D \to K_S^0\pi^+\pi^-$, are also sensitive to s_i:

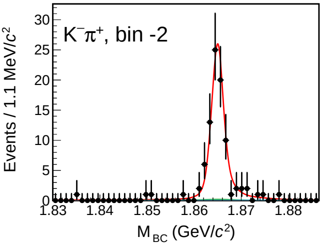
$$\frac{N_{ij}^{\mathrm{DT}}}{N^{\mathrm{ST}}} = \mathcal{B}(KK\pi\pi) \times \left(K_iK'_{-j} + K_{-i}K'_j - 2\sqrt{K_iK_{-i}K'_jK'_{-j}}(c_ic'_j + s_is'_j)\right)$$

Some double tag yield results

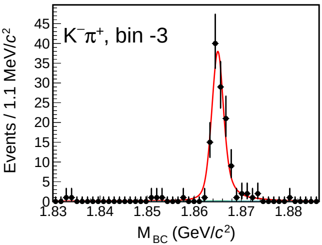
Double tag fit of $KK\pi\pi$ vs $K\pi$



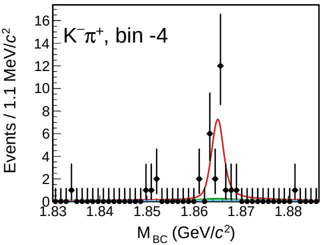
(a) Bin -1 yield: $32.4^{+6.2}_{-5.5}$



(b) Bin -2 yield: $82.7^{+9.7}_{-9.0}$

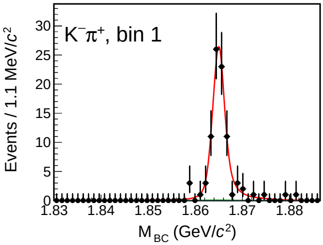


(c) Bin -3 yield: $120.3^{+11.6}_{-10.9}$

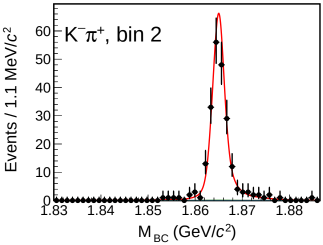


(d) Bin -4 yield: $22.4^{+5.2}_{-4.6}$

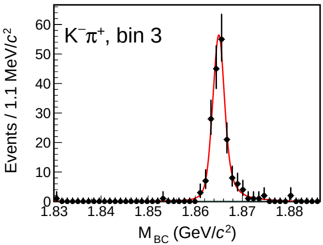
Double tag fit of $KK\pi\pi$ vs $K\pi$



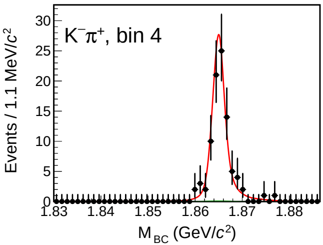
(a) Bin 1 yield: $84.5^{+9.8}_{-9.1}$



(b) Bin 2 yield: $211.2^{+15.4}_{-14.8}$

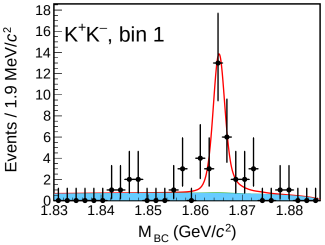


(c) Bin 3 yield: $181.0^{+14.0}_{-13.3}$

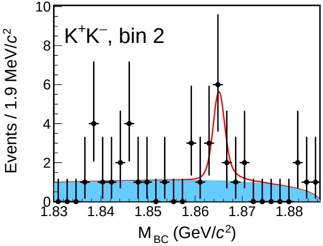


(d) Bin 4 yield: $88.6^{+9.7}_{-9.0}$

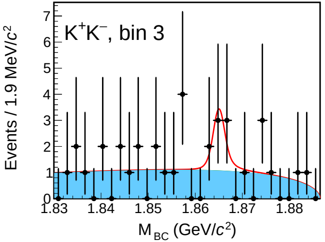
Double tag fit of $KK\pi\pi$ vs KK



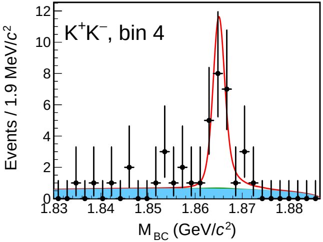
(a) Bin 1 yield: $25.3^{+6.2}_{-5.5}$



(b) Bin 2 yield: $8.8^{+4.0}_{-3.3}$

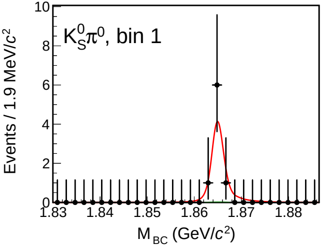


(c) Bin 3 yield: $4.5^{+3.3}_{-2.6}$

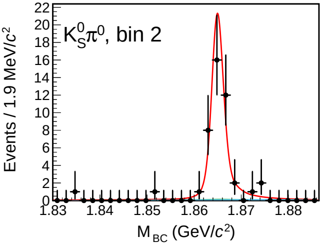


(d) Bin 4 yield: $21.1^{+5.5}_{-4.8}$

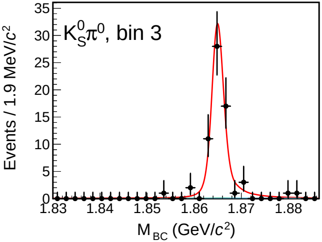
Double tag fit of $KK\pi\pi$ vs $K_S\pi^0$



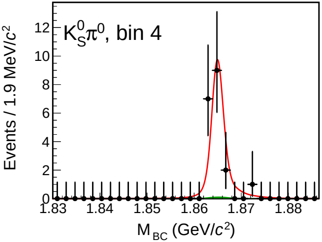
(a) Bin 1 yield: $7.9^{+3.1}_{-2.5}$



(b) Bin 2 yield: $40.4^{+6.8}_{-6.3}$

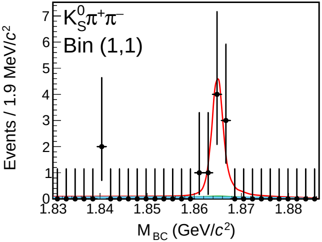


(c) Bin 3 yield: $61.1^{+8.3}_{-7.8}$

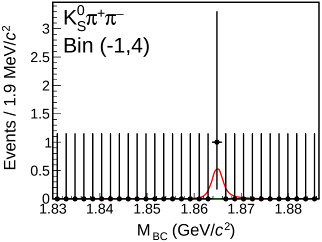


(d) Bin 4 yield: $18.3^{+4.5}_{-3.9}$

Double tag fit of $KK\pi\pi$ vs $K_S\pi^+\pi^-$



(a) Bin $(1, 1)$ yield: $8.2^{+3.3}_{-2.7}$



(b) Bin $(-1, 4)$ yield: $0.9^{+1.3}_{-0.7}$

Qualitative features of double tag yields

What do we observe in the double tag yields?

1. In CP even tags, bin 1 and 4 are enhanced, while bin 2 and 3 are suppressed (and vice versa for CP odd tags)
   - Expect bin 1 and 4 to have a CP odd behaviour, while bin 2 and 3 are more CP even
   - Bin 1 and 4 should have $c_i < 0$, while bin 2 and 3 will have $c_i > 0$
2. Uncertainties can be very asymmetric
3. In multi-body tags some bins have very low yields

Strong phase fit setup

Strong phase fit setup

What goes into the fit?

1. $8 \times 4 = 32$ flavour tag yields
2. $4 \times 12 = 48$ CP tag yields
3. $8 \times 8 \times 3 = 192$ multi-body tag yields
4. In total: 272 measured yields
5. Fixed parameters:
   - Single tag yields
   - Efficiency matrices
   - External strong phase parameters

Master equations

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}\left(K_{-j} + r_D^2 K_j - 2r_D\sqrt{K_j K_{-j}}\left(c_j\cos(\delta_D) + s_j\sin(\delta_D)\right)\right)$$

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}(K_j + K_{-j} \mp 2\sqrt{K_j K_{-j}}c_j)$$

$$\hat{N}_{ij}^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ijkl}\left(K_k K'_{-l} + K_{-k}K'_l - 2\sqrt{K_k K_{-k} K'_l K'_{-l}}(c_k c'_l + s_k s'_l)\right)$$

Strong phase fit setup

What comes out of the fit?

1. The $D^0 \to KK\pi\pi$ branching fraction $\mathcal{B}$ (1 nominal, 1 for $K_L\pi\pi$)
2. c_i and s_i (8 parameters)
3. K_i (7 parameters with recursive fraction parameterisation R_i)
4. $r_D^{K\pi}\cos\left(\delta_D^{K\pi}\right)$ and $r_D^{K\pi}\sin\left(\delta_D^{K\pi}\right)$
5. In total: 19 free parameters

Master equations

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}\left(K_{-j} + r_D^2 K_j - 2r_D\sqrt{K_j K_{-j}}\left(c_j\cos(\delta_D) + s_j\sin(\delta_D)\right)\right)$$

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}(K_j + K_{-j} \mp 2\sqrt{K_j K_{-j}}c_j)$$

$$\hat{N}_{ij}^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ijkl}\left(K_k K'_{-l} + K_{-k}K'_l - 2\sqrt{K_k K_{-k} K'_l K'_{-l}}(c_k c'_l + s_k s'_l)\right)$$

Likelihood fit

Master equations

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}} \mathcal{B} \epsilon_{ij} \left(K_{-j} + r_D^2 K_j - 2 r_D \sqrt{K_j K_{-j}} (c_j \cos(\delta_D) + s_j \; sin(\delta_D)) \right)$$

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}} \mathcal{B} \epsilon_{ij} (K_j + K_{-j} \mp 2\sqrt{K_j K_{-j}} c_j)$$

$$\hat{N}_{ij}^{\mathrm{DT}} = N^{\mathrm{ST}} \mathcal{B} \epsilon_{ijkl} (K_k K'_{-l} + K_{-k} K'_l - 2\sqrt{K_k K_{-k} K'_l K'_{-l}} (c_k c'_l + s_k s'_l))$$

Ordinarily, we would construct a Gaussian (log)likelihood function $\implies$ Obtain $\mathcal{B}$, K_i, c_i and s_i by minimising the following function:

$$-\ln(\mathcal{L}) = \frac{1}{2} \sum_{\mathrm{Tag}} \sum_{ij} (V^{-1})_{ij} (N_i^{\mathrm{DT}} - \hat{N}_i^{\mathrm{DT}})(N_j^{\mathrm{DT}} - \hat{N}_j^{\mathrm{DT}})$$

$$V_{ij} = \rho_{ij} \sigma_i \sigma_j$$

[0] ρ are correlation coefficients

Likelihood fit

Master equations

$$
\begin{aligned}
\hat{N}_i^{\mathrm{DT}} &= N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}\Big(K_{-j} + r_D^2 K_j - 2r_D\sqrt{K_j K_{-j}}\big(c_j\cos(\delta_D) + s_j\sin(\delta_D)\big)\Big) \\
\hat{N}_i^{\mathrm{DT}} &= N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}(K_j + K_{-j} \mp 2\sqrt{K_j K_{-j}}c_j) \\
\hat{N}_{ij}^{\mathrm{DT}} &= N^{\mathrm{ST}}\mathcal{B}\epsilon_{ijkl}\big(K_k K'_{-l} + K_{-k}K'_l - 2\sqrt{K_k K_{-k} K'_l K'_{-l}}(c_k c'_l + s_k s'_l)\big)
\end{aligned}
$$

Our DT yields are very small, so their uncertainties are asymmetric $\implies$ Approximate covariance matrix from the asymmetric uncertainties[1]:

$$
\begin{aligned}
-\ln(\mathcal{L}) &= \frac{1}{2}\sum_{\mathrm{Tag}}\sum_{ij}(V^{-1})_{ij}(N_i^{\mathrm{DT}} - \hat{N}_i^{\mathrm{DT}})(N_j^{\mathrm{DT}} - \hat{N}_j^{\mathrm{DT}}) \\
V_{ij} &= \rho_{ij}\sigma_i\sigma_j, \quad \sigma = \sqrt{\sigma_-\sigma_+ - (\sigma_+ - \sigma_-)(N^{\mathrm{DT}} - \hat{N}^{\mathrm{DT}})}
\end{aligned}
$$

[1] arXiv:physics/0406120

Likelihood fit

$$-\ln(\mathcal{L}) = \frac{1}{2}\sum_{\mathrm{Tag}}\sum_{ij}(V^{-1})_{ij}(N_i^{\mathrm{DT}} - \hat{N}_i^{\mathrm{DT}})(N_j^{\mathrm{DT}} - \hat{N}_j^{\mathrm{DT}})$$

$$V_{ij} = \rho_{ij}\sigma_i\sigma_j, \quad \sigma = \sqrt{\sigma_-\sigma_+ - (\sigma_+ - \sigma_-)(N^{\mathrm{DT}} - \hat{N}^{\mathrm{DT}})}$$

- The above likelihood has good coverage for flavour and *CP* tags...
- ... but not for multi-body decays
  - Bins with $\sigma_- \approx 0$ make the fit unstable
  - Fit convergence was found to be less than 60%
- In multi-body decays, use the full unbinned likelihood directly
  - Fit convergence improves to over 95%
  - Much slower, but much more accurate

Toy studies

Toy studies

Aim of toy studies:

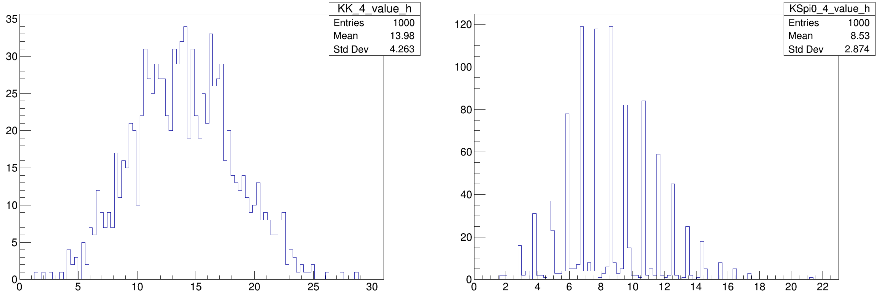
1. Check fit convergence
2. Check error coverage
3. Correct any biases in fitted parameters

Unfortunately, in this analysis we cannot simply generate Poisson-distributed DT yields:

1. Asymmetric uncertainties
2. Large background-to-signal
3. Multi-body tags with low yields require a full unbinned likelihood

Solution: Generate toy datasets for each double tag fit

Toy studies



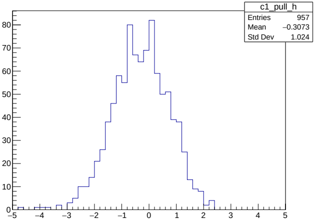
(a) Input yield: 13.9

(b) Input yield: 8.6

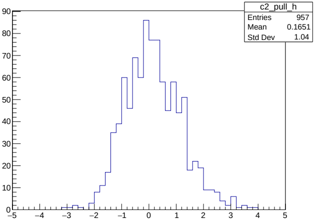
Figure 7: Fitted yields in toy datasets for the (left) KK and (right) $K_S\pi^0$ tags

1. No biases are observed
2. Distributions are asymmetric $\implies$ uncertainties are asymmetric
3. KK uncertainty is non-Poisson because of large backgrounds from $q\bar{q}$
4. $K_S\pi^0$ has small backgrounds, so observed yields Poisson distributed

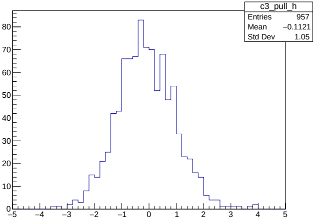
Toy studies



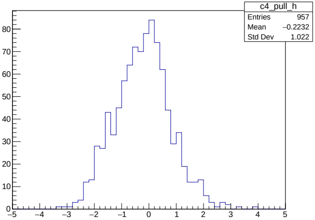
(a) c_1 pulls



(b) c_2 pulls

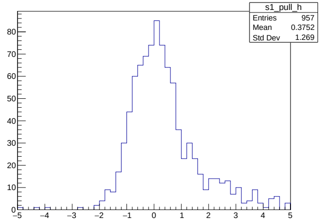


(c) c_3 pulls

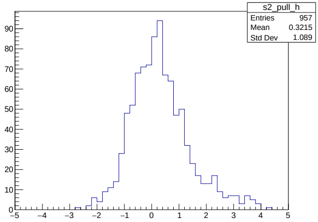


(d) c_4 pulls

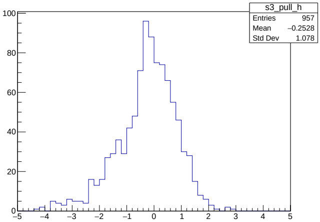
Toy studies



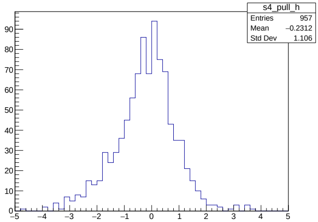
(a) s_1 pulls



(b) s_2 pulls



(c) s_3 pulls



(d) s_4 pulls

Toy studies

What do the toy fits tell us?

1. Both c_i and s_i pulls has asymmetric tails
2. Effect is small in c_i and a small bias correction will be sufficient
3. s_i has a small overcoverage, but a large tail, so special care is required

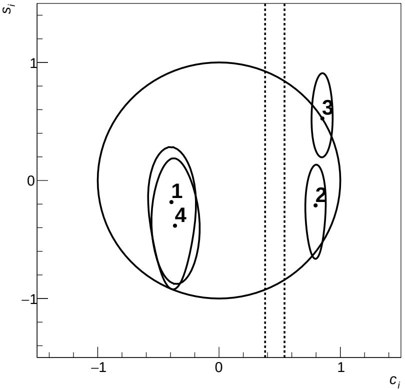
Fit results

Cross check: Fitted K_i parameters

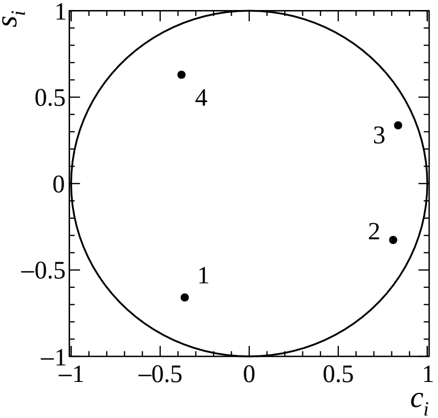
Variable	Fit result	Model prediction
R_{-4}	0.092 ± 0.005	0.086
R_{-3}	0.284 ± 0.009	0.297
R_{-2}	0.398 ± 0.012	0.398
R_{-1}	0.259 ± 0.012	0.267
R_1	0.122 ± 0.011	0.110
R_2	0.412 ± 0.020	0.401
R_3	0.801 ± 0.019	0.833

Naive χ^2 check: $\chi^2 = 8.3/7 = 1.2 \implies$ Excellent agreement!

c_i and s_i measurement



(a) Data



(b) LHCb model

Figure 10: Comparison between model and model-independent measurements

$\delta_{K\pi}$ measurement

- K_i, which are constrained by flavour tags, are free parameters
- Corrections for DCS decays, which depend on the strong phases δ_D, are part of the fit
- We could instead treat δ_D as a free parameter, and make a simultaneous measurement
- LHCb γ and charm combination: $\delta_D^{K\pi} = (190.2^{+2.8}_{2.8})^\circ$
- Free parameters: $r_D\cos(\delta_{K\pi})$ and $r_D\sin(\delta_{K\pi})$

$$\hat{N}_i^{\mathrm{DT}} = N^{\mathrm{ST}}\mathcal{B}\epsilon_{ij}\Big(K_{-j} + r_D^2 K_j - 2r_D\sqrt{K_j K_{-j}}\big(c_j\cos(\delta_D) + s_j\sin(\delta_D)\big)\Big)$$

$\delta_{K\pi}$ measurement

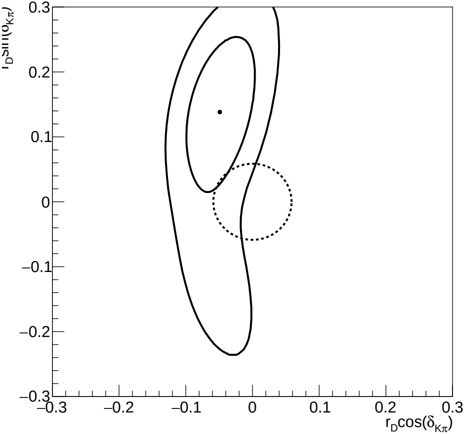


Figure 11: Contours of $r_D\cos(\delta_{K\pi})$ vs $r_D\sin(\delta_{K\pi})$, corresponding to $\Delta\log(\mathcal{L}) = 2.30$ and 6.18, or 68% and 95% confidence level. Dashed line indicates measured value of r_D.

In addition, we have another nuisance parameter: The $D^0 \to K^+K^-\pi^+\pi^-$ branching fraction

Current PDG value:
$\mathcal{B} = (2.47 \pm 0.11) \times 10^{-3}$ (FOCUS experiment, 2005)

Fit result:
$\mathcal{B} = (2.765 \pm 0.046) \times 10^{-3}$

Much better precision than the current world average!

Assume 0.3% for each track and 1.0% (0.3%) for each charged kaon (pion)

Branching fraction measurement: $\mathcal{B} = (2.76 \pm 0.05 \pm 0.05) \times 10^{-3}$

Cross check with single tag yield and integrated luminosity:
$N^{\mathrm{ST}} = 29227 \pm 268 \implies \mathcal{B} = 2.70 \pm 0.04$

Systematics

Systematics

	BF	c_1	c_2	c_3	c_4	s_1	s_2	s_3	s_4
ST yield	0.2	1.4	0.5	0.5	1.1	1.1	0.7	1.1	0.4
$K_L^0\pi^0$ ST yield	0.1	4.3	4.8	2.8	3.7	0.3	0.3	0.4	0.6
$K^-e^+\nu_e$ ST yield	0.7	0.3	0.1	0.4	0.6	3.3	1.1	3.2	0.9
External strong phases	0.2	5.7	3.7	5.0	3.1	18.4	25.4	36.4	24.5
Finite MC size	0.6	5.9	2.9	2.4	5.5	71.1	20.6	112.1	104.4
Single and double tag fit	0.3	3.1	4.6	3.0	5.2	4.8	4.2	2.5	4.3
K_S^0 veto	0.0	0.2	4.3	1.9	5.8	0.4	6.2	1.7	2.8
Tracking and PID efficiency	4.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Total systematic	4.5	9.8	9.3	7.2	10.8	73.6	33.6	117.9	107.3
Statistical	4.6	130.2	55.5	57.2	131.0	319.6	239.9	239.7	312.4

All systematic uncertainties on c_i and s_i are an order of magnitude smaller than the statistical uncertainties

The exception is "Finite MC size", but this can easily be reduced further

Summary

1. I have presented a complete analysis of c_i and s_i for $D^0 \to KK\pi\pi$
2. I have developed a new fit procedure, inspired by the γ analyses of $D \to K_S hh$ and $D \to K\pi\pi\pi$
3. There is limited sensitivity to $\delta_D^{K\pi}$, but we expect a more competitive measurements when using this method on $K_S\pi\pi$
4. The $D^0 \to KK\pi\pi$ BF, which is nuisance parameter, turns out to be better than the PDG value
5. Systematic uncertainties are an order of magnitude smaller than the statistical uncertainties
6. Analysis note (MEMO) has been written

Next steps

What's next?

1. MEMO review by a charm convener and a charm WG reader
2. Approval talk at weekly Physics & Software meeting
3. Review committee (3 reviewers)
4. Write up paper

Thanks for your attention!