

# Update of $\gamma$ in $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D h^\pm$ with external strong-phase inputs

Martin Tat

University of Oxford

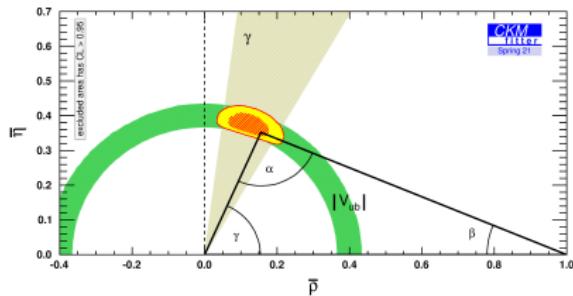
LHCb-UK annual meeting, RAL

8th-10th January 2024



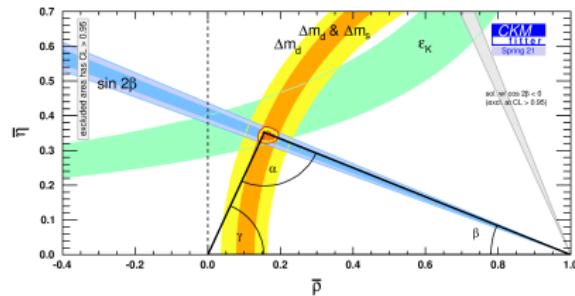
# Introduction to $\gamma$ and $CP$ violation

- CPV in SM is described by the Unitary Triangle, with angles  $\alpha, \beta, \gamma$
- The angle  $\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$  is very important:
  - ① Negligible theoretical uncertainties: Ideal SM benchmark
  - ② Accessible at tree level: Indirectly probe New Physics that enter loops
  - ③ Compare with a global CKM fit: Is the Unitary Triangle a triangle?



(a) Tree level:  $\gamma = (72.1^{+5.4}_{-5.7})^\circ$

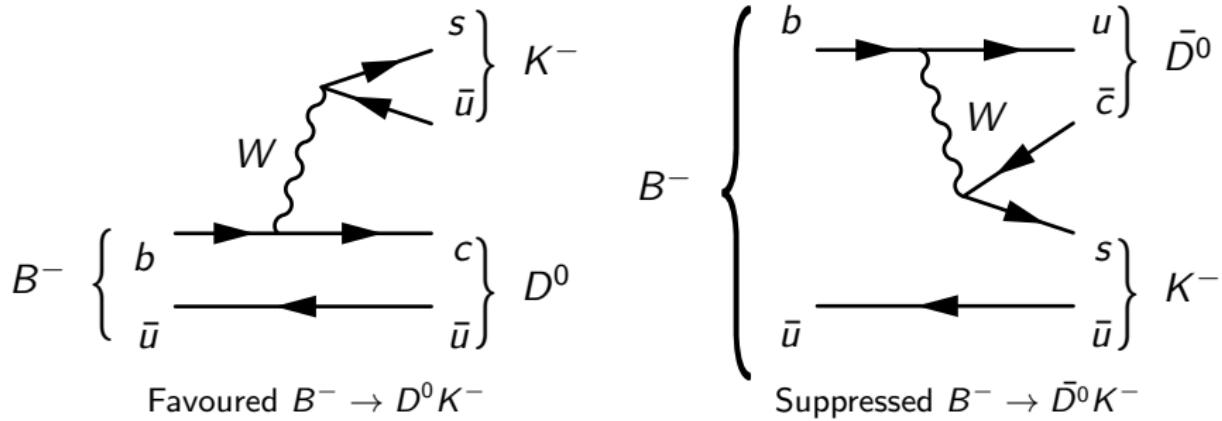
CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005), updated results and plots available at:  
<http://ckmfitter.in2p3.fr>



(b) Loop level:  $\gamma = (65.5^{+1.1}_{-2.7})^\circ$

# Sensitivity through interference

Measure  $\gamma$  through interference effects in  $B^\pm \rightarrow DK^\pm$



- Superposition of  $D^0$  and  $\bar{D}^0$ 
  - Consider  $D^0/\bar{D}^0$  decays to the same final state, such as  $D \rightarrow K^+K^-$
- $b \rightarrow u\bar{c}s$  and  $b \rightarrow c\bar{u}s$  interference  $\rightarrow$  Sensitivity to  $\gamma$

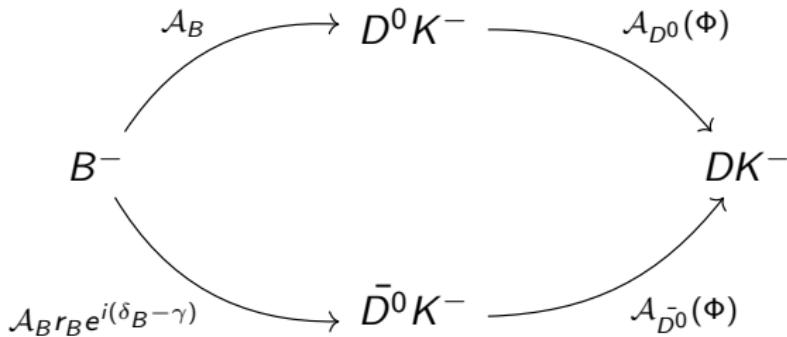
$$\mathcal{A}(B^-) = \mathcal{A}_B \left( \mathcal{A}_{D^0} + r_B e^{i(\delta_B - \gamma)} \mathcal{A}_{\bar{D}^0} \right)$$

$$\mathcal{A}(B^+) = \mathcal{A}_B \left( \mathcal{A}_{\bar{D}^0} + r_B e^{i(\delta_B + \gamma)} \mathcal{A}_{D^0} \right)$$

# Multi-body $D$ decays

This talk: Discuss  $D \rightarrow K^+K^-\pi^+\pi^-$ , where interference effects vary across phase space

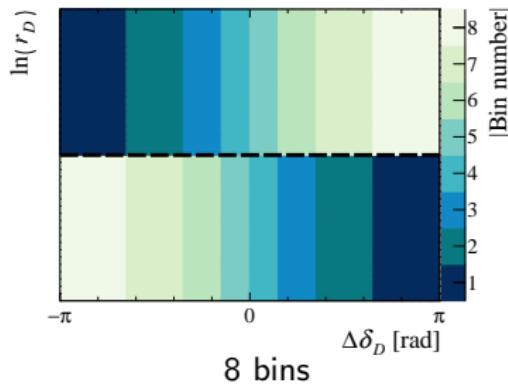
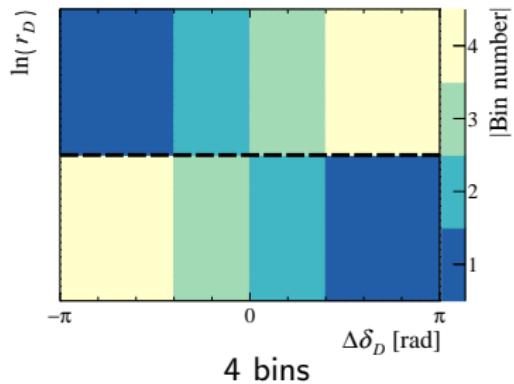
- Strong-phase difference  $\delta_D$  is a function of phase space
- Compare yields of  $B^+$  and  $B^-$  and determine the asymmetry in local phase space regions



$$|\mathcal{A}(B^-)|^2 \propto |\mathcal{A}_{D^0}(\Phi)|^2 + r_B^2 |\mathcal{A}_{\bar{D}^0}(\Phi)|^2 + 2r_B |\mathcal{A}_{D^0}(\Phi)| |\mathcal{A}_{\bar{D}^0}(\Phi)| \cos(\delta_B - \gamma + \delta_D)$$

# Multi-body $D$ decays

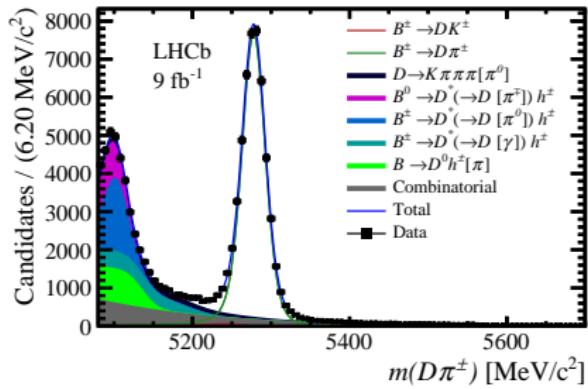
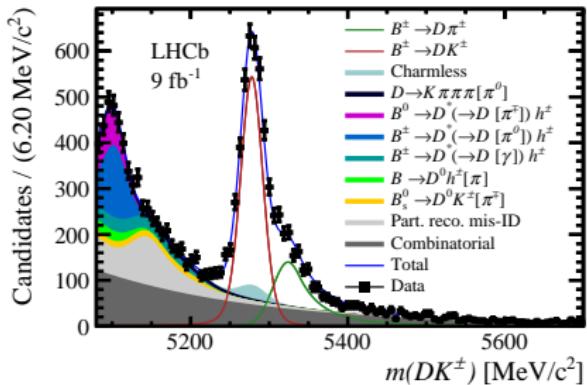
- Interpretation of  $\gamma$  from the multi-body charm decays require external inputs of the charm strong-phase differences
- Measure model-independent strong-phases at a charm factory, such as BESIII, using an optimised binning scheme



Eur. Phys. J. C 83, 547 (2023)

# Phase-space binned $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$

Fully charged final state  $\implies$  Highly suitable for LHCb



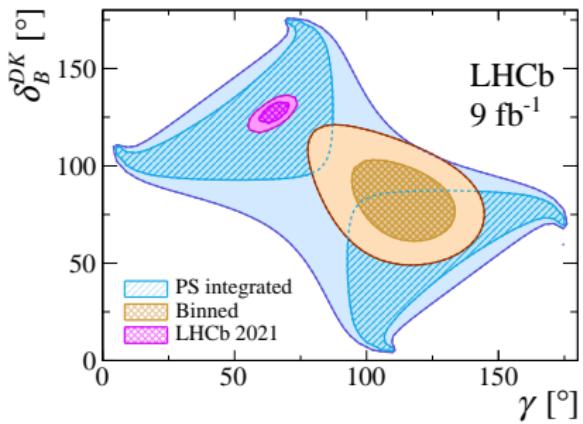
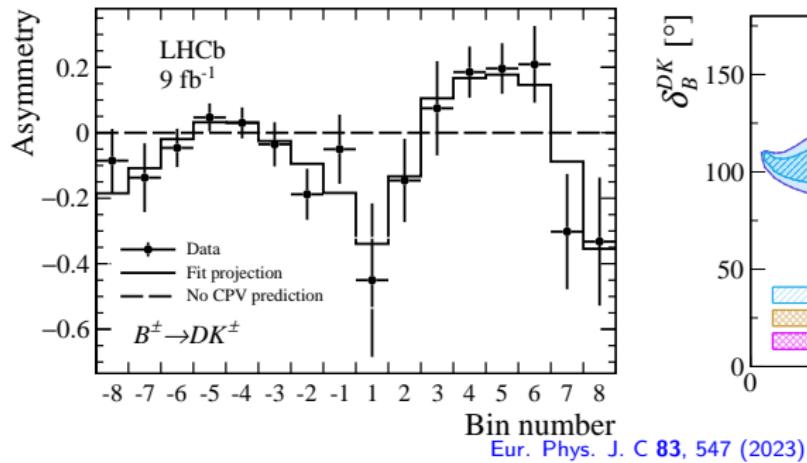
Eur. Phys. J. C 83, 547 (2023)

- $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D h^\pm$  signal yield:
  - $B^\pm \rightarrow DK^\pm$ :  $3026 \pm 38$
  - $B^\pm \rightarrow D\pi^\pm$ :  $44349 \pm 218$

# Phase-space binned $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$

From the phase-space binned asymmetries, we obtain:

$$\gamma = (116^{+12}_{-14})^\circ$$



How will this evolve with model-independent BESIII inputs?

## Reminder of formalism

Key free parameters in the fit:

- $\gamma$  (obviously)
- $r_B, \delta_B$ : Hadronic parameters of  $B^\pm \rightarrow DK^\pm$
- $c_i, s_i$ : Charm strong-phase parameters

$B^\pm \rightarrow DK^\pm$  yield in bin  $i$

$$\hat{N}_{\pm i}^\pm = h_{B^\pm} \left( F_i + r_B^2 F_{+i} + 2r_B \sqrt{F_+ F_-} (\cos(\delta_B \pm \gamma) c_i - \sin(\delta_B \pm \gamma) s_i) \right)$$

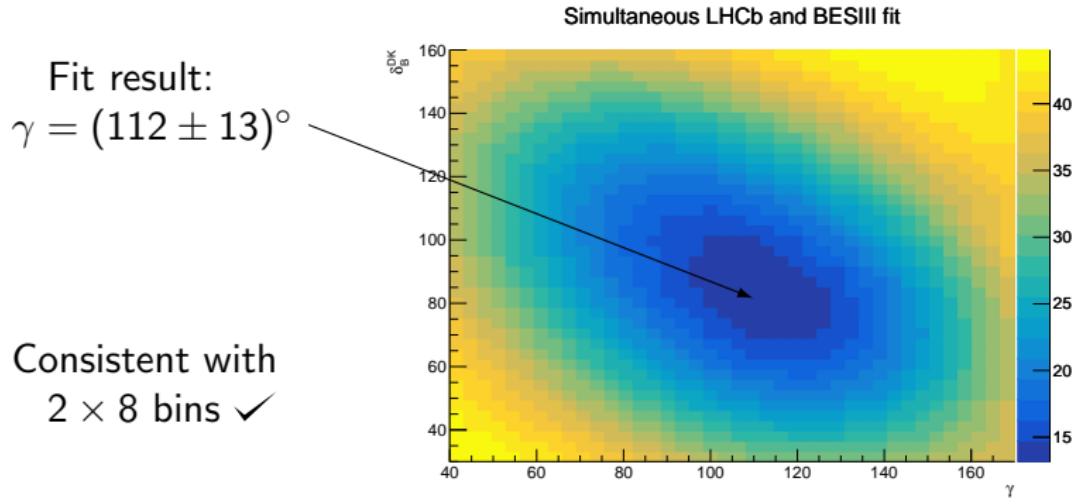
In principle straightforward: Fit  $B^\pm$  yields and extract  $\gamma$

Reduce binning to  $2 \times 4$  bins to accommodate BESIII statistics:  
Current analysis uses  $8 \text{ fb}^{-1}$  at  $\psi(3770)$ , expect  $20 \text{ fb}^{-1}$  in the near future

## Cross check: Model-dependent fit of $\gamma$

Construct log-likelihood function using  $B^\pm \rightarrow DK^\pm$  yields and model-predicted  $c_i$  and  $s_i$ :

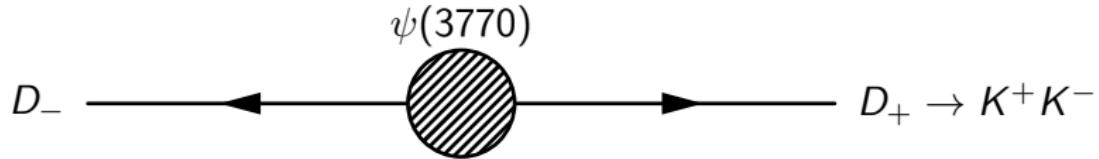
$$\mathcal{L} = \frac{1}{2} \sum_i \left( \frac{N_i - \hat{N}_i}{\sigma_i} \right)^2$$



## Quick digression: Charm factories 101

Consider charm production at threshold:  $e^+e^- \rightarrow \psi(3770) \rightarrow D^0\bar{D}^0$

- $\psi(3770) \rightarrow D^0\bar{D}^0$  decay conserves  $\mathcal{C} = -1$



- If, for example, the tag is CP-even,  $D_+ \rightarrow K^+K^-$ , the other  $D$  meson is forced into an CP-odd state

## Quick digression: Charm factories 101

Consider charm production at threshold:  $e^+e^- \rightarrow \psi(3770) \rightarrow D^0\bar{D}^0$

- CP-odd wave function and decay rate:

$$\mathcal{A}(D_-) = \mathcal{A}(D^0) - \mathcal{A}(\bar{D}^0) \implies$$

$$|\mathcal{A}(D_-)|^2 = |\mathcal{A}(D^0)|^2 + |\mathcal{A}(\bar{D}^0)|^2 - 2|\mathcal{A}(D^0)||\mathcal{A}(\bar{D}^0)| \cos(\delta_D)$$

With CP tags, one can directly access  $\cos(\delta_D)$

# Strong-phase measurement at charm factories

## Quick digression: Charm factories 101

### CP tags

$$N_i \propto K_i + K_{-i} \mp 2\sqrt{K_i K_{-i}} c_i$$

### Self-conjugate multi-body tags

$$N_{ij} \propto K_i K'_{-j} + K_{-i} K'_j - 2\sqrt{K_i K_{-i} K'_j K'_{-j}} (c_i c'_j + s_i s'_j)$$

- ① More than 10 different CP tags  $\implies$  Can measure  $c_i$  precisely
- ② Only  $D \rightarrow K_S^0 h^+ h^-$  tag is sensitive to  $s_i$   $\implies$  Worse  $s_i$  precision

## Simultaneous fit of LHCb and BESIII bin yields

To include the effect of  $c_i$  and  $s_i$  from the BESIII measurement, perform a simultaneous fit:

$$\mathcal{L} = \frac{1}{2} \sum_i \left( \frac{N_i - \hat{N}_i}{\sigma_i} \right)^2 + \mathcal{L}_{BESIII}$$

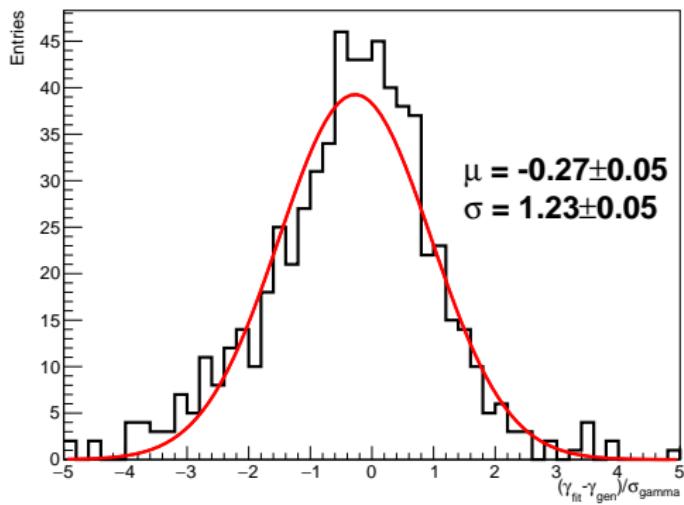
Why not simply assign a systematic uncertainty?

- ① Contribution of  $\gamma$  uncertainty from BESIII could be large, and may move the central value of  $\gamma$
- ② Uncertainties of  $s_i$  are expected to be very non-Gaussian, which could propagate into non-Gaussian uncertainties of  $\gamma$

# Simultaneous fit of LHCb and BESIII bin yields

Run toys using expected BESIII yields and bin yields from  
 $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D h^\pm$  paper:

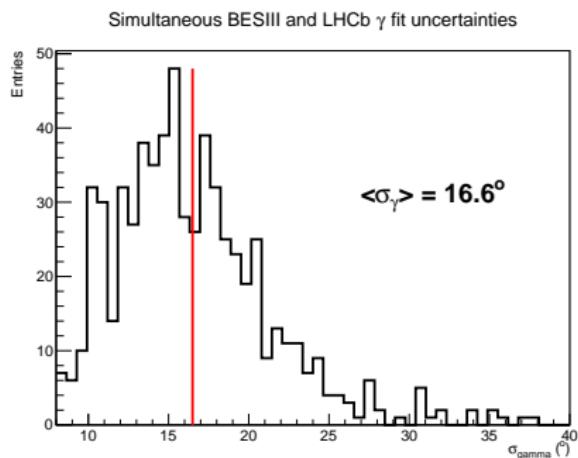
Simultaneous BESIII and LHCb  $\gamma$  fit pulls



Stable fit with minimal bias and small undercoverage

# Simultaneous fit of LHCb and BESIII bin yields

Study expected  $\gamma$  uncertainty, after correcting for coverage:

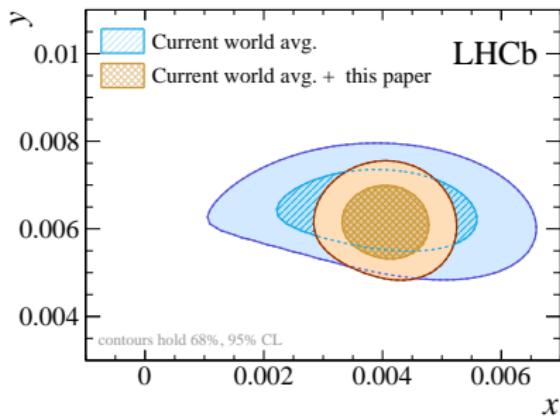
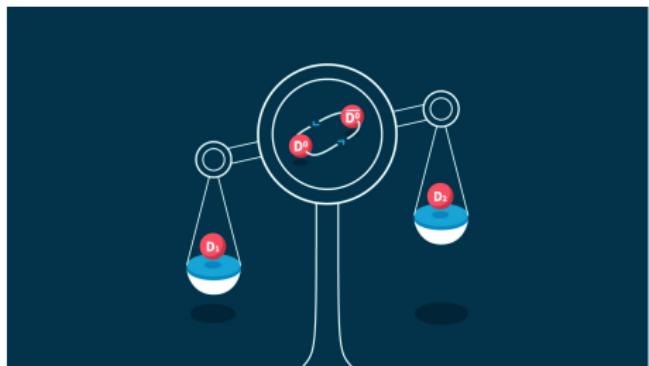


Conclusion from toy studies:

- ① Well behaved fit, with expected sensitivity of  $\sigma(\gamma) = 16.6^\circ$
- ② Only small corrections to bias and coverage required
- ③ Will update  $\gamma$  result once BESIII measurement is released

# Charm mixing studies with multi-body decays

The non-zero mass difference between  $D^0$  and  $\bar{D}^0$  was measured using the multi-body decay  $D \rightarrow K_S^0 \pi^+ \pi^-$



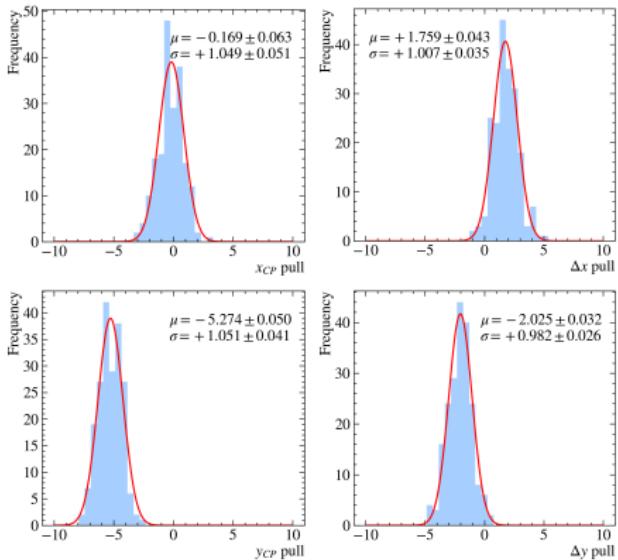
Phys. Rev. Lett 127, 111801 (2021)

Charm strong-phase differences were crucial for this measurement!

# Charm mixing studies with multi-body decays

## Ongoing charm mixing study of $D \rightarrow h^+ h^- \pi^+ \pi^-$ by Jairus Tristan Patoc

- Prompt  $D^{*+} \rightarrow D^0 \pi^+$  sample
- Study  $D^0 \rightarrow h^+ h^- \pi^+ \pi^-$
- Run 1 and 2 ( $9 \text{ fb}^{-1}$ )
- Expected yields are
  - $K^+ K^- \pi^+ \pi^-$ : 4 million
  - $\pi^+ \pi^- \pi^+ \pi^-$ : 12 million
- Current work:
  - ① Sensitivity and bias studies
  - ② Optimised selection



**Fig 3:** Pull distributions in  $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$  from 200 AmpGen toys.

# Charm mixing studies with multi-body decays

Mixing equations depend on  $x$  and  $y$ , but also  $c_i$  and  $s_i$ :

## 1. Charm mixing equations

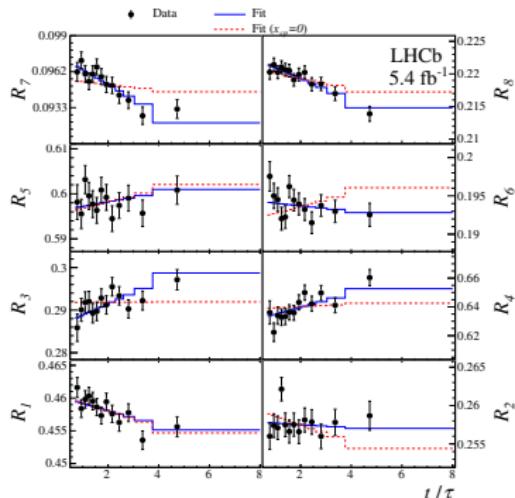
$$N_{D^0}(+i, \langle t \rangle_j) = K_{+i} - \sqrt{K_{+i}K_{-i}}\langle t \rangle_j(yc_i + xs_i)$$
$$N_{D^0}(-i, \langle t \rangle_j) = K_{-i} - \sqrt{K_{+i}K_{-i}}\langle t \rangle_j(yc_i - xs_i)$$

① Fit the mixing equations

② Fit the ratio of mixing equations

## 2. Charm mixing ratio (bin-flip)

$$R_i = \frac{K_{+i} - \sqrt{K_{+i}K_{-i}}\langle t \rangle_j(yc_i + xs_i)}{K_{-i} - \sqrt{K_{+i}K_{-i}}\langle t \rangle_j(yc_i - xs_i)}$$



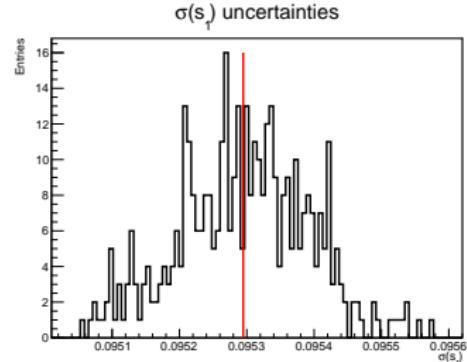
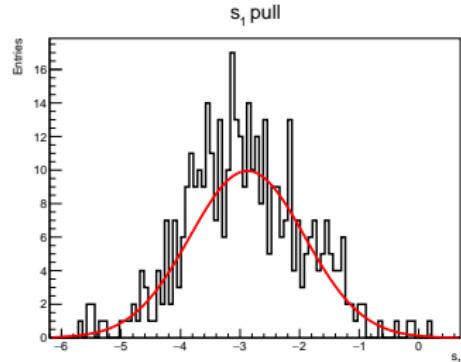
# Charm mixing studies with multi-body decays

Alternative strategy: Fix  $x$  and  $y$ , and measure  $c_i$  and  $s_i$

## 1. Charm mixing equations

$$N_{D^0}(+i, \langle t \rangle_j) = K_{+i} - \sqrt{K_{+i}K_{-i}} \langle t \rangle_j (yc_i + xs_i)$$
$$N_{D^0}(-i, \langle t \rangle_j) = K_{-i} - \sqrt{K_{+i}K_{-i}} \langle t \rangle_j (yc_i - xs_i)$$

- Two independent equations per bin, two observables per bin
- Similar statistical sensitivity to  $c_i$  and  $s_i$ , in contrast to BESIII



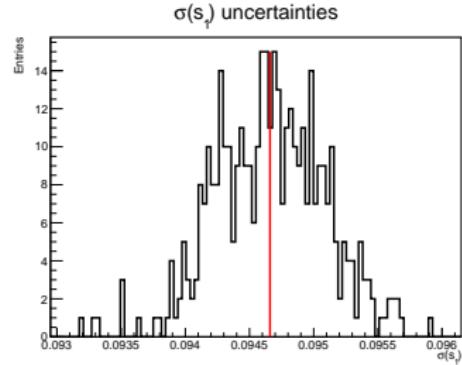
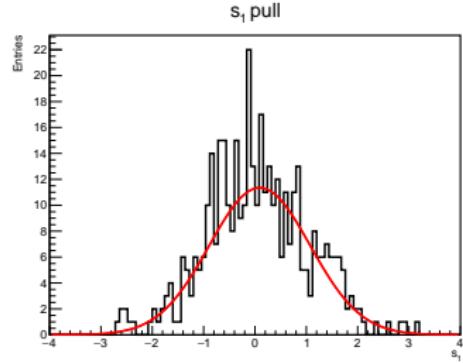
# Charm mixing studies with multi-body decays

Can also use bin-flip method to fit  $s_i$ , but  $c_i$  must be fixed

## 2. Charm mixing ratio (bin-flip)

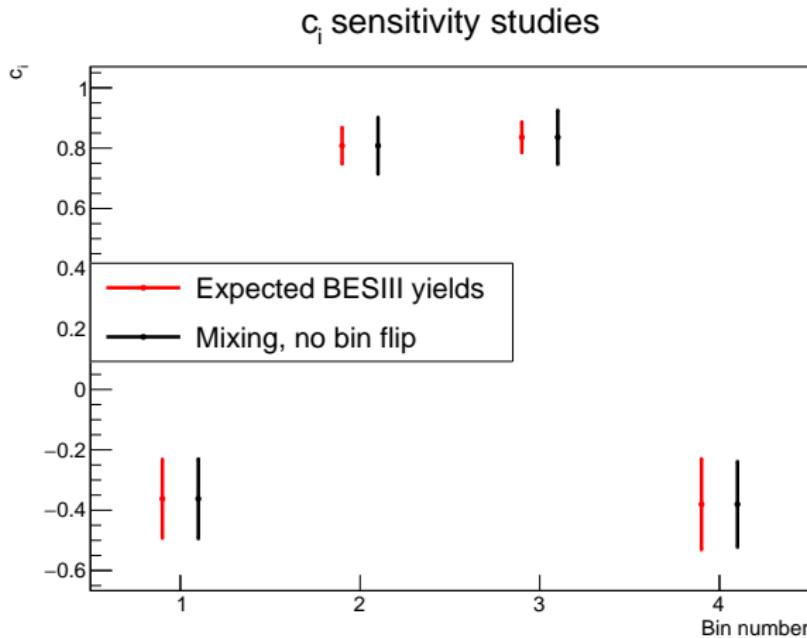
$$R_i = \frac{K_{+i} - \sqrt{K_{+i} K_{-i}} \langle t \rangle_j (yc_i + xs_i)}{K_{-i} - \sqrt{K_{+i} K_{-i}} \langle t \rangle_j (yc_i - xs_i)}$$

- Only one independent equation per bin
- Sensitivity found to be similar to fitting mixing equations directly



# Charm mixing studies with multi-body decays

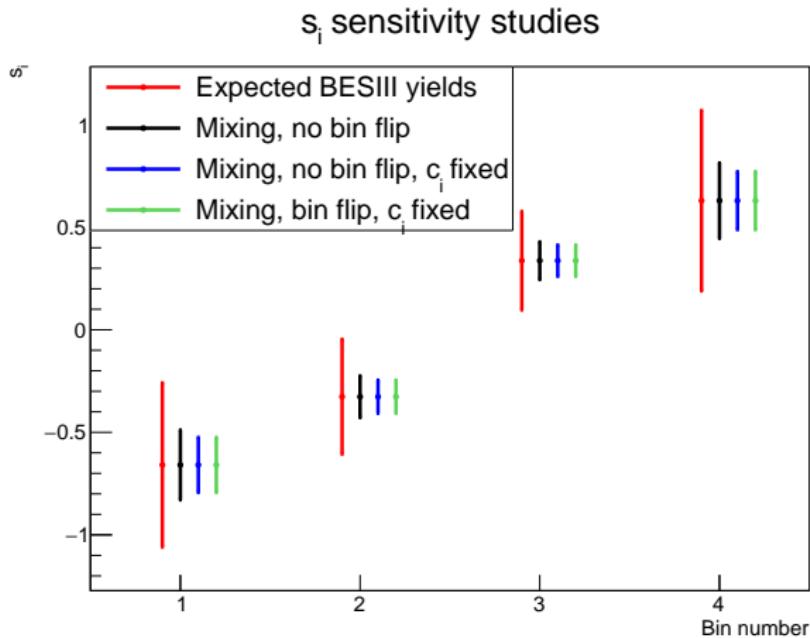
Sensitivity to  $c_i$ : Similar between BESIII and charm mixing at LHCb



- BESIII yields equivalent to  $8 \text{ fb}^{-1}$  of  $\psi(3770)$
- 4 million  $D \rightarrow K^+K^-\pi^+\pi^-$  candidates in mixing analysis

# Charm mixing studies with multi-body decays

Sensitivity to  $s_i$ : Significant improvements expected!



- BESIII yields equivalent to  $8 \text{ fb}^{-1}$  of  $\psi(3770)$
- 4 million  $D \rightarrow K^+K^-\pi^+\pi^-$  candidates in mixing analysis

# Summary and future prospects

## Summary:

- ① Measurement of  $\gamma$  in  $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D h^\pm$  is ready to be combined with **model-independent** strong-phase inputs
- ② BESIII strong-phase inputs can be further constrained using charm-mixing measurements at LHCb, and provide comparable sensitivity to  $s_i$

# Summary and future prospects

## Future prospects:

- ① Measurement is still statistically limited, and will be significantly improved with LHCb Upgrade I
- ② Additional BESIII data and charm-mixing measurements from LHCb will bring strong-phase systematics down further
- ③ Can extend this strategy to many more four-body modes
  - Studies of  $D \rightarrow \pi^+ \pi^- \pi^+ \pi^-$  show similar results

Thanks for your attention!