

# Model independent measurement of the CKM angle $\gamma$ with $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D h^\pm$ at LHCb and BESIII

Martin Tat

University of Oxford

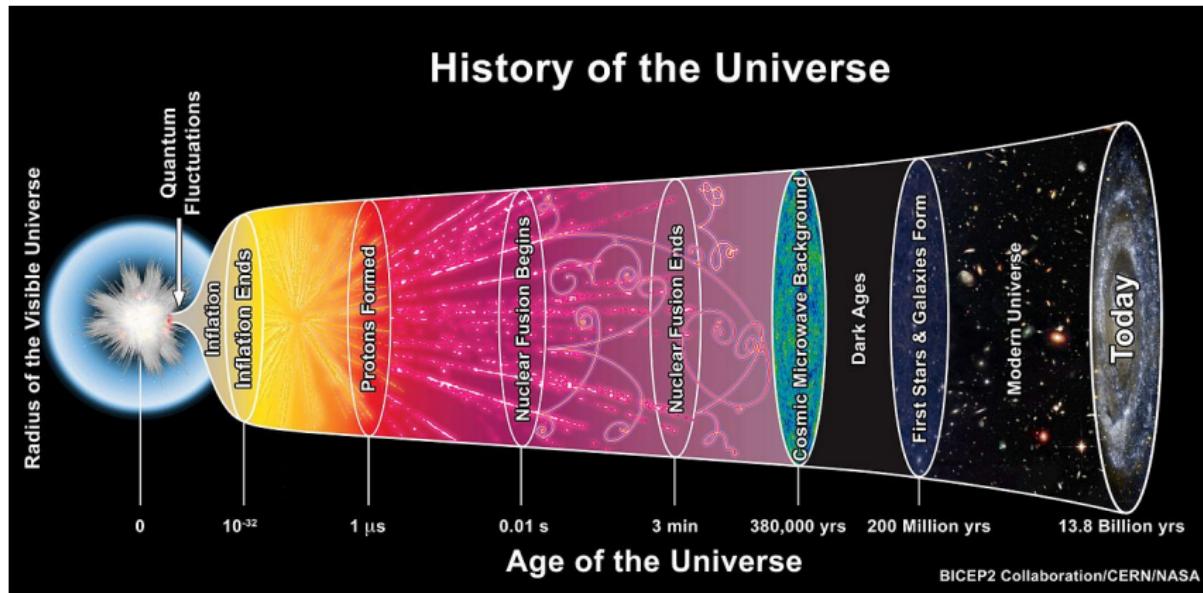
Warwick EPP Seminar

9th February 2023



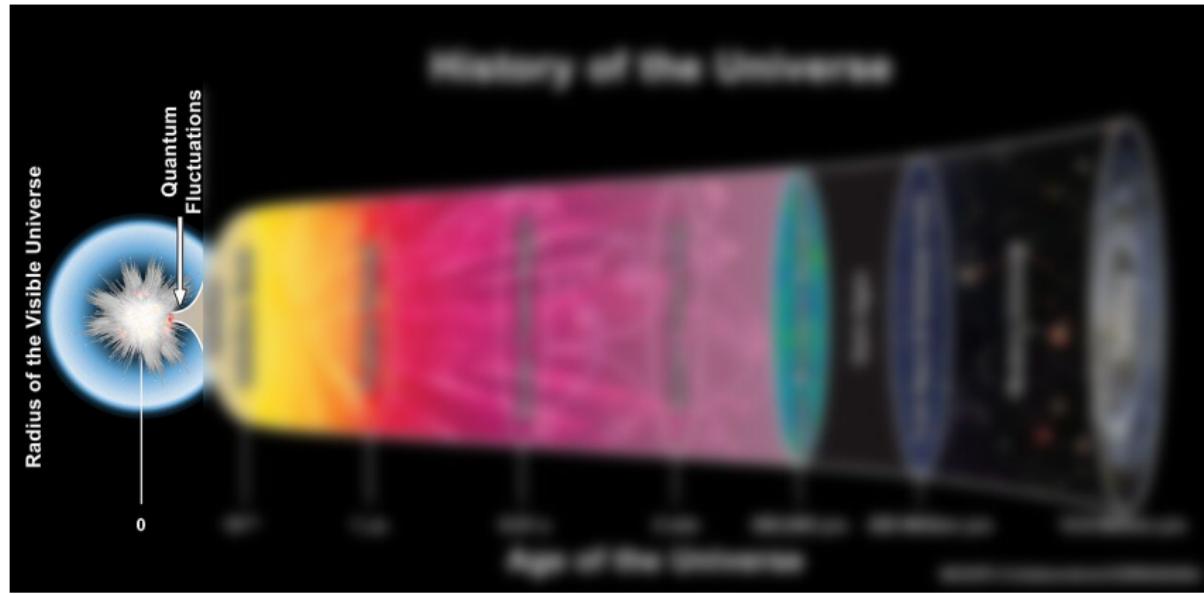
# Introduction to $CP$ violation

# Big Bang and matter-antimatter asymmetry



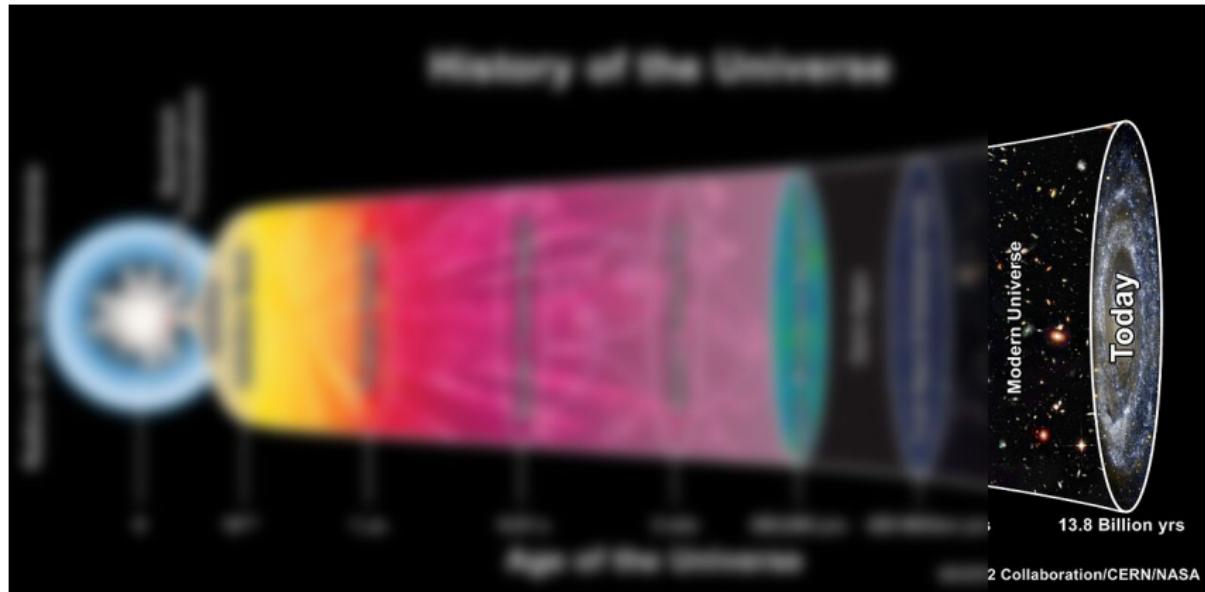
Where is the antimatter in the universe?

# Big Bang and matter-antimatter asymmetry



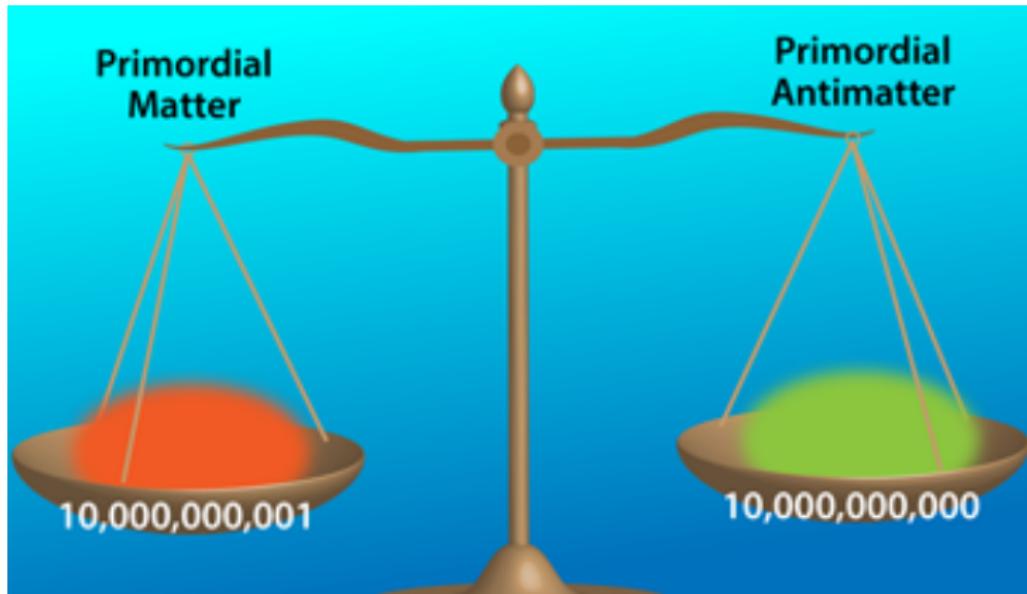
Initially equal amounts of matter and antimatter...

# Big Bang and matter-antimatter asymmetry



... but today we only see matter!

# Big Bang and matter-antimatter asymmetry



APS/Alan Stonebraker

The difference is very small...

# Big Bang and matter-antimatter asymmetry



Quantum Diaries: "Why B physics? Why not A Physics?"

... but the effects we observe today are obviously huge!  
How can we explain this?

# $CP$ violation

## The Nobel Prize in Physics 1980



Photo from the Nobel Foundation archive.  
James Watson Cronin  
Prize share: 1/2

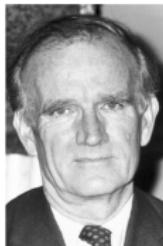


Photo from the Nobel Foundation archive.  
Val Logsdon Fitch  
Prize share: 1/2

The Nobel Prize in Physics 1980 was awarded jointly to James Watson Cronin and Val Logsdon Fitch "for the discovery of violations of fundamental symmetry principles in the decay of neutral K-mesons"

- $CP$  violation discovery in 1964
- Phys. Rev. Lett. **13**, 138
- Observed  $K_L^0 \rightarrow \pi^+ \pi^-$
- Since,  $CP$  violation has also been observed in the  $B$ ,  $B_s$  and  $D$  systems

Can Standard Model CPV explain the matter-antimatter asymmetry?  
Or, could it be physics beyond the SM?

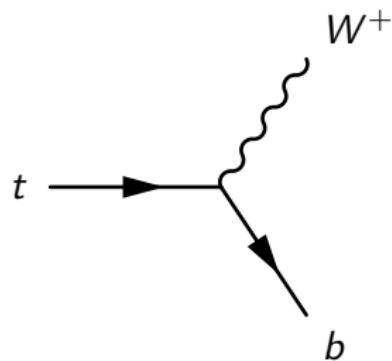
# The CKM matrix and the Unitary Triangle

## The CKM matrix and the Unitary Triangle

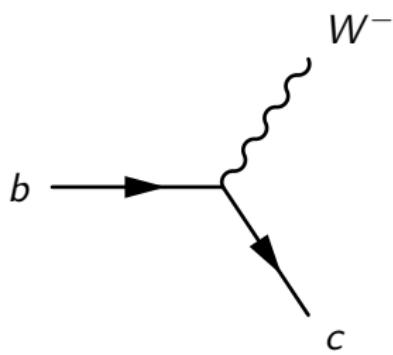
# The CKM matrix and the Unitary Triangle

In SM, the charged current  $W^\pm$  interactions couple (left-handed) up- and down-type quarks, given by

$$\frac{-g}{\sqrt{2}} \begin{bmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{bmatrix} \gamma^\mu W_\mu V_{CKM} \begin{bmatrix} d_L \\ s_L \\ b_L \end{bmatrix} + \text{h.c.}$$



(a)  $t \rightarrow bW^+$



(b)  $b \rightarrow cW^-$

# The CKM matrix and the Unitary Triangle

The Cabibbo-Kobayashi-Maskawa matrix  $V_{\text{CKM}}$ ,

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

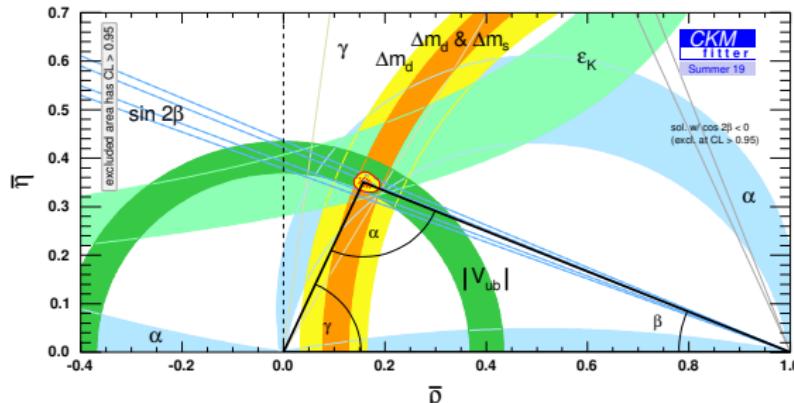
must be a unitary matrix:  $V_{\text{CKM}}^\dagger V_{\text{CKM}} = I \implies$

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

Represent this constraint as a triangle in the complex plane:  
Unitary Triangle

# The CKM matrix and the Unitary Triangle

- CPV in SM is described by the Unitary Triangle, with angles  $\alpha, \beta, \gamma$
- The angle  $\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$  is very important:
  - ① Negligible theoretical uncertainties: Ideal SM benchmark
  - ② Accessible at tree level: Indirectly probe New Physics that enter loops
  - ③ Compare with  $\alpha, \beta$  measurements: Is the Unitary Triangle a triangle?



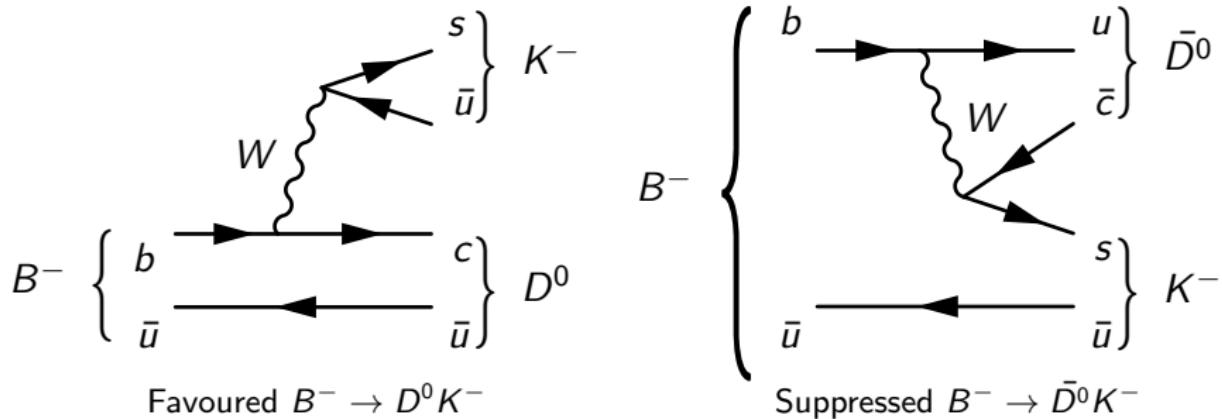
CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005)

How to measure  $\gamma$ ?

How to measure  $\gamma$ ?

# Sensitivity through interference

Measure  $\gamma$  through interference effects in  $B^\pm \rightarrow DK^\pm$



- Superposition of  $D^0$  and  $\bar{D}^0$

- $b \rightarrow u \bar{c} s$  and  $b \rightarrow c \bar{u} s$  interference  $\rightarrow$  Sensitivity to  $\gamma$

$$\mathcal{A}(B^-) = \mathcal{A}_B \left( \mathcal{A}_{D^0} + r_B e^{i(\delta_B - \gamma)} \mathcal{A}_{\bar{D}^0} \right)$$

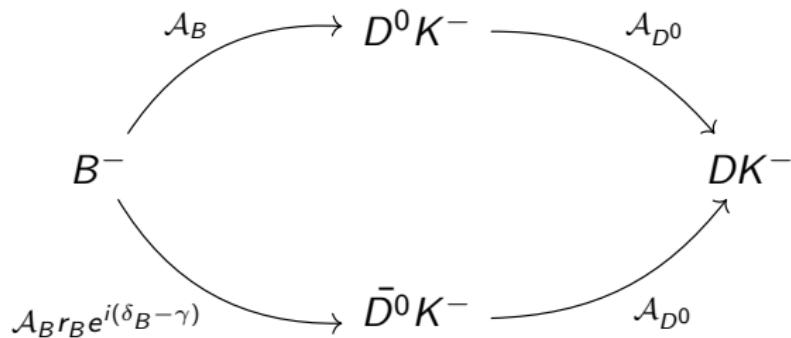
$$\mathcal{A}(B^+) = \mathcal{A}_B \left( \mathcal{A}_{\bar{D}^0} + r_B e^{i(\delta_B + \gamma)} \mathcal{A}_{D^0} \right)$$

- The magnitude of interference effects governed by  $r_B \approx 0.1$

## $D$ decays to a $CP$ eigenstate

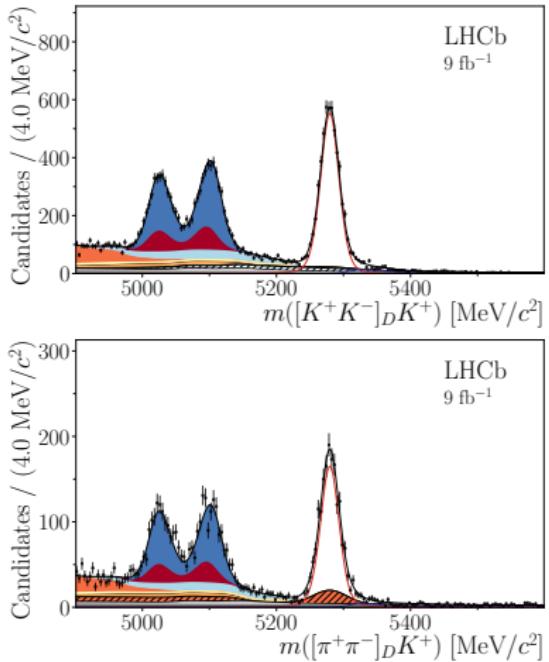
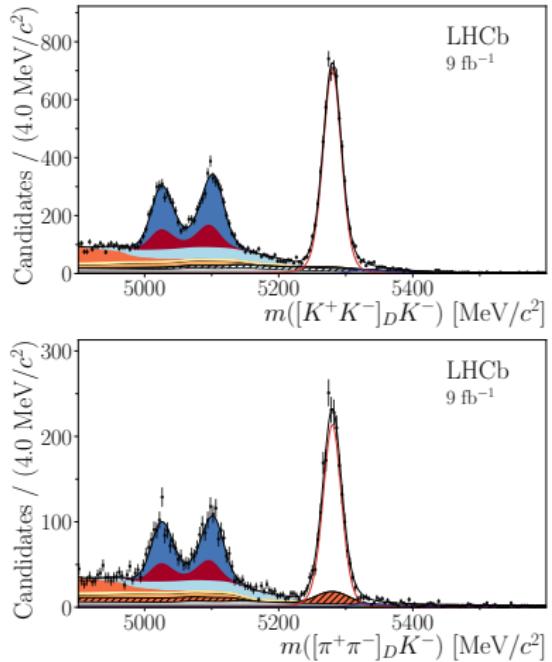
A well known strategy is to consider  $D$  decays to a  $CP$  eigenstate

For  $CP$  eigenstates,  $\mathcal{A}_{D^0} = \mathcal{A}_{\bar{D}^0}$



$$|\mathcal{A}(B^-)|^2 \propto 1 + r_B^2 + 2r_B \cos(\delta_B - \gamma)$$

# $D$ decays to a $CP$ eigenstate



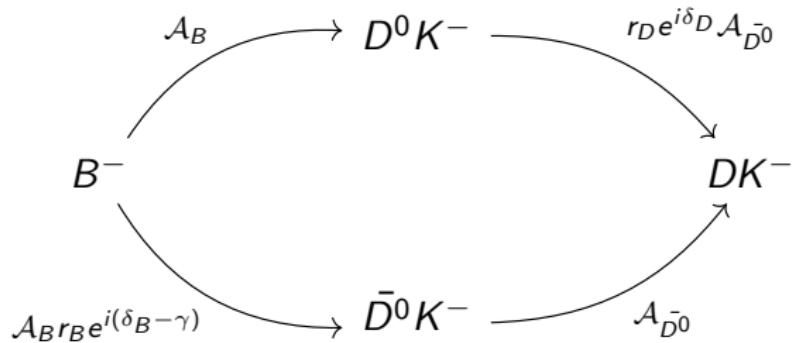
JHEP 04 (2021) 081

In  $B^\pm \rightarrow [h^+ h^-]_D K^\pm$ , we see significant CPV effects

# Doubly Suppressed Cabibbo $D$ decays

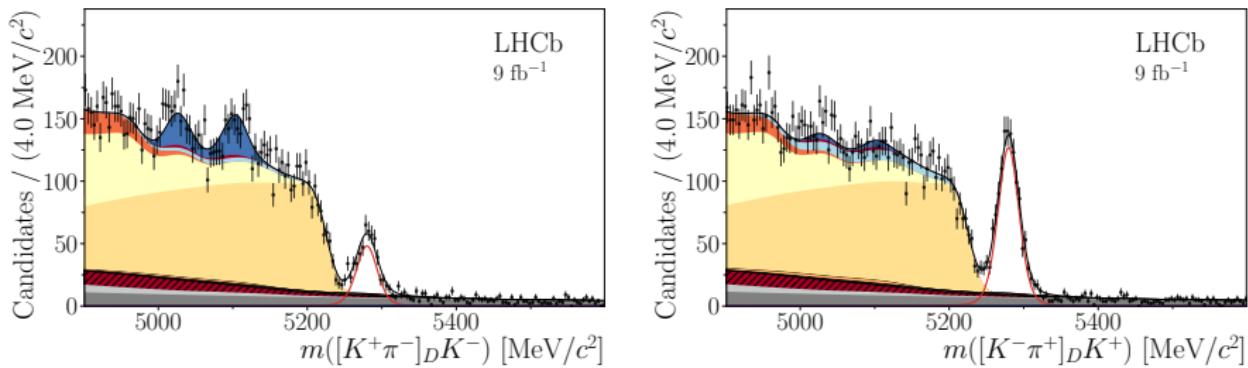
Can we enhance the interference effects?

Yes! Use a Doubly Suppressed Cabibbo decay:  $\mathcal{A}_{D^0} = r_D e^{i\delta_D} \mathcal{A}_{\bar{D}^0}$



$$|\mathcal{A}(B^-)|^2 \propto r_D^2 + r_B^2 + 2r_B r_D \cos(\delta_B - \gamma + \delta_D)$$

# Doubly Suppressed Cabibbo $D$ decays



JHEP 04 (2021) 081

$B^\pm \rightarrow [K^\mp\pi^\pm]_D K^\pm$  has lower statistics, but a spectacular asymmetry!

Additionally, the partially reconstructed background has an equal but opposite asymmetry

The  $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$  decay mode

The  $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$  decay mode

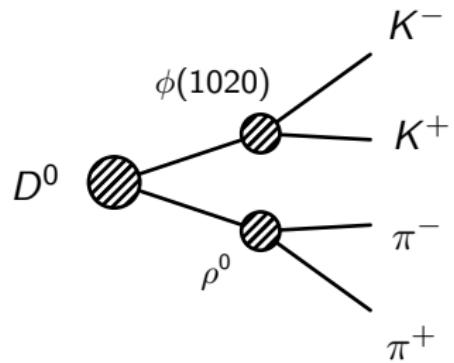
# The $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$ decay mode

The mode  $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$  has been proposed as a powerful channel for a measurement of  $\gamma$

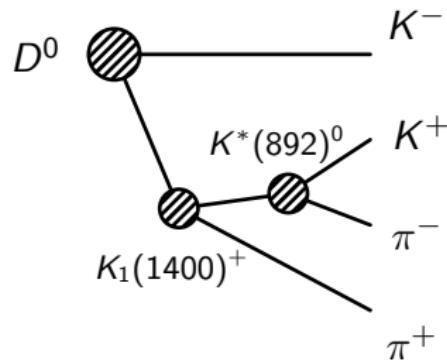
- $D \rightarrow K^+ K^- \pi^+ \pi^-$  has the best of both worlds:
  - ① Singly Cabibbo Suppressed decay: Larger branching fraction
  - ② Interference effects from over 25 resonance components
- Large interference effects in local regions of the 5D phase space
- First proposed by J. Rademacker and G. Wilkinson
  - Phys. Lett. **B647** (2007) 400
  - FOCUS amplitude model predicts a  $14^\circ$  precision with 1000 candidates
- State of the art amplitude analysis by LHCb:
  - JHEP **02** (2019) 126
  - Exploits the huge dataset of charm decays collected by LHCb

# The $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$ decay mode

Why do four-body decays have large local interferences?



$$\phi(1020)(\rightarrow K^+ K^-) \rho^0(\rightarrow \pi^+ \pi^-)$$



$$K_1(1400)^+(\rightarrow K^*(892)^0(\rightarrow K^+ \pi^-)) K^-$$

Many possible decay paths, in different phase space locations,  
contribute to the total decay amplitude...

# The $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$ decay mode

Amplitude	$ c_k $	$\arg(c_k)$ [rad]	Fit fraction [%]
$D^0 \rightarrow [\phi(1020)(\rho - \omega)^0]_{L=0}$	1 (fixed)	0 (fixed)	$23.82 \pm 0.38 \pm 0.50$
$D^0 \rightarrow K_1(1400)^+ K^-$	$0.614 \pm 0.011 \pm 0.031$	$1.05 \pm 0.02 \pm 0.05$	$19.08 \pm 0.60 \pm 1.46$
$D^0 \rightarrow [K^- \pi^+]_{L=0} [K^+ \pi^-]_{L=0}$	$0.282 \pm 0.004 \pm 0.008$	$-0.60 \pm 0.02 \pm 0.10$	$18.46 \pm 0.35 \pm 0.94$
$D^0 \rightarrow K_1(1270)^+ K^-$	$0.452 \pm 0.011 \pm 0.017$	$2.02 \pm 0.03 \pm 0.05$	$18.05 \pm 0.52 \pm 0.98$
$D^0 \rightarrow [K^*(892)^0 \bar{K}^*(892)^0]_{L=0}$	$0.259 \pm 0.004 \pm 0.018$	$-0.27 \pm 0.02 \pm 0.03$	$9.18 \pm 0.21 \pm 0.28$
$D^0 \rightarrow K^*(1680)^0 [K^- \pi^+]_{L=0}$	$2.359 \pm 0.036 \pm 0.624$	$0.44 \pm 0.02 \pm 0.03$	$6.61 \pm 0.15 \pm 0.37$
$D^0 \rightarrow [K^*(892)^0 \bar{K}^*(892)^0]_{L=1}$	$0.249 \pm 0.005 \pm 0.017$	$1.22 \pm 0.02 \pm 0.03$	$4.90 \pm 0.16 \pm 0.18$
$D^0 \rightarrow K_1(1270)^- K^+$	$0.220 \pm 0.006 \pm 0.011$	$2.09 \pm 0.03 \pm 0.07$	$4.29 \pm 0.18 \pm 0.41$
$D^0 \rightarrow [K^+ K^-]_{L=0} [\pi^+ \pi^-]_{L=0}$	$0.120 \pm 0.003 \pm 0.018$	$-2.49 \pm 0.03 \pm 0.16$	$3.14 \pm 0.17 \pm 0.72$
$D^0 \rightarrow K_1(1400)^- K^+$	$0.236 \pm 0.008 \pm 0.018$	$0.04 \pm 0.04 \pm 0.09$	$2.82 \pm 0.19 \pm 0.39$
$D^0 \rightarrow [K^*(1680)^0 \bar{K}^*(892)^0]_{L=0}$	$0.823 \pm 0.023 \pm 0.218$	$2.99 \pm 0.03 \pm 0.05$	$2.75 \pm 0.15 \pm 0.19$
$D^0 \rightarrow [\bar{K}^*(1680)^0 K^*(892)^0]_{L=1}$	$1.009 \pm 0.022 \pm 0.276$	$-2.76 \pm 0.02 \pm 0.03$	$2.70 \pm 0.11 \pm 0.09$
$D^0 \rightarrow \bar{K}^*(1680)^0 [K^+ \pi^-]_{L=0}$	$1.379 \pm 0.029 \pm 0.373$	$1.06 \pm 0.02 \pm 0.03$	$2.41 \pm 0.09 \pm 0.27$
$D^0 \rightarrow [\phi(1020)(\rho - \omega)^0]_{L=2}$	$1.311 \pm 0.031 \pm 0.018$	$0.54 \pm 0.02 \pm 0.02$	$2.29 \pm 0.08 \pm 0.08$
$D^0 \rightarrow [K^*(892)^0 \bar{K}^*(892)^0]_{L=2}$	$0.652 \pm 0.018 \pm 0.043$	$2.85 \pm 0.03 \pm 0.04$	$1.85 \pm 0.09 \pm 0.10$
$D^0 \rightarrow \phi(1020)[\pi^+ \pi^-]_{L=0}$	$0.049 \pm 0.001 \pm 0.004$	$-1.71 \pm 0.04 \pm 0.37$	$1.49 \pm 0.09 \pm 0.33$
$D^0 \rightarrow [K^*(1680)^0 \bar{K}^*(892)^0]_{L=1}$	$0.747 \pm 0.021 \pm 0.203$	$0.14 \pm 0.03 \pm 0.04$	$1.48 \pm 0.08 \pm 0.10$
$D^0 \rightarrow [\phi(1020)\rho(1450)^0]_{L=1}$	$0.762 \pm 0.035 \pm 0.068$	$1.17 \pm 0.04 \pm 0.04$	$0.98 \pm 0.09 \pm 0.05$
$D^0 \rightarrow a_0(980)^0 f_2(1270)^0$	$1.524 \pm 0.058 \pm 0.189$	$0.21 \pm 0.04 \pm 0.19$	$0.70 \pm 0.05 \pm 0.08$
$D^0 \rightarrow a_1(1260)^+ \pi^-$	$0.189 \pm 0.011 \pm 0.042$	$-2.84 \pm 0.07 \pm 0.38$	$0.46 \pm 0.05 \pm 0.22$
$D^0 \rightarrow a_1(1260)^- \pi^+$	$0.188 \pm 0.014 \pm 0.031$	$0.18 \pm 0.06 \pm 0.43$	$0.45 \pm 0.06 \pm 0.16$
$D^0 \rightarrow [\phi(1020)(\rho - \omega)^0]_{L=1}$	$0.160 \pm 0.011 \pm 0.005$	$0.28 \pm 0.07 \pm 0.03$	$0.43 \pm 0.05 \pm 0.03$
$D^0 \rightarrow [K^*(1680)^0 \bar{K}^*(892)^0]_{L=2}$	$1.218 \pm 0.089 \pm 0.354$	$-2.44 \pm 0.08 \pm 0.15$	$0.33 \pm 0.05 \pm 0.06$
$D^0 \rightarrow [K^+ K^-]_{L=0} (\rho - \omega)^0$	$0.195 \pm 0.015 \pm 0.035$	$2.95 \pm 0.08 \pm 0.29$	$0.27 \pm 0.04 \pm 0.05$
$D^0 \rightarrow [\phi(1020)f_2(1270)^0]_{L=1}$	$1.388 \pm 0.095 \pm 0.257$	$1.71 \pm 0.06 \pm 0.37$	$0.18 \pm 0.02 \pm 0.07$
$D^0 \rightarrow [K^*(892)^0 \bar{K}_2^*(1430)^0]_{L=1}$	$1.530 \pm 0.086 \pm 0.131$	$2.01 \pm 0.07 \pm 0.09$	$0.18 \pm 0.02 \pm 0.02$
Sum of fit fractions		$129.32 \pm 1.09 \pm 2.38$	
$\chi^2/\text{ndf}$		$9242/8121 = 1.14$	

JHEP 02 (2019) 126

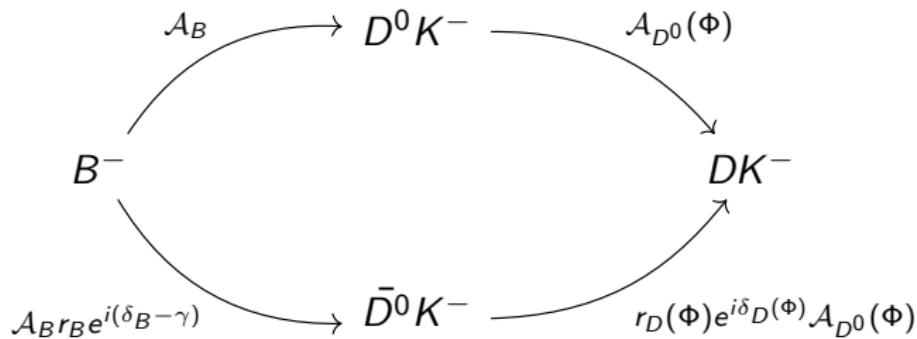
... and I really mean a lot of resonances!

# The $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$ decay mode

Our equations suddenly become a lot more complicated

$\mathcal{A}_{D^0}(\Phi)$  now depends on a 5D phase space point  $\Phi$

Defining  $\mathcal{A}_{\bar{D}^0} = r_D e^{i\delta_D} \mathcal{A}_{D^0}$ ,  $r_D$  and  $\delta_D$  are now also functions of  $\Phi$ !



$$|\mathcal{A}(B^-)|^2 \propto 1 + r_B^2 r_D^2(\Phi) + 2r_B r_D(\Phi) \cos(\delta_B - \gamma + \delta_D(\Phi))$$

## The $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$ decay mode

$r_D(\Phi)$  and  $\delta_D(\Phi)$  can be predicted using the LHCb amplitude model

However, there are many reasons why we should **not** do this:

- ①  $r_D(\Phi)$  can be measured directly in data at LHCb
- ② Amplitude models are just models, which may not reflect reality
- ③ In fact, the model is fitted to data that knows nothing about  $\delta_D(\Phi)$
- ④ It is impossible to assign an objective error to a model!

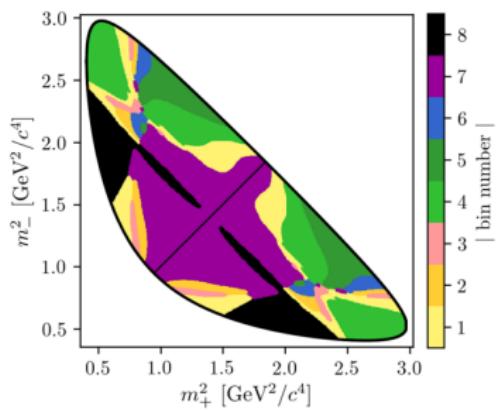
We wish to do a model independent measurement

Binned analysis of the  $D \rightarrow K^+K^-\pi^+\pi^-$  mode

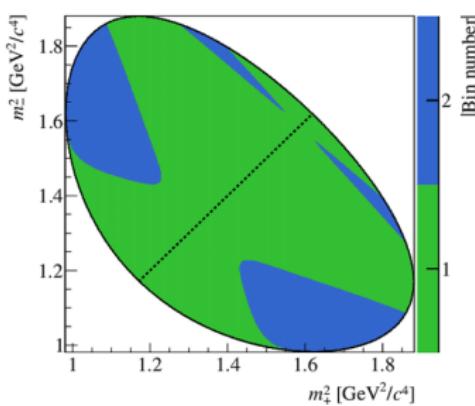
Binned analysis of the  $D \rightarrow K^+K^-\pi^+\pi^-$   
mode

# Binned analysis of the $D \rightarrow K^+K^-\pi^+\pi^-$ mode

- Solution: Split phase space into bins, labelled by  $i = 1, 2, \dots$
- Study the CP asymmetry separately in each bin
- For the decays  $D^0 \rightarrow K_S^0\pi^+\pi^-$  and  $K_S^0K^+K^-$ , the binning scheme may be visualised on a Dalitz plot

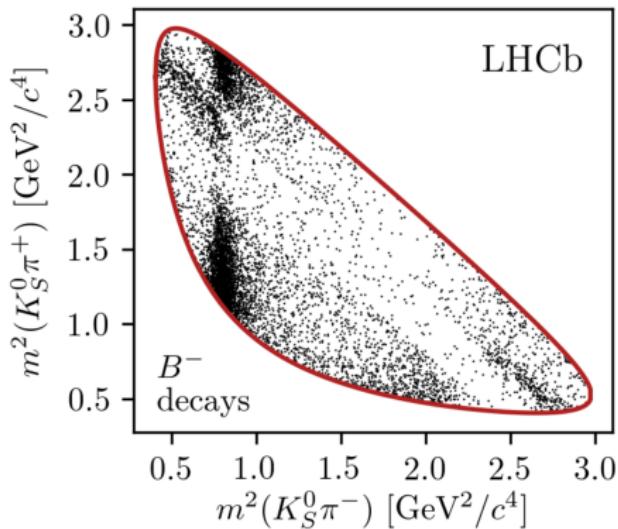


$K_S^0\pi^+\pi^-$

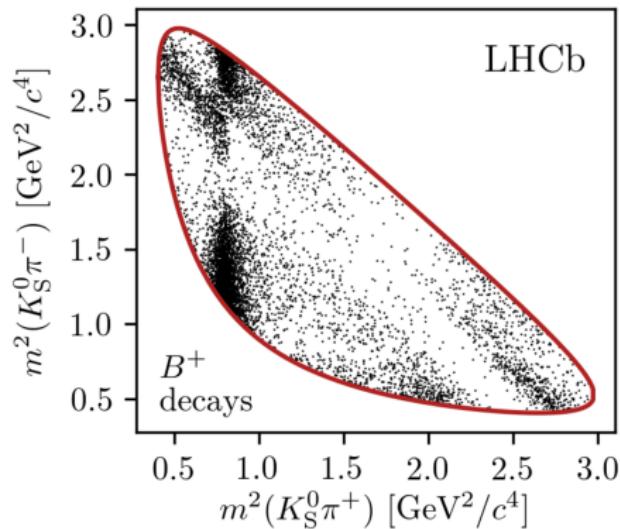


$K_S^0K^+K^-$

# Binned analysis of the $D \rightarrow K^+K^-\pi^+\pi^-$ mode



$$B^- \rightarrow [K_S^0\pi^+\pi^-]_D K^-$$



$$B^+ \rightarrow [K_S^0\pi^+\pi^-]_D K^+$$

Can you find the asymmetries?

# Binned analysis of the $D \rightarrow K^+K^-\pi^+\pi^-$ mode

Back to rate equation:

$$|\mathcal{A}(B^-)|^2 \propto 1 + r_B^2 r_D^2 + 2r_B r_D (\cos(\delta_B - \gamma) \cos(\delta_D) - \sin(\delta_B - \gamma) \sin(\delta_D))$$

Integrate rate over a local region  $\Phi_i$ , which we call bin  $i$ :

$$N_i^- \propto F_i + r_B^2 \bar{F}_i + 2r_B \sqrt{F_i \bar{F}_i} (\cos(\delta_B - \gamma) c_i - \sin(\delta_B - \gamma) s_i)$$

Amplitude averaged strong phase

$$c_i \equiv \frac{\int_i d\Phi |\mathcal{A}_{D^0}| |\mathcal{A}_{\bar{D}^0}| \cos(\delta_D(\Phi))}{\sqrt{\int d\Phi |\mathcal{A}_{D^0}|^2 \int d\Phi |\mathcal{A}_{\bar{D}^0}|^2}}$$

# Binned analysis of the $D \rightarrow K^+K^-\pi^+\pi^-$ mode

To “decouple” the interference effects in  $B^+$  and  $B^-$ ,  
define the  $CP$  violating observables

$$x_{\pm} \equiv r_B \cos(\delta_B \pm \gamma), \quad y_{\pm} \equiv r_B \sin(\delta_B \pm \gamma)$$

Our final equation, which relates the  $CP$  observables to  
experimentally measured yields, is

$$N_i^- \propto F_i + r_B^2 \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_- c_i - y_- s_i)$$

Amplitude averaged strong phase

$$c_i \equiv \frac{\int_i d\Phi |\mathcal{A}_{D^0}| |\mathcal{A}_{\bar{D}^0}| \cos(\delta_D(\Phi))}{\sqrt{\int d\Phi |\mathcal{A}_{D^0}|^2 \int d\Phi |\mathcal{A}_{\bar{D}^0}|^2}}$$

# Binned analysis of the $D \rightarrow K^+K^-\pi^+\pi^-$ mode

Bin yield

$$N_i^- \propto F_i + r_B^2 \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_- c_i - y_- s_i)$$

The strategy for measuring  $\gamma$  is now clear:

- ① Measure bin yields  $N_i^\pm$  in  $B^\pm \rightarrow [K^+K^-\pi^+\pi^-]_D K^\pm$  decays
- ② Do a likelihood maximisation to determine  $F_i$ ,  $\bar{F}_i$ ,  $c_i$ ,  $s_i$ ,  $x_\pm$  and  $y_\pm$
- ③ From  $x_\pm$  and  $y_\pm$ , extract  $r_B$ ,  $\delta_B$  and  $\gamma$
- ④ Publish new measurement of  $\gamma$ !

## Strong phase input from charm factories

Strong phase input from charm factories

## Strong phase input from charm factories

Unfortunately, it is unlikely that this fit will converge...

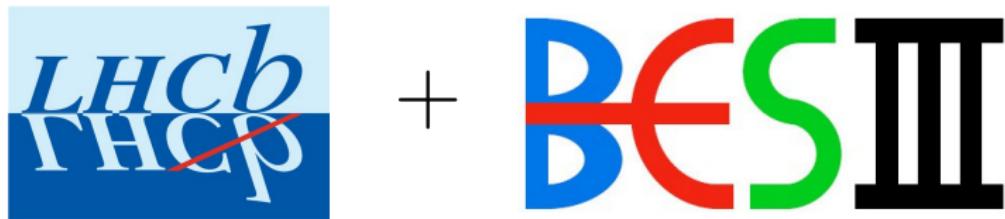
Sensitivity to  $c_i$  and  $s_i$  is very limited with current statistics

# Strong phase input from charm factories

Unfortunately, it is unlikely that this fit will converge...

Sensitivity to  $c_i$  and  $s_i$  is very limited with current statistics

Instead, we can join forces with BESIII and measure  $c_i$  and  $s_i$  directly



This has never been done for  $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$   
More on this later!

Constraining  $F_i$  with  $B^\pm \rightarrow D\pi^\pm$

Constraining  $F_i$  with  $B^\pm \rightarrow D\pi^\pm$

# Constraining $F_i$ with $B^\pm \rightarrow D\pi^\pm$

- The fractional bin yields  $F_i$  are yields in the absence of  $CP$  violation
- In principle we can measure these directly at both LHCb and BESIII

Four strategies:

- ① Calculate from amplitude model
- ② Measure in  $B^- \rightarrow D^0 \mu^- \bar{\nu}_\mu$  at LHCb
- ③ Measure with flavour tagged  $D^0$  decays at BESIII
- ④ Measure in  $B^\pm \rightarrow D\pi^\pm$

# Constraining $F_i$ with $B^\pm \rightarrow D\pi^\pm$

- The fractional bin yields  $F_i$  are yields in the absence of  $CP$  violation
- In principle we can measure these directly at both LHCb and BESIII

Four strategies:

- ① ~~Calculate from amplitude model~~ Avoid model dependence
- ② Measure in  $B^- \rightarrow D^0 \mu^- \bar{\nu}_\mu$  at LHCb
- ③ Measure with flavour tagged  $D^0$  decays at BESIII
- ④ Measure in  $B^\pm \rightarrow D\pi^\pm$

# Constraining $F_i$ with $B^\pm \rightarrow D\pi^\pm$

- The fractional bin yields  $F_i$  are yields in the absence of  $CP$  violation
- In principle we can measure these directly at both LHCb and BESIII

Four strategies:

- ① ~~Calculate from amplitude model~~ Avoid model dependence
- ② ~~Measure in  $B^- \rightarrow D^0 \mu^- \bar{\nu}_\mu$  at LHCb~~ Different acceptance effects
- ③ ~~Measure with flavour tagged  $D^0$  decays at BESIII~~
- ④ Measure in  $B^\pm \rightarrow D\pi^\pm$

# Constraining $F_i$ with $B^\pm \rightarrow D\pi^\pm$

- The fractional bin yields  $F_i$  are yields in the absence of  $CP$  violation
- In principle we can measure these directly at both LHCb and BESIII

Four strategies:

- ① ~~Calculate from amplitude model~~ Avoid model dependence
- ② ~~Measure in  $B^- \rightarrow D^0 \mu^- \bar{\nu}_\mu$  at LHCb~~ Different acceptance effects
- ③ ~~Measure with flavour tagged  $D^0$  decays at BESIII~~
- ④ Measure in  $B^\pm \rightarrow D\pi^\pm$  Small CPV effects?

# Constraining $F_i$ with $B^\pm \rightarrow D\pi^\pm$

- The fractional bin yields  $F_i$  are yields in the absence of  $CP$  violation
- In principle we can measure these directly at both LHCb and BESIII

Four strategies:

- ① ~~Calculate from amplitude model~~ Avoid model dependence
- ② ~~Measure in  $B^- \rightarrow D^0 \mu^- \bar{\nu}_\mu$  at LHCb~~ Different acceptance effects
- ③ ~~Measure with flavour tagged  $D^0$  decays at BESIII~~
- ④ Measure in  $B^\pm \rightarrow D\pi^\pm$  Small CPV effects?

No problem, include  $B^\pm \rightarrow D\pi^\pm$  as a signal channel

## Constraining $F_i$ with $B^\pm \rightarrow D\pi^\pm$

- $B^\pm \rightarrow D\pi^\pm$  has an identical topology to  $B^\pm \rightarrow DK^\pm$
- CPV effects are highly suppressed because  $r_B^{D\pi} \approx 0.005$
- Branching fraction more than 10 times larger
- As a signal channel, we add another 4 free parameters to our fit:

$$x_\pm^{D\pi} = r_B^{D\pi} \cos(\delta_B^{D\pi} - \gamma), \quad y_\pm^{D\pi} = r_B^{D\pi} \sin(\delta_B^{D\pi} - \gamma)$$

# Constraining $F_i$ with $B^\pm \rightarrow D\pi^\pm$

To avoid degeneracy, reduce this to 2 additional parameters using this parameterisation:

$$x_\xi = \text{Re}(\xi), \quad y_\xi = \text{Im}(\xi), \quad \xi = \frac{r_B^{D\pi} e^{i\delta_B^{D\pi}}}{r_B^{DK} e^{i\delta_B^{DK}}}$$

In summary:

- ① Both  $B^\pm \rightarrow DK^\pm$  and  $B^\pm \rightarrow D\pi^\pm$  are signal channels, with  $x_\pm^{DK}$ ,  $y_\pm^{DK}$ ,  $x_\xi$  and  $y_\xi$  as CP observables
- ②  $B^\pm \rightarrow DK^\pm$  has lower statistics, but higher CPV effects
- ③  $B^\pm \rightarrow D\pi^\pm$  has higher statistics and constrain  $F_i$  in the fit, but sensitivity to CPV is limited

## Binning scheme

# Binning scheme

We need to split the phase space into bins

But how do we navigate through a 5D space? How do we decide on the bin boundaries?



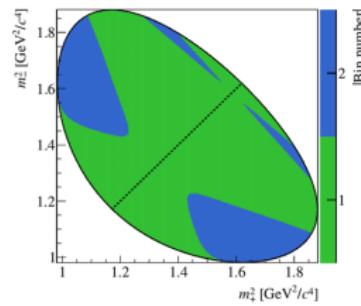
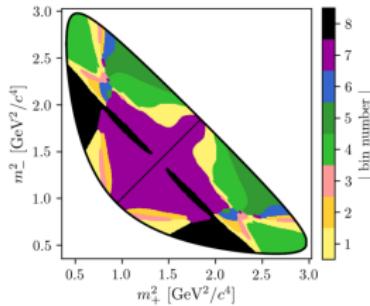
Let the amplitude model guide us!

# Binning scheme

Back to the amplitude averaged strong phase:

$$c_i \equiv \frac{\int_i d\Phi |\mathcal{A}_{D^0}| |\mathcal{A}_{\bar{D}^0}| \cos(\delta_D(\Phi))}{\sqrt{\int d\Phi |\mathcal{A}_{D^0}|^2 \int d\Phi |\mathcal{A}_{\bar{D}^0}|^2}}$$

- If the strong phase varies significantly within a bin, the interference effects will be diluted when integrating
- We need to group regions of similar strong phase into the same bin
- This was done for  $K_S^0 h^+ h^-$ , resulting in colourful “butterfly” plots

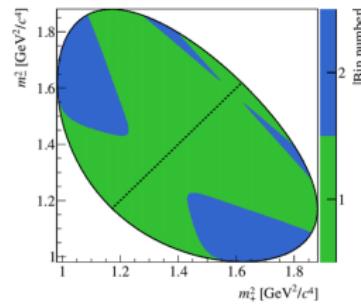
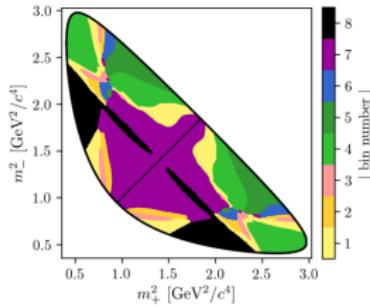


# Binning scheme

Back to our yield formula:

$$N_i^- \propto F_i + r_B^2 \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_- c_i - y_- s_i)$$

- In the charm system,  $CP$  is (approximately) conserved, so each  $D^0$  decay has a corresponding identical  $CP$  conjugated decay
- Under  $CP$ ,  $\delta_D \rightarrow -\delta_D$ , so  $c_i \rightarrow c_i$  and  $s_i \rightarrow -s_i$
- Split each bin  $i$  into two “ $CP$  mirror bins”, labelled by  $\pm i$
- In  $K_S^0 h^+ h^-$ , this is indicated by the black symmetry line



## Binning scheme

A binning scheme must satisfy the following:

- Minimal dilution of strong phases when integrating over bins
- Enhance interference between  $B^\pm \rightarrow D^0 K^\pm$  and  $B^\pm \rightarrow \bar{D}^0 K^\pm$

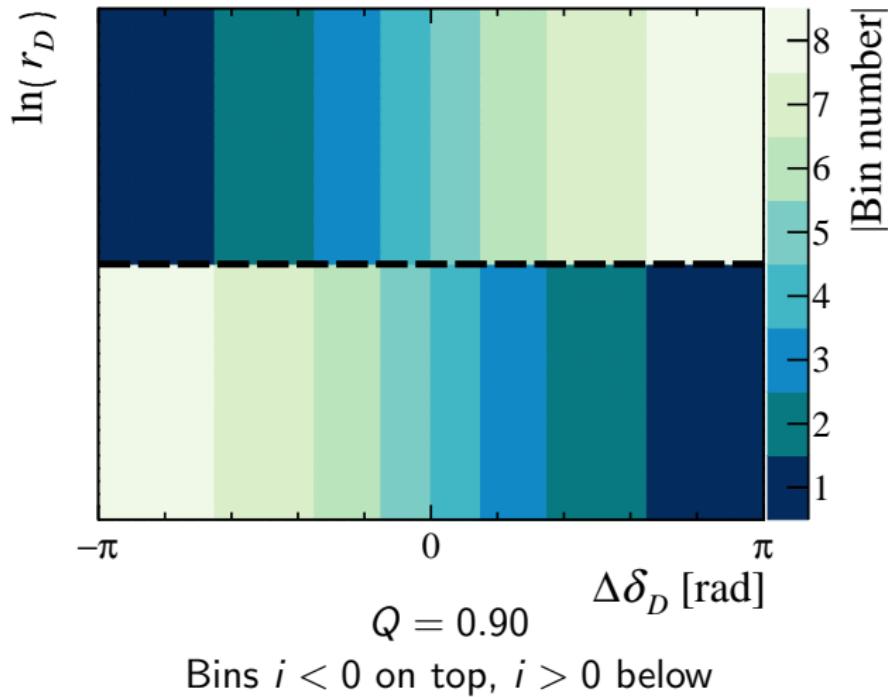
How to bin a 5-dimensional phase space?

- ① For each  $B^\pm$  candidate, use the amplitude model to calculate

$$\frac{\mathcal{A}(D^0)}{\mathcal{A}(\bar{D}^0)} = r_D e^{i\delta_D}$$

- ② Split  $\delta_D$  into uniformly spaced bins
- ③ Use the symmetry line  $r_D = 1$  to separate bin  $+i$  from  $-i$
- ④ Optimise the binning scheme by adjusting the bin boundaries in  $\delta_D$

# Binning scheme



## Mass fits and yield extraction

# Mass fits and yield extraction

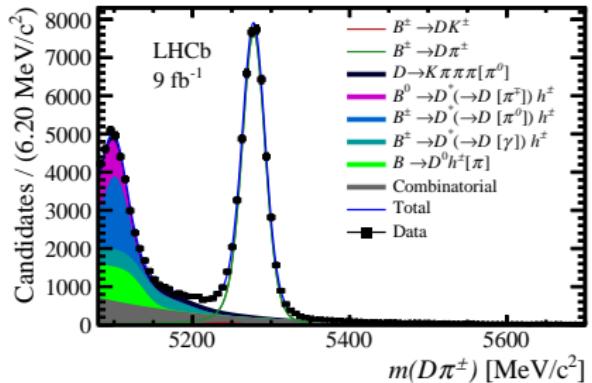
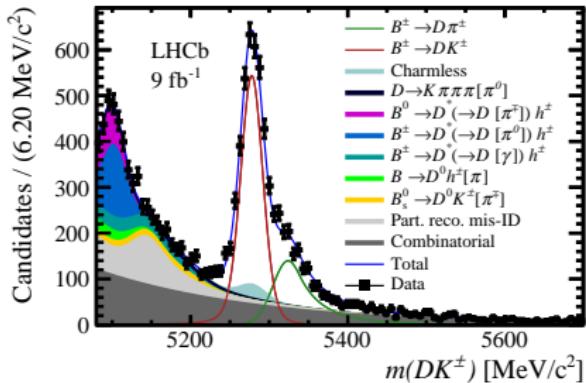
# Mass fits and yield extraction

In the end, this analysis is a counting experiment

Counting strategy:

- ① Perform a “global fit” of all  $B^\pm$  candidates
- ② Fix all shape parameters
- ③ Sort  $B^\pm$  candidates by charge and bins
- ④ Perform a “ $CP$  fit” simultaneously, but only let bin yields float
- ⑤ From the bin yields, determine  $x_\pm^{DK}$ ,  $y_\pm^{DK}$ ,  $x_\xi$  and  $y_\xi$

# Mass fits and yield extraction



Signal yield:

$$B^\pm \rightarrow DK^\pm : 3026 \pm 38$$

$$B^\pm \rightarrow D\pi^\pm : 44\,349 \pm 218$$

## $CP$ fit setup

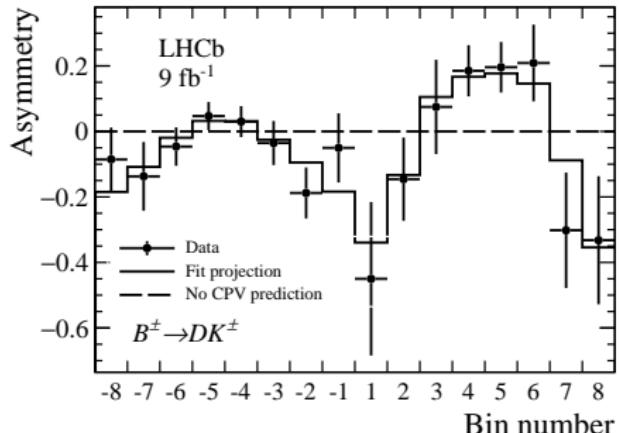
- No measurement of  $c_i$  and  $s_i$  available yet, use model predictions
  - Fix mass shape from global fit
  - Split by  $B^\pm$  charge and  $D$  phase space bins (64 categories)
- 
- ① CP observables  $x_{\pm}^{DK}$ ,  $y_{\pm}^{DK}$   $x_{\xi}^{D\pi}$ ,  $y_{\xi}^{D\pi}$  (6 parameters )
  - ② Fractional bin yields  $F_i$  (15 parameters)
  - ③ Low mass and combinatorial background (128 parameters)
  - ④ Yield normalisation (4 parameters)

In total: 153 free parameters

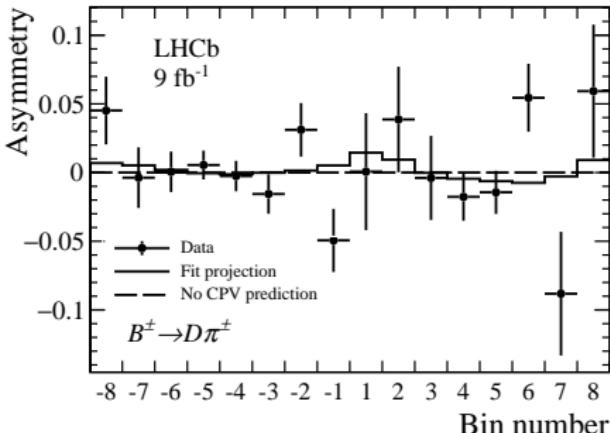
*CP* fit results and  $\gamma$

*CP* fit results and  $\gamma$

# Fractional bin asymmetries



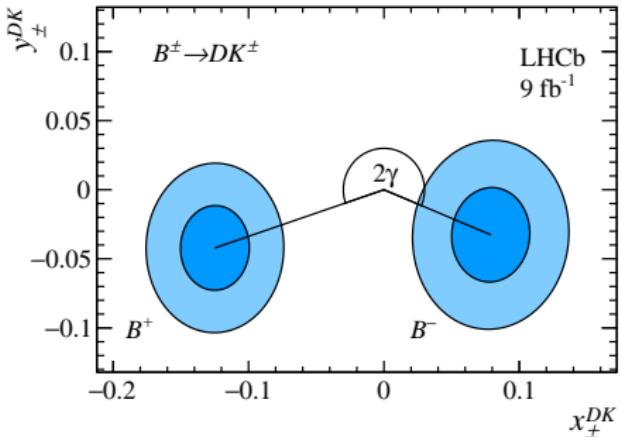
(a)  $B^\pm \rightarrow DK^\pm$



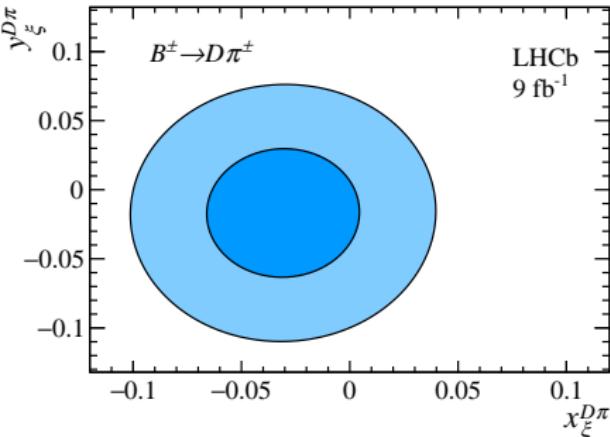
(b)  $B^\pm \rightarrow D\pi^\pm$

- Useful cross check to compare measured bin asymmetries against bin asymmetries predicted by the fitted CP observables
- The  $B^\pm \rightarrow DK^\pm$  mode show non-zero bin asymmetries, and the non-trivial distribution is driven by the change in strong phases across phase space

# CP fit results



(a)  $x_{\pm}^{DK}$  vs  $y_{\pm}^{DK}$



(b)  $x_{\xi}^{D\pi}$  vs  $y_{\xi}^{D\pi}$

$$x_{\pm}^{DK} = r_B \cos(\delta_B \pm \gamma)$$

$$y_{\pm}^{DK} = r_B \sin(\delta_B \pm \gamma)$$

- The  $B^{\pm} \rightarrow DK^{\pm}$  contours are distinct, indicating CP violation
- The  $B^{\pm} \rightarrow D\pi^{\pm}$  mode has very low sensitivity to CP violation

## Interpretation of $\gamma$

We can interpret our  $CP$  observables in terms of the physics parameters  $\gamma$ ,  $r_B^{DK}$ ,  $\delta_B^{DK}$ ,  $r_B^{D\pi}$ ,  $\delta_B^{D\pi}$

$$\gamma = (116_{-14}^{+12})^\circ,$$

$$\delta_B^{DK} = (81_{-13}^{+14})^\circ,$$

$$r_B^{DK} = 0.110_{-0.020}^{+0.020},$$

$$\delta_B^{D\pi} = (298_{-118}^{+62})^\circ,$$

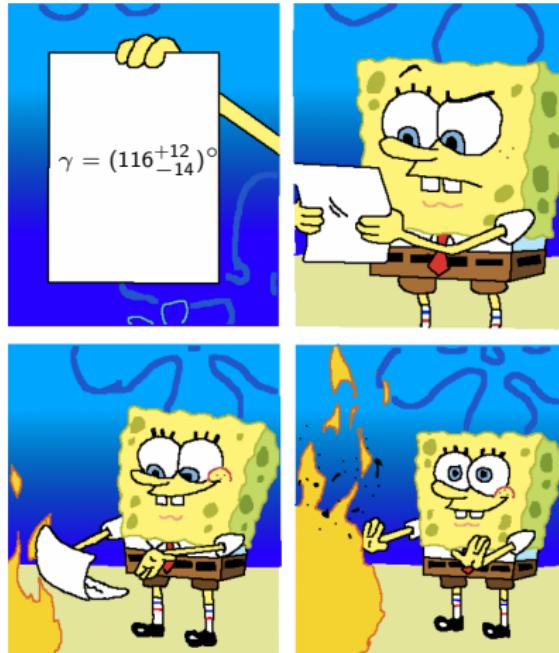
$$r_B^{D\pi} = 0.0041_{-0.0041}^{+0.0054},$$

However, the latest  $\gamma$  and charm combination result is:

$$\gamma = (63.8_{-3.7}^{+3.5})^\circ$$

What went wrong?!

# Interpretation of $\gamma$



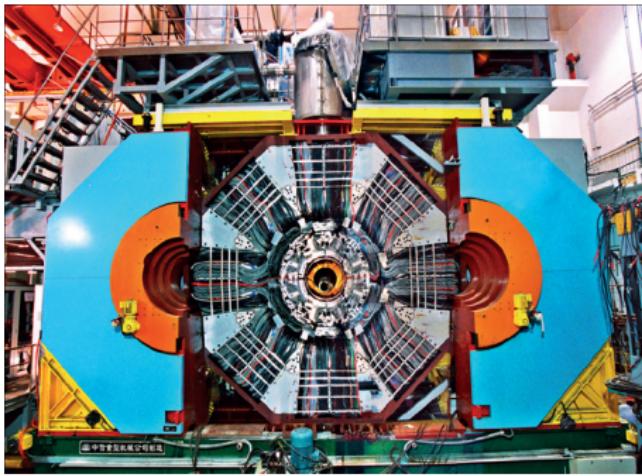
Do we trust the model predicted  $c_i$  and  $s_i$ , or their uncertainties? No!  
Let's go and measure  $c_i$  and  $s_i$  at BESIII!

# Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

## Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

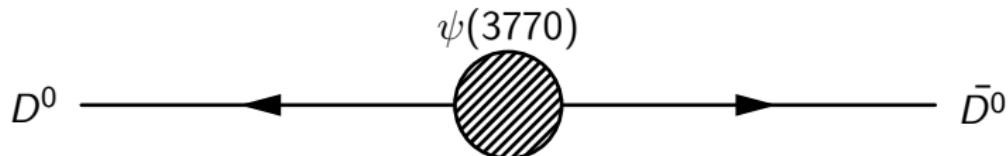
# Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

- BESIII: Beijing Spectrometer III, a detector at the Beijing Electron-Position Collider II, located at IHEP
- $e^+e^-$  collider at the  $\psi(3770) \rightarrow D^0\bar{D}^0$  threshold
  - 2010-2011:  $3\text{ fb}^{-1}$
  - 2022:  $5\text{ fb}^{-1}$
  - Expect  $20\text{ fb}^{-1}$  in total by end of 2024

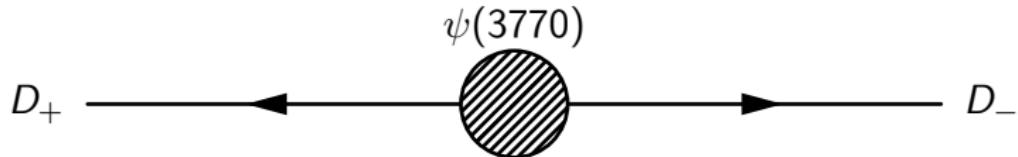


# Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

- Double-tag analysis: Reconstruct signal ( $KK\pi\pi$ ) and tag mode
- $D^0\bar{D}^0$  pair is quantum correlated



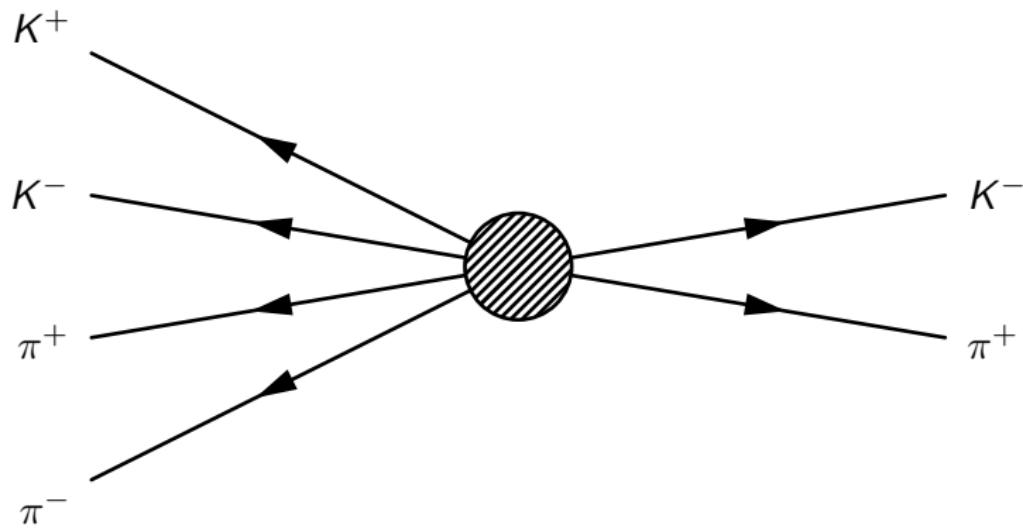
- Equivalently, we can consider  $D_+D_-$ 
  - $D_{\pm} = \frac{1}{\sqrt{2}}(D^0 \pm \bar{D}^0)$  are CP eigenstates



The  $DD$  pair is quantum correlated, spooky action at a distance!

# Strong-phase in quantum correlated $D^0\bar{D}^0$ decays

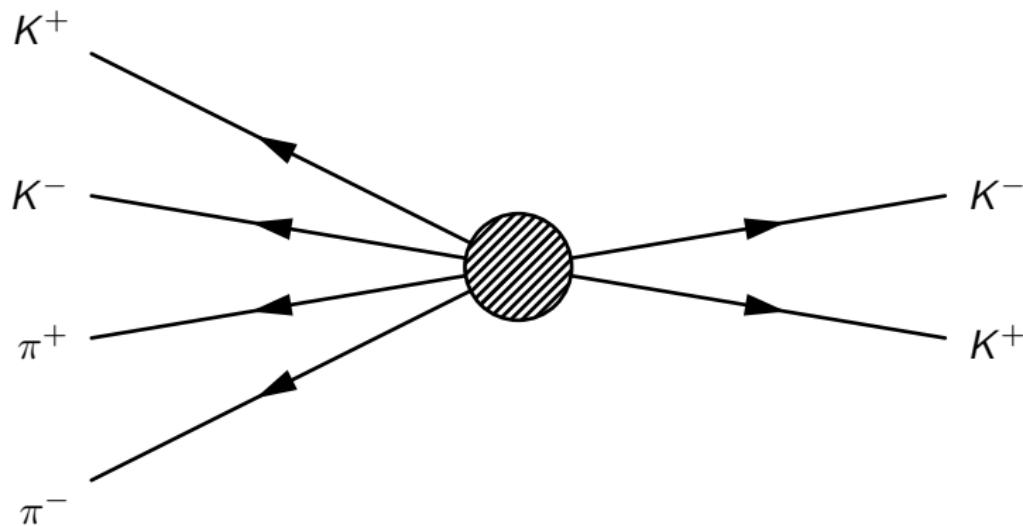
- Tag mode can be a flavour tag
  - $K^-\pi^+$ ,  $K^-\pi^+\pi^0$ ,  $K^-\pi^+\pi^-\pi^+$ ,  $K^-e^+\nu_e$



Flavour tags do not exhibit quantum correlation effects

# Strong-phases in quantum correlated $D^0\bar{D}^0$ decays

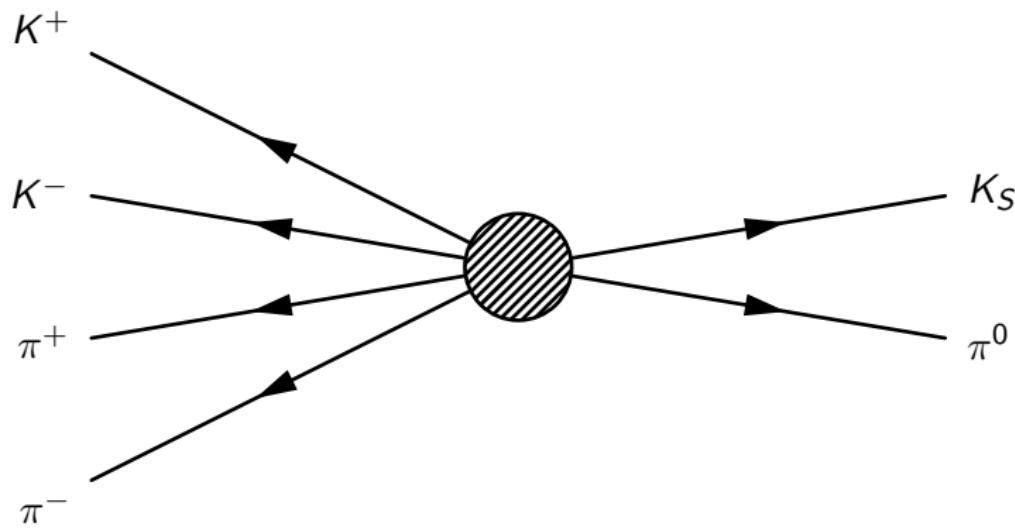
- Tag mode can be a CP even tag
  - $KK, \pi\pi, \pi\pi\pi^0, K_S\pi^0\pi^0, K_L\pi^0, K_L\omega$



$D \rightarrow K^+K^-$ , which is CP even, forces  $D \rightarrow K^+K^-\pi^+\pi^-$  to be CP odd

# Strong-phase in quantum correlated $D^0\bar{D}^0$ decays

- Tag mode can be a CP odd tag
  - $K_S\pi^0, K_S\omega, K_S\eta, K_S\eta', K_L\pi^0\pi^0$

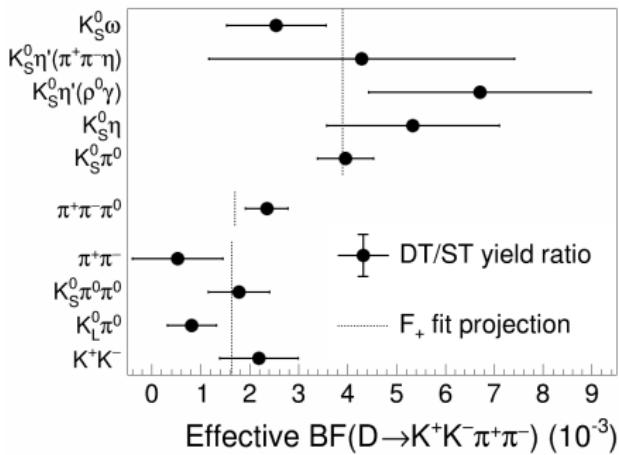


$D \rightarrow K_S^0\pi^0$ , which is  $CP$  odd, forces  $D \rightarrow K^+K^-\pi^+\pi^-$  to be  $CP$  even

# Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

Quantum correlation can modify the effective branching fraction:

$$\frac{N^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi)(1 \pm c_1)$$



arXiv:2212.06489

$c_1$  is the cosine of the strong phase, averaged over the whole phase space

# Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

Our next task is to change the phase space inclusive analysis,

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi) \quad (\text{flavour tag})$$

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi)(1 \pm c_i) \quad (\text{CP tag})$$

into a binned phase space analysis:

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi)F_i \quad (\text{flavour tag})$$

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi)(F_i + \bar{F}_i \pm 2\sqrt{F_i \bar{F}_i}c_i) \quad (\text{CP tag})$$

- ①  $F_i$ : Measure using flavour tags
- ②  $c_i$ : Determine from asymmetry of CP even and odd tags
- ③  $s_i$ : Analogous to  $c_i$ , but requires binning of tag mode

# Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

Our next task is to change the phase space inclusive analysis,

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi) \quad (\text{f' tag})$$

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi) \quad (\text{CP tag})$$

into a binned phase space analysis:

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KV) \quad (\text{flavour tag})$$

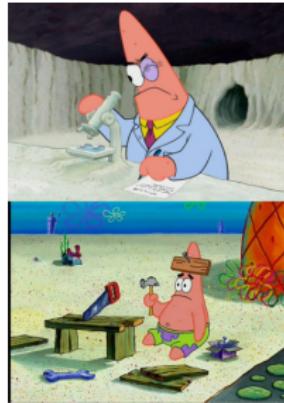
$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow K\bar{K}) \langle F_i + \bar{F}_i \pm 2\sqrt{F_i\bar{F}_i}c_i \rangle \quad (\text{CP tag})$$

- ①  $F_i$ : Measure using our tags
- ②  $c_i$ : Determine from asymmetry of  $CP$  even and odd tags
- ③  $s_i$ : Analogous to  $c_i$ , but requires binning of tag mode

## Summary and conclusion

- ① I have presented a CPV study of  $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D h^\pm$
- ② Multi-body decays, such as  $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ , have a great potential for measuring  $\gamma$
- ③ The optimised binning scheme, developed with an amplitude model, successfully identified regions with large, local  $CP$  asymmetries

- ④ However, amplitude model predictions of  $\delta_D$  are wrong



Making binning  
scheme with  
amplitude model

Predicting strong  
phases with  
amplitude model

## Summary and conclusion

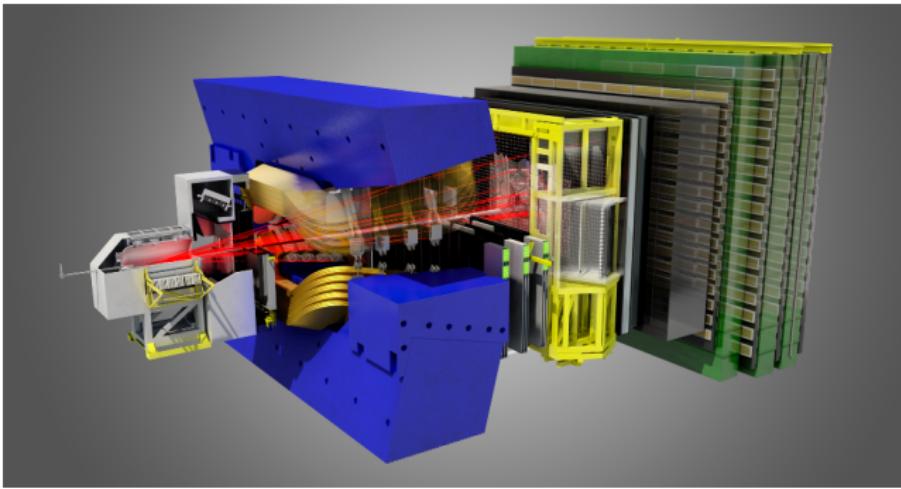
- ⑤ The fit results, using model predicted strong phases, were found to have a  $3\sigma$  tension with the current LHCb combination
- ⑥ External inputs from charm factories, such as BESIII, are crucial to constrain charm strong phases
- ⑦ Combined, the LHCb and BESIII measurement will lead to the first measurement of  $\gamma$  in this channel
- ⑧ Work is ongoing in similar four-body modes:
  - $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$
  - $D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0$

Thanks for your attention!

# Backup slides

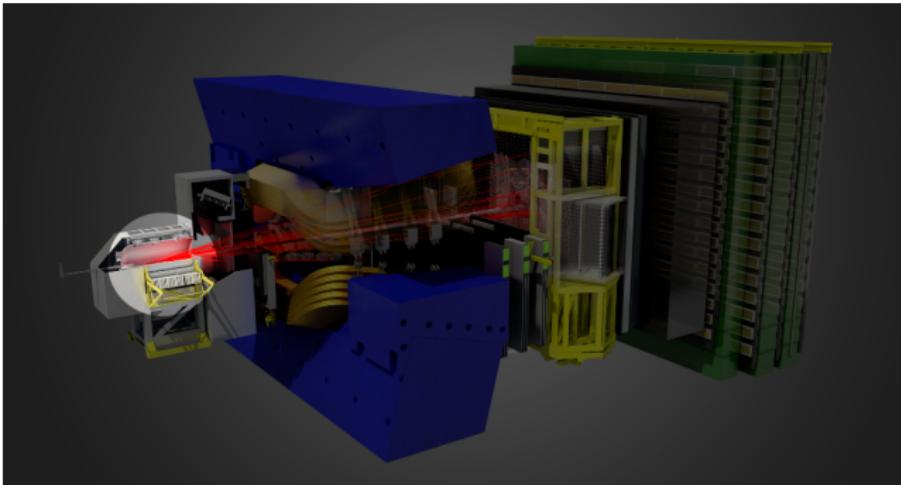
# The LHCb detector

# The LHCb detector



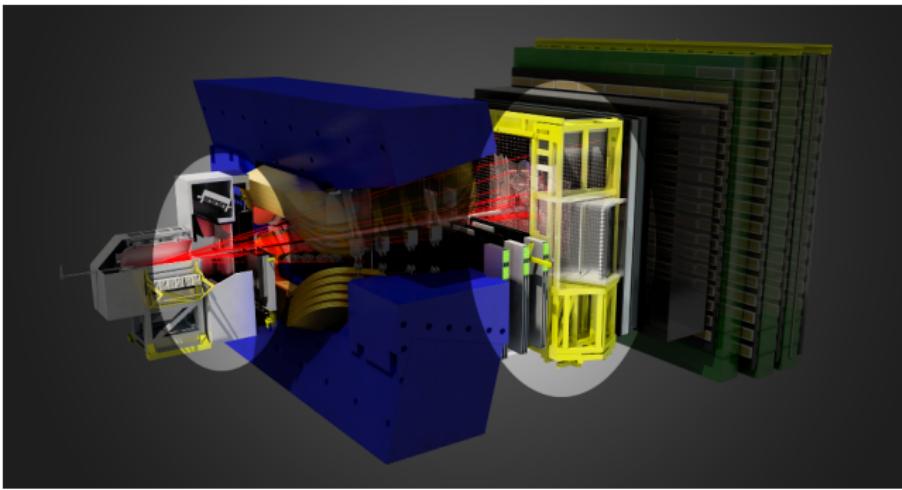
LHCb: A beauty experiment with a lot of charm

# The LHCb detector



VELO: Vertex locator to reconstruct  $B$  and  $D$  vertices

# The LHCb detector



RICH: Identify  $B$  and  $D$  daughter particles

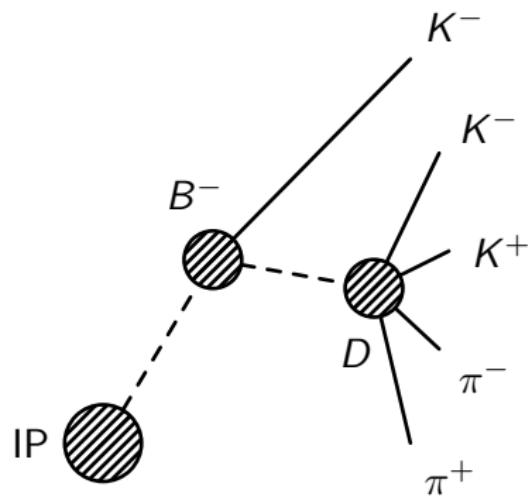
# Event selection

# Event selection

## Decay topology

Look for:

- ① 5 charged tracks
- ② Displaced  $B$  vertex
- ③ 1 bachelor track with good PID information
- ④ Displaced  $D$  vertex with invariant mass within 25 MeV of the  $D^0$  mass



# Event selection

Offline selection has 3 stages

Initial cuts:

- ① Invariant  $D$  and  $B$  mass cuts
- ② Momentum and RICH requirements

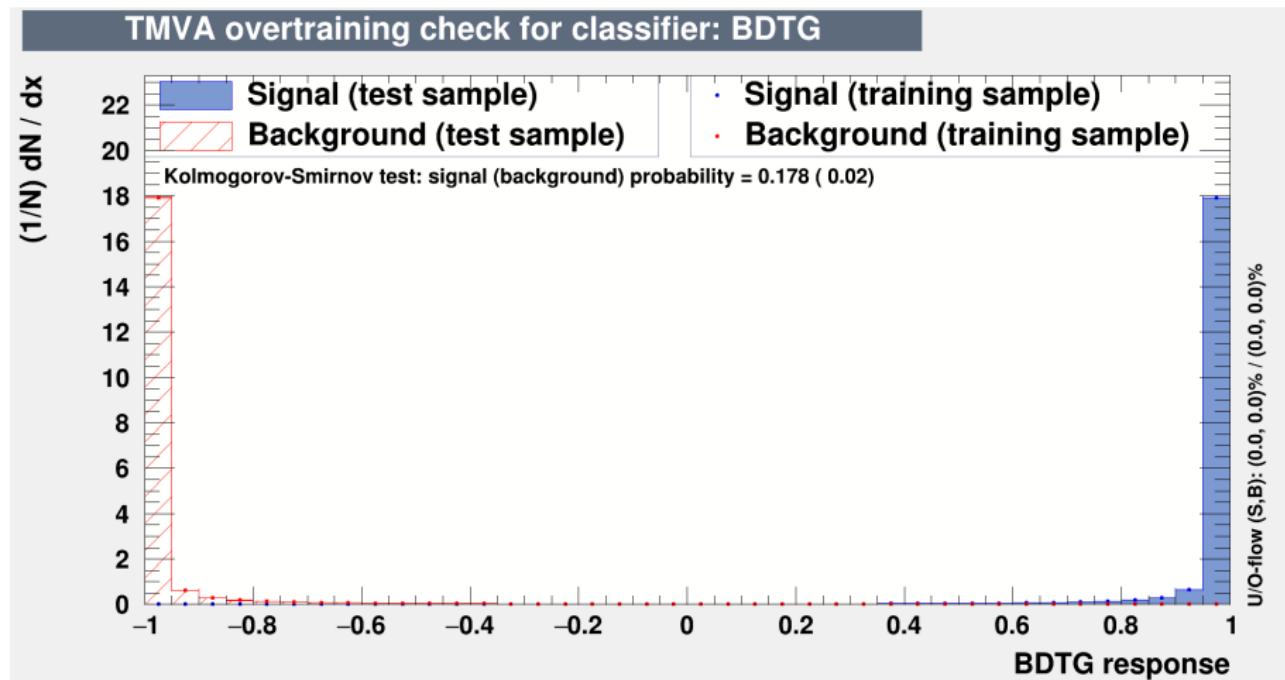
Boosted Decision Tree (BDT)

- Signal sample: Simulation samples
- Background sample: Upper  $B$  mass sideband
- 28 variables describing kinematics, impact parameters, vertex quality

Final selection

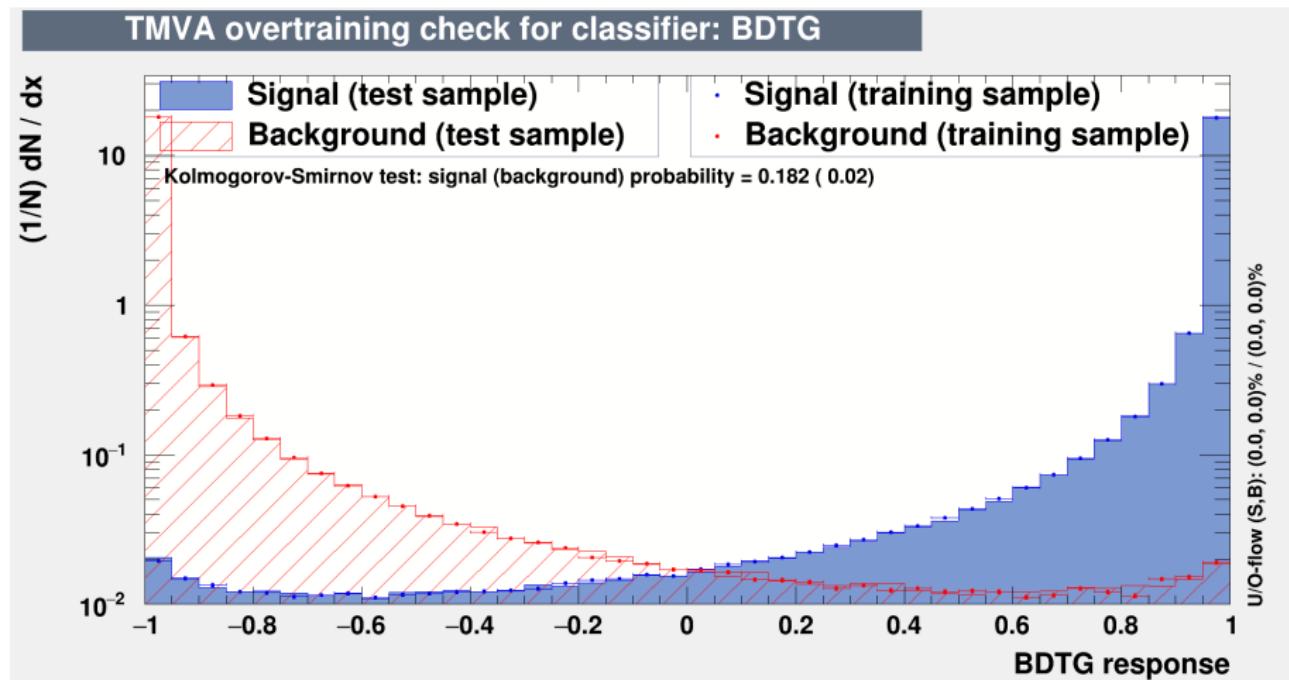
- ①  $D$  Flight distance
- ② Particle Identification of bachelor
- ③  $K_S^0$  veto

# Event selection



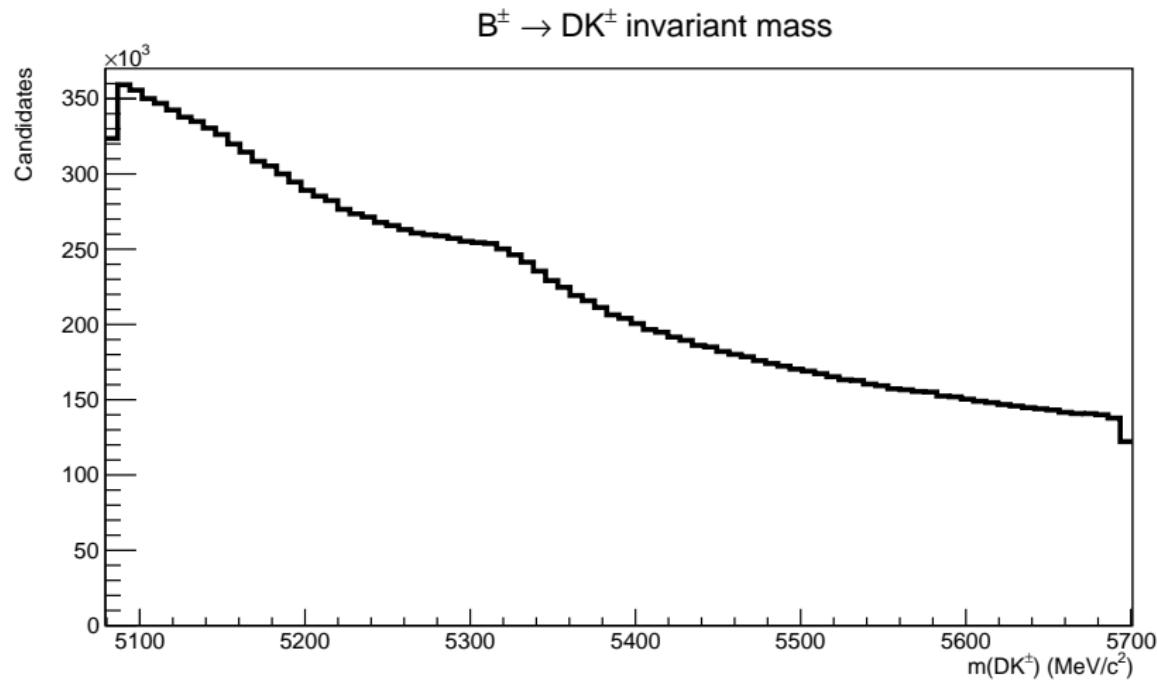
BDT is highly efficient at rejecting combinatorial background

# Event selection



Very important, combinatorial background is large in multi-body decays

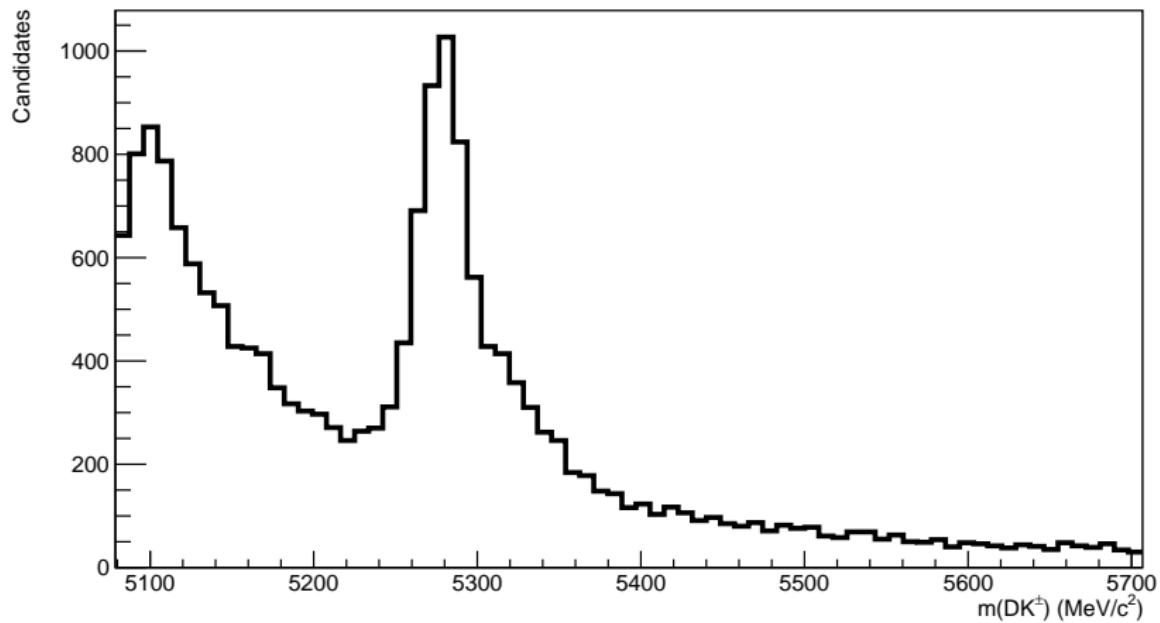
## Event selection



The invariant  $B$  mass, after online selection, show no visible signal...

# Event selection

$B^\pm \rightarrow D K^\pm$  invariant mass



... but the BDT does a great job cleaning this up!