

Model independent measurement of the CKM angle γ with $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D h^\pm$ at LHCb and BESIII

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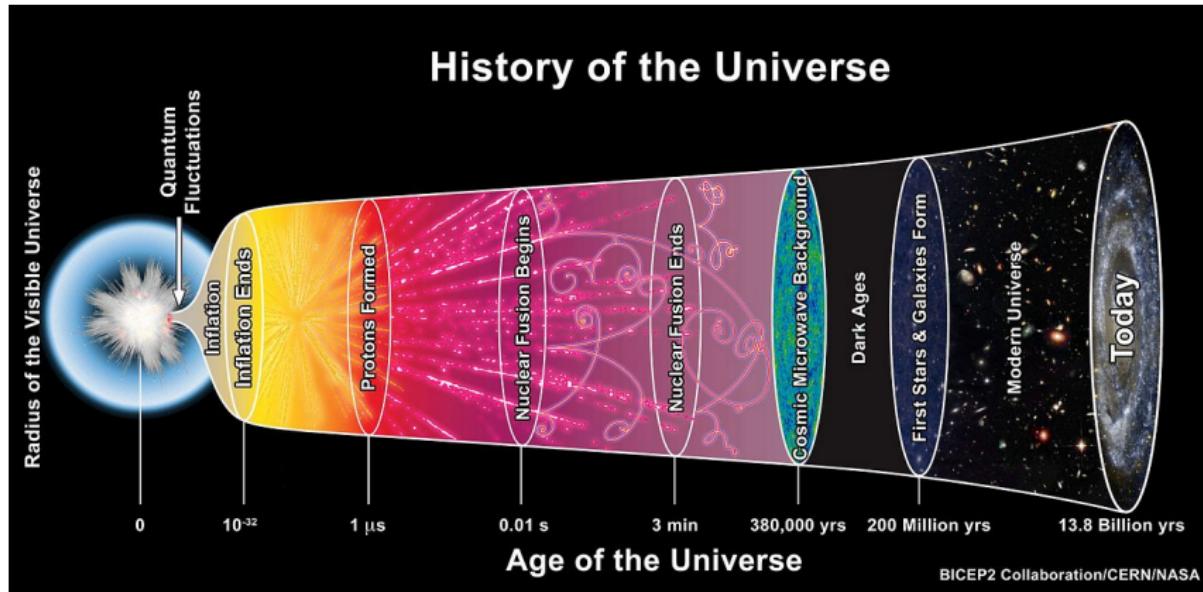
9th February 2023



Introduction to CP violation

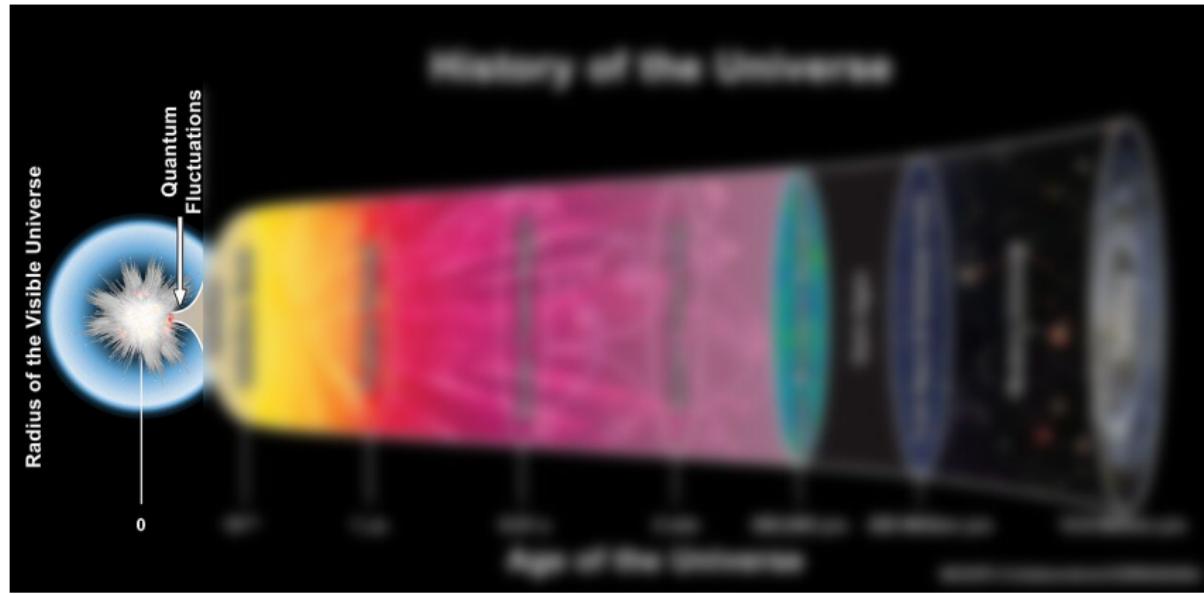
Introduction to CP violation

Big Bang and matter-antimatter asymmetry



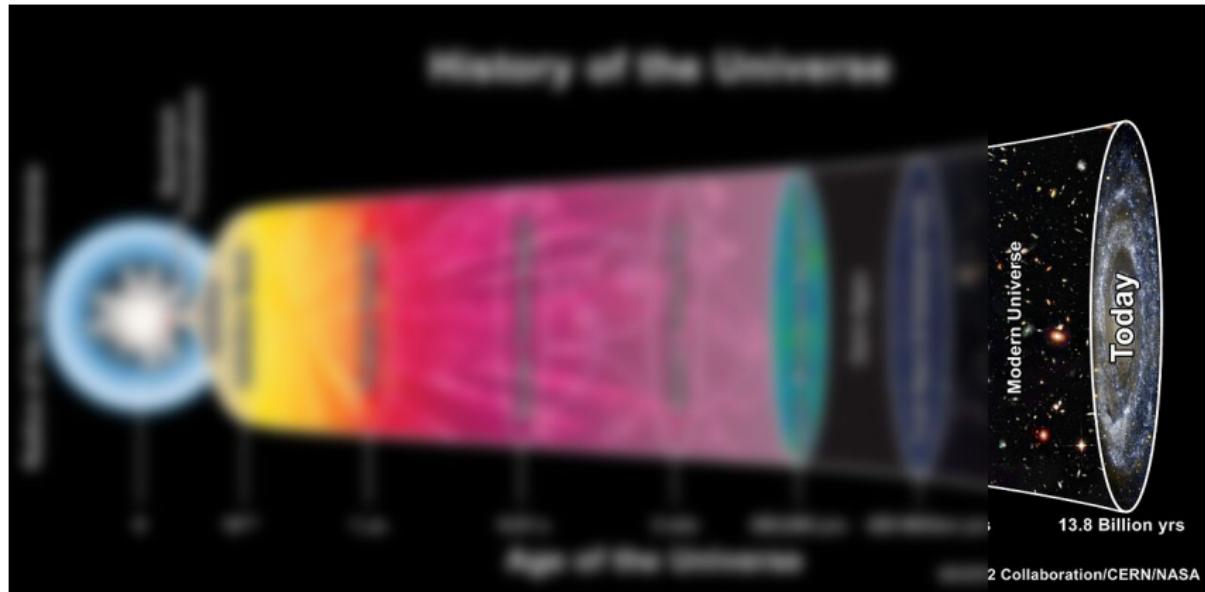
Where is the antimatter in the universe?

Big Bang and matter-antimatter asymmetry



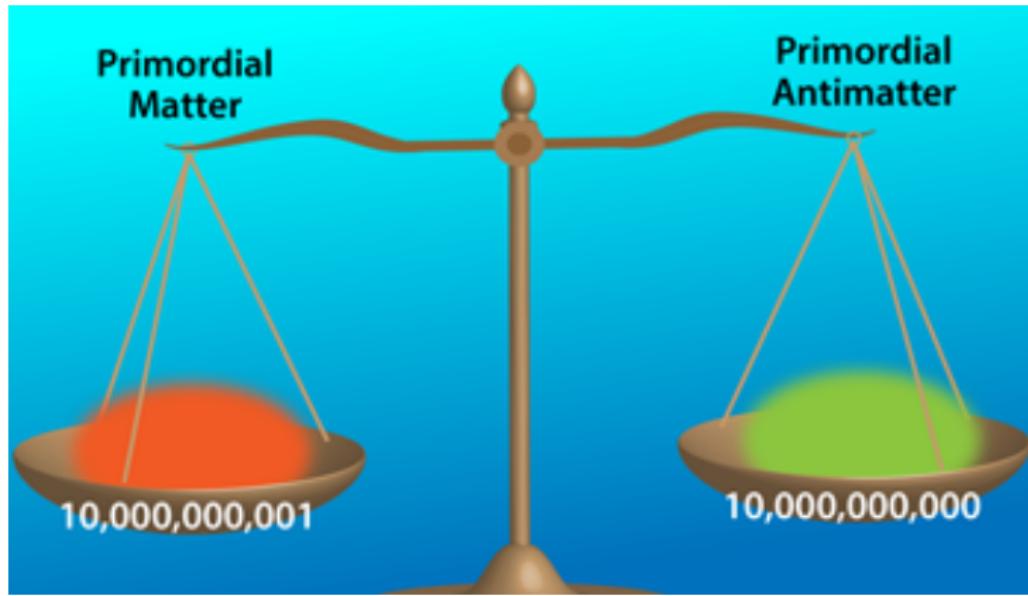
Initially equal amounts of matter and antimatter...

Big Bang and matter-antimatter asymmetry



... but today we only see matter!

Big Bang and matter-antimatter asymmetry



APS/Alan Stonebraker

The difference is very small...

Big Bang and matter-antimatter asymmetry



Quantum Diaries: "Why B physics? Why not A Physics?"

... but the effects we observe today are obviously huge!
How can we explain this?

CP violation

The Nobel Prize in Physics 1980



Photo from the Nobel Foundation archive.
James Watson Cronin
Prize share: 1/2



Photo from the Nobel Foundation archive.
Val Logsdon Fitch
Prize share: 1/2

The Nobel Prize in Physics 1980 was awarded jointly to James Watson Cronin and Val Logsdon Fitch "for the discovery of violations of fundamental symmetry principles in the decay of neutral K-mesons"

- CP violation discovery in 1964
- Phys. Rev. Lett. **13**, 138
- Observed $K_L^0 \rightarrow \pi^+ \pi^-$
- Since, CP violation has also been observed in the B , B_s and D systems

Can Standard Model CPV explain the matter-antimatter asymmetry?
Or, could it be physics beyond the SM?

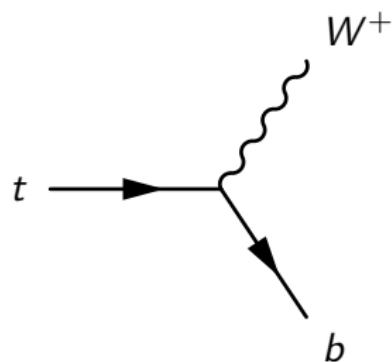
The CKM matrix and the Unitary Triangle

The CKM matrix and the Unitary Triangle

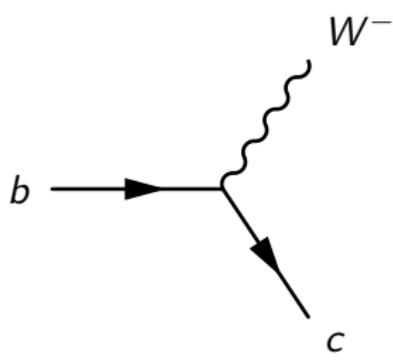
The CKM matrix and the Unitary Triangle

In SM, the charged current W^\pm interactions couple (left-handed) up- and down-type quarks, given by

$$\frac{-g}{\sqrt{2}} \begin{bmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{bmatrix} \gamma^\mu W_\mu V_{CKM} \begin{bmatrix} d_L \\ s_L \\ b_L \end{bmatrix} + \text{h.c.}$$



(a) $t \rightarrow bW^+$



(b) $b \rightarrow cW^-$

The CKM matrix and the Unitary Triangle

The Cabibbo-Kobayashi-Maskawa matrix V_{CKM} ,

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

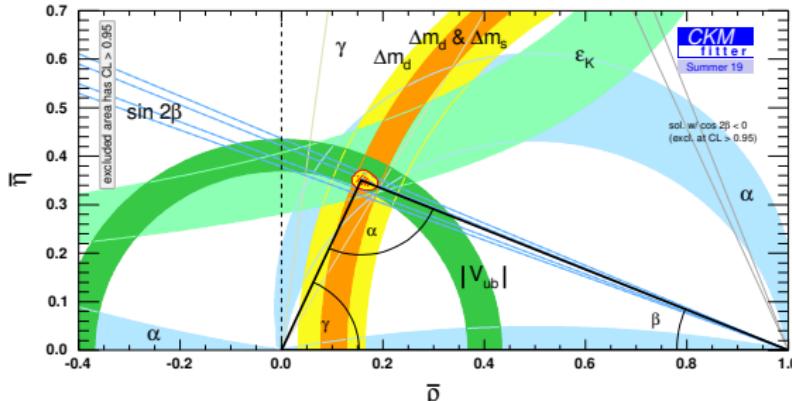
must be a unitary matrix: $V_{\text{CKM}}^\dagger V_{\text{CKM}} = I \implies$

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

Represent this constraint as a triangle in the complex plane:
Unitary Triangle

The CKM matrix and the Unitary Triangle

- CPV in SM is described by the Unitary Triangle, with angles α, β, γ
- The angle $\gamma = \arg\left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}\right)$ is very important:
 - ① Negligible theoretical uncertainties: Ideal SM benchmark
 - ② Accessible at tree level: Indirectly probe New Physics that enter loops
 - ③ Compare with α, β measurements: Is the Unitary Triangle a triangle?



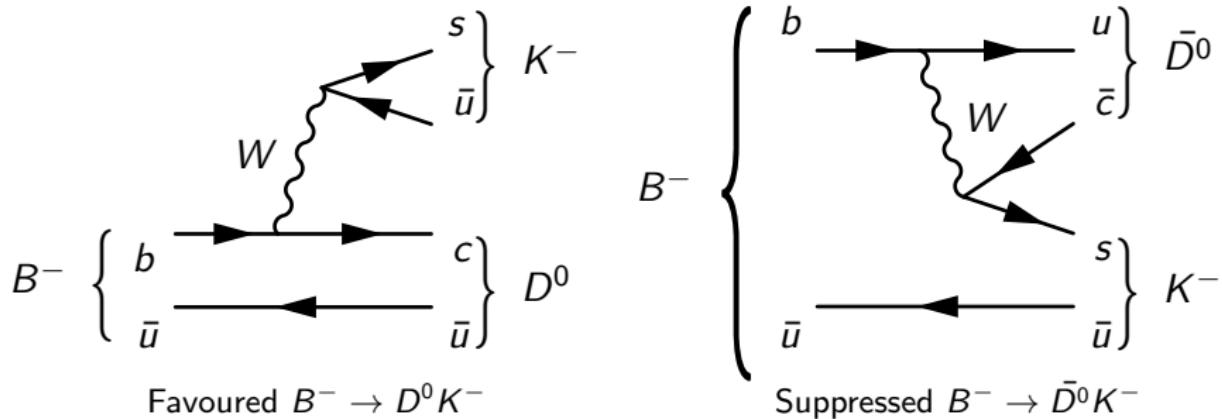
CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005)

How to measure γ ?

How to measure γ ?

Sensitivity through interference

Measure γ through interference effects in $B^\pm \rightarrow DK^\pm$



- Superposition of D^0 and \bar{D}^0
- $b \rightarrow u\bar{c}s$ and $b \rightarrow c\bar{u}s$ interference \rightarrow Sensitivity to γ

$$\mathcal{A}(B^-) = \mathcal{A}_B \left(\mathcal{A}_{D^0} + r_B e^{i(\delta_B - \gamma)} \mathcal{A}_{\bar{D}^0} \right)$$

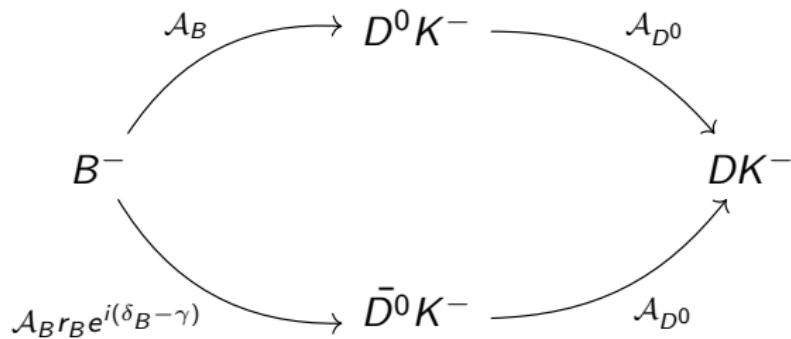
$$\mathcal{A}(B^+) = \mathcal{A}_B \left(\mathcal{A}_{\bar{D}^0} + r_B e^{i(\delta_B + \gamma)} \mathcal{A}_{D^0} \right)$$

- The magnitude of interference effects governed by $r_B \approx 0.1$

D decays to a CP eigenstate

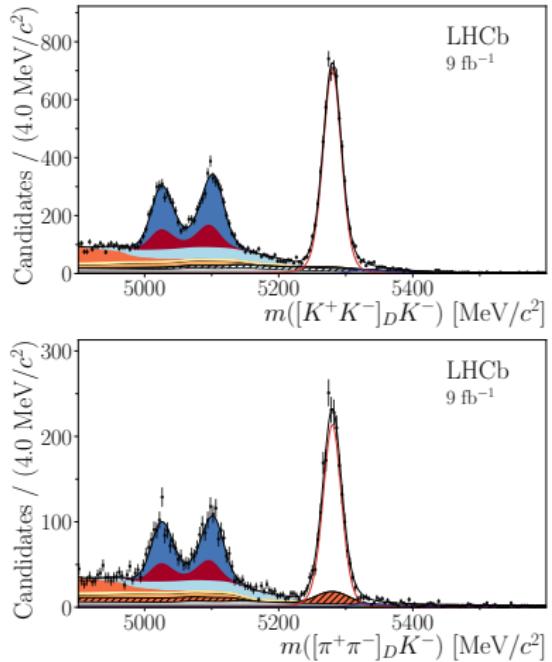
A well known strategy is to consider D decays to a CP eigenstate

For CP eigenstates, $\mathcal{A}_{D^0} = \mathcal{A}_{\bar{D}^0}$



$$|\mathcal{A}(B^-)|^2 \propto 1 + r_B^2 + 2r_B \cos(\delta_B - \gamma)$$

D decays to a CP eigenstate



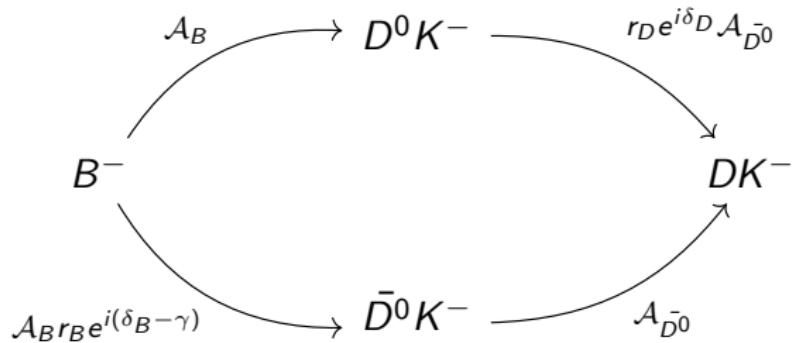
JHEP 04 (2021) 081

In $B^\pm \rightarrow [h^+ h^-]_D K^\pm$, we see significant CPV effects

Doubly Suppressed Cabibbo D decays

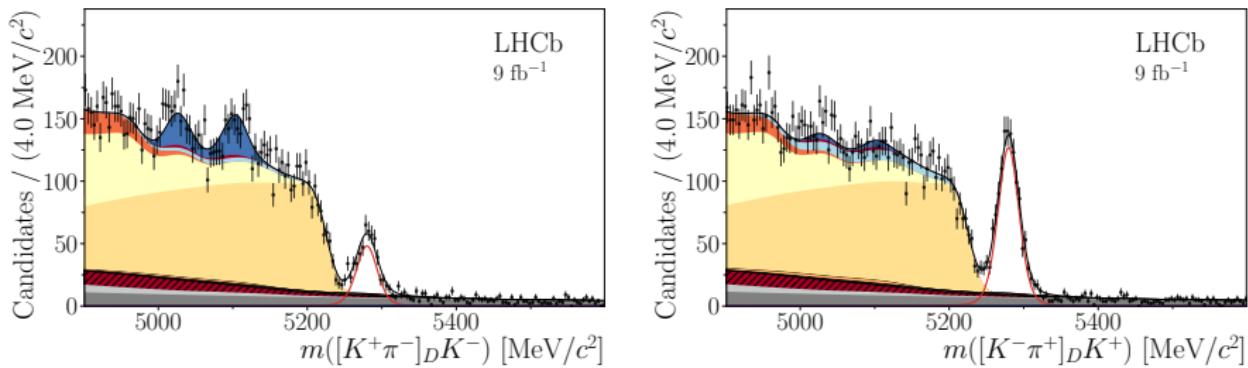
Can we enhance the interference effects?

Yes! Use a Doubly Suppressed Cabibbo decay: $\mathcal{A}_{D^0} = r_D e^{i\delta_D} \mathcal{A}_{\bar{D}^0}$



$$|\mathcal{A}(B^-)|^2 \propto r_D^2 + r_B^2 + 2r_B r_D \cos(\delta_B - \gamma + \delta_D)$$

Doubly Suppressed Cabibbo D decays



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$B^\pm \rightarrow [K^\mp\pi^\pm]_D K^\pm$ has lower statistics, but a spectacular asymmetry!

Additionally, the partially reconstructed background has an equal but opposite asymmetry

The $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$ decay mode

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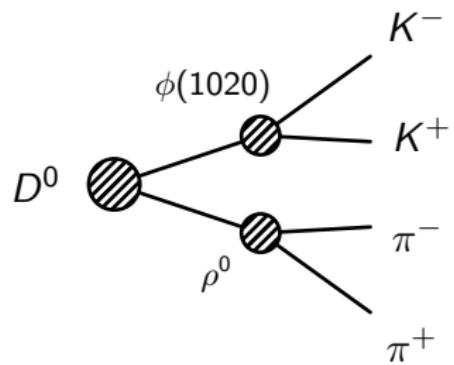
The $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$ decay mode

The mode $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$ has been proposed as a powerful channel for a measurement of γ

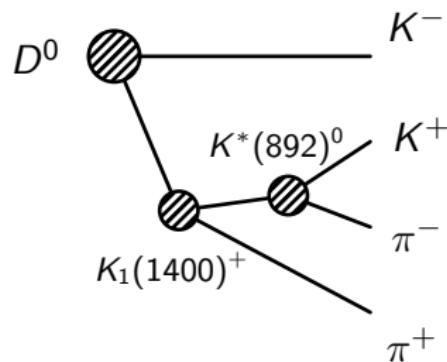
- $D \rightarrow K^+ K^- \pi^+ \pi^-$ has the best of both worlds:
 - ① Singly Cabibbo Suppressed decay: Larger branching fraction
 - ② Interference effects from over 25 resonance components
- Large interference effects in local regions of the 5D phase space
- First proposed by J. Rademacker and G. Wilkinson
 - Phys. Lett. **B647** (2007) 400
 - FOCUS amplitude model predicts a 14° precision with 1000 candidates
- State of the art amplitude analysis by LHCb:
 - JHEP **02** (2019) 126
 - Exploits the huge dataset of charm decays collected by LHCb

The $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$ decay mode

Why do four-body decays have large local interferences?



$$\phi(1020)(\rightarrow K^+ K^-) \rho^0(\rightarrow \pi^+ \pi^-)$$



$$K_1(1400)^+(\rightarrow K^*(892)^0(\rightarrow K^+ \pi^-)) K^-$$

Many possible decay paths, in different phase space locations,
contribute to the total decay amplitude...

The $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$ decay mode

Amplitude	$ c_k $	$\arg(c_k)$ [rad]	Fit fraction [%]
$D^0 \rightarrow [\phi(1020)(\rho - \omega)^0]_{L=0}$	1 (fixed)	0 (fixed)	$23.82 \pm 0.38 \pm 0.50$
$D^0 \rightarrow K_1(1400)^+ K^-$	$0.614 \pm 0.011 \pm 0.031$	$1.05 \pm 0.02 \pm 0.05$	$19.08 \pm 0.60 \pm 1.46$
$D^0 \rightarrow [K^- \pi^+]_{L=0} [K^+ \pi^-]_{L=0}$	$0.282 \pm 0.004 \pm 0.008$	$-0.60 \pm 0.02 \pm 0.10$	$18.46 \pm 0.35 \pm 0.94$
$D^0 \rightarrow K_1(1270)^+ K^-$	$0.452 \pm 0.011 \pm 0.017$	$2.02 \pm 0.03 \pm 0.05$	$18.05 \pm 0.52 \pm 0.98$
$D^0 \rightarrow [K^*(892)^0 \bar{K}^*(892)^0]_{L=0}$	$0.259 \pm 0.004 \pm 0.018$	$-0.27 \pm 0.02 \pm 0.03$	$9.18 \pm 0.21 \pm 0.28$
$D^0 \rightarrow K^*(1680)^0 [K^- \pi^+]_{L=0}$	$2.359 \pm 0.036 \pm 0.624$	$0.44 \pm 0.02 \pm 0.03$	$6.61 \pm 0.15 \pm 0.37$
$D^0 \rightarrow [K^*(892)^0 \bar{K}^*(892)^0]_{L=1}$	$0.249 \pm 0.005 \pm 0.017$	$1.22 \pm 0.02 \pm 0.03$	$4.90 \pm 0.16 \pm 0.18$
$D^0 \rightarrow K_1(1270)^- K^+$	$0.220 \pm 0.006 \pm 0.011$	$2.09 \pm 0.03 \pm 0.07$	$4.29 \pm 0.18 \pm 0.41$
$D^0 \rightarrow [K^+ K^-]_{L=0} [\pi^+ \pi^-]_{L=0}$	$0.120 \pm 0.003 \pm 0.018$	$-2.49 \pm 0.03 \pm 0.16$	$3.14 \pm 0.17 \pm 0.72$
$D^0 \rightarrow K_1(1400)^- K^+$	$0.236 \pm 0.008 \pm 0.018$	$0.04 \pm 0.04 \pm 0.09$	$2.82 \pm 0.19 \pm 0.39$
$D^0 \rightarrow [K^*(1680)^0 \bar{K}^*(892)^0]_{L=0}$	$0.823 \pm 0.023 \pm 0.218$	$2.99 \pm 0.03 \pm 0.05$	$2.75 \pm 0.15 \pm 0.19$
$D^0 \rightarrow [\bar{K}^*(1680)^0 K^*(892)^0]_{L=1}$	$1.009 \pm 0.022 \pm 0.276$	$-2.76 \pm 0.02 \pm 0.03$	$2.70 \pm 0.11 \pm 0.09$
$D^0 \rightarrow \bar{K}^*(1680)^0 [K^+ \pi^-]_{L=0}$	$1.379 \pm 0.029 \pm 0.373$	$1.06 \pm 0.02 \pm 0.03$	$2.41 \pm 0.09 \pm 0.27$
$D^0 \rightarrow [\phi(1020)(\rho - \omega)^0]_{L=2}$	$1.311 \pm 0.031 \pm 0.018$	$0.54 \pm 0.02 \pm 0.02$	$2.29 \pm 0.08 \pm 0.08$
$D^0 \rightarrow [K^*(892)^0 \bar{K}^*(892)^0]_{L=2}$	$0.652 \pm 0.018 \pm 0.043$	$2.85 \pm 0.03 \pm 0.04$	$1.85 \pm 0.09 \pm 0.10$
$D^0 \rightarrow \phi(1020)[\pi^+ \pi^-]_{L=0}$	$0.049 \pm 0.001 \pm 0.004$	$-1.71 \pm 0.04 \pm 0.37$	$1.49 \pm 0.09 \pm 0.33$
$D^0 \rightarrow [K^*(1680)^0 \bar{K}^*(892)^0]_{L=1}$	$0.747 \pm 0.021 \pm 0.203$	$0.14 \pm 0.03 \pm 0.04$	$1.48 \pm 0.08 \pm 0.10$
$D^0 \rightarrow [\phi(1020)\rho(1450)^0]_{L=1}$	$0.762 \pm 0.035 \pm 0.068$	$1.17 \pm 0.04 \pm 0.04$	$0.98 \pm 0.09 \pm 0.05$
$D^0 \rightarrow a_0(980)^0 f_2(1270)^0$	$1.524 \pm 0.058 \pm 0.189$	$0.21 \pm 0.04 \pm 0.19$	$0.70 \pm 0.05 \pm 0.08$
$D^0 \rightarrow a_1(1260)^+ \pi^-$	$0.189 \pm 0.011 \pm 0.042$	$-2.84 \pm 0.07 \pm 0.38$	$0.46 \pm 0.05 \pm 0.22$
$D^0 \rightarrow a_1(1260)^- \pi^+$	$0.188 \pm 0.014 \pm 0.031$	$0.18 \pm 0.06 \pm 0.43$	$0.45 \pm 0.06 \pm 0.16$
$D^0 \rightarrow [\phi(1020)(\rho - \omega)^0]_{L=1}$	$0.160 \pm 0.011 \pm 0.005$	$0.28 \pm 0.07 \pm 0.03$	$0.43 \pm 0.05 \pm 0.03$
$D^0 \rightarrow [K^*(1680)^0 \bar{K}^*(892)^0]_{L=2}$	$1.218 \pm 0.089 \pm 0.354$	$-2.44 \pm 0.08 \pm 0.15$	$0.33 \pm 0.05 \pm 0.06$
$D^0 \rightarrow [K^+ K^-]_{L=0} (\rho - \omega)^0$	$0.195 \pm 0.015 \pm 0.035$	$2.95 \pm 0.08 \pm 0.29$	$0.27 \pm 0.04 \pm 0.05$
$D^0 \rightarrow [\phi(1020)f_2(1270)^0]_{L=1}$	$1.388 \pm 0.095 \pm 0.257$	$1.71 \pm 0.06 \pm 0.37$	$0.18 \pm 0.02 \pm 0.07$
$D^0 \rightarrow [K^*(892)^0 \bar{K}_2^*(1430)^0]_{L=1}$	$1.530 \pm 0.086 \pm 0.131$	$2.01 \pm 0.07 \pm 0.09$	$0.18 \pm 0.02 \pm 0.02$
Sum of fit fractions		$129.32 \pm 1.09 \pm 2.38$	
χ^2/ndf		$9242/8121 = 1.14$	

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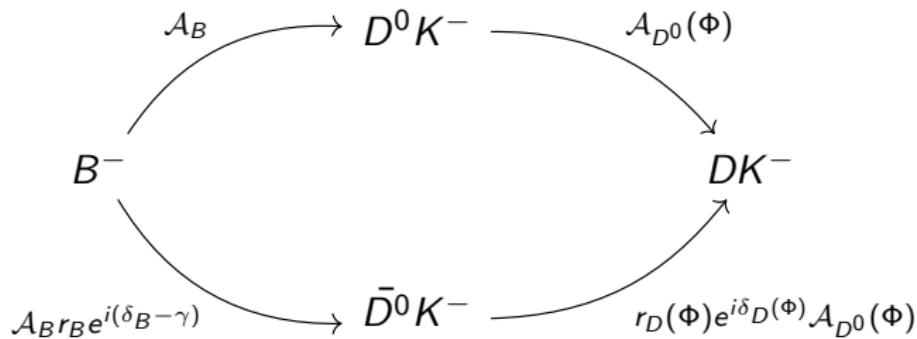
... and I really mean a lot of resonances!

The $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$ decay mode

Our equations suddenly become a lot more complicated

$\mathcal{A}_{D^0}(\Phi)$ now depends on a 5D phase space point Φ

Defining $\mathcal{A}_{\bar{D}^0} = r_D e^{i\delta_D} \mathcal{A}_{D^0}$, r_D and δ_D are now also functions of Φ !



$$|\mathcal{A}(B^-)|^2 \propto 1 + r_B^2 r_D^2(\Phi) + 2r_B r_D(\Phi) \cos(\delta_B - \gamma + \delta_D(\Phi))$$

The $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D K^\pm$ decay mode

$r_D(\Phi)$ and $\delta_D(\Phi)$ can be predicted using the LHCb amplitude model

However, there are many reasons why we should **not** do this:

- ① $r_D(\Phi)$ can be measured directly in data at LHCb
- ② Amplitude models are just models, which may not reflect reality
- ③ In fact, the model is fitted to data that knows nothing about $\delta_D(\Phi)$
- ④ It is impossible to assign an objective error to a model!

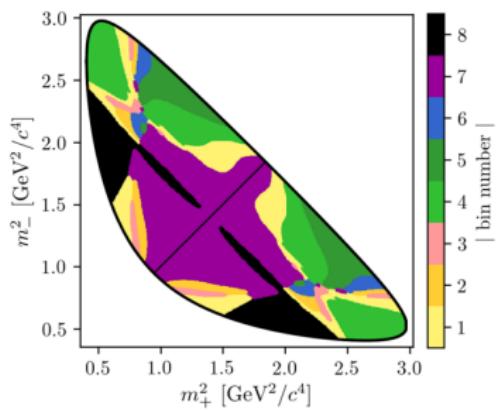
We wish to do a model independent measurement

Binned analysis of the $D \rightarrow K^+K^-\pi^+\pi^-$ mode

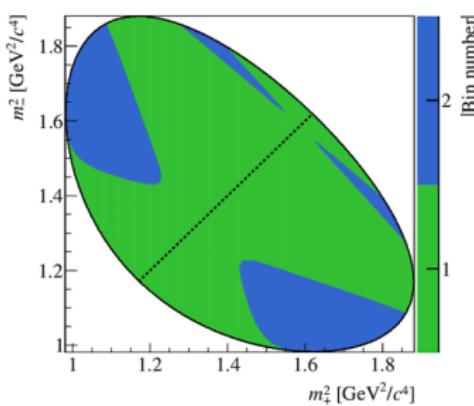
Binned analysis of the $D \rightarrow K^+K^-\pi^+\pi^-$
mode

Binned analysis of the $D \rightarrow K^+K^-\pi^+\pi^-$ mode

- Solution: Split phase space into bins, labelled by $i = 1, 2, \dots$
- Study the CP asymmetry separately in each bin
- For the decays $D^0 \rightarrow K_S^0\pi^+\pi^-$ and $K_S^0K^+K^-$, the binning scheme may be visualised on a Dalitz plot

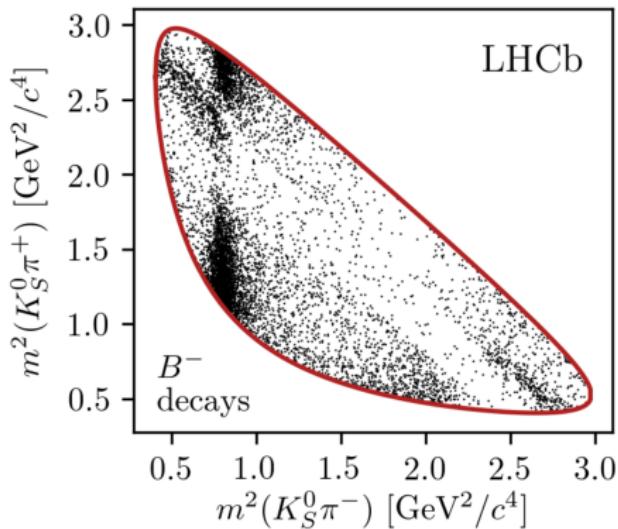


$K_S^0\pi^+\pi^-$

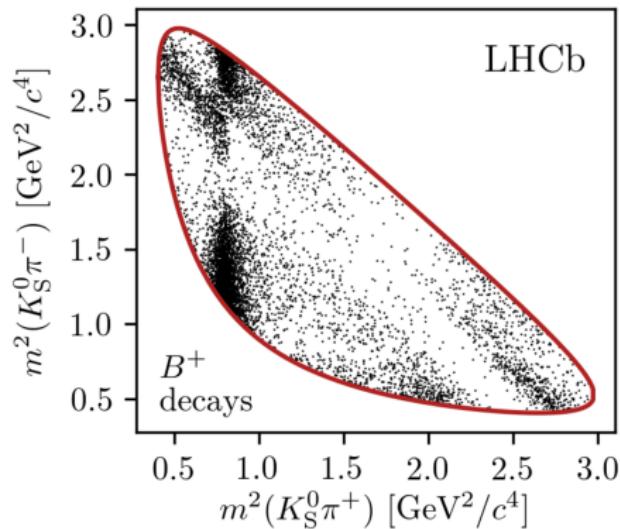


$K_S^0K^+K^-$

Binned analysis of the $D \rightarrow K^+K^-\pi^+\pi^-$ mode



$$B^- \rightarrow [K_S^0\pi^+\pi^-]_D K^-$$



$$B^+ \rightarrow [K_S^0\pi^+\pi^-]_D K^+$$

Can you find the asymmetries?

Binned analysis of the $D \rightarrow K^+K^-\pi^+\pi^-$ mode

Back to rate equation:

$$|\mathcal{A}(B^-)|^2 \propto 1 + r_B^2 r_D^2 + 2r_B r_D (\cos(\delta_B - \gamma) \cos(\delta_D) - \sin(\delta_B - \gamma) \sin(\delta_D))$$

Integrate rate over a local region Φ_i , which we call bin i :

$$N_i^- \propto F_i + r_B^2 \bar{F}_i + 2r_B \sqrt{F_i \bar{F}_i} (\cos(\delta_B - \gamma) c_i - \sin(\delta_B - \gamma) s_i)$$

Amplitude averaged strong phase

$$c_i \equiv \frac{\int_i d\Phi |\mathcal{A}_{D^0}| |\mathcal{A}_{\bar{D}^0}| \cos(\delta_D(\Phi))}{\sqrt{\int d\Phi |\mathcal{A}_{D^0}|^2 \int d\Phi |\mathcal{A}_{\bar{D}^0}|^2}}$$

Binned analysis of the $D \rightarrow K^+K^-\pi^+\pi^-$ mode

To “decouple” the interference effects in B^+ and B^- ,
define the CP violating observables

$$x_{\pm} \equiv r_B \cos(\delta_B \pm \gamma), \quad y_{\pm} \equiv r_B \sin(\delta_B \pm \gamma)$$

Our final equation, which relates the CP observables to
experimentally measured yields, is

$$N_i^- \propto F_i + r_B^2 \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_- c_i - y_- s_i)$$

Amplitude averaged strong phase

$$c_i \equiv \frac{\int_i d\Phi |\mathcal{A}_{D^0}| |\mathcal{A}_{\bar{D}^0}| \cos(\delta_D(\Phi))}{\sqrt{\int d\Phi |\mathcal{A}_{D^0}|^2 \int d\Phi |\mathcal{A}_{\bar{D}^0}|^2}}$$

Binned analysis of the $D \rightarrow K^+K^-\pi^+\pi^-$ mode

Bin yield

$$N_i^- \propto F_i + r_B^2 \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_- c_i - y_- s_i)$$

The strategy for measuring γ is now clear:

- ① Measure bin yields N_i^\pm in $B^\pm \rightarrow [K^+K^-\pi^+\pi^-]_D K^\pm$ decays
- ② Do a likelihood maximisation to determine F_i , \bar{F}_i , c_i , s_i , x_\pm and y_\pm
- ③ From x_\pm and y_\pm , extract r_B , δ_B and γ
- ④ Publish new measurement of γ !

Strong phase input from charm factories

Strong phase input from charm factories

Strong phase input from charm factories

Unfortunately, it is unlikely that this fit will converge...

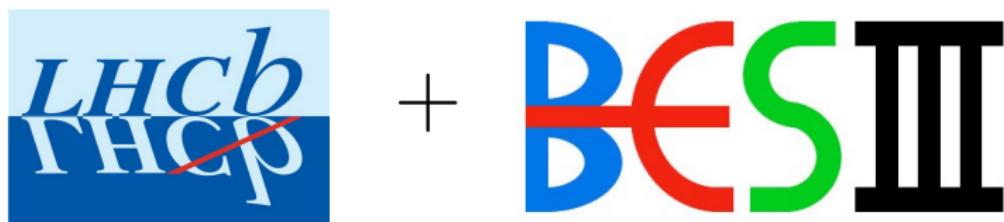
Sensitivity to c_i and s_i is very limited with current statistics

Strong phase input from charm factories

Unfortunately, it is unlikely that this fit will converge...

Sensitivity to c_i and s_i is very limited with current statistics

Instead, we can join forces with BESIII and measure c_i and s_i directly



This has never been done for $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$
More on this later!

Constraining F_i with $B^\pm \rightarrow D\pi^\pm$

Constraining F_i with $B^\pm \rightarrow D\pi^\pm$

Constraining F_i with $B^\pm \rightarrow D\pi^\pm$

- The fractional bin yields F_i are yields in the absence of CP violation
- In principle we can measure these directly at both LHCb and BESIII

Four strategies:

- ① Calculate from amplitude model
- ② Measure in $B^- \rightarrow D^0 \mu^- \bar{\nu}_\mu$ at LHCb
- ③ Measure with flavour tagged D^0 decays at BESIII
- ④ Measure in $B^\pm \rightarrow D\pi^\pm$

Constraining F_i with $B^\pm \rightarrow D\pi^\pm$

- The fractional bin yields F_i are yields in the absence of CP violation
- In principle we can measure these directly at both LHCb and BESIII

Four strategies:

- ① ~~Calculate from amplitude model~~ Avoid model dependence
- ② Measure in $B^- \rightarrow D^0 \mu^- \bar{\nu}_\mu$ at LHCb
- ③ Measure with flavour tagged D^0 decays at BESIII
- ④ Measure in $B^\pm \rightarrow D\pi^\pm$

Constraining F_i with $B^\pm \rightarrow D\pi^\pm$

- The fractional bin yields F_i are yields in the absence of CP violation
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Four strategies:

- ① ~~Calculate from amplitude model~~ Avoid model dependence
- ② ~~Measure in $B^- \rightarrow D^0 \mu^- \bar{\nu}_\mu$ at LHCb~~ Different acceptance effects
- ③ ~~Measure with flavour tagged D^0 decays at BESIII~~
- ④ Measure in $B^\pm \rightarrow D\pi^\pm$

Constraining F_i with $B^\pm \rightarrow D\pi^\pm$

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- ③ ~~Measure with flavour tagged D^0 decays at BESIII~~
- ④ Measure in $B^\pm \rightarrow D\pi^\pm$ Small CPV effects?

Constraining F_i with $B^\pm \rightarrow D\pi^\pm$

- The fractional bin yields F_i are yields in the absence of CP violation
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Four strategies:

- ① ~~Calculate from amplitude model~~ Avoid model dependence
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- ③ ~~Measure with flavour tagged D^0 decays at BESIII~~
- ④ Measure in $B^\pm \rightarrow D\pi^\pm$ Small CPV effects?

No problem, include $B^\pm \rightarrow D\pi^\pm$ as a signal channel

Constraining F_i with $B^\pm \rightarrow D\pi^\pm$

- $B^\pm \rightarrow D\pi^\pm$ has an identical topology to $B^\pm \rightarrow DK^\pm$
- CPV effects are highly suppressed because $r_B^{D\pi} \approx 0.005$
- Branching fraction more than 10 times larger
- As a signal channel, we add another 4 free parameters to our fit:

$$x_\pm^{D\pi} = r_B^{D\pi} \cos(\delta_B^{D\pi} - \gamma), \quad y_\pm^{D\pi} = r_B^{D\pi} \sin(\delta_B^{D\pi} - \gamma)$$

Constraining F_i with $B^\pm \rightarrow D\pi^\pm$

To avoid degeneracy, reduce this to 2 additional parameters using this parameterisation:

$$x_\xi = \text{Re}(\xi), \quad y_\xi = \text{Im}(\xi), \quad \xi = \frac{r_B^{D\pi} e^{i\delta_B^{D\pi}}}{r_B^{DK} e^{i\delta_B^{DK}}}$$

In summary:

- ① Both $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ are signal channels, with x_\pm^{DK} , y_\pm^{DK} , x_ξ and y_ξ as CP observables
- ② $B^\pm \rightarrow DK^\pm$ has lower statistics, but higher CPV effects
- ③ $B^\pm \rightarrow D\pi^\pm$ has higher statistics and constrain F_i in the fit, but sensitivity to CPV is limited

Binning scheme

Binning scheme

Binning scheme

We need to split the phase space into bins

But how do we navigate through a 5D space? How do we decide on the bin boundaries?



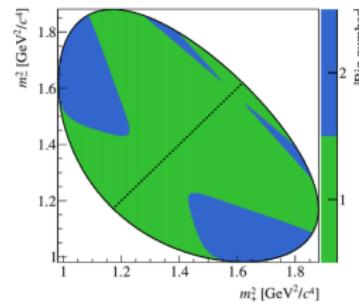
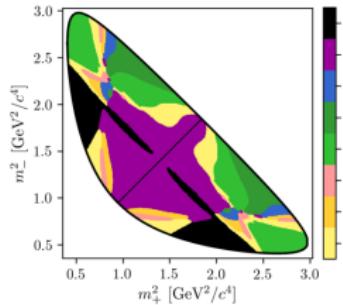
Let the amplitude model guide us!

Binning scheme

Back to the amplitude averaged strong phase:

$$c_i \equiv \frac{\int_i d\Phi |\mathcal{A}_{D^0}| |\mathcal{A}_{\bar{D}^0}| \cos(\delta_D(\Phi))}{\sqrt{\int d\Phi |\mathcal{A}_{D^0}|^2 \int d\Phi |\mathcal{A}_{\bar{D}^0}|^2}}$$

- If the strong phase varies significantly within a bin, the interference effects will be diluted when integrating
- We need to group regions of similar strong phase into the same bin
- This was done for $K_S^0 h^+ h^-$, resulting in colourful “butterfly” plots

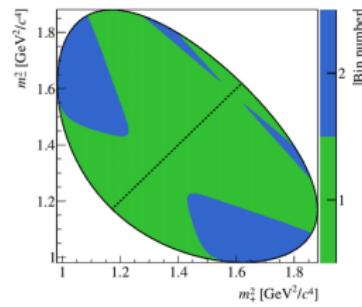
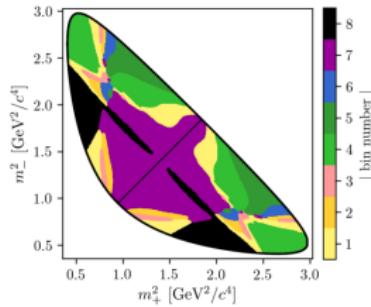


Binning scheme

Back to our yield formula:

$$N_i^- \propto F_i + r_B^2 \bar{F}_i + 2\sqrt{F_i \bar{F}_i} (x_- c_i - y_- s_i)$$

- In the charm system, CP is (approximately) conserved, so each D^0 decay has a corresponding identical CP conjugated decay
- Split each bin i into two “ CP mirror bins”, labelled by $\pm i$
- In $K_S^0 h^+ h^-$, this is indicated by the black symmetry line
- Under CP , $\delta_D \rightarrow -\delta_D$, so $c_i \rightarrow c_i$ and $s_i \rightarrow -s_i$



Binning scheme

A binning scheme must satisfy the following:

- Minimal dilution of strong phases when integrating over bins
- Enhance interference between $B^\pm \rightarrow D^0 K^\pm$ and $B^\pm \rightarrow \bar{D}^0 K^\pm$

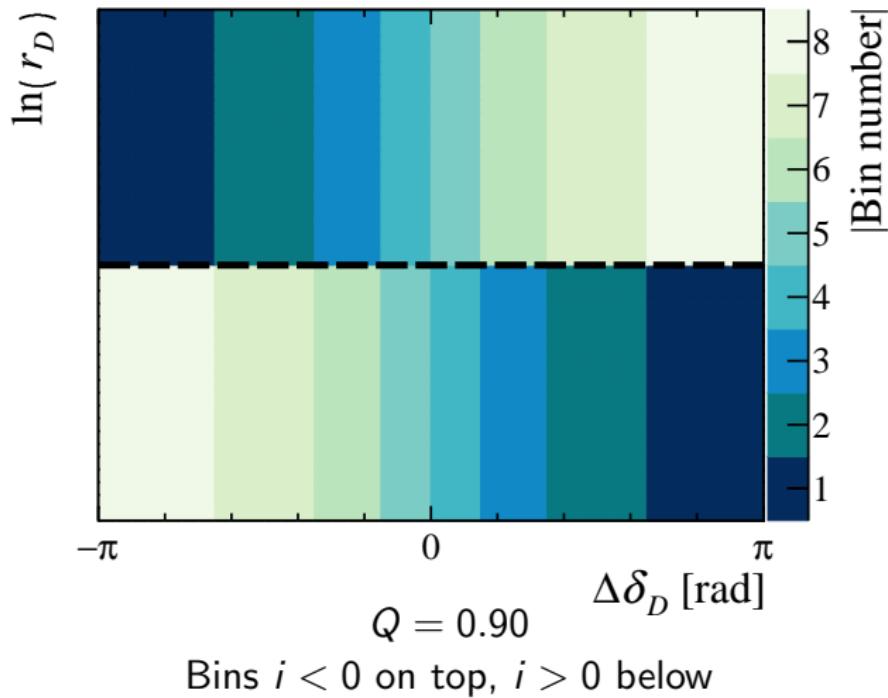
How to bin a 5-dimensional phase space?

- ① For each B^\pm candidate, use the amplitude model to calculate

$$\frac{\mathcal{A}(D^0)}{\mathcal{A}(\bar{D}^0)} = r_D e^{i\delta_D}$$

- ② Split δ_D into uniformly spaced bins
- ③ Use the symmetry line $r_D = 1$ to separate bin $+i$ from $-i$
- ④ Optimise the binning scheme by adjusting the bin boundaries in δ_D

Binning scheme



Mass fits and yield extraction

Mass fits and yield extraction

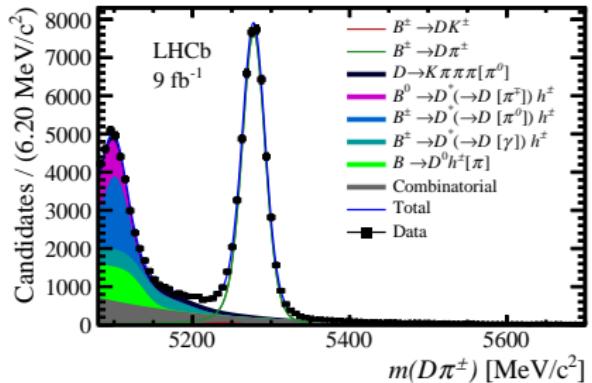
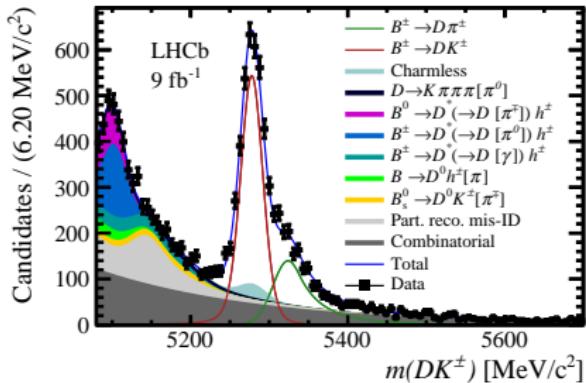
Mass fits and yield extraction

In the end, this analysis is a counting experiment

Counting strategy:

- ① Perform a “global fit” of all B^\pm candidates
- ② Fix all shape parameters
- ③ Sort B^\pm candidates by charge and bins
- ④ Perform a “ CP fit” simultaneously, but only let bin yields float
- ⑤ From the bin yields, determine x_\pm^{DK} , y_\pm^{DK} , x_ξ and y_ξ

Mass fits and yield extraction



Signal yield:

$$B^\pm \rightarrow DK^\pm : 3026 \pm 38$$

$$B^\pm \rightarrow D\pi^\pm : 44\,349 \pm 218$$

CP fit setup

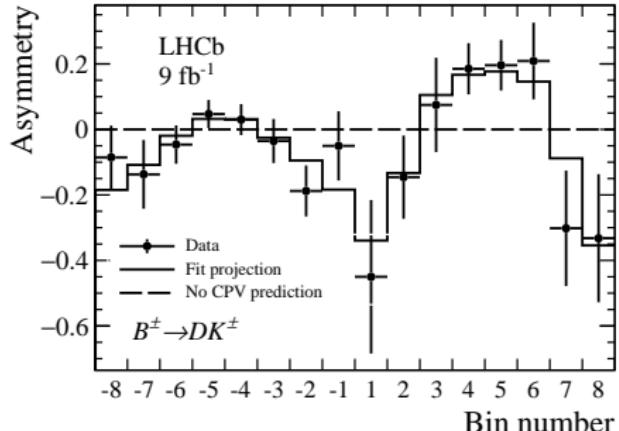
- No measurement of c_i and s_i available yet, use model predictions
 - Fix mass shape from global fit
 - Split by B^\pm charge and D phase space bins (64 categories)
-
- ① CP observables x_{\pm}^{DK} , y_{\pm}^{DK} $x_{\xi}^{D\pi}$, $y_{\xi}^{D\pi}$ (6 parameters)
 - ② Fractional bin yields F_i (15 parameters)
 - ③ Low mass and combinatorial background (128 parameters)
 - ④ Yield normalisation (4 parameters)

In total: 153 free parameters

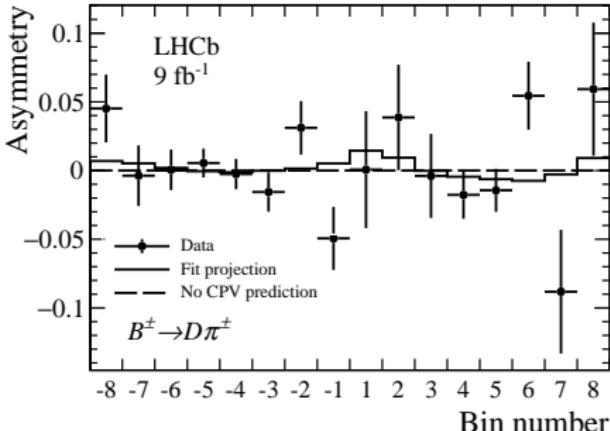
CP fit results and γ

CP fit results and γ

Fractional bin asymmetries



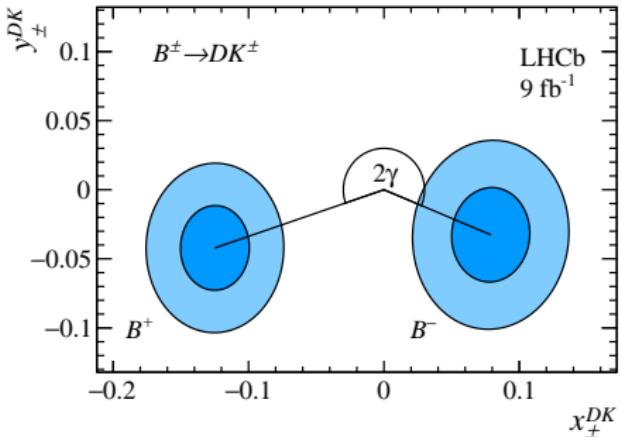
(a) $B^\pm \rightarrow DK^\pm$



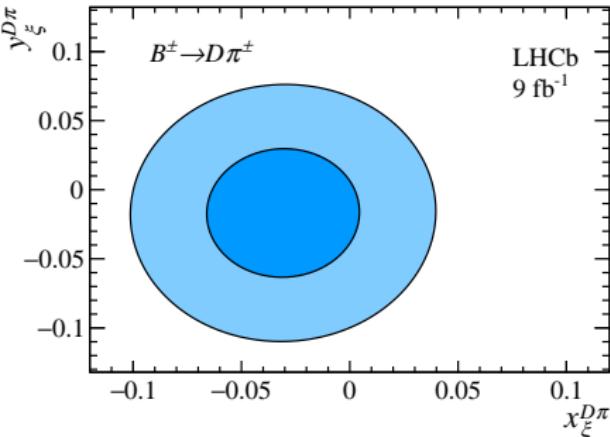
(b) $B^\pm \rightarrow D\pi^\pm$

- Useful cross check to compare measured bin asymmetries against bin asymmetries predicted by the fitted CP observables
- The $B^\pm \rightarrow DK^\pm$ mode show non-zero bin asymmetries, and the non-trivial distribution is driven by the change in strong phases across phase space

CP fit results



(a) x_{\pm}^{DK} vs y_{\pm}^{DK}



(b) $x_{\xi}^{D\pi}$ vs $y_{\xi}^{D\pi}$

$$x_{\pm}^{DK} = r_B \cos(\delta_B \pm \gamma)$$

$$y_{\pm}^{DK} = r_B \sin(\delta_B \pm \gamma)$$

- The $B^{\pm} \rightarrow DK^{\pm}$ contours are distinct, indicating CP violation
- The $B^{\pm} \rightarrow D\pi^{\pm}$ mode has very low sensitivity to CP violation

Interpretation of γ

We can interpret our CP observables in terms of the physics parameters γ , r_B^{DK} , δ_B^{DK} , $r_B^{D\pi}$, $\delta_B^{D\pi}$

$$\gamma = (116_{-14}^{+12})^\circ,$$

$$\delta_B^{DK} = (81_{-13}^{+14})^\circ,$$

$$r_B^{DK} = 0.110_{-0.020}^{+0.020},$$

$$\delta_B^{D\pi} = (298_{-118}^{+62})^\circ,$$

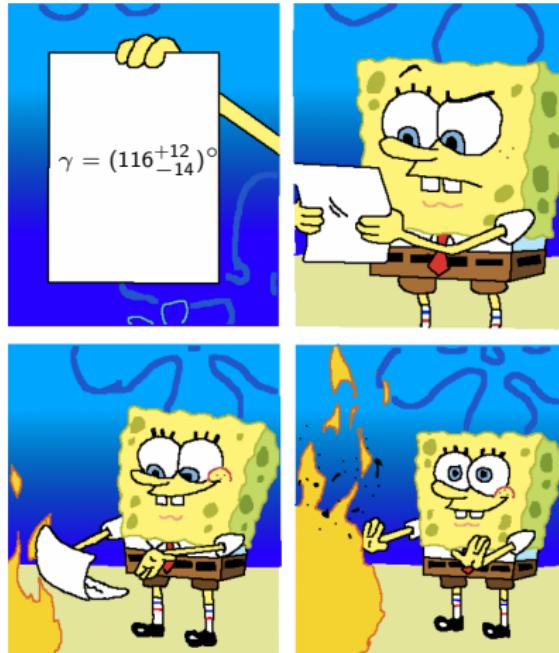
$$r_B^{D\pi} = 0.0041_{-0.0041}^{+0.0054},$$

However, the latest γ and charm combination result is:

$$\gamma = (63.8_{-3.7}^{+3.5})^\circ$$

What went wrong?!

Interpretation of γ



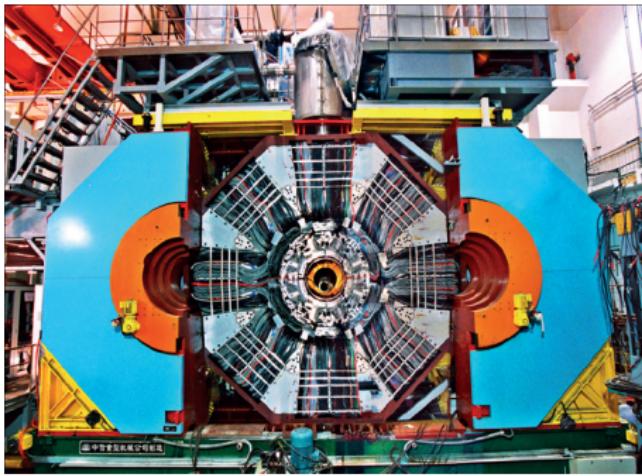
Do we trust the model predicted c_i and s_i , or their uncertainties? No!
Let's go and measure c_i and s_i at BESIII!

Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

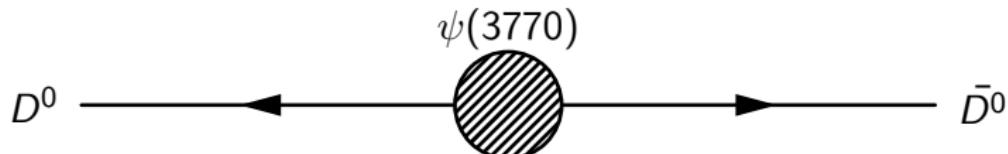
Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

- BESIII: Beijing Spectrometer III, a detector at the Beijing Electron-Positron Collider II, located at IHEP
- e^+e^- collider at the $\psi(3770) \rightarrow D^0\bar{D}^0$ threshold
 - 2010-2011: 3 fb^{-1}
 - 2022: 5 fb^{-1}
 - Expect 20 fb^{-1} in total by end of 2024

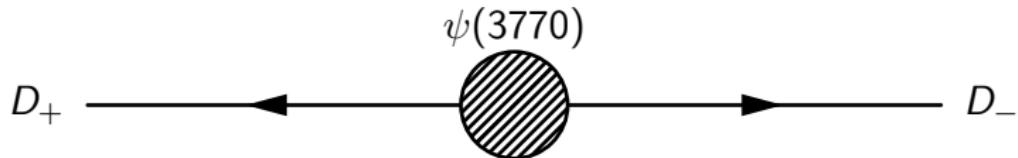


Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

- Double-tag analysis: Reconstruct signal ($KK\pi\pi$) and tag mode
- $D^0\bar{D}^0$ pair is quantum correlated



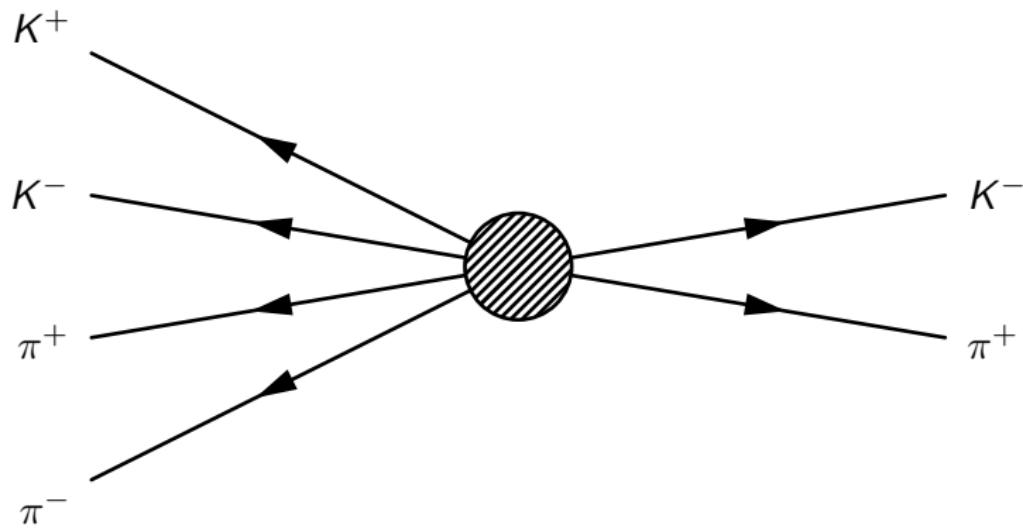
- Equivalently, we can consider D_+D_-
 - $D_{\pm} = \frac{1}{\sqrt{2}}(D^0 \pm \bar{D}^0)$ are CP eigenstates



The DD pair is quantum correlated, spooky action at a distance!

Strong-phase in quantum correlated $D^0\bar{D}^0$ decays

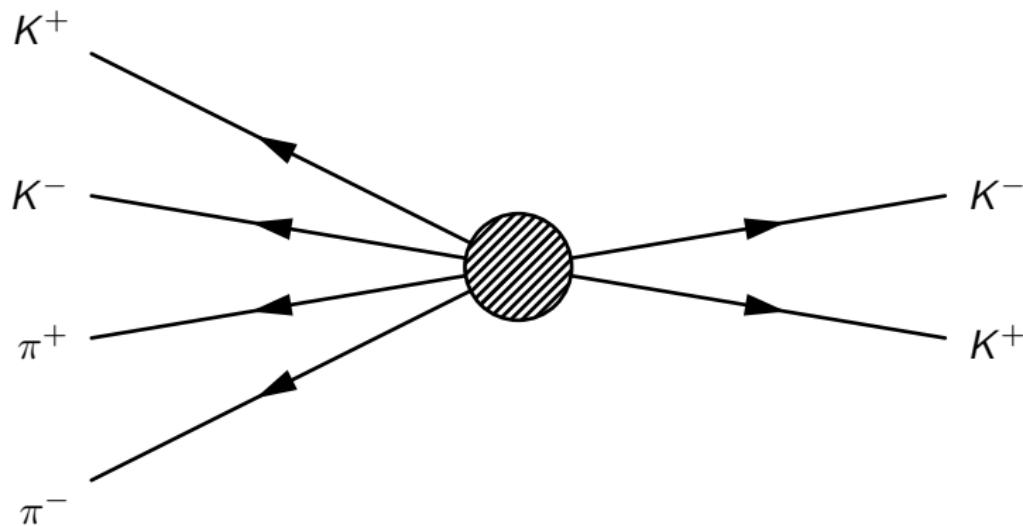
- Tag mode can be a flavour tag
 - $K^-\pi^+$, $K^-\pi^+\pi^0$, $K^-\pi^+\pi^-\pi^+$, $K^-e^+\nu_e$



Flavour tags do not exhibit quantum correlation effects

Strong-phases in quantum correlated $D^0\bar{D}^0$ decays

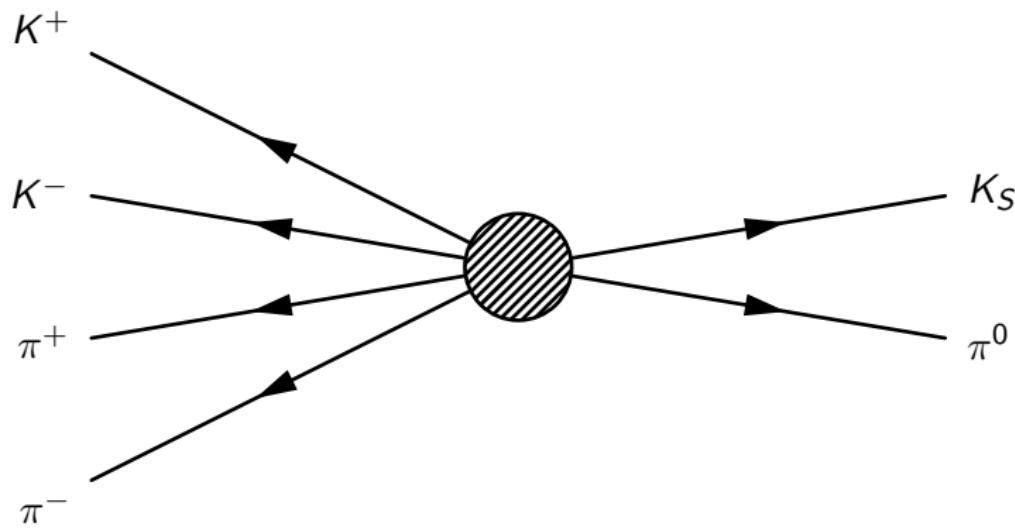
- Tag mode can be a CP even tag
 - $KK, \pi\pi, \pi\pi\pi^0, K_S\pi^0\pi^0, K_L\pi^0, K_L\omega$



$D \rightarrow K^+K^-$, which is CP even, forces $D \rightarrow K^+K^-\pi^+\pi^-$ to be CP odd

Strong-phase in quantum correlated $D^0\bar{D}^0$ decays

- Tag mode can be a CP odd tag
 - $K_S\pi^0, K_S\omega, K_S\eta, K_S\eta', K_L\pi^0\pi^0$

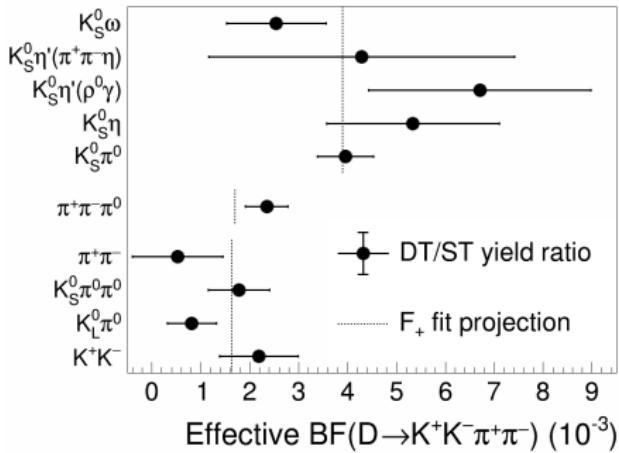


$D \rightarrow K_S^0\pi^0$, which is CP odd, forces $D \rightarrow K^+K^-\pi^+\pi^-$ to be CP even

Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

Quantum correlation can modify the effective branching fraction:

$$\frac{N^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi)(1 \pm c_1)$$



arXiv:2212.06489

c_1 is the cosine of the strong phase, averaged over the whole phase space

Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

Our next task is to change the phase space inclusive analysis,

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi) \quad (\text{flavour tag})$$

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi)(1 \pm c_i) \quad (\text{CP tag})$$

into a binned phase space analysis:

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi)F_i \quad (\text{flavour tag})$$

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi)(F_i + \bar{F}_i \pm 2\sqrt{F_i \bar{F}_i}c_i) \quad (\text{CP tag})$$

- ① F_i : Measure using flavour tags
- ② c_i : Determine from asymmetry of CP even and odd tags

Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

Our next task is to change the phase space inclusive analysis,

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi) \quad (\text{flavour tag})$$

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi)(1 \pm c_i) \quad (\text{CP tag})$$

into a binned phase space analysis:

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$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi)(F_i + \bar{F}_i \pm 2\sqrt{F_i \bar{F}_i}c_i) \quad (\text{CP tag})$$

- ① F_i : Measure using flavour tags
- ② c_i : Determine from asymmetry of CP even and odd tags
- ③ s_i : Analogous to c_i , but requires binning of tag mode

Strong phase analysis of $D^0 \rightarrow K^+K^-\pi^+\pi^-$ at BESIII

Our next task is to change the phase space inclusive analysis,

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi\pi) \quad (\text{f' tag})$$

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KK\pi) \quad (\text{CP tag})$$

into a binned phase space analysis:

$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow KV) \quad (\text{flavour tag})$$

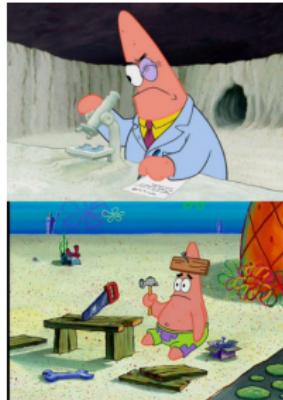
$$\frac{N_i^{\text{DT}}}{N^{\text{ST}}} = \mathcal{B}(D^0 \rightarrow K\bar{K}) \langle F_i + \bar{F}_i \pm 2\sqrt{F_i\bar{F}_i}c_i \rangle \quad (\text{CP tag})$$

- Work in progress!**
- ① F_i : Measure using our tags
 - ② c_i : Determine from asymmetry of CP even and odd tags
 - ③ s_i : Analogous to c_i , but requires binning of tag mode

Summary and conclusion

- ① I have presented a CPV study of $B^\pm \rightarrow [K^+ K^- \pi^+ \pi^-]_D h^\pm$
- ② Multi-body decays, such as $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$, have a great potential for measuring γ
- ③ The optimised binning scheme, developed with an amplitude model, successfully identified regions with large, local CP asymmetries

- ④ However, amplitude model predictions of δ_D should not be trusted



Making binning
scheme with
amplitude model

Predicting strong
phases with
amplitude model

Summary and conclusion

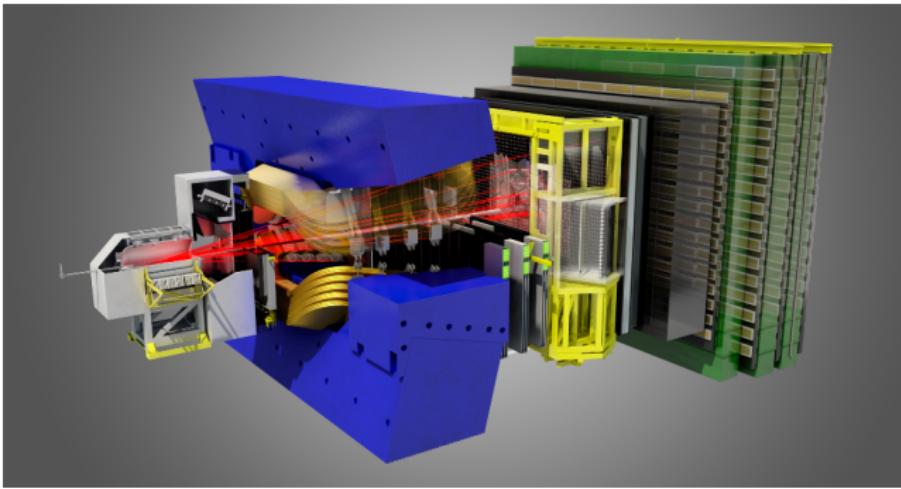
- ⑤ The fit results, using model predicted strong phases, were found to have a 3σ tension with the current LHCb combination
- ⑥ External inputs from charm factories, such as BESIII, are crucial to constrain charm strong phases
- ⑦ Combined, the LHCb and BESIII analyses will lead to the first model independent measurement of γ in this channel
- ⑧ Work is ongoing in similar four-body modes:
 - $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$
 - $D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0$

Thanks for your attention!

Backup slides

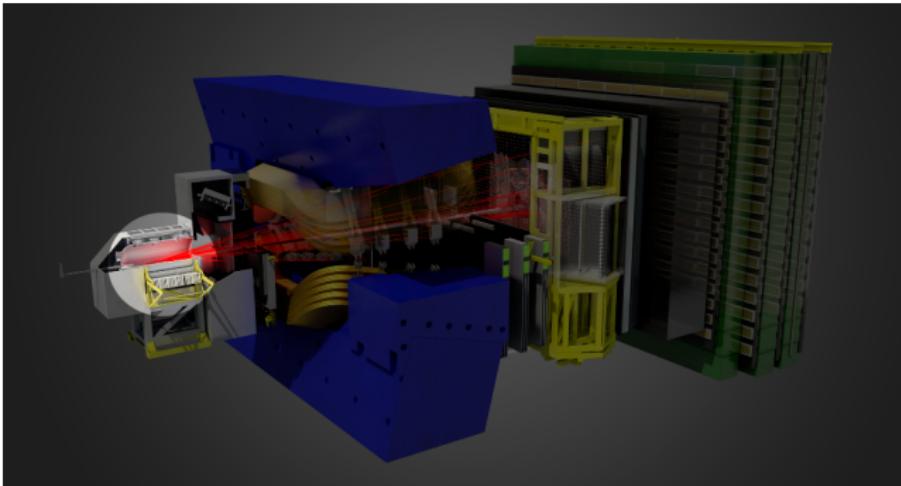
The LHCb detector

The LHCb detector



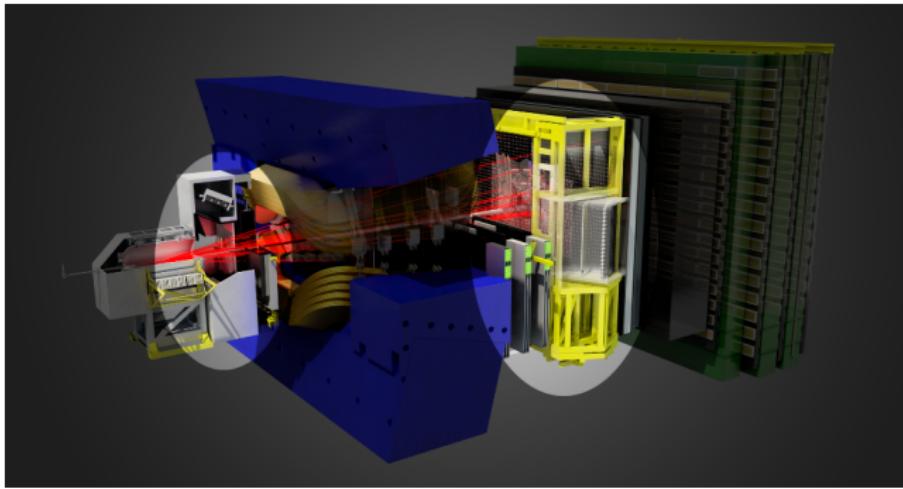
LHCb: A beauty experiment with a lot of charm

The LHCb detector



VELO: Vertex locator to reconstruct B and D vertices

The LHCb detector



RICH: Identify B and D daughter particles

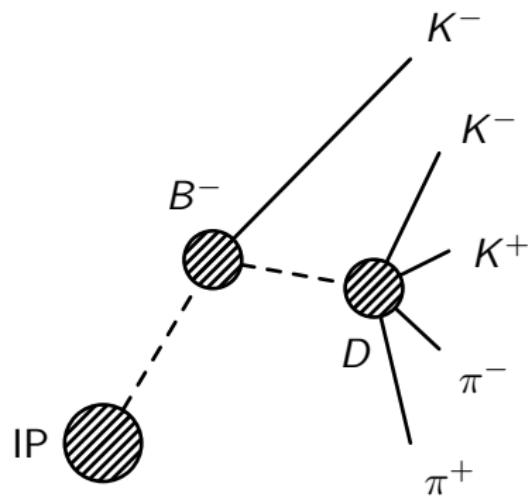
Event selection

Event selection

Decay topology

Look for:

- ① 5 charged tracks
- ② Displaced B vertex
- ③ 1 bachelor track with good PID information
- ④ Displaced D vertex with invariant mass within 25 MeV of the D^0 mass



Event selection

Offline selection has 3 stages

Initial cuts:

- ① Invariant D and B mass cuts
- ② Momentum and RICH requirements

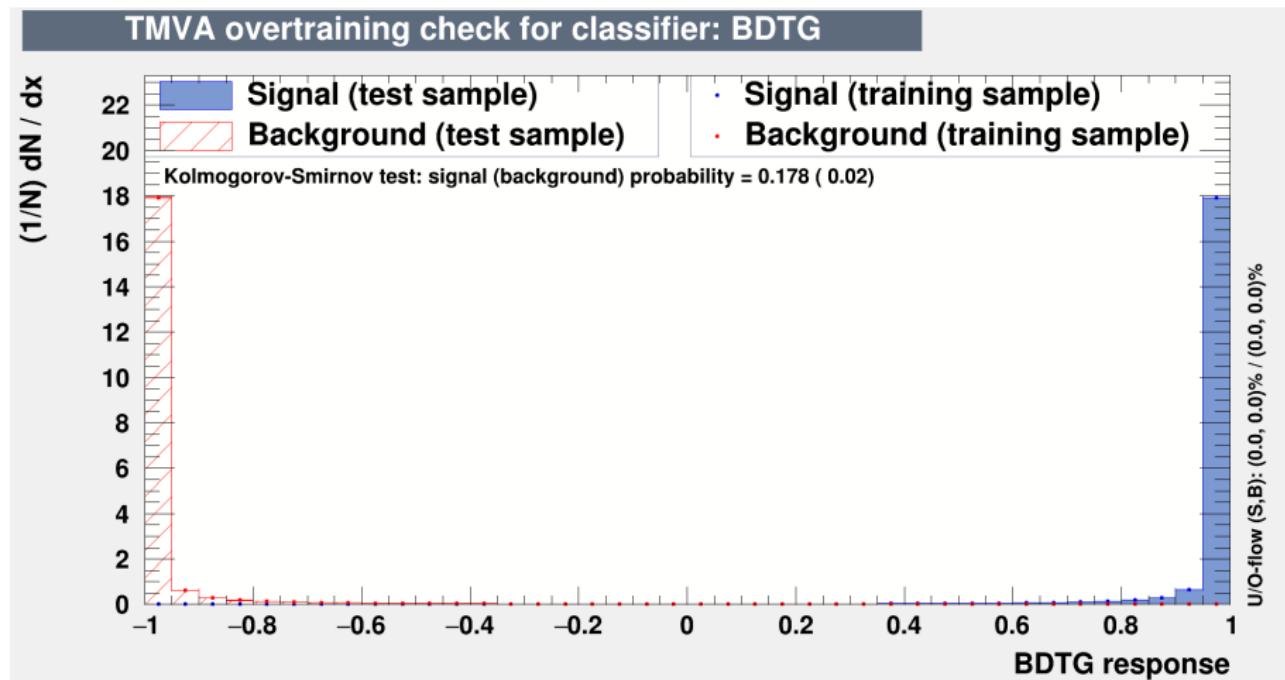
Boosted Decision Tree (BDT)

- Signal sample: Simulation samples
- Background sample: Upper B mass sideband
- 28 variables describing kinematics, impact parameters, vertex quality

Final selection

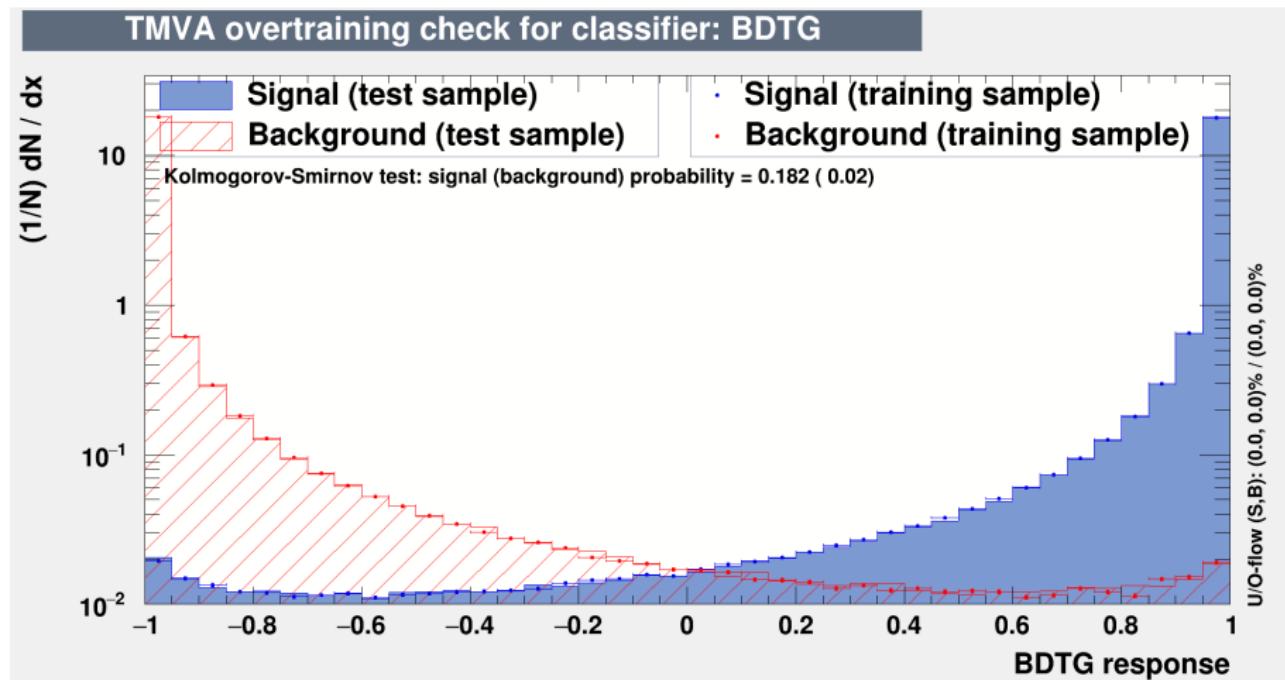
- ① D Flight distance
- ② Particle Identification of bachelor
- ③ K_S^0 veto

Event selection



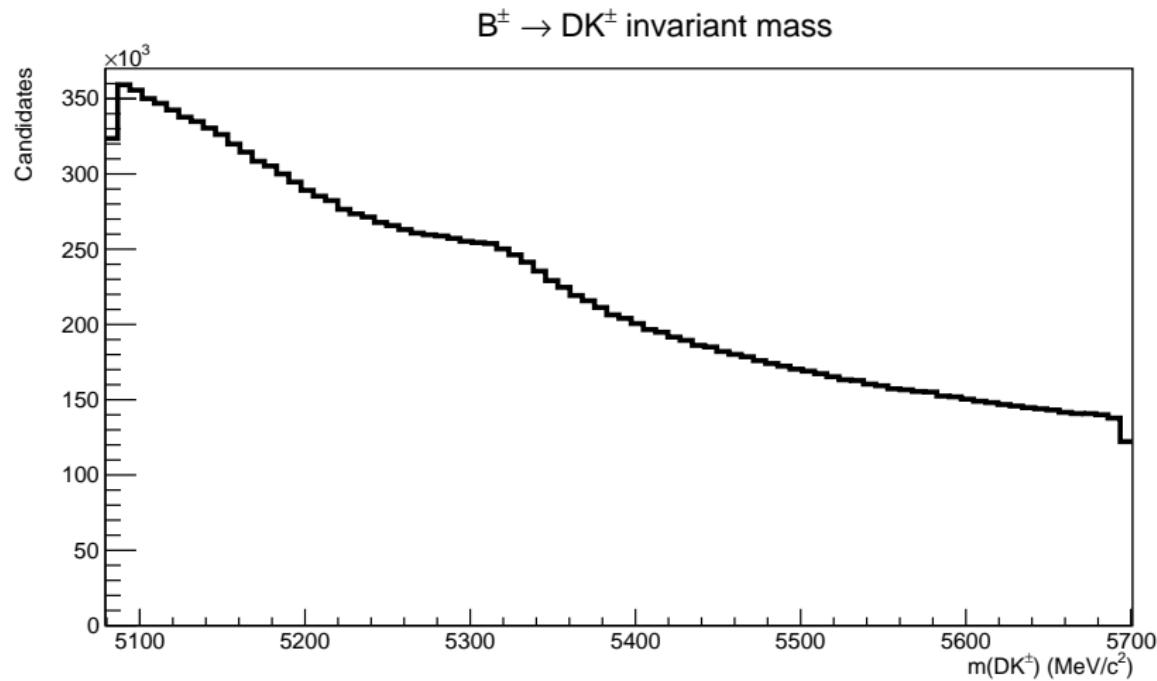
BDT is highly efficient at rejecting combinatorial background

Event selection



Very important, combinatorial background is large in multi-body decays

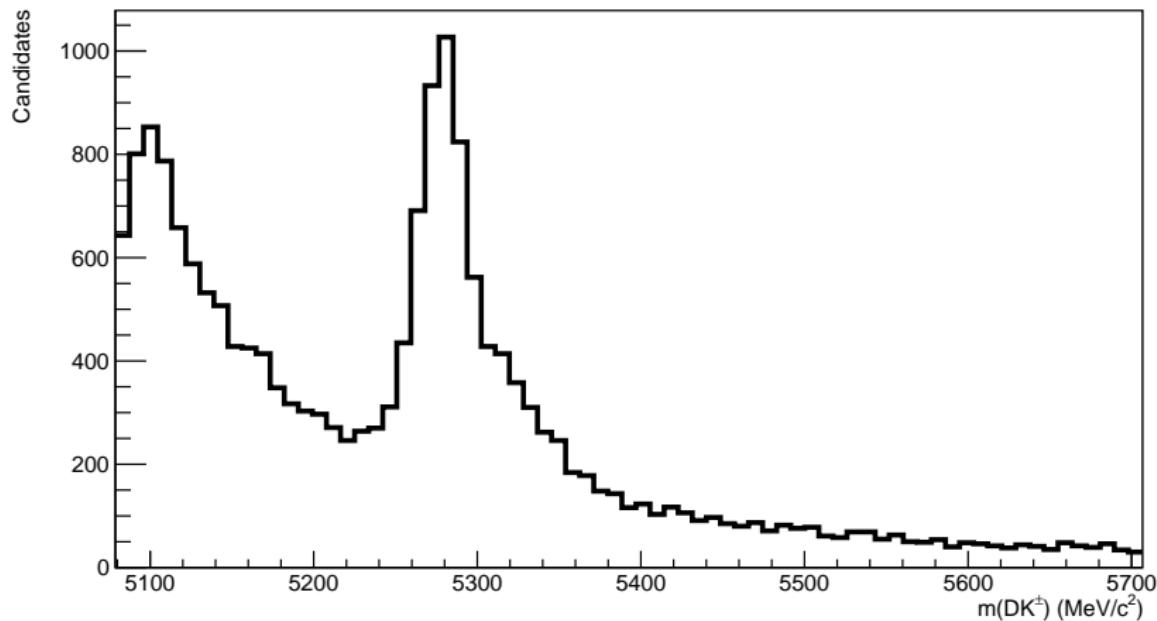
Event selection



The invariant B mass, after online selection, show no visible signal...

Event selection

$B^\pm \rightarrow D K^\pm$ invariant mass



... but the BDT does a great job cleaning this up!