SUPPLEMENTARY INFORMATION FOR
DO MELT RATE DYNAMICS OF INNER GREENLAND APPROACH THOSE OF COASTAL ONES?

PREPRINT SUPPLEMENT

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This pdf includes:

- Supplementary text
- Figure S1
- SI references

Other supplementary materials for this manuscript include the following:

- The empirical Greenland melt rate time series are freely available at `https://www.nature.com/articles/s41586-018-0752-4#Sec14`.
- The open-source package *antiCPy* is available at `https://github.com/MartinHessler/antiCPy` under a *GNU General Public License v3.0* and documented at `https://anticpy.readthedocs.io`.

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Supplementary Text and Figure

S1 Skewness Hypothesis Test

The Python function *scipy.stats.skewtest* [1, 2] is utilized to conduct a two-sided test for skewness in the distribution of the GrIS melt rate data. This test examines the hypothesis $\mathcal{H}_0$: *The data are drawn from the corresponding Gaussian distribution* by comparing the estimated skewness to zero, which is expected for a Gaussian distribution. If $\mathcal{H}_0$ is not rejected, the test does not assert the probability in favour of $\mathcal{H}_0$. Instead, it serves as supporting evidence for weak effects of the absence of significant skewness in the data distribution.

S2 Melt Rate Analysis With Log Transformation to Account for Skewness

We round out our analysis of the GrIS melt rates by presenting the BLE results for the CWG and NU melt rates after preprocessing, including a log transformation to address the skewness in the raw data. The transformation follows the methods described in the original article [3] to guarantee an easy comparison of the results. The BLE analysis is shown in Figure S1. Aside from a quantitative shift, the overall findings remain consistent with those obtained without the log transformation. This consistency is expected because the BLE primarily depends on the PDF of the increments, which are not significantly skewed within each rolling window. Nevertheless, the first differences of the whole CWG melt rates are found to be significantly skewed (p-value $p < 0.001$) at the 95 % confidence level (cf. Section S1) for details on the skewness test). The log transformation accounts for this fact. After the transformation, the p-value is only slightly significant at $p = 0.048$. However, apart from a quantitative shift, the overall BLE results of both melt rate time series remain nearly identical. Note that the significant increase in the NU BLE noise level observed during the early 2010s is suppressed by the logarithmic transformation. Since the increments of the NU time series are already not skewed without the transformation, computing the logarithm predominantly dampens the overall fluctuations. Essentially, this transformation reduces the noise level increase observed without it.

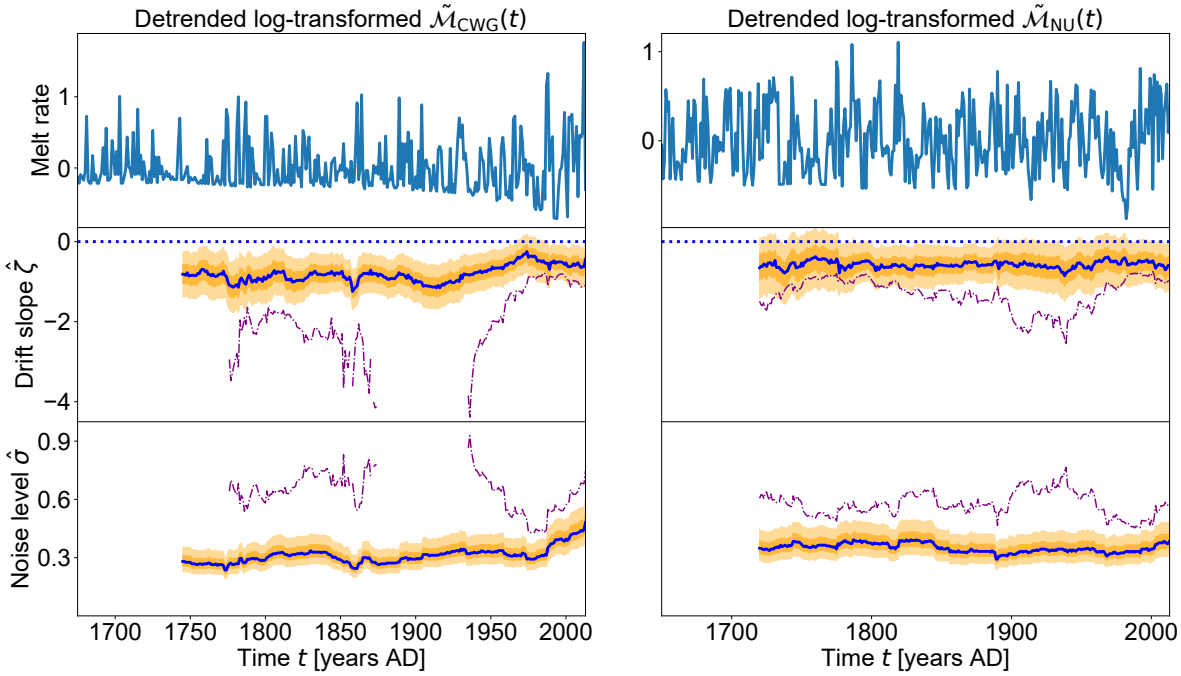

Figure S1: BLE analysis of log-transformed GrIS melt rates for an easy comparison with the results from Ref. [3]. Apart from an expected quantitative shift, the results remain largely unchanged. However, note that the logarithmic transformation of the NU melt rates, which is not necessary for the BLE, almost completely suppresses the significant increase in the noise level that can be observed without it. Additionally, the purple dashed-dotted lines show the OUE estimates for comparison. The log transformation does not enhance their overall applicability.

SI References

[1] Skipper Seabold and Josef Perktold. statsmodels: Econometric and statistical modeling with python. In *9th Python in Science Conference*, 2010.

[2] Ralph B. D'Agostino and Albert J. Belanger. A suggestion for using powerful and informative tests of normality. *The American Statistician*, 44:316–321, 1990.

[3] Niklas Boers and Martin Rypdal. Critical slowing down suggests that the western greenland ice sheet is close to a tipping point. *Proceedings of the National Academy of Sciences*, 118(21):e2024192118, 5 2021.