Supplementary material: Quantifying resilience and the risk of regime shifts under realistic noise conditions

Preprint Supplement

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In order to isolate the influence of the seasonality of the data regarding the leading indicator estimation, the Bayesian model comparison is repeated for the measures computed on the datasets detrended by subtracting a Gaussian kernel smoothing with kernelwidth of 150. The smoothing is performed by the Python function *scipy.ndimage.filters.gaussian_filter* Virtanen et al. [2020]. The results considering a linear model $\mathcal{M}_1$ with positive slope are presented in table 1 and complemented by the Bayes factors of the skewness based on a linear model $\mathcal{M}_1$ with negative slope in table 2. By comparing the tables in the main article without any preprocessing to the detrended ones shown here, we can deduce that the influence of the trend component of the time series is negligible in contrast to its seasonality.

For completeness of the presented analysis, the Bayesian model comparison of the skewness is also performed assuming negative slopes for the linear model $\mathcal{M}_1$ without and with deseasonalization. The results, shown in table 3 and 4, confirm the calculations stated in the main article.

Furthermore, we present the statistical measures of each considered case for inspection by eye in the figures 1, 2 and 3 without deseasonalization, with deseasonalization and with detrending, respectively. Most of the cases under study do not exhibit a clear trend of the kurtosis which is therefore excluded from further analysis in the main article. In the end, the generally late increase of the standard deviation as well as the improved clear trends of autocorrelation and skewness in the deseasonalized cases is underlined by the results of figures 1, 2 and 3.

References

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	white noise			pink noise ($\phi = 0.53$)			red noise ($\phi = 0.92$)		
noise level	$\sigma = 0.1$	$\sigma = 2.2$	$\sigma = 4.5$	$\sigma = 0.1$	$\sigma = 2.2$	$\sigma = 4.5$	$\sigma = 0.1$	$\sigma = 2.2$	$\sigma = 4.5$
indicator $\underline{\mathcal{I}}$	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}
slope ζ	∞ 0	∞ 0	∞ 0	∞ 0	$2.3 \cdot 10^{140}$ $4.4 \cdot 10^{-141}$	$1.5 \cdot 10^{145}$ $6.5 \cdot 10^{-146}$	∞ 0	∞ 0	$1.0 \cdot 10^{114}$ 9.8^{-115}
AR1	14 $7.0 \cdot 10^{-2}$	$7.6 \cdot 10^{7}$ $1.3 \cdot 10^{-8}$	$2.4 \cdot 10^{7}$ $4.1 \cdot 10^{-8}$	11 $9.0 \cdot 10^{-2}$	4.8 0.21	1.2 0.85	11 $9.0 \cdot 10^{-2}$	5.3 0.19	2.3 0.43
std $\hat{\sigma}$	15 $6.6 \cdot 10^{-2}$	15 $6.9 \cdot 10^{-2}$	3.6 $2.8 \cdot 10^{-2}$	15 $6.7 \cdot 10^{-2}$	9.3 0.11	$2.4 \cdot 10^{3}$ $4.1 \cdot 10^{-4}$	15 $6.8 \cdot 10^{-2}$	53 $1.9 \cdot 10^{-2}$	19 $5.4 \cdot 10^{-2}$
skewness γ	$6.5 \cdot 10^{6}$ $1.5 \cdot 10^{-7}$	$5.4 \cdot 10^{5}$ $1.9 \cdot 10^{-6}$	0.67 1.5	$1.0 \cdot 10^{7}$ $9.8 \cdot 10^{-8}$	97 $1.0 \cdot 10^{-2}$	270 $3.7 \cdot 10^{-3}$	$1.7 \cdot 10^{7}$ $6.0 \cdot 10^{-8}$	54 $1.9 \cdot 10^{-2}$	0.99 1.0

Table 1: Summary of the Bayes factors comparing a linear model $\mathcal{M}_1$ with positive slope to a constant model $\mathcal{M}_2$ for the drift slope ζ, the AR1, the std $\hat{\sigma}$ and the skewness γ with detrending of the data. The kurtosis is excluded because of its non-monotone behaviour. Green tiles mark a $BF_{12} > 100$ which is the threshold for a significant leading indicator trend. Grey tiles mark insignificant results. The constant model $\mathcal{M}_2$ is never preferred in the analysis. Infinite Bayes factors result from one model with evidence zero which leads to preferring the finite evidence model. Apart from slightly different values the significance does not change compared to the Bayes factors of the original data. This confirms the conclusion that the seasonality has a predominant importance for the calculation of the standard leading indicators in the considered ecological model.

	white noise			pink noise ($\phi = 0.53$)			red noise ($\phi = 0.92$)		
noise level	$\sigma = 0.1$	$\sigma = 2.2$	$\sigma = 4.5$	$\sigma = 0.1$	$\sigma = 2.2$	$\sigma = 4.5$	$\sigma = 0.1$	$\sigma = 2.2$	$\sigma = 4.5$
indicator $\underline{\mathcal{I}}$	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}
skewness γ	$1.7 \cdot 10^{-2}$ 60	$2.0 \cdot 10^{-2}$ 50	0.14 6.8	$1.5 \cdot 10^{-2}$ 67	$3.7 \cdot 10^{-2}$ 27	$2.0 \cdot 10^{-2}$ 49	$1.5 \cdot 10^{-2}$ 69	$2.8 \cdot 10^{-2}$ 36	$4.4 \cdot 10^{-2}$ 23

Table 2: Summary of the Bayes factors comparing a linear model $\mathcal{M}_1$ with negative slope to a constant model $\mathcal{M}_2$ for the skewness γ for various noise types and levels with detrending of the data. Green tiles mark a $BF_{12} > 100$ which is the threshold for a significant leading indicator trend. Grey tiles mark insignificant results.

	white noise			pink noise ($\phi = 0.53$)			red noise ($\phi = 0.92$)		
noise level	$\sigma = 0.1$	$\sigma = 2.2$	$\sigma = 4.5$	$\sigma = 0.1$	$\sigma = 2.2$	$\sigma = 4.5$	$\sigma = 0.1$	$\sigma = 2.2$	$\sigma = 4.5$
indicator $\underline{\mathcal{I}}$	BF_{12}	BF_{12}	BF_{12}	BF_{12}	BF_{12}	BF_{12}	BF_{12}	BF_{12}	BF_{12}
	BF_{21}	BF_{21}	BF_{21}	BF_{21}	BF_{21}	BF_{21}	BF_{21}	BF_{21}	BF_{21}
skewness γ	$1.5 \cdot 10^{-2}$	$1.9 \cdot 10^{-2}$	0.12	$1.4 \cdot 10^{-2}$	$3.6 \cdot 10^{-2}$	$1.5 \cdot 10^{-2}$	$1.4 \cdot 10^{-2}$	$2.6 \cdot 10^{-2}$	$6.5 \cdot 10^{-2}$
	65	52	8.6	70	28	67	71	39	15

Table 3: Bayesian model comparison with a linear model $\mathcal{M}_1$ with negative slope and a constant model $\mathcal{M}_2$ for the skewness computed on the data without deseasonalization. Grey tiles mark insignificant results, i.e. $BF_{ij} \leq 100$.

	white noise			pink noise ($\phi = 0.53$)			red noise ($\phi = 0.92$)		
noise level	$\sigma = 0.1$	$\sigma = 2.2$	$\sigma = 4.5$	$\sigma = 0.1$	$\sigma = 2.2$	$\sigma = 4.5$	$\sigma = 0.1$	$\sigma = 2.2$	$\sigma = 4.5$
indicator $\mathcal{I}$	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}	BF_{12} BF_{21}
skewness γ	$5.3 \cdot 10^{-2}$ 19	0.16 6.1	0.19 5.2	$4.7 \cdot 10^{-3}$ 210	$5.4 \cdot 10^{-3}$ 180	$4.8 \cdot 10^{-3}$ 210	$4.7 \cdot 10^{-3}$ 210	$5.1 \cdot 10^{-3}$ 200	$4.7 \cdot 10^{-3}$ 210

Table 4: Bayesian model comparison with a linear model $\mathcal{M}_1$ with negative slope and a constant model $\mathcal{M}_2$ for the skewness computed on the data with deseasonalization. Orange tiles mark a $BF_{21} > 100$ which is the threshold for preferring the constant model. Grey tiles mark insignificant results.

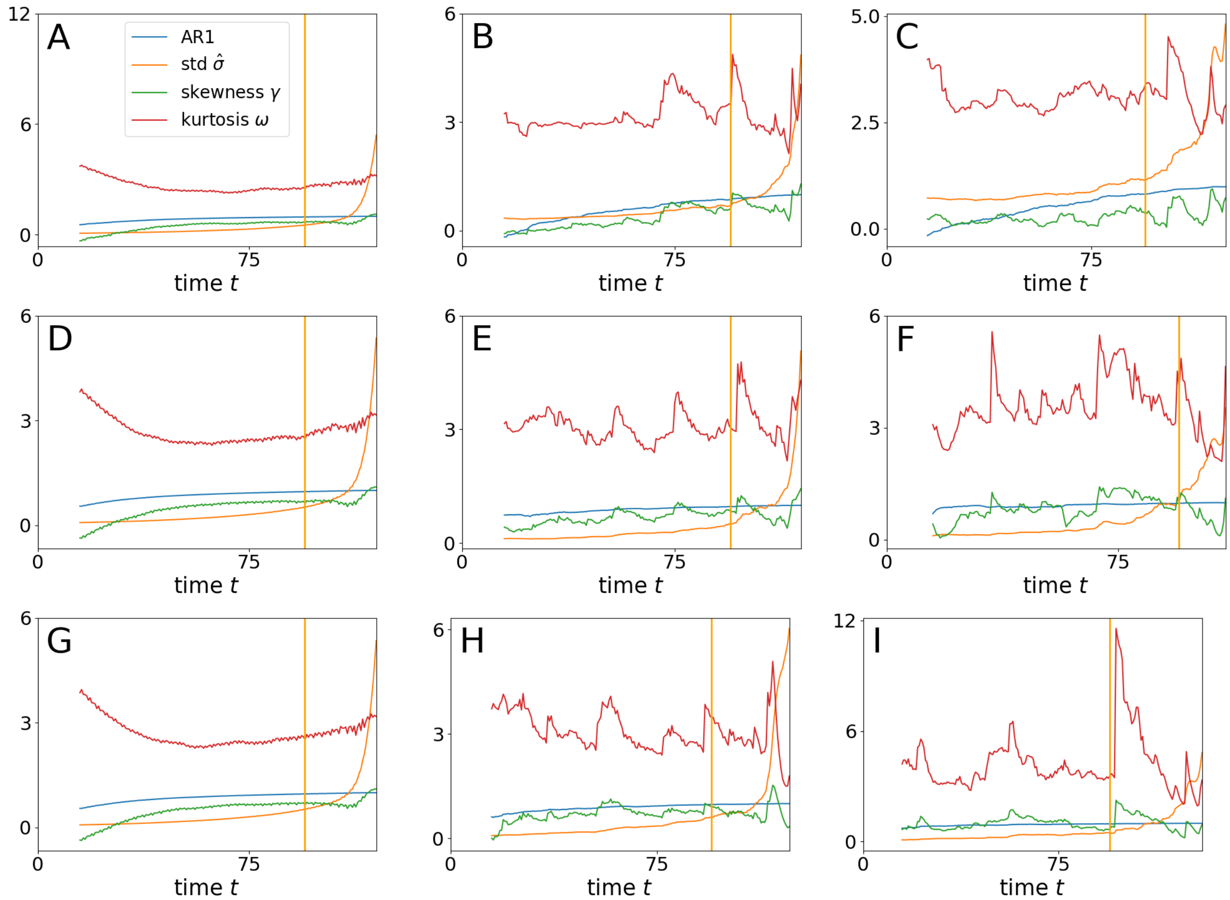

Figure 1: Statistical measures calculated for the datasets without preprocessing, i.e. without deseasonalization or detrending. (A-C) White noise cases, (D-F) pink noise cases and (G-I) red noise cases with increasing noise levels $\sigma = \{0.1, 2.2, 4.5\}$. The solid orange vertical line marks the "point of no return" up to which the data is used for the Bayesian model comparison in analogy to Perretti and Munch [2012] and Biggs et al. [2009].

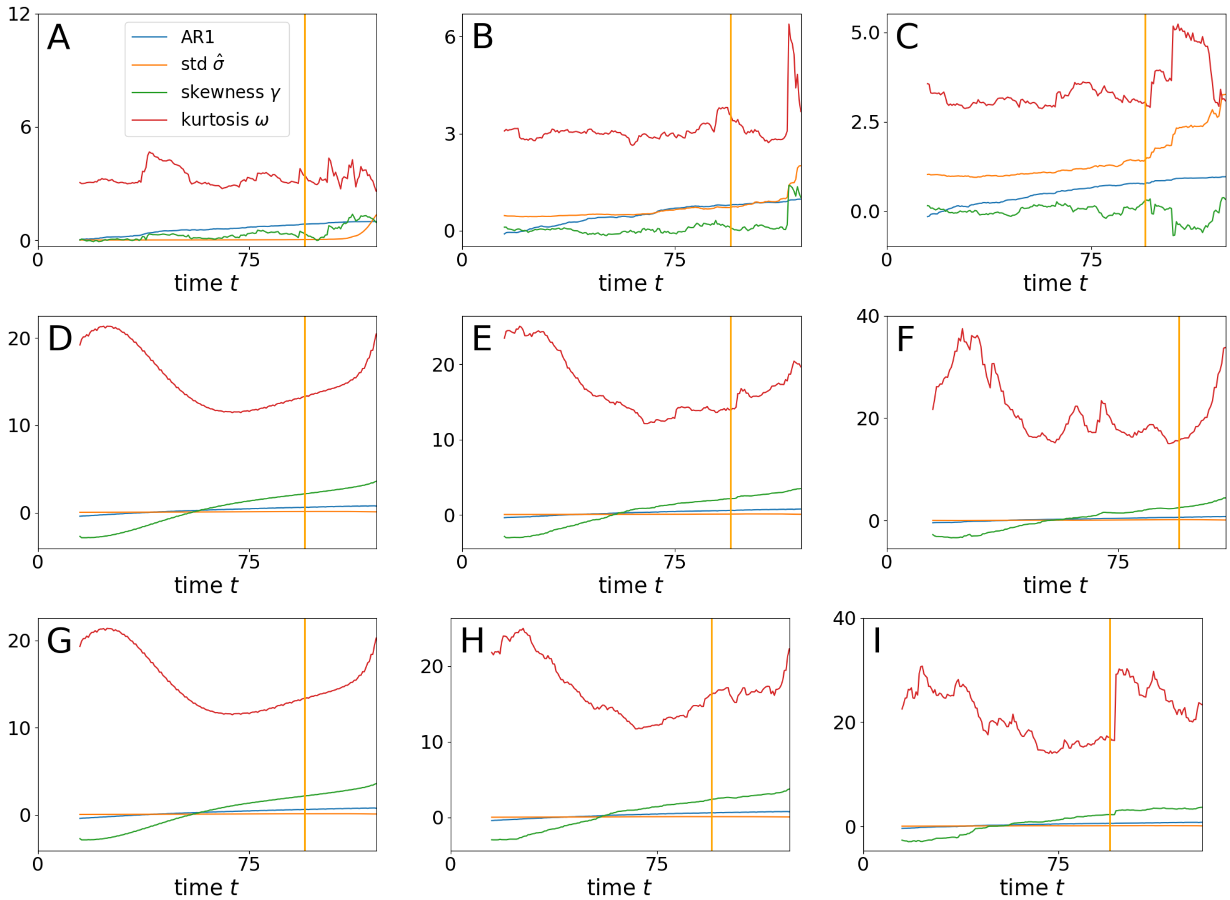

Figure 2: Statistical measures calculated for the datasets with deseasonalization. (A-C) White noise cases, (D-F) pink noise cases and (G-I) red noise cases with increasing noise levels $\sigma = \{0.1, 2.2, 4.5\}$. The solid orange vertical line marks the "point of no return" up to which the data is used for the Bayesian model comparison in analogy to Perretti and Munch [2012] and Biggs et al. [2009].

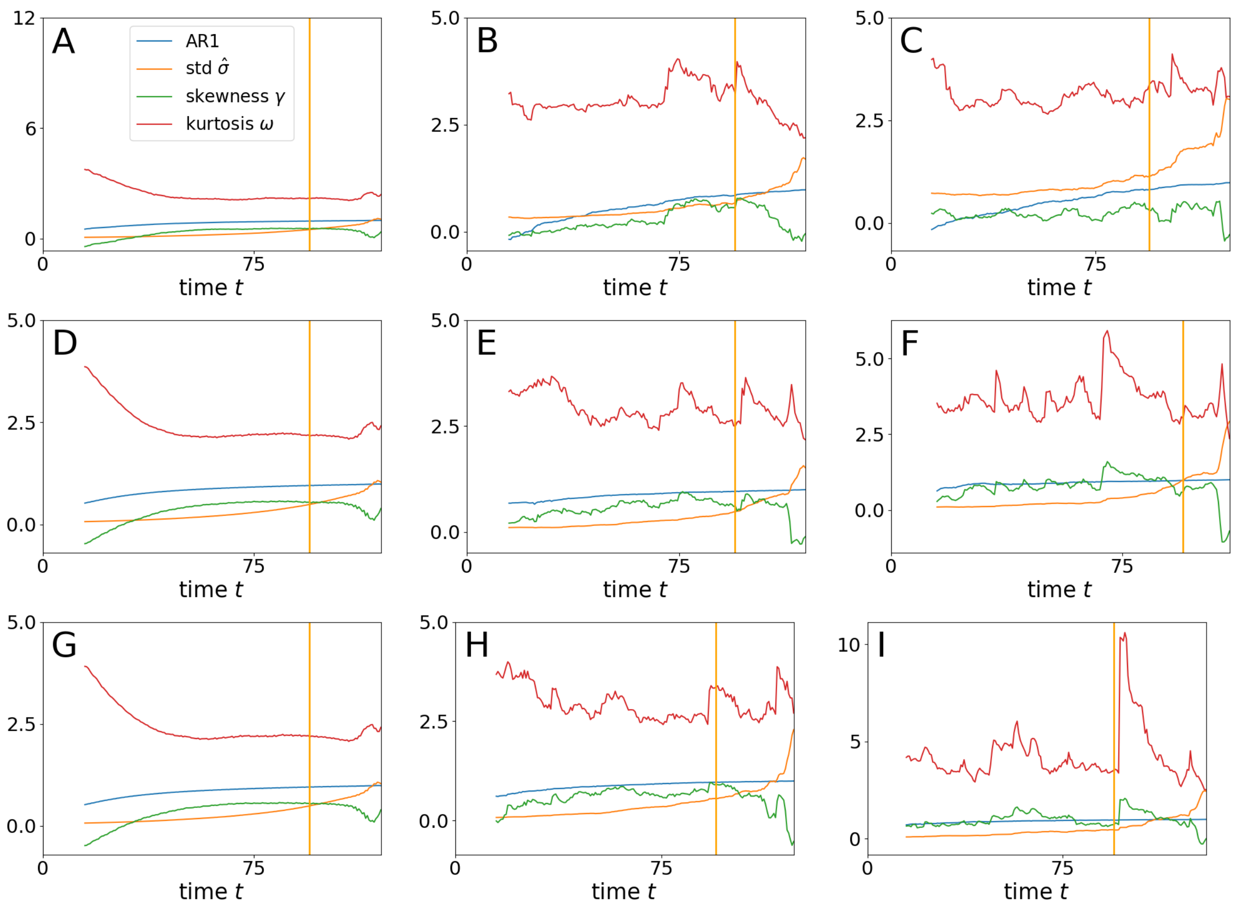

Figure 3: Statistical measures calculated for the datasets with detrending. (A-C) White noise cases, (D-F) pink noise cases and (G-I) red noise cases with increasing noise levels $\sigma = \{0.1, 2.2, 4.5\}$. The detrending is performed by subtracting a Gaussian kernel smoothing with kernelwidth of 150 from the original data. The Python function *scipy.ndimage.filters.gaussian_filter* Virtanen et al. [2020] is used for this procedure. The solid orange vertical line marks the "point of no return" up to which the data is used for the Bayesian model comparison in analogy to Perretti and Munch [2012] and Biggs et al. [2009].