

Table 2.1 Summary of methods for the solution of ordinary differential equations. The second- and third-order Runge-Kutta methods are low-storage variants; $h = \Delta t$

Method	Order	Formulae
Forward	1	$\phi_{n+1} = \phi_n + hF(\phi_n)$
Backward	1	$\phi_{n+1} = \phi_n + hF(\phi_{n+1})$
Asselin leapfrog	1	$\begin{aligned}\phi_{n+1} &= \overline{\phi_{n-1}} + 2hF(\phi_n), \\ \overline{\phi_n} &= \phi_n + \gamma(\overline{\phi_{n-1}} - 2\phi_n + \phi_{n+1})\end{aligned}$
Leapfrog	2	$\phi_{n+1} = \phi_{n-1} + 2hF(\phi_n)$
Adams-Bashforth	2	$\phi_{n+1} = \phi_n + \frac{h}{2} [3F(\phi_n) - F(\phi_{n-1})]$
Trapezoidal	2	$\phi_{n+1} = \phi_n + \frac{h}{2} [F(\phi_{n+1}) + F(\phi_n)]$
Runge-Kutta (2-step explicit)	2	$\begin{aligned}q_1 &= hF(\phi_n), & \phi_1 &= \phi_n + q_1, \\ q_2 &= hF(\phi_1) - q_1, & \phi_{n+1} &= \phi_1 + q_2/2\end{aligned}$
Magazenkov	2	$\begin{aligned}\phi_n &= \phi_{n-2} + 2hF(\phi_{n-1}) \\ \phi_{n+1} &= \phi_n + \frac{h}{2} [3F(\phi_n) - F(\phi_{n-1})]\end{aligned}$
Leapfrog-trapezoidal	2	$\begin{aligned}\phi_1 &= \phi_{n-1} + 2hF(\phi_n), \\ \phi_{n+1} &= \phi_n + \frac{h}{2} [F(\phi_1) + F(\phi_n)]\end{aligned}$
Adams-Bashforth	3	$\phi_{n+1} = \phi_n + \frac{h}{12} [23F(\phi_n) - 16F(\phi_{n-1}) + 5F(\phi_{n-2})]$
Adams-Moulton	3	$\phi_{n+1} = \phi_n + \frac{h}{12} [5F(\phi_{n+1}) + 8F(\phi_n) - F(\phi_{n-1})]$
Adams-Bashforth-Moulton predictor corrector	3	$\begin{aligned}\phi_1 &= \phi_n + \frac{h}{2} [3F(\phi_n) - F(\phi_{n-1})], \\ \phi_{n+1} &= \phi_n + \frac{h}{12} [5F(\phi_1) + 8F(\phi_n) - F(\phi_{n-1})]\end{aligned}$
Runge-Kutta (3-step explicit)	3	$\begin{aligned}q_1 &= hF(\phi_n), & \phi_1 &= \phi_n + q_1/3, \\ q_2 &= hF(\phi_1) - 5q_1/9, & \phi_2 &= \phi_1 + 15q_2/16, \\ q_3 &= hF(\phi_2) - 153q_2/128, & \phi_{n+1} &= \phi_2 + 8q_3/15\end{aligned}$
Runge-Kutta (4-step explicit)	4	$\begin{aligned}q_1 &= hF(\phi_n), & q_2 &= hF(\phi_n + q_1/2), \\ q_3 &= hF(\phi_n + q_2/2), & q_4 &= hF(\phi_n + q_3), \\ \phi_{n+1} &= \phi_n + (q_1 + 2q_2 + 2q_3 + q_4)/6\end{aligned}$