Even solutions for Richard H. Hammack's Book of Proof

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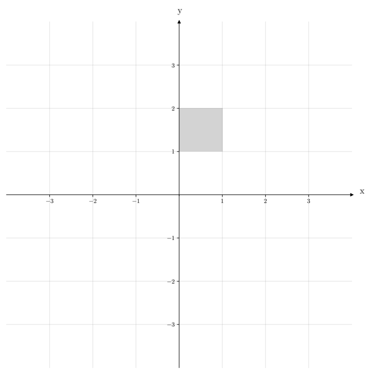
1 Chapter

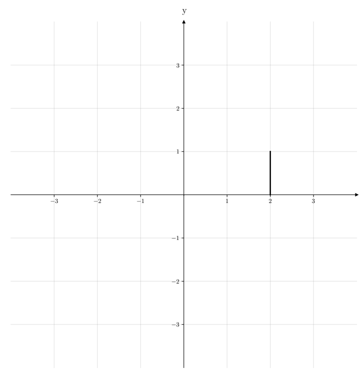
1.1 Section

2. $\{3x+2 : x \in \mathbb{Z}\} = \{..., -4, -1, 2, 5, 8, ...\}$
4. $\{x \in \mathbb{N} : -2 < x \leq 7\} = \{1, 2, 3, 4, 5, 6, 7\}$
6. $\{x \in \mathbb{R} : x^2 = 9\} = \{-3, 3\}$
8. $\{x \in \mathbb{R} : x^3 + 5x^2 = -6x\} = \{0, -2, -3\}$
10. $\{x \in \mathbb{R} : cosx = 1\} = \{..., -2\pi, 0, 2\pi, ...\}$
12. $\{x \in \mathbb{Z} : |2x| < 5\} = \{-2, -1, 0, 1, 2\}$
14. $\{5x : x \in \mathbb{Z}, |2x| \leq 8\} = \{-20, -15, -10, -5, 0, 5, 10, 15, 20\}$
16. $\{6a + 2b : a, b \in \mathbb{Z}\} = \{..., -4, -2, 0, 2, 4, ...\}$
18. $\{0, 4, 16, 36, 64, 100, ...\} = \{x^2 : x \in \mathbb{W}, \text{x is even}\}$
20. $\{..., -8, -3, 2, 7, 12, 17, ...\} = \{5x + 2 : x \in \mathbb{Z}\}$
22. $\{3, 6, 11, 18, 27, 38, ...\} = \{x^2 + 2 : x \in \mathbb{Z}\}$
24. $\{-4, -3, -2, -1, 0, 1, 2\} = \{x : x \in \mathbb{Z}, -4 \leq x \leq 2\}$
26. $\{..., \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, ...\} = \{3^x : x \in \mathbb{Z}\}$
28. $\{..., -\frac{3}{2}, -\frac{3}{4}, 0, \frac{3}{4}, \frac{3}{2}, \frac{9}{4}, 3, \frac{15}{4}, \frac{9}{2}, ...\} = \{x * \frac{3}{4} : x \in \mathbb{Z}\}$
30. $|\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\emptyset\}\}| = 5$
32. $|\{\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\emptyset\}\}\}| = 1$
34. $|\{x \in \mathbb{N} : |x| < 10\}| = 9$
36. $|\{x \in \mathbb{N} : x^2 < 10\}| = 3$
38. $|\{x \in \mathbb{N} : 5x \leq 20\}| = 4$

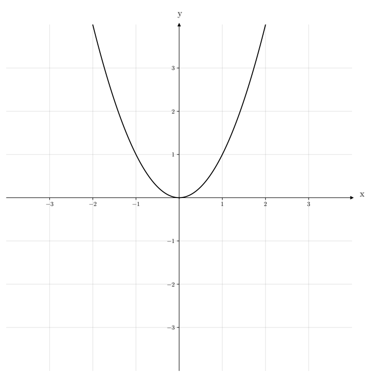
40. $\{(x,y) : x \in [0,1], y \in [1,2]\}$



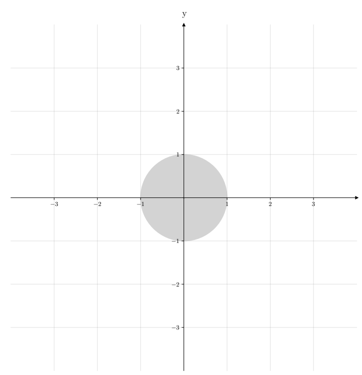
42. $\{(x,y) : x = 2, y \in [0,1]\}$



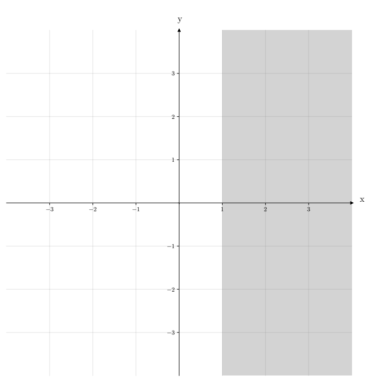
44. $\{(x,x^2) : x \in \mathbb{R}\}$



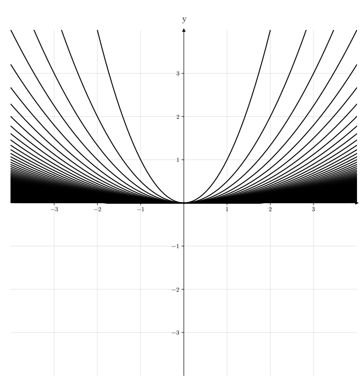
46. $\{(x,y) : x, y \in \mathbb{R}, x^2 + y^2 \leq 1\}$



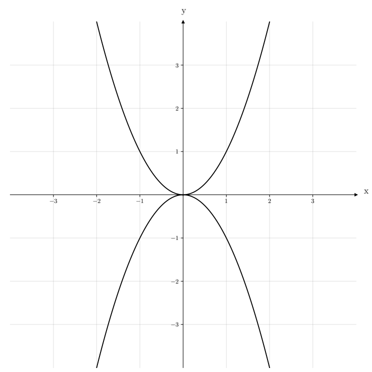
48. $\{(x,y) : x, y \in \mathbb{R}, x > 1\}$



50. $\{(x, \frac{x^2}{y}) : x \in \mathbb{R}, y \in \mathbb{N}\}$



52. $\{(x,y) \in \mathbb{R}^2 : (y-x^2)(y+x^2) = 0\}$



1.2 Section

2. $A = \{\pi, e, 0\}, B = \{0, 1\}$

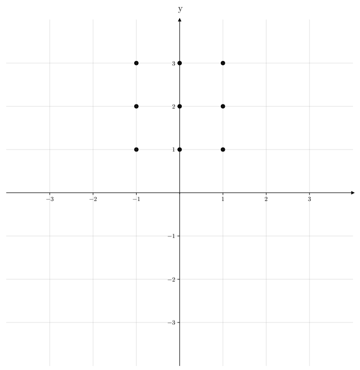
   a) $A \times B = \{(\pi,0),(\pi,1),(e,0),(e,1),(0,0),(0,1)\}$
   b) $B \times A = \{(0,\pi),(0,e),(0,0),(1,\pi),(1,e),(1,0)\}$
   c) $A \times A = \{(\pi,\pi),(\pi,e),(\pi,0),(e,\pi),(e,e),(e,0),(0,\pi),(0,e),(0,0)\}$
   d) $B \times B = \{(0,0),(0,1),(1,0),(1,1)\}$
   e) $A \times \emptyset = \emptyset$
   f) $(A \times B) \times B =$
   $\{((\pi,0),0),((\pi,0),1),((\pi,1),0),((\pi,1),1),((e,0),0),((e,0),1),$
   $((e,1),0),((e,1),1),((0,0),0),((0,0),1),((0,1),0),((0,1),1)\}$
   g) $A \times (B \times B) =$
   $\{(\pi,(0,0)),(\pi,(0,1)),(\pi,(1,0)),(\pi,(1,1)),(e,(0,0)),(e,(0,1)),$
   $(e,(1,0)),(e,(1,1)),(0,(0,0)),(0,(0,1)),(0,(1,0)),(0,(1,1))\}$
   h) $A \times B \times B =$
   $\{(\pi,0,0),(\pi,0,1),(\pi,1,0),(\pi,1,1),(e,0,0),(e,0,1),$
   $(e,1,0),(e,1,1),(0,0,0),(0,0,1),(0,1,0),(0,1,1)\}$

4. $\{n \in \mathbb{Z} : 2 < n < 5\} \times \{n \in \mathbb{Z} : |n| = 5\} = \{(3,5),(3,-5),(4,5),(4,-5)\}$

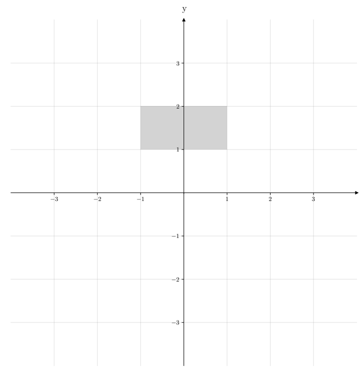
6. $\{x \in \mathbb{R} : x^2 = x\} \times \{x \in \mathbb{N} : x^2 = x\} = \{(0,1),(1,1)\}$

8. $\{0,1\}^4 = \{(0,0,0,0),(0,0,0,1),(0,0,1,0),(0,0,1,1),(0,1,0,0),(0,1,0,1),(0,1,1,0),$
   $(0,1,1,1),(1,0,0,0),(1,0,0,1),(1,0,1,0),(1,0,1,1),(1,1,0,0),(1,1,0,1),(1,1,1,0),(1,1,1,1)\}$

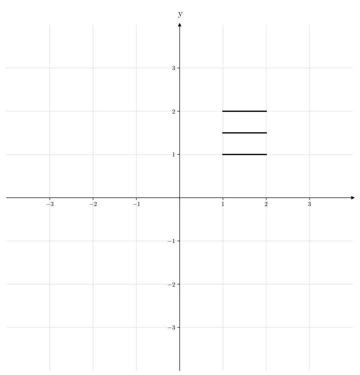
10. $\{-1, 0, 1\} \times \{1, 2, 3\}$



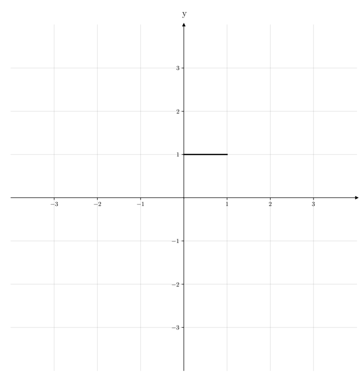
12. $[-1, 1] \times [1, 2]$



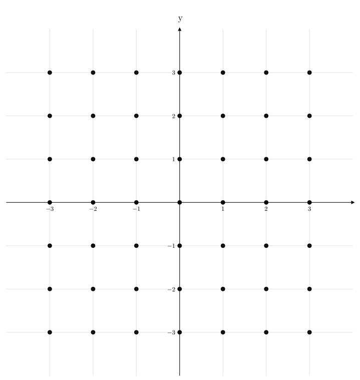
14. $[1, 2] \times \{1, 1.5, 2\}$



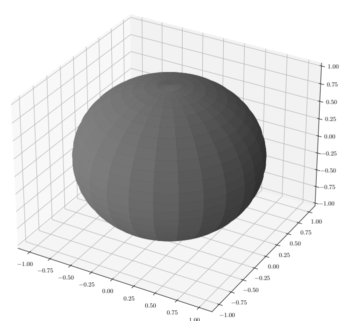
16. $[0, 1] \times \{1\}$



18. $\mathbb{Z} \times \mathbb{Z}$



20. $\{(x, y) :\in \mathbb{R}^2 : x^2 + y^2 \leq 1\} \times [0, 1]$



1.3 Section

2 $\mathcal{P}(\{1,2,\emptyset\}) = \{\{\},\{1\},\{2\},\{\emptyset\},\{1,2\},\{1,\emptyset\},\{2,\emptyset\},\{1,2,\emptyset\}\}$

4 $\mathcal{P}(\emptyset) = \{\emptyset\}$

6 $\mathcal{P}(\{\mathbb{R},\mathbb{Q},\mathbb{N}\}) = \{\{\},\{\mathbb{R}\},\{\mathbb{Q}\},\{\mathbb{N}\},\{\mathbb{R},\mathbb{Q}\},\{\mathbb{R},\mathbb{N}\},\{\mathbb{Q},\mathbb{N}\},\{\mathbb{R},\mathbb{Q},\mathbb{N}\}\}$

8 $\mathcal{P}(\{\{0,1\},\{0,1,\{2\}\},\{0\}\}) = \{\{\},\{\{0,1\}\},\{\{0,1,\{2\}\}\},\{\{0\}\},$
$\{\{0,1\},\{0,1,\{2\}\}\},\{\{0,1\},\{0\}\},\{\{0,1,\{2\}\},\{0\}\},\{\{0,1\},\{0,1,\{2\}\},\{0\}\}\}$

10 $\{X \subseteq \mathbb{N} : |X| \leq 1\} = \{\emptyset,\{1\},\{2\},\{3\},...\}$

12 $\{X : X \subseteq \{3,2,a\}$ and $|X| = 1\} = \{\emptyset,\{3\},\{2\},\{a\}\}$

14 $\mathbb{R}^2 \subseteq \mathbb{R}^3$ is false, because the point $(1,1)$ of the 2-dimensional plane $\mathbb{R}^2$ is not part of the 3-dimensional space $\mathbb{R}^3$, which has points such as $(1,1,1)$

16 $\{(x,y) \in \mathbb{R}^2 : x^2 - x = 0\} \subseteq \{(x,y) \in \mathbb{R}^2 : x - 1 = 0\}$ is false, because the set on the right does not contain point $(-1,y)$

1.4 Section

2 $\mathcal{P}(\{1,2,3,4\}) = \{\{\},\{1\},\{2\},\{1,2\},\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{4\},\{1,4\},\{2,4\},$
$\{1,2,4\},\{3,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\}$

4 $\mathcal{P}(\{\mathbb{R},\mathbb{Q}\}) = \{\{\},\{\mathbb{R}\},\{\mathbb{Q}\},\{\mathbb{R},\mathbb{Q}\}\}$

6 $\mathcal{P}(\{1,2\}) \times \mathcal{P}(\{3\}) = \{(\{\},\{\}),(\{\},\{3\}),(\{1\},\{\}),(\{1\},\{3\}),(\{2\},\{\}),(\{2\},\{3\}),$
$(\{1,2\},\{\}),(\{1,2\},\{3\})\}$

8 $\mathcal{P}(\{1,2\} \times \{3\}) = \{\{\},\{(1,3)\},\{(2,3)\},\{(1,3),(2,3)\}$

10 $\{X \in \mathcal{P}(\{1,2,3\}) : |X| \leq 1\} = \{\{\},\{1\},\{2\},\{3\}\}$

12 $\{X \in \mathcal{P}(\{1,2,3\}) : 2 \in X\} = \{\{2\},\{1.2\},\{2,3\},\{1,2,3\}\}$

Suppose $|A| = m, |B| = n$

14 $|\mathcal{P}(\mathcal{P}(A))| = 2^{2^m}$

16 $|\mathcal{P}(A) \times \mathcal{P}(B)| = 2^m * 2^n$

18 $|\mathcal{P}(A \times \mathcal{P}(B))| = 2^{m*2^n}$

20 $|\{X \subseteq \mathcal{P}(A) : |X| \leq 1\}| = m + 2$

1.5 Section

10 The statement $(\mathbb{R} - \mathbb{Q}) \times \mathbb{N} = (\mathbb{R} \times \mathbb{N}) - (\mathbb{Z} \times \mathbb{N})$ is true, because each point has a y value $\in \mathbb{N}$ and both side remove all $\mathbb{Z}$ values from x.