

# Even solutions for Richard H. Hammack's Book of Proof

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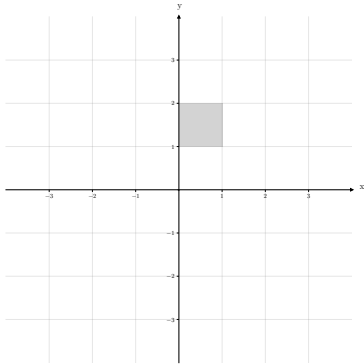
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## 1 Chapter

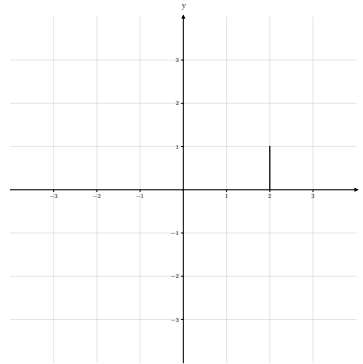
### 1.1 Section

2.  $\{3x + 2 : x \in \mathbb{Z}\} = \{\dots, -4, -1, 2, 5, 8, \dots\}$
4.  $\{x \in \mathbb{N} : -2 < x \leq 7\} = \{1, 2, 3, 4, 5, 6, 7\}$
6.  $\{x \in \mathbb{R} : x^2 = 9\} = \{-3, 3\}$
8.  $\{x \in \mathbb{R} : x^3 + 5x^2 = -6x\} = \{0, -2, -3\}$
10.  $\{x \in \mathbb{R} : \cos x = 1\} = \{\dots, -2\pi, 0, 2\pi, \dots\}$
12.  $\{x \in \mathbb{Z} : |2x| < 5\} = \{-2, -1, 0, 1, 2\}$
14.  $\{5x : x \in \mathbb{Z}, |2x| \leq 8\} = \{-20, -15, -10, -5, 0, 5, 10, 15, 20\}$
16.  $\{6a + 2b : a, b \in \mathbb{Z}\} = \{\dots, -4, -2, 0, 2, 4, \dots\}$
18.  $\{0, 4, 16, 36, 64, 100, \dots\} = \{x^2 : x \in \mathbb{W}, x \text{ is even}\}$
20.  $\{\dots, -8, -3, 2, 7, 12, 17, \dots\} = \{5x + 2 : x \in \mathbb{Z}\}$
22.  $\{3, 6, 11, 18, 27, 38, \dots\} = \{x^2 + 2 : x \in \mathbb{Z}\}$
24.  $\{-4, -3, -2, -1, 0, 1, 2\} = \{x : x \in \mathbb{Z}, -4 \leq x \leq 2\}$
26.  $\{\dots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, \dots\} = \{3^x : x \in \mathbb{Z}\}$
28.  $\{\dots, -\frac{3}{2}, -\frac{3}{4}, 0, \frac{3}{4}, \frac{3}{2}, \frac{9}{4}, 3, \frac{15}{4}, \frac{9}{2}, \dots\} = \{x * \frac{3}{4} : x \in \mathbb{Z}\}$
30.  $|\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\emptyset\}\}| = 5$
32.  $|\{\{\{1, 4\}, a, b, \{\{3, 4\}\}, \{\emptyset\}\}\}| = 1$
34.  $|\{x \in \mathbb{N} : |x| < 10\}| = 9$
36.  $|\{x \in \mathbb{N} : x^2 < 10\}| = 3$
38.  $|\{x \in \mathbb{N} : 5x \leq 20\}| = 4$

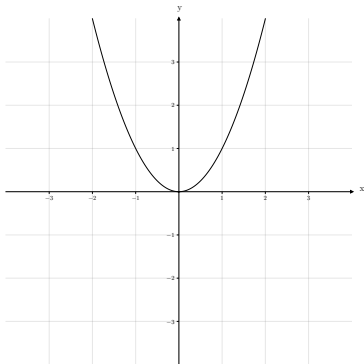
40.  $\{(x, y) : x \in [0, 1], y \in [1, 2]\}$



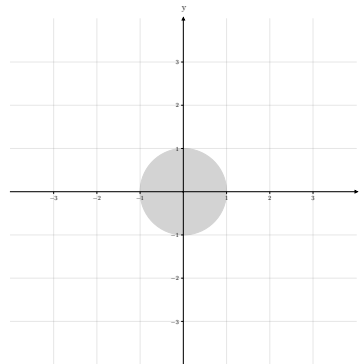
42.  $\{(x, y) : x = 2, y \in [0, 1]\}$



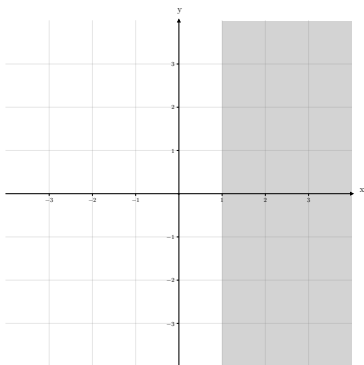
44.  $\{(x, x^2) : x \in \mathbb{R}\}$



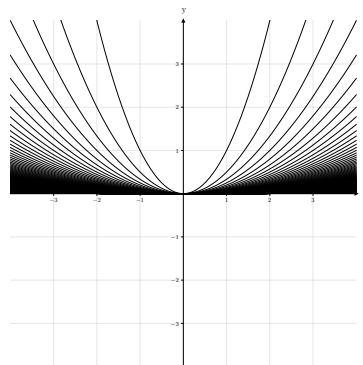
46.  $\{(x, y) : x, y \in \mathbb{R}, x^2 + y^2 \leq 1\}$



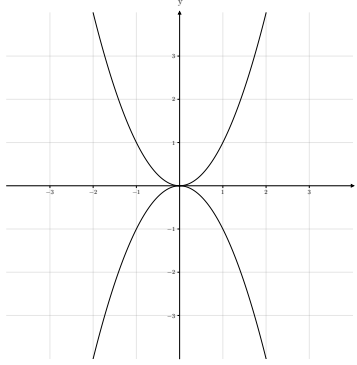
48.  $\{(x, y) : x, y \in \mathbb{R}, x > 1\}$



50.  $\{(x, \frac{x^2}{y}) : x \in \mathbb{R}, y \in \mathbb{N}\}$



52.  $\{(x, y) \in \mathbb{R}^2 : (y - x^2)(y + x^2) = 0\}$



## 1.2 Section

2.  $A = \{\pi, e, 0\}, B = \{0, 1\}$

a)  $A \times B = \{(\pi, 0), (\pi, 1), (e, 0), (e, 1), (0, 0), (0, 1)\}$

b)  $B \times A = \{(0, \pi), (0, e), (0, 0), (1, \pi), (1, e), (1, 0)\}$

c)  $A \times A = \{(\pi, \pi), (\pi, e), (\pi, 0), (e, \pi), (e, e), (e, 0), (0, \pi), (0, e), (0, 0)\}$

d)  $B \times B = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$

e)  $A \times \emptyset = \emptyset$

f)  $(A \times B) \times B =$

$$\{((\pi, 0), 0), ((\pi, 0), 1), ((\pi, 1), 0), ((\pi, 1), 1), ((e, 0), 0), ((e, 0), 1), ((e, 1), 0), ((e, 1), 1), ((0, 0), 0), ((0, 0), 1), ((0, 1), 0), ((0, 1), 1)\}$$

g)  $A \times (B \times B) =$

$$\{(\pi, (0, 0)), (\pi, (0, 1)), (\pi, (1, 0)), (\pi, (1, 1)), (e, (0, 0)), (e, (0, 1)), (e, (1, 0)), (e, (1, 1)), (0, (0, 0)), (0, (0, 1)), (0, (1, 0)), (0, (1, 1))\}$$

h)  $A \times B \times B =$

$$\{(\pi, 0, 0), (\pi, 0, 1), (\pi, 1, 0), (\pi, 1, 1), (e, 0, 0), (e, 0, 1), (e, 1, 0), (e, 1, 1), (0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1)\}$$

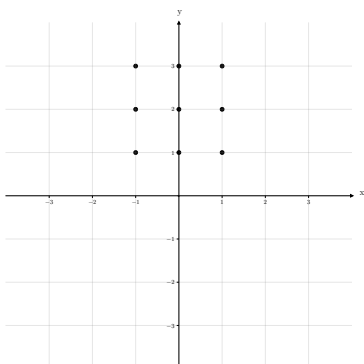
4.  $\{n \in \mathbb{Z} : 2 < n < 5\} \times \{n \in \mathbb{Z} : |n| = 5\} = \{(3, 5), (3, -5), (4, 5), (4, -5)\}$

6.  $\{x \in \mathbb{R} : x^2 = x\} \times \{x \in \mathbb{N} : x^2 = x\} = \{(0, 1), (1, 1)\}$

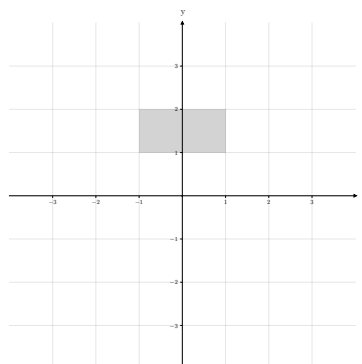
8.  $\{0, 1\}^4 = \{(0, 0, 0, 0), (0, 0, 0, 1), (0, 0, 1, 0), (0, 0, 1, 1), (0, 1, 0, 0), (0, 1, 0, 1), (0, 1, 1, 0),$

$$(0, 1, 1, 1), (1, 0, 0, 0), (1, 0, 0, 1), (1, 0, 1, 0), (1, 0, 1, 1), (1, 1, 0, 0), (1, 1, 0, 1), (1, 1, 1, 0), (1, 1, 1, 1)\}$$

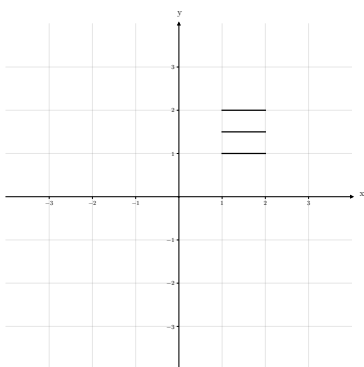
10.  $\{-1, 0, 1\} \times \{1, 2, 3\}$



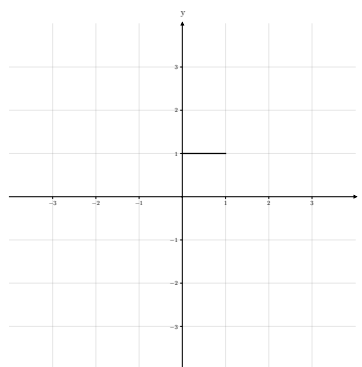
12.  $[-1, 1] \times [1, 2]$



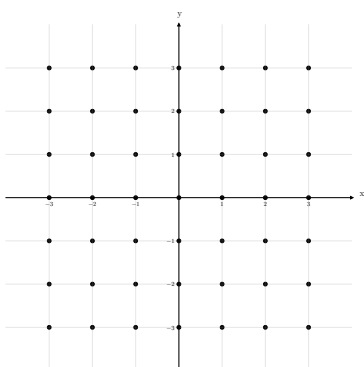
14.  $[1, 2] \times \{1, 1.5, 2\}$



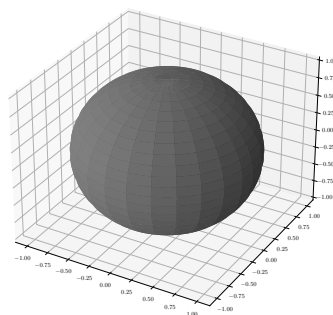
16.  $[0, 1] \times \{1\}$



18.  $\mathbb{Z} \times \mathbb{Z}$



20.  $\{(x, y) : x^2 + y^2 \leq 1\} \times [0, 1]$



### 1.3 Section

$$2 \quad \mathcal{P}(\{1, 2, \emptyset\}) = \{\{\}, \{1\}, \{2\}, \{\emptyset\}, \{1, 2\}, \{1, \emptyset\}, \{2, \emptyset\}, \{1, 2, \emptyset\}\}$$

$$4 \quad \mathcal{P}(\emptyset) = \{\emptyset\}$$

$$6 \quad \mathcal{P}(\{\mathbb{R}, \mathbb{Q}, \mathbb{N}\}) = \{\{\}, \{\mathbb{R}\}, \{\mathbb{Q}\}, \{\mathbb{N}\}, \{\mathbb{R}, \mathbb{Q}\}, \{\mathbb{R}, \mathbb{N}\}, \{\mathbb{Q}, \mathbb{N}\}, \{\mathbb{R}, \mathbb{Q}, \mathbb{N}\}\}$$

$$8 \quad \mathcal{P}(\{\{0, 1\}, \{0, 1, \{2\}\}, \{0\}\}) = \{\{\}, \{\{0, 1\}\}, \{\{0, 1, \{2\}\}\}, \{\{0\}\}, \\ \{\{0, 1\}, \{0, 1, \{2\}\}\}, \{\{0, 1\}, \{0\}\}, \{\{0, 1, \{2\}\}, \{0\}\}, \{\{0, 1\}, \{0, 1, \{2\}\}, \{0\}\}\}$$

$$10 \quad \{X \subseteq \mathbb{N} : |X| \leq 1\} = \{\emptyset, \{1\}, \{2\}, \{3\}, \dots\}$$

$$12 \quad \{X : X \subseteq \{3, 2, a\} \text{ and } |X| = 1\} = \{\emptyset, \{3\}, \{2\}, \{a\}\}$$

$$14 \quad \mathbb{R}^2 \subseteq \mathbb{R}^3 \text{ is false, because the point } (1, 1) \text{ of the 2-dimensional plane } \mathbb{R}^2 \text{ is} \\ \text{not part of the 3-dimensional space } \mathbb{R}^3, \text{ which has points such as } (1, 1, 1)$$

$$16 \quad \{(x, y) \in \mathbb{R}^2 : x^2 - x = 0\} \subseteq \{(x, y) \in \mathbb{R}^2 : x - 1 = 0\} \text{ is false, because the} \\ \text{set on the right does not contain point } (-1, y)$$