Project 2: Orbital Perturbations

AERO 452

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Introduction

As part of the delta-v analysis for a possible replacement mission, we chose a sun synchronous LEO cubesat and a geostationary weather satellite. Our LEO object is the DELFI-C3 3U CubeSat from TU Delft launched in 2008. We chose this LEO satellite because it satisfies the semi major axis requirement and readily has the physical properties and TLE available. The other object is one of the Geostationary Operational Environmental Satellites, GOES-16 or GOES-R. The GOES-R satellite is part of a four satellite series in a partnership between NOAA and NASA. It was chosen for its geostationary orbit and accessibility of information [1].

Assumptions

- Cr = 1.2
- Cd = 2.2
- No perturbations for Lamberts trajectory
- Exponential Drag model sufficient
- Satellite modeled as a sphere with longest dimension as the hydraulic diameter

Part 1

Object 1: DELFI-C3

Table 1

Orbital Parameters of DELFI-C3	
Semi Major Axis	6922.7 km
Eccentricity	.0011
Inclination	97.4474°
RAAN	5.6562°
Argument of Perigee	335.1825°
True Anomaly	190.4478°
Period	1.59 hrs
JD0	2458805.666651000 days

A TLE was obtained on November 18, 2019 at 23:36 UTC, and the orbital parameters from this time are presented in Table 1 [2], [3].

Table 2

Physical Properties of DELFI-C3	
Mass	3 kg
Cr	1.2
Hydraulic Diameter	.34 m

Table 3

Perturbations acting on DELFI-C3	
Perturbation	**Reasoning**
J2-J6	The high inclination of the satellite requires that J2 through J6 perturbations are all included to ensure that earth oblateness effects at higher latitudes are taken into account.
Lunar 3-body	Not a dominant perturbation but still necessary to include.
SRP	Although not the largest perturbation on such a small surface area object, SRP was still necessary.
Drag	As a LEO satellite, aerodynamic drag is one of the most prevalent perturbing effects.

We used Gaussian Variation of Parameters (VOP) to propagate DELFI-C3 because the eccentricity was not too close to 0. Below, Figures 1 through 4 present the orbital elements and radii for long term and short term propagations of DELFI-C3. The CubeSat is at a sun synchronous inclination, therefore the RAAN rotates at 1 degree per day as shown in both the short and long term plots. The short term results show seemingly oscillatory and secular progression of eccentricity and argument of perigee. However, from the long term plots it is clear that eccentricity is strictly periodic while the argument of perigee is periodic and secular, eliciting the predominant effect of Earth's oblateness. Both the short and long term plots show that inclination is periodic with a very small amplitude but high frequency. The long term propagation demonstrates the oscillatory and conservative perturbation acceleration contributed by earth oblateness but the gradually decreasing orbital radii show that the non conservative forces such as SRP and drag are also having a noticeable effect. The effect of lunar gravity is non distinguishable with the implementation of all the aforementioned perturbations. In LEO, lunar gravity should affect a gradually decreasing RAAN and argument of perigee with a gradually increasing inclination, however these effects are overpowered by the other perturbations, especially earth oblateness.

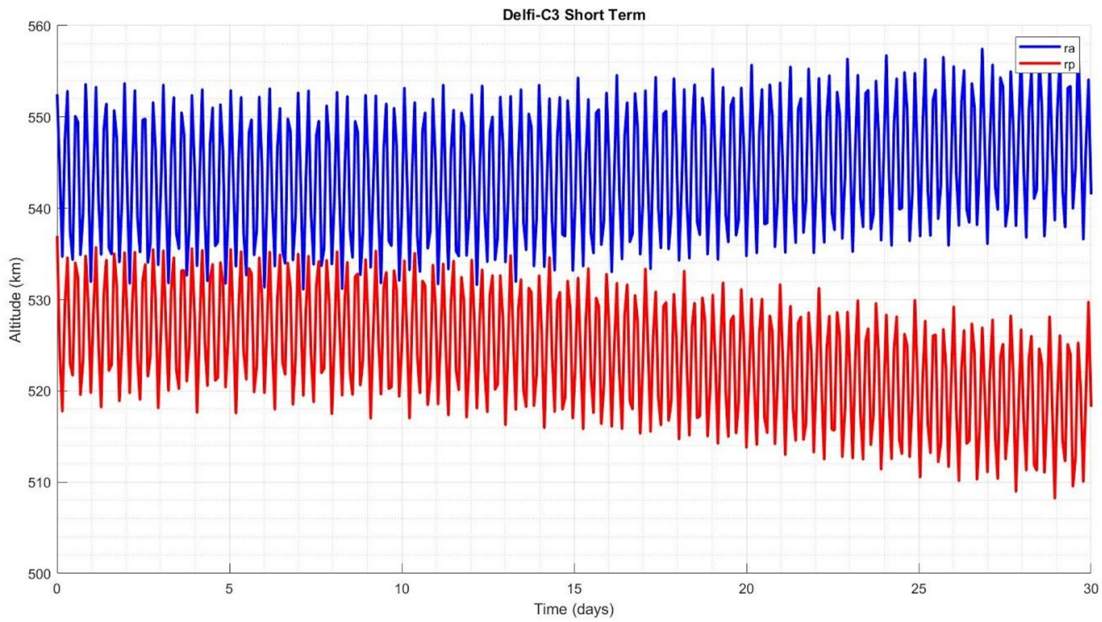

Fig. 1 DELFI-C3 Short Term Orbital Radii

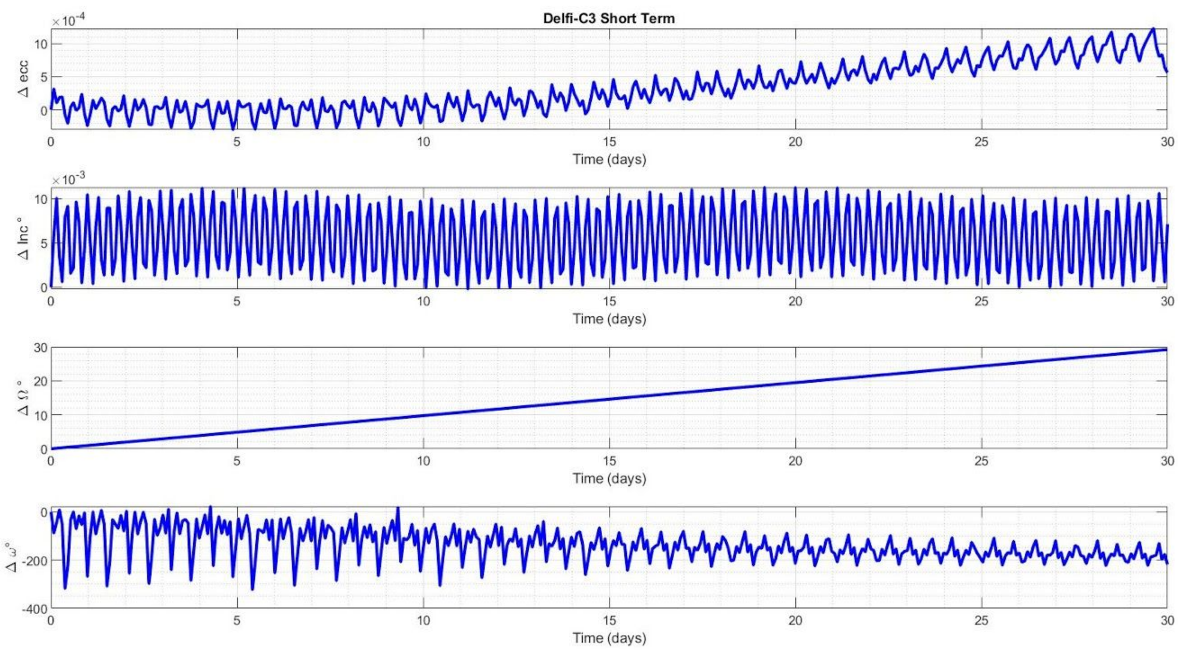

Fig. 2 DELFI-C3 Short Term Orbital Elements

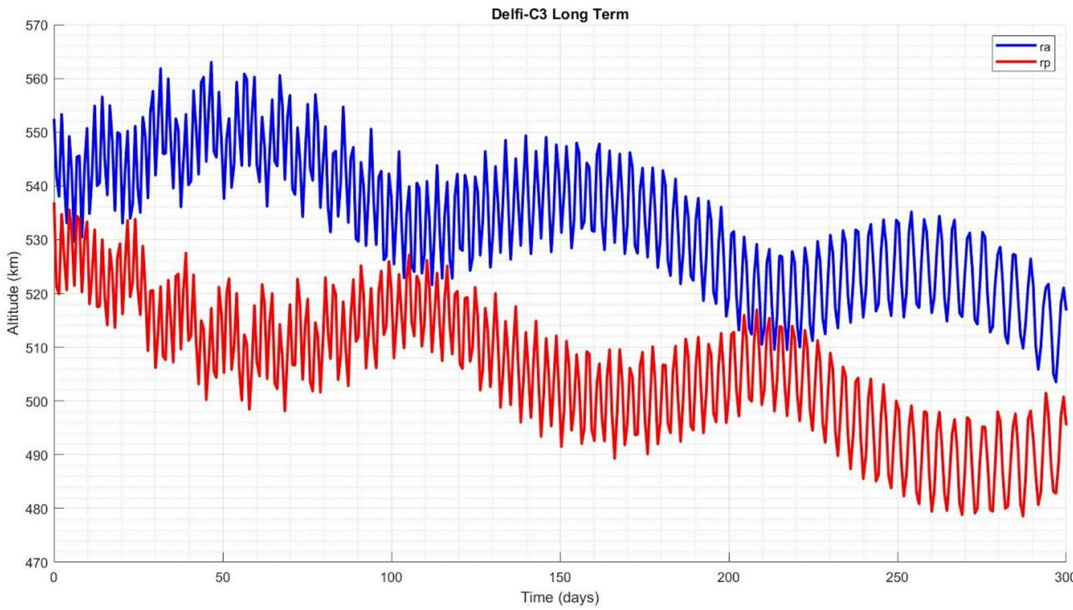

Fig. 3 DELFI-C3 Long Term Orbital Radii

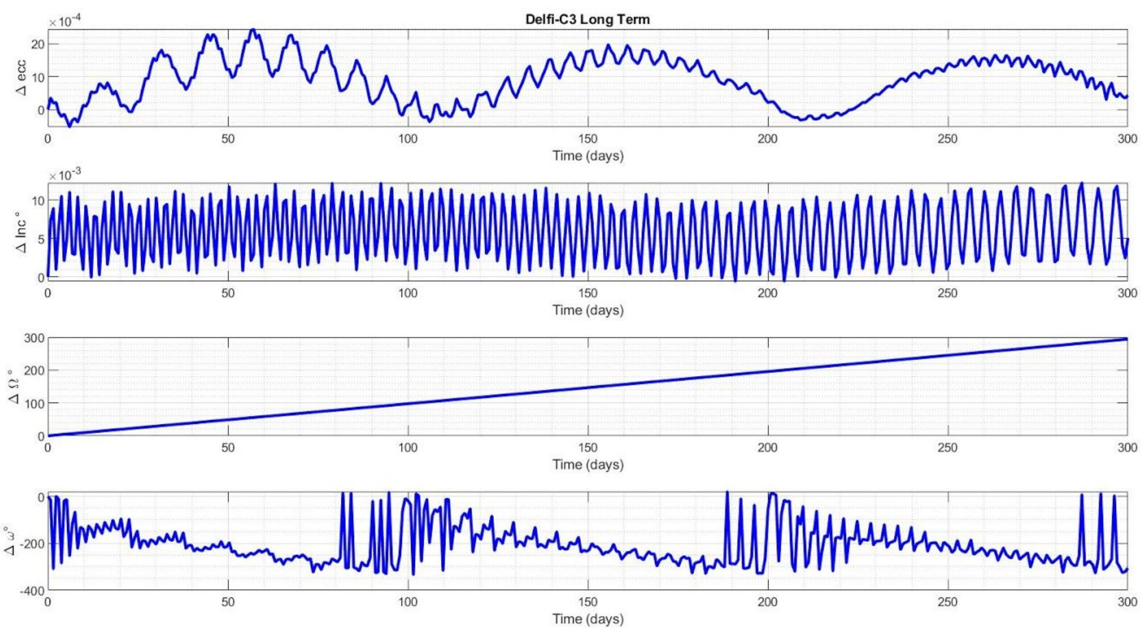

Fig. 4 DELFI-C3 Long Term Orbital Elements

Object 2: GOES-R

A TLE was obtained on November 18, 2019 at 23:36 UTC, and the orbital parameters from this time are presented in Table 4 [2], [3]. We decided to propagate the Geo satellite with Gaussian VOP as well, despite the very small eccentricity. We wanted to use the same propagation method for all satellites with the intent of eliminating differences introduced by different propagation methods.

Table 4.

Orbital Parameters of GOES-R	
Semi-Major Axis	42,165 km
Eccentricity	0.0000817
Inclination	0.0014°
RAAN	16.6683°
Argument of Perigee	267.90°
True Anomaly	230.69°
Period	23.92 hrs
JD0	2458806.4838 days

GOES-R specifically serves as GOES-East and was designed for a 10 year lifetime. It was launched in 2016 with a wet mass of 5,192 kg, and a dry mass of 2,857 kg. This results in 2,335 kg of propellant, which was assumed to be enough propellant to support 15 years on orbit for margin. The satellite has been in orbit for three years, and a linear use of propellant was assumed over the 15 years, so 467 kg of propellant has already been expended so the mass of the satellite at this time is assumed to be 4725 kg. The size of the satellite is 6.1 m x 5.6 m x 3.9 m, but to understand the worst case effect of solar radiation pressure, the area normal to the sun was assumed to be a circle with a hydraulic diameter of 6.1 m. These physical properties are summarized below in table 5.

Table 5.

Physical Properties of GOES-R	
Mass	4725 kg
Cr	1.2
Hydraulic Diameter	6.1 m

The geostationary position of the GOES-R also influences the perturbations that are applied to the analysis. Table 6 describes which perturbing accelerations were included in this analysis.

Table 6.

Perturbations acting on GOES-R	
Perturbation	**Reasoning**
J2-J6	Earth's oblateness continues to affect GEO orbits about, although the influence is lessened with the higher the altitude.
Lunar n-body	The moon has an effect on the GOES-R satellite and an effect on the Earth that indirectly influences its orbit as well.
SRP	Solar radiation is also significant enough to include in the analysis of GOES-R.

To understand the short term effects, the orbit was propagated for six months; Figures 5 and 6 shows the effect on the altitude and orbital elements, respectively. The eccentricity and inclination both increase over time but on a very small scale, 2.5 x 10^{-4} and 0.3° respectively. This is reasonable for a geostationary orbit where the effect of the perturbations is much more gradual than in LEO. Due to the circular nature of the orbit, the right ascension of ascending node is initially oscillatory before stabilizing around a specific value. The argument of perigee increases due to the secular and periodic effect of Earth Oblateness.

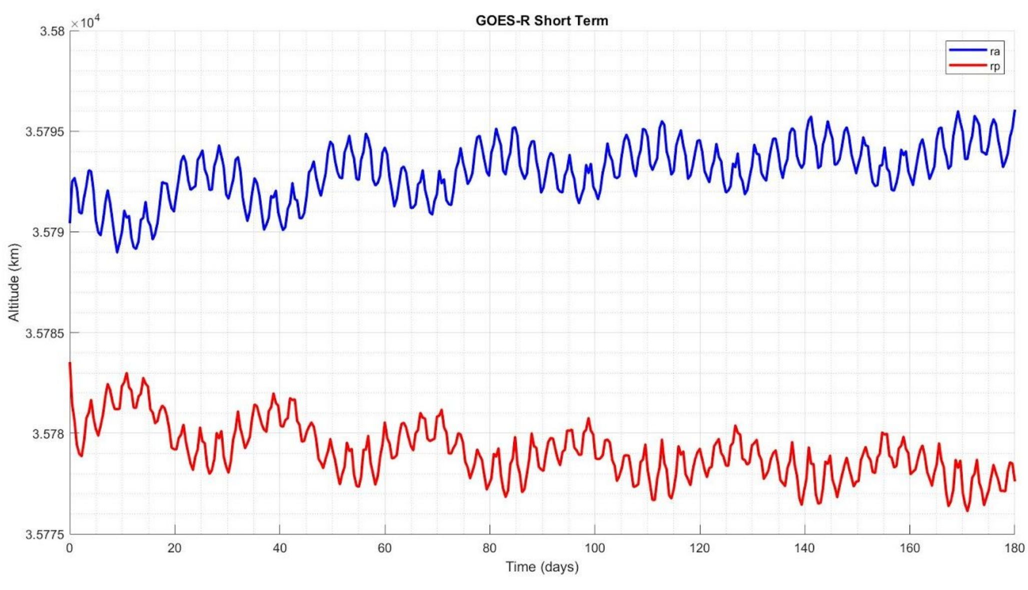

Fig. 5 GOES-R Short Term Orbital Radii

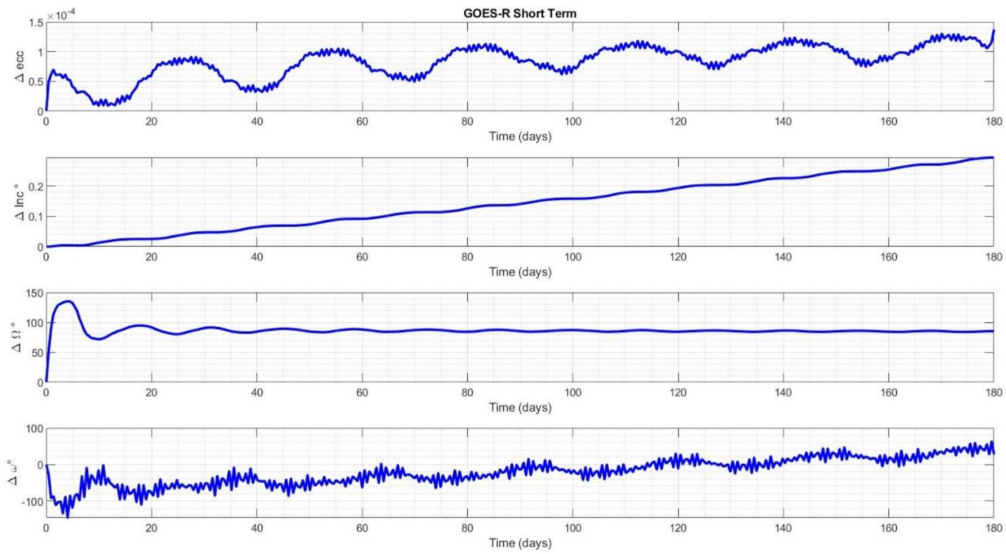

Fig. 6 GOES-R Short Term Orbital Elements

To examine the long term effects of the perturbations, the orbit was propagated for ten years; Figure 7 and Figure 8 show the effects on the altitude and the orbital elements. This is in the scenario that the GOES-R does not correct for perturbations over its lifetime. In the long term effect, the behavior altitude of the satellite is oscillatory and only changes about 10 km, and this effect is mirrored in the eccentricity. The inclination increases six degrees over ten years, while the RAAN decreases after the initial stabilization. The argument of perigee is erratic overtime resulting from the near circular nature of the orbit.

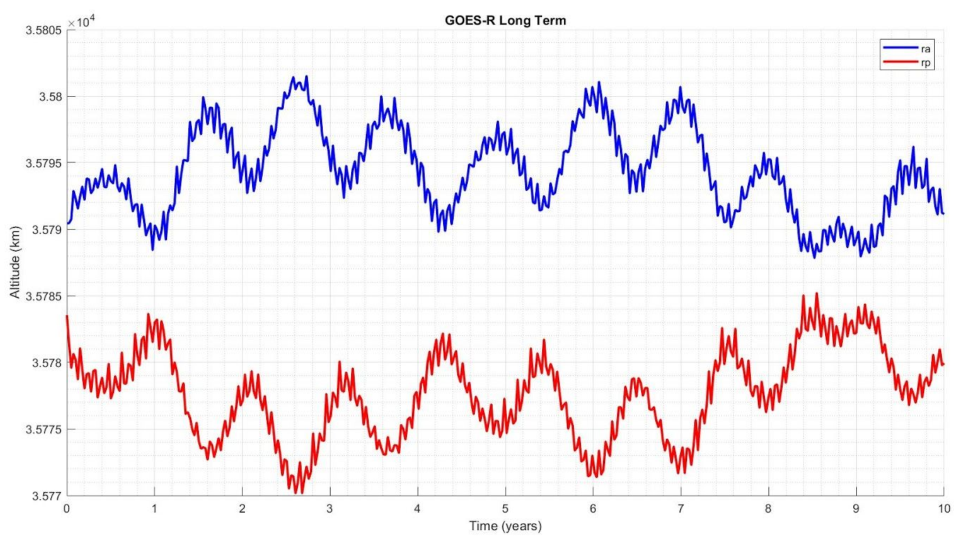

Fig. 7 GOES-R Long Term Orbital Radii

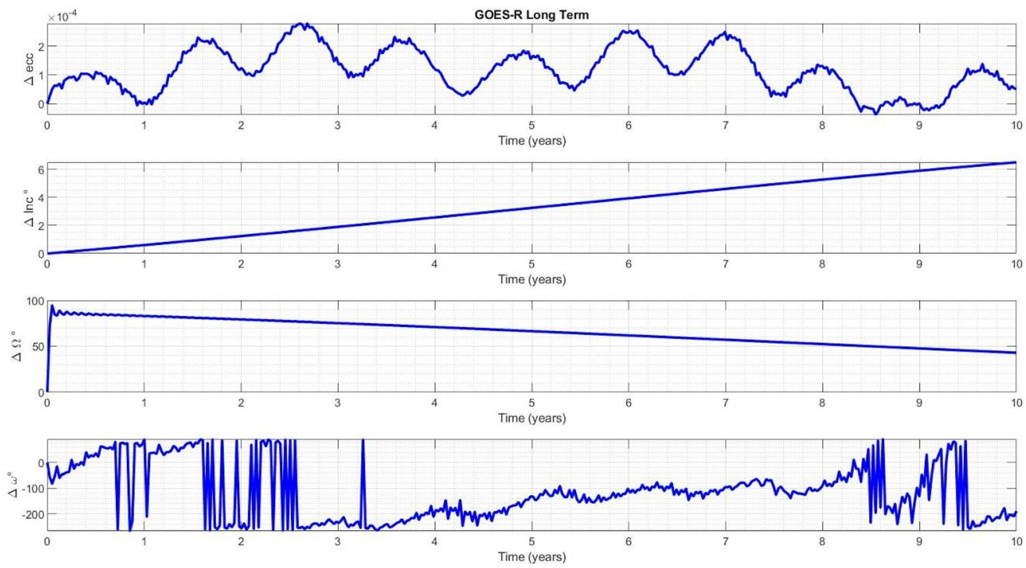

Fig. 8 GOES-R Long Term Orbital Elements

Part 2

Table 7.

	DELFI-C3 (Sun Synchronous)	GOES-R (GEO)	NROL - 35 (Molinya)
DeltaV [km/s]	312.05	147.5	134.35

To calculate the deltaV required for orbit correction each satellite was allowed to propagate for one week with the aforementioned perturbations. At the beginning of this propagation, the osculating orbit was propagated with only two body and this was used as the desired orbit. At the end of the week, the orbit was corrected with a Lamberts burn forced to take half of the original period. This process was repeated for 12 weeks to come up with an estimate of the total orbit correction cost. Figure 9 visualizes this burn correction method for DELFI-C3 with the Lamberts trajectories for each week. Table 7 contains a comparison of the deltaV costs for the sun synchronous satellite, the geostationary satellite, and a Molynia satellite. The orbit correction cost is so absurdly high partly because of the time restriction on the Lamberts burn. In addition, the Lamberts burn corrects for heavy burns such as inc and RAAN changes, thereby further contributing to the large required deltaV for orbit correction.

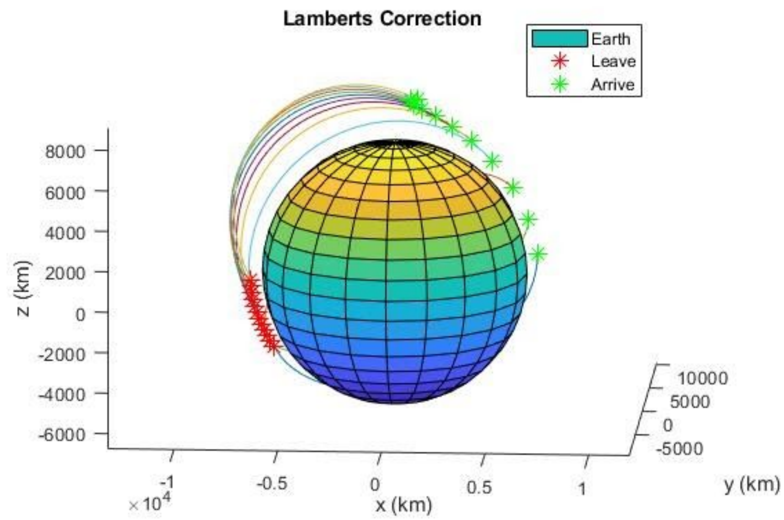

Fig. 9 Lamberts Correction Burns for DELFI-C3

Part 3

Table 8.

Orbital Parameters of NROL-35	
Semi-Major Axis	42,165 km
Eccentricity	.7185
Inclination	63.4408°
RAAN	358.09°
Argument of Perigee	269.57°
True Anomaly	37.7058°
Period	11.96 hrs
JD0	2458806.4838 days

We chose the NROL-35 satellite that was launched in December 2014 and placed into a highly elliptical Molynia orbit, shown in Figure 10 [4], [5]. This type of orbit is special since its near critical inclination causes the argument of perigee to not change over time. The NROL-35 TLE was obtained from October 18, 2019 at 10:58 UTC [2], [3]. The NROL-35 satellite is a spy satellite so the size and dimensions are not public information [5], so the the GOES-R physical properties were assumed.

Table 9.

Perturbations acting on NROL-35	
Perturbation	**Reasoning**
J2-J6	Earth oblateness is the strongest perturbing factor for a Molynia satellite. [8]
Lunar 3-body	Lunar n-body effects have an increasing effect on Molynia satellites as its eccentricity and semi major axis increases. [8]
SRP	Solar radiation shows becomes significant in long term propagations of Molynia orbits. Affects all orbital elements except the semi major axis. [8]
Drag	Although the satellite spends most of its orbital period outside of the drag, this perturbation was still included for perigee passage.

The NROL-35 was propagated using Gaussian VOP with perturbations listed in table 9. We chose NROL-35 satellite to show that it uses some of the perturbations to its advantage and Table 7 compares its deltaV cost to the two other satellites. The cost for the correction of the NROL-35 is less than that of the DELIF-C3 and GOES-R even with no optimization of the lamberts correction burn. Although the DELFI-C3 is in a specialized sun synchronous orbit, it requires significantly more orbit correction because the perturbations are significantly larger in magnitude than at Moynia altitudes.

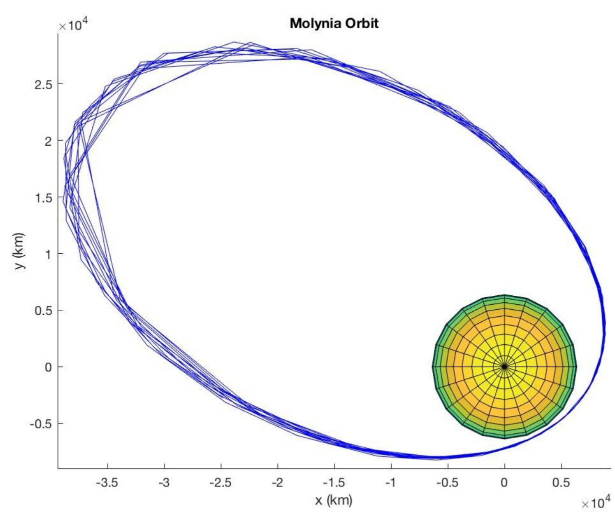

Fig. 10 Orbit of NROL-35 in ECI with Earth

Figure 10 show the gradual perturbation effects on the orbit, and Figures 11 and 12 show the short and long term changes in orbital radii. The radii of apogee and perigee were split into separate plots because the change was too small to see when both trends were included on a single plot. The radius of apogee is showing larger changes than perigee because stronger perturbing accelerations affect the satellite at perigee but show their effect in the radius of apogee.

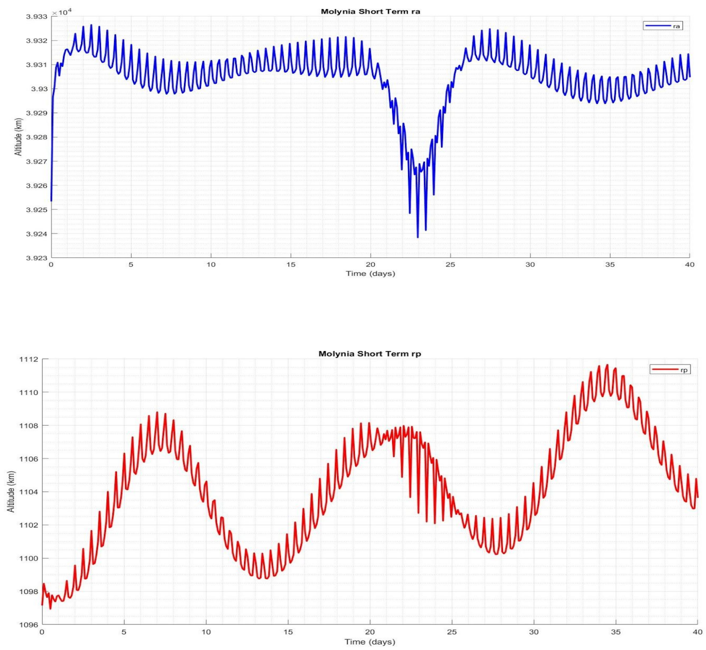

Fig. 11 The variation of apogee and perigee for Molynia orbit over 40 days.

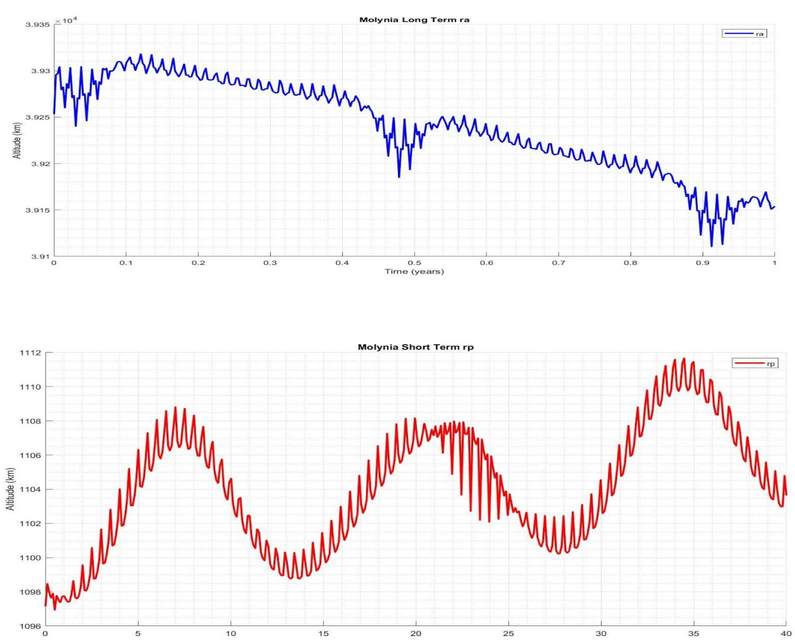

Fig. 12 The apogee and perigee of a Molynia orbit over one year with perturbations.

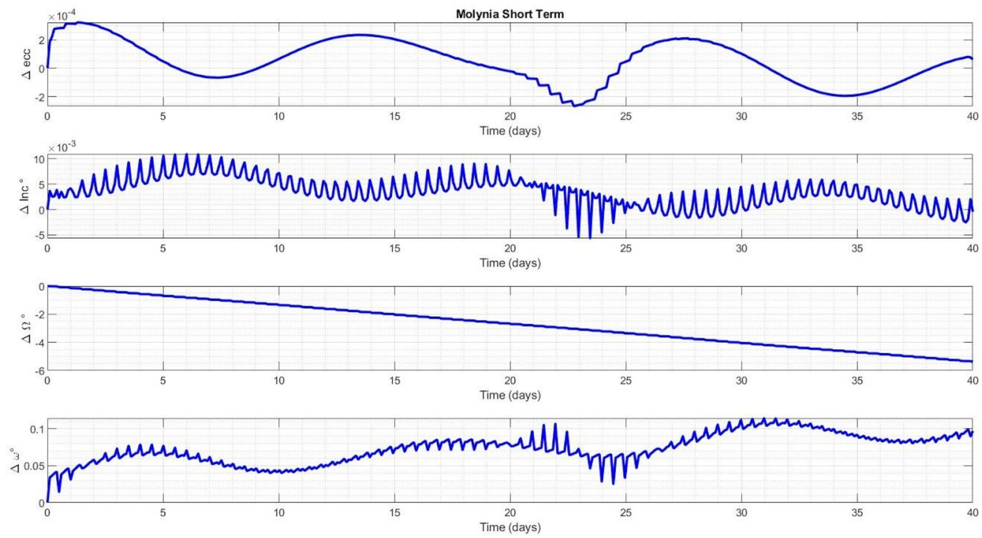

Fig. 13 The variation of orbital elements of Molynia orbit over 40 days.

Inclination and eccentricity change on the order of 10^{-3} and 10^{-4} while RAAN changes by single digits over 40 days but by the tens over one year. This very small change in eccentricity shows that despite the variation of the radii of apogee and perigee, the shape of the orbit remains nearly constant. This RAAN change is reasonable as the ground track for the orbit is expected to change over the lifetime of the orbit. Molynia orbits are designed to have no change in argument of perigee, confirmed by the fact that even with the perturbations taken into consideration, it only increases about half a degree over the course of one year.

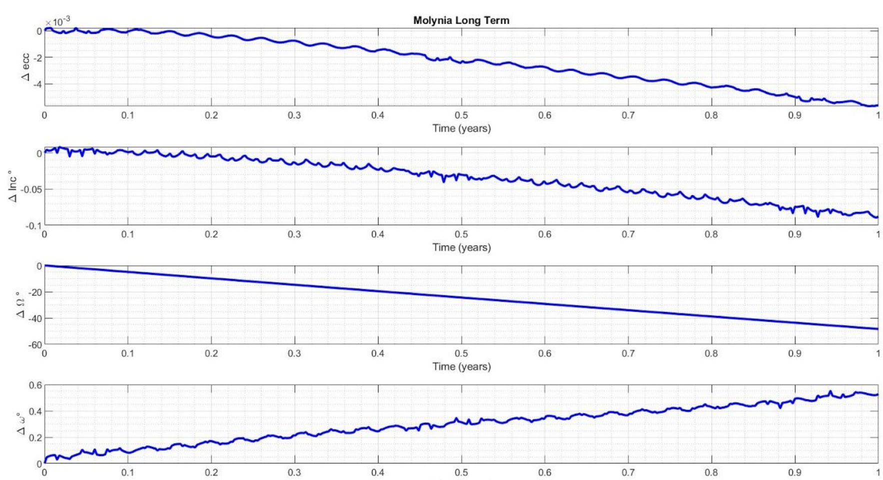

Fig. 14 The change of orbital elements from the original values over one year with perturbations.

Extra

For our extra we chose to repeat the long-term analysis of the GOES-R satellite while including the N-Body effect of the sun.

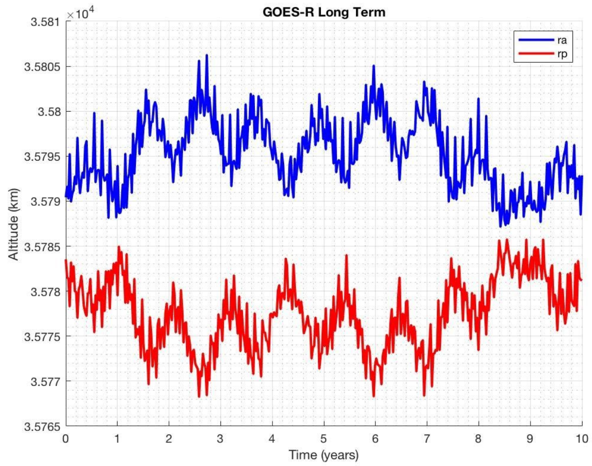

Fig. 15 The N-Body effects of both the Sun and the Moon over ten years in a geostationary orbit

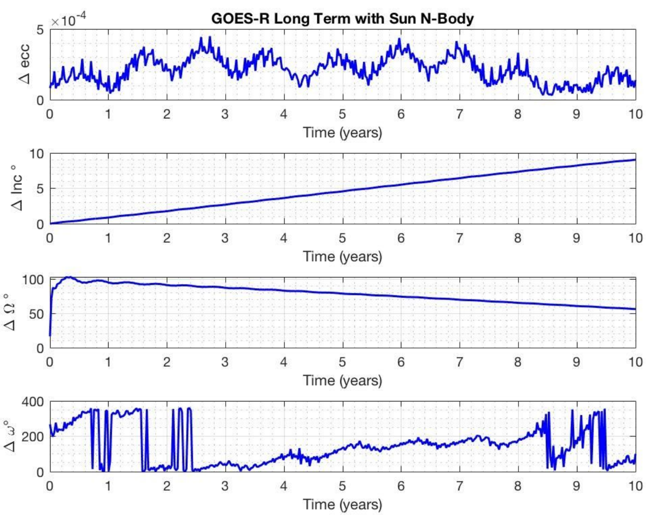

Fig. 16 The N-Body effects of both the Sun and the Moon over ten years in a geostationary orbit

Figure 15 shows the effect of both N-Body perturbations on altitude for ten years, while Figure 16 shows the change in orbital elements. For the GOES-R orbit, the lunar N-Body has a larger contribution to perturbation than solar, but combined there is a noticeable difference. The apogee of the orbit has more variance than in Figure 7, but the perigee is more constant, a result of greater variation in the eccentricity. The inclination of the orbit increases to nearly 10° rather than 6°, and the RAAN decreases at a greater rate. The argument of perigee exhibits the same behavior, which is reasonable as the orbit remains near circular. This shows that although small, the effect of the sun on satellites orbiting the earth is significant if an accurate representation of the effect of perturbations is necessary. In this case it may be advantageous to also include the effect of the sun on the moon in the analysis, but for understanding the general trends or magnitude of how the orbit will change over the lifetime of the satellite, including only lunar n-body is sufficient.

References

[1] https://www.goes-r.gov/
[2] https://www.n2yo.com/
[3] http://www.onlineconversion.com/julian_date.htm
[4] https://www.nasaspaceflight.com/2014/12/atlas-v-launch-nrol-35-vandenberg/
[5] http://spaceflight101.com/spacecraft/nrol-35/
[6] https://issfd.org/ISSFD_2009/CollisionRiskII/Kolyuka.pdf

Affidavit

Kate:

- Propagated GOES-R using Gaussian Variation of Parameters
- Coded solar radiation pressure perturbation
- Coded solar n body perturbation for extra
- Worked together to find objects and relevant information
- Worked together on deciding how to correct for perturbations
- Worked together on debugging code
- Worked together on amalgamating all required functions
- Worked together on producing plots and commenting on trends
- Worked together on organizing the final write up

Martin:

- Propagated DELFI-C3 using Gaussian Variation of Parameters
- Coded lunar n body perturbation
- Coded J2-J6
- Calculated deltaV to correct for perturbations
- Worked together to find objects and relevant information
- Worked together on deciding how to correct for perturbations
- Worked together on debugging code
- Worked together on amalgamating all required functions
- Worked together on producing plots and commenting on trends
- Worked together on organizing the final write up

We are in agreement with the work presented above is an accurate, fair and equal amount of the work completed for this project.