ArrayStack: Amortized Analysis

Open Data Structures

ArrayStack

Amortized analysis of resize()

Theorem

An ArrayStack implements the List interface.

Ignoring the time spent in calls to resize(),

- get(i) and set(i, x) each run in $O(1)$ time; and
- add(i, x) and remove(i) each run in $O(1 + n - i)$ time.

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Furthermore, if we start with an empty ArrayStack and perform any sequence of m add(i, x) and remove(i) operations, then the total time spent in all calls to resize() is $O(m)$.

The Plan

resize() during add(i, x)

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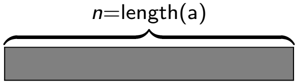

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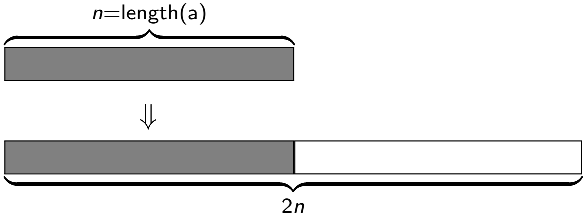

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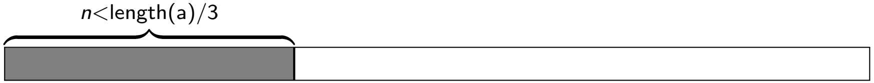

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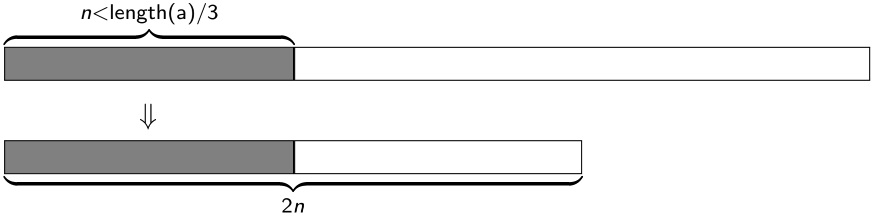

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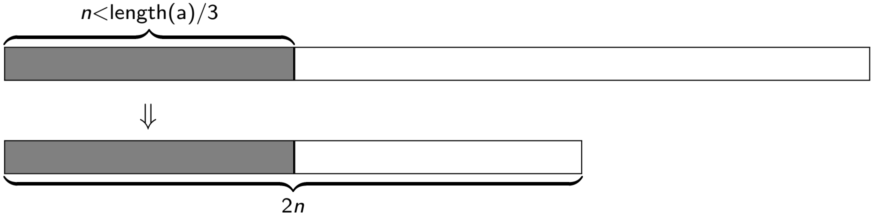
resize() during remove(i)

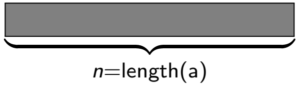


Plan

Show that the total number of items copied by resize() is at most $2m$.

Time Between Resizes

Resize triggered by add(i, x)

Now: 

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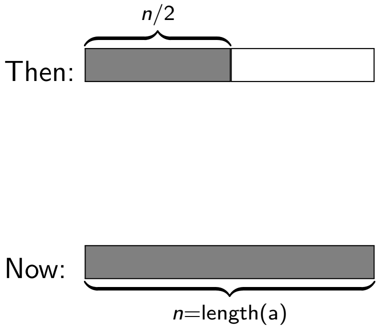

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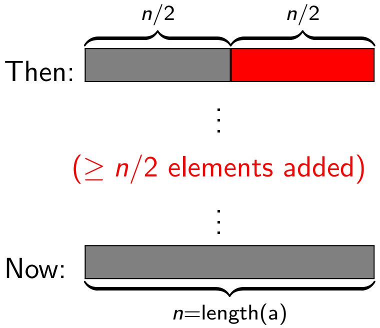

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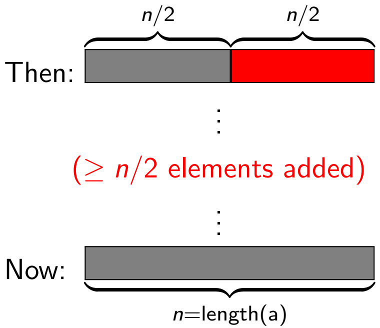
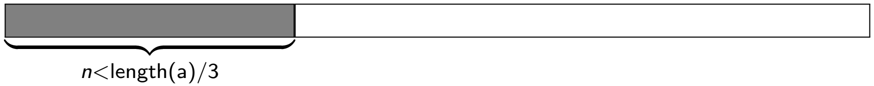


- At least $n/2$ add(i, x) operations between then and now

Time Between Resizes

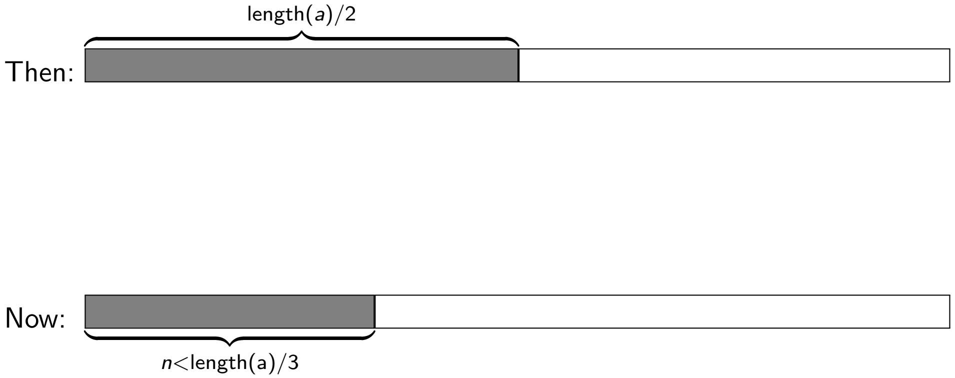
Resize triggered by remove(i)

Now:



Time Between Resizes

Resize triggered by remove(i)



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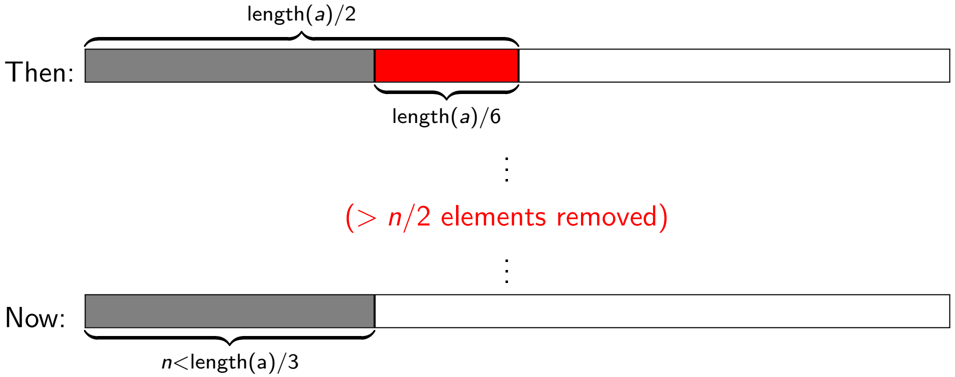
Resize triggered by remove(i)



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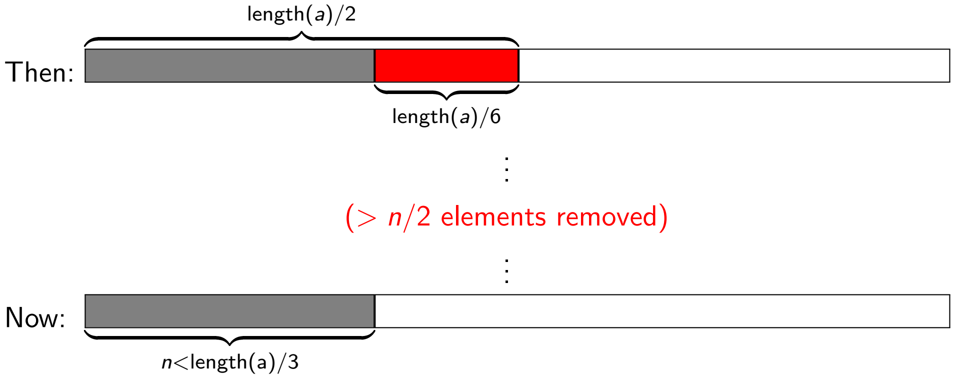
Resize triggered by remove(i)



- At least $n/2$ remove(i) operations between then and now

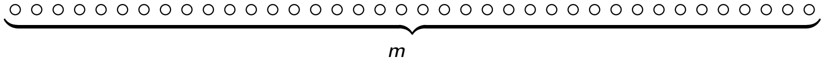
Time Between Resizes

Summary

Operations Between Resizes

If a resize() operation copies n elements, then there have been at least $n/2$ add(i, x) or remove(i) operations since the preceding resize() operation.

Overview



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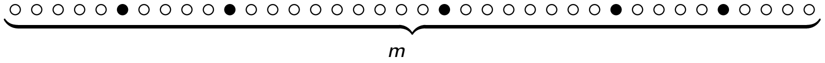
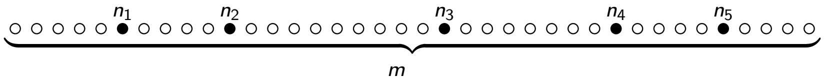


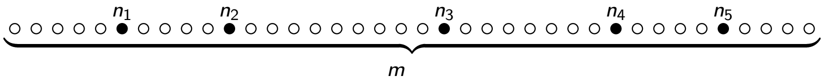
Overview

- $n_i =$ number of items copied by the i^{th} resize() operation



Overview

- $n_i =$ number of items copied by the i^{th} resize() operation
- $m_i =$ number of add/remove operations between the $(i-1)^{\text{th}}$ and i^{th} resize()



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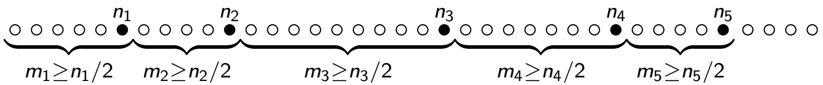

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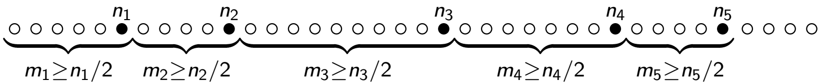

- Total number of items copied: $N = n_1 + n_2 + n_3 + \cdots$

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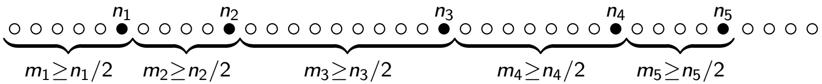

- Total number of items copied: $N = n_1 + n_2 + n_3 + \cdots$
- Total number of add/remove operations: $m \geq m_1 + m_2 + m_3 + \cdots$

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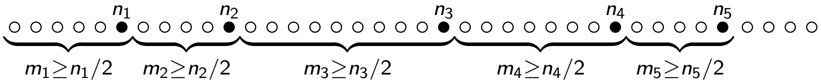

- Total number of items copied: $N = n_1 + n_2 + n_3 + \cdots$
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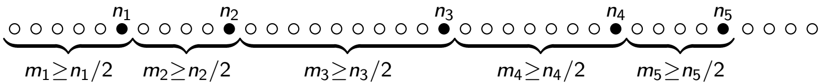

- Total number of items copied: $N = n_1 + n_2 + n_3 + \cdots$
- Total number of add/remove operations: $m \geq N/2$

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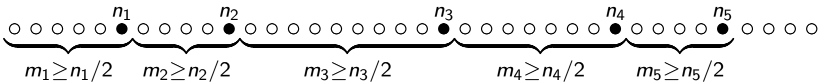

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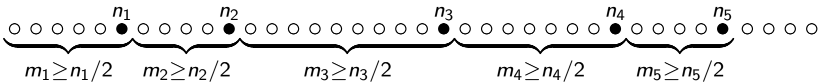

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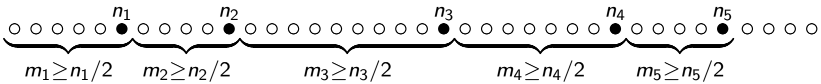

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End of Lesson