Winter Fruits New Problems from OEIS

December 2016 - January 2017

Neil J.A. Sloane

The OEIS Foundation, and Rutgers University

Experimental Mathematics Seminar Rutgers University, January 26 2017

Outline

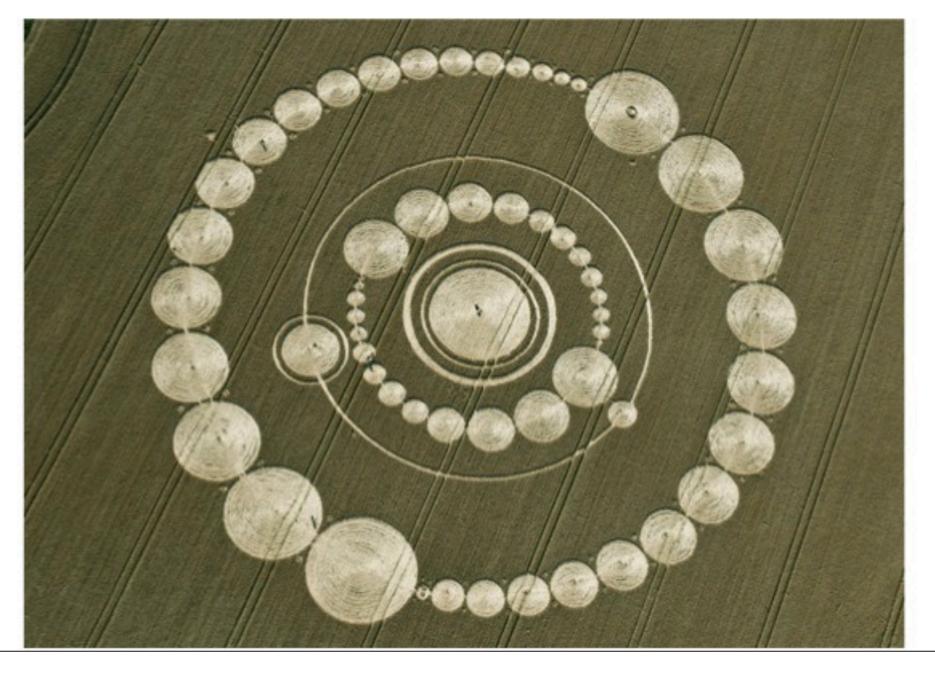
- Crop circles / What not to submit / pau
- Graphs of Chaotic Cousin of Hofstadter-Conway
- Richard Guy's 1971 letter
- Fibonachos
- Fibonacci digital sums
- Carryless problems
- Tisdale's sieve
- Square permutations, square words
- Remy Sigrist's new recurrences
- Michael Nyvang's musical compositions based on OEIS

Dec 28 2016, Question in email:

What is the significance of 11,12,14,18?

Prime numbers?
The Windmill Hill crop circle, July 26, 2011

This elegant and massive display (over 400 feet in diameter) is composed of circles that ascend from small to large or descend from large to small. In the inner ring there are 11 circles followed by 12 circles. In the outer ring there are 14 circles followed by 18.



Answer: Sorry, I cannot help you

WHAT NOT TO SUBMIT

A-numbers of sequences contributed by [your name]

```
271***, 275***, 275***, 275***, 276***, 276***, 276***, 276***, 276***, 276***, 276***, 276***, 276***, 276***, 276***, 276***, 276***, 278***, ...
```

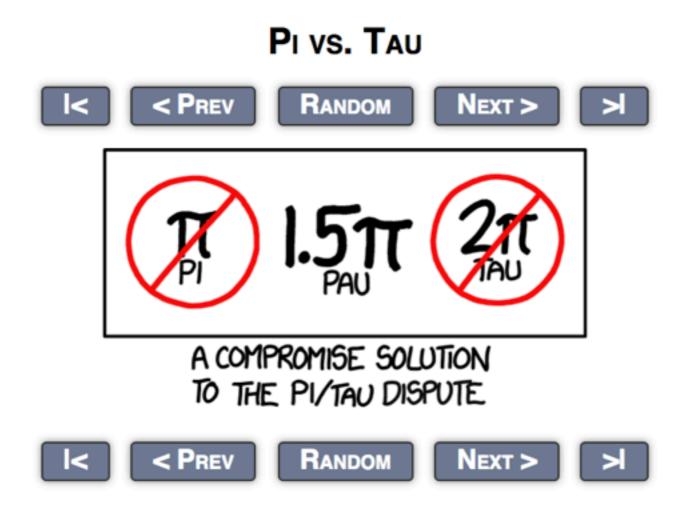
Added immediately to OEIS Wiki page "Examples of What Not to Submit"

"NOGI" = Not of General Interest

The number pau

Comment on A197723, Jan 8 2017: Decimal expansion of 3 Pi / 2 = 4.712388980384...

Randall Munroe suggests the name pau as a compromise between pi and tau.



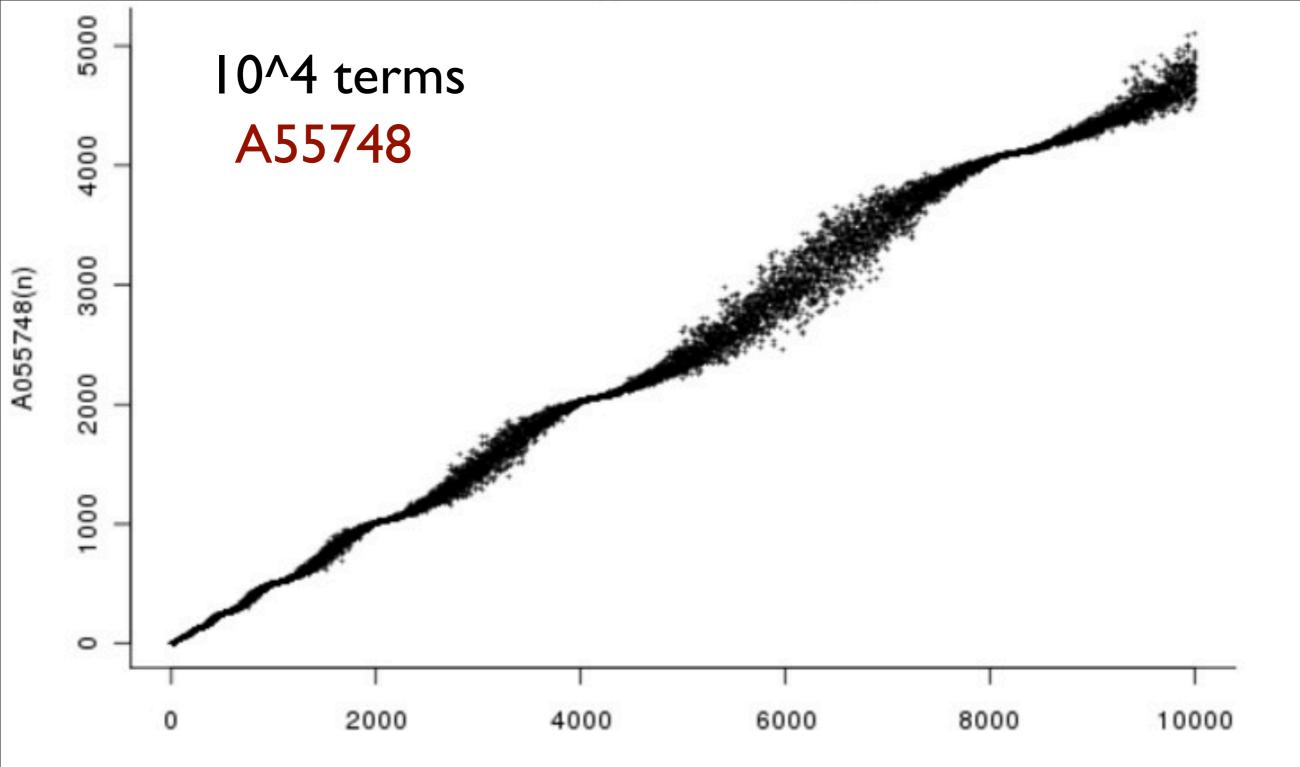
Permanent link to this comic: http://xkcd.com/1292/
Image URL (for hotlinking/embedding): http://imgs.xkcd.com/comics/pi_vs_tau.png

New Graphs of A55748 Chaotic Cousin of Hofstadter-Conway

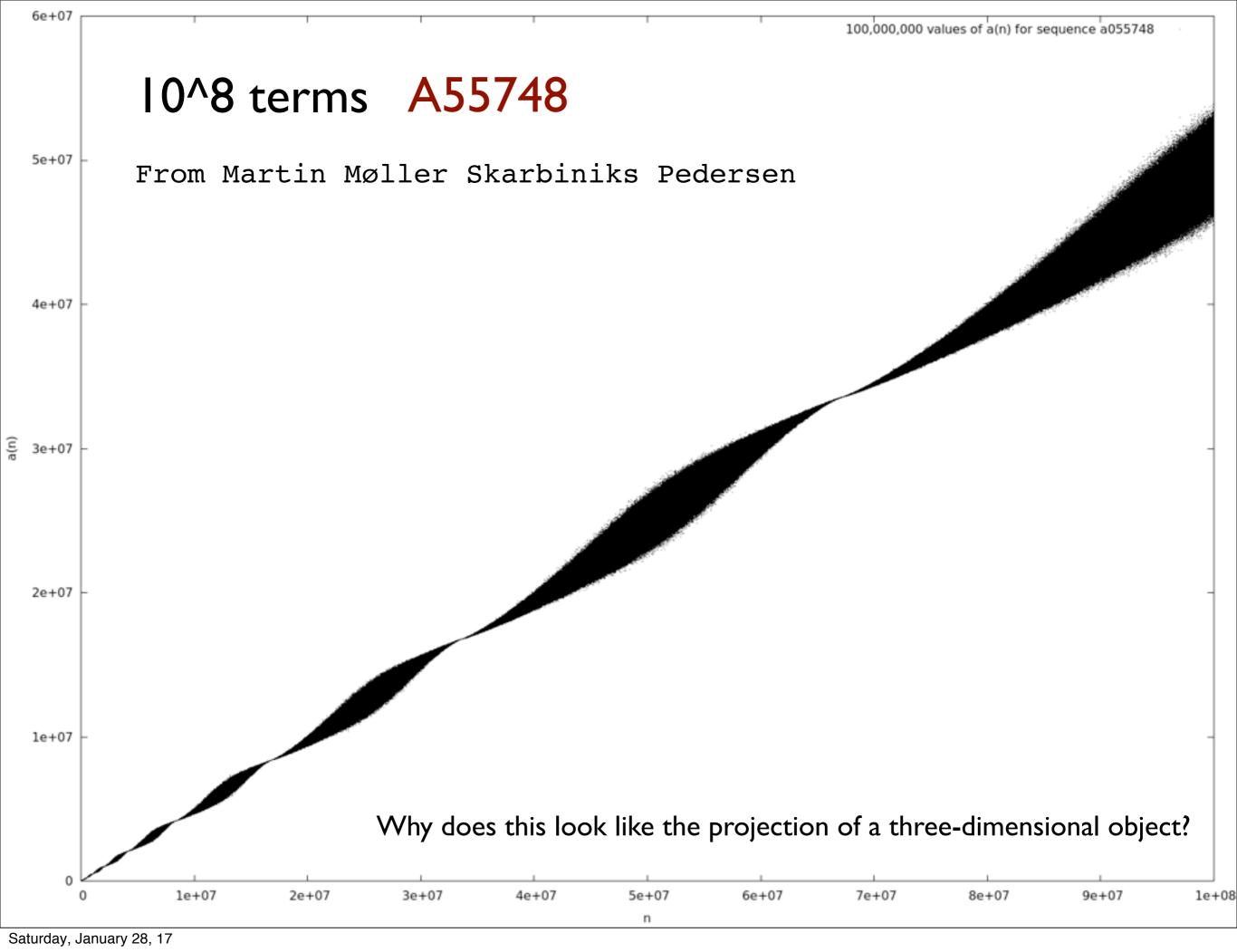
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A4001 (the $10,000 sequence):

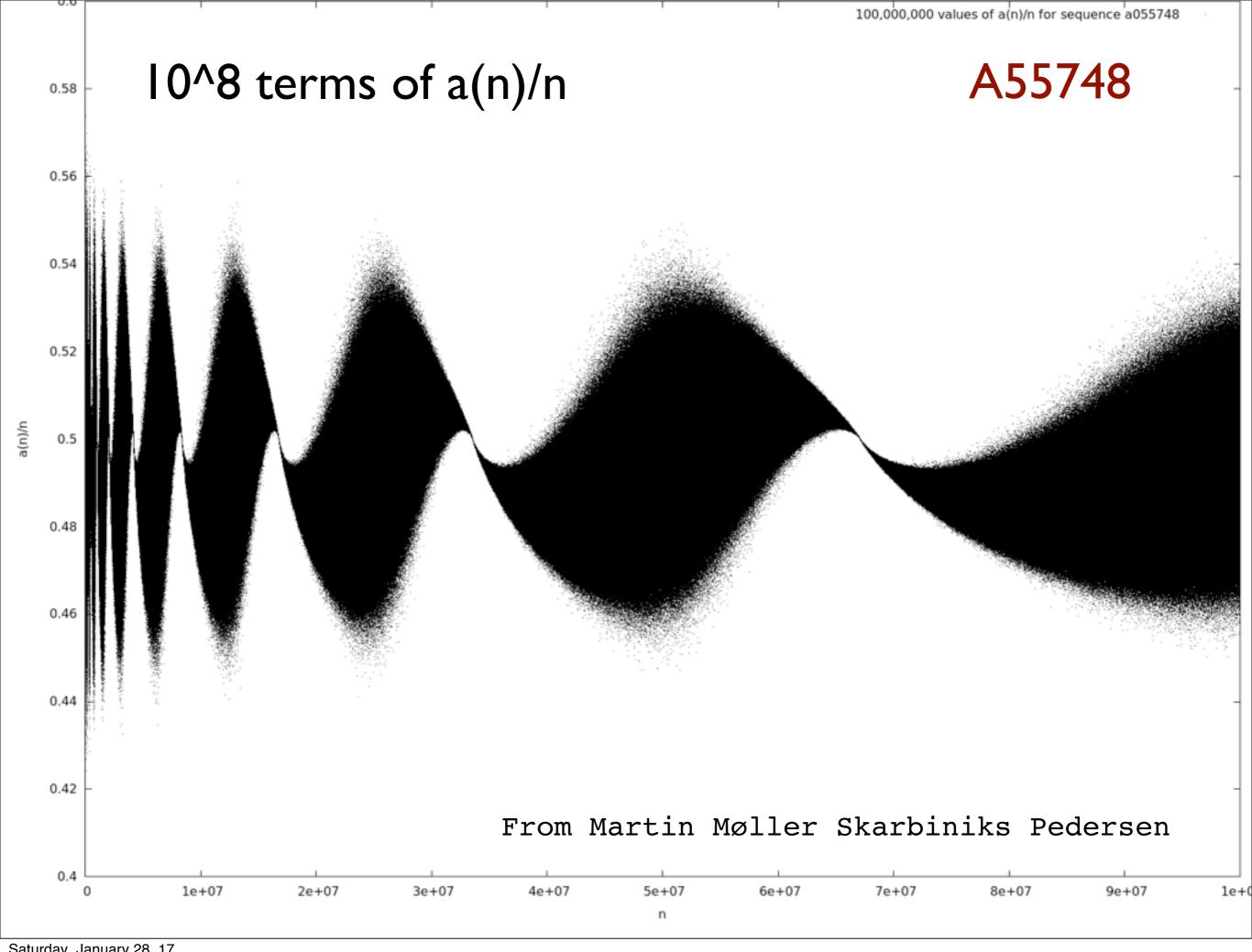
a(n) = a(a(n-1)) + a(n-a(n-1))
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A55748: a(n) = a(a(n-1)) + a(n-a(n-2)-1)



From Martin Møller Skarbiniks Pedersen



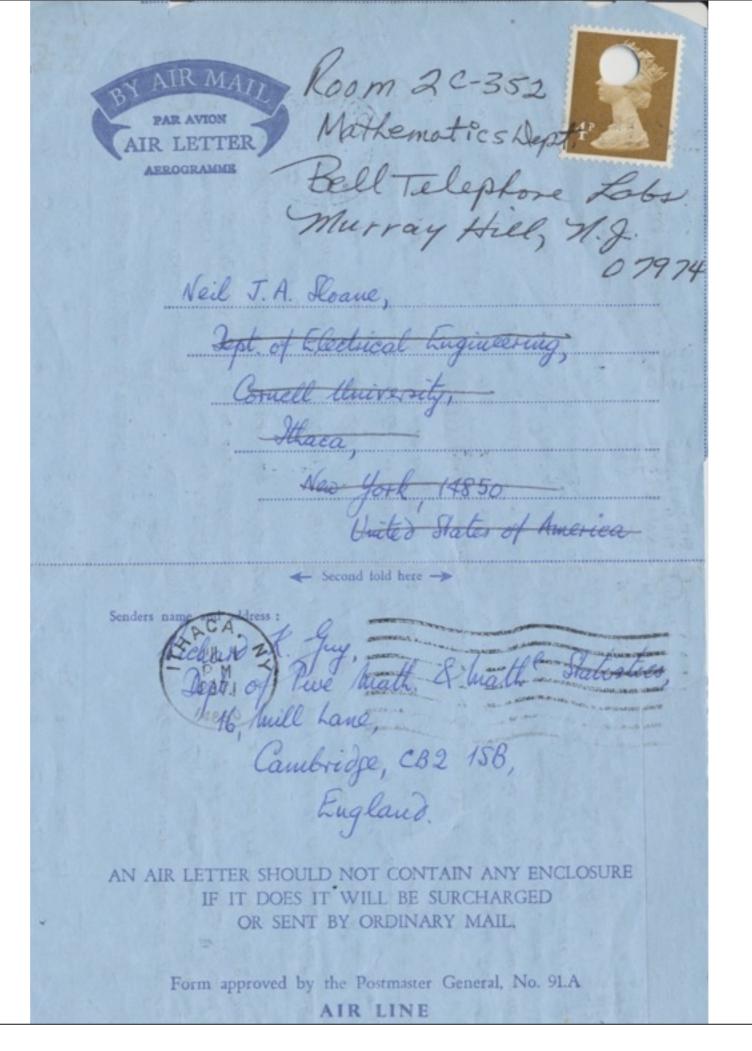


Richard Guy's letter June 24 1971

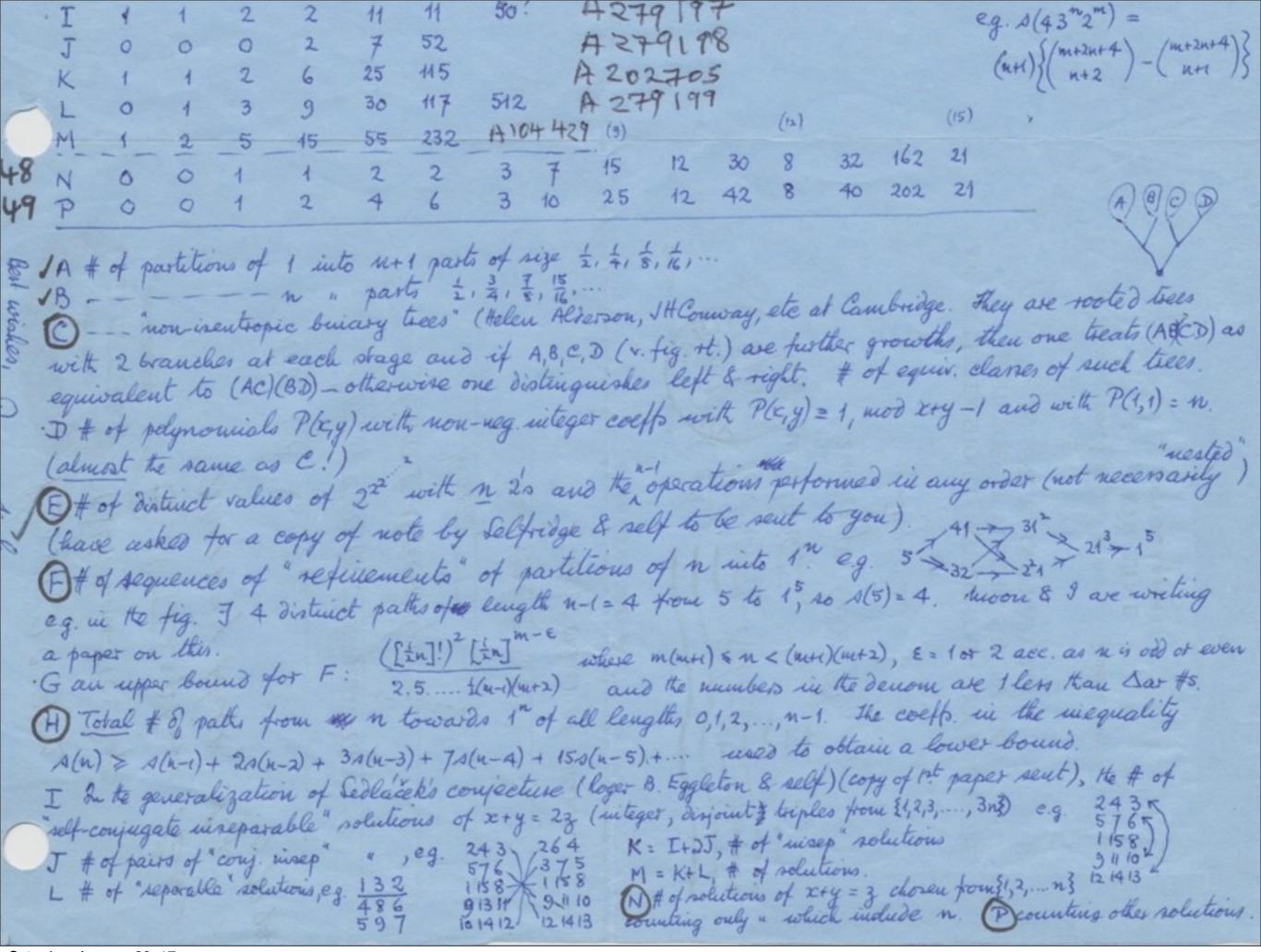
(15 sequences, many still need extending, 46 years later)

One of many letters from Richard Guy

June 24 1971



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Sequences C and D from Guy's letter need more terms and clearer definition

C: A2844

Number of non-isentropic binary rooted trees with n nodes.

1, 1, 2, 5, 13, 36, 102, 296, 871, 2599

Studied by Helen Alderson, J. H. Conway, etc. at Cambridge. These are rooted trees with two branches at each stage and if A,B,C,D (see drawing in letter) are further growths, then one treats (AB)(CD) as equivalent to (AC)(BD) - otherwise one distinguishes left and right. The sequence gives the number of equivalence classes of such trees.

D: A279196

December 15 2016

Number of polynomials P(x,y) with non-negative integer coefficients such that

 $P(x,y) == I \mod x+y-I \text{ and } P(I,I) = n.$

1, 1, 2, 5, 13, 36, 102, 295, 864

(both have offset I)

Postscript, Jan 28 2017: Doron Zeilberger informs me he has a Maple program that implements the definition of sequence C, and he is extending the sequence.

See A002844 for details.

Guy's sequences I, J, K, L, M also need more terms

M: A202705

Number of irreducible ways to split 1...3n into n 3-term arithmetic progressions

1, 1, 2, 6, 25, 115, 649, 4046, 29674, ...

Offset I. Only 14 terms known, extended by Alois Heinz in 2011.

3 papers by Richard Guy, 1971-1976 Calgary thesis by Richard Nowakowski 1975, not online

Are there any applications here of modern "additive combinatorics" (Gowers et al.)?

Definition not clear, need better examples, formulas?

I: A279197

Number of self-conjugate inseparable solutions of X + Y = 2Z (integer, disjoint triples from $\{1,2,3,...,3n\}$).

```
1, 1, 2, 2, 11, 11, 50 (offset I)
```

```
Example of solutions X,Y,Z for n=5: 2,4,3 5,7,6 1,15,8 9,11,10 12,14,13
```

Fibonachos and generalizations

Fibonachos Numbers



Based on Reddit page created by "Teblefer"

A280521, contributed by Peter Kagey, Jan 4 2017

Fibonachos, cont. Start with pile of n nachos.

```
Successively remove I, I, 2, 3, 5, 8, ..., F_i until number left is \{i+1\}
```

Then successively remove $I, I, 2, 3, 5, 8, ..., F_j$ until number left is $\{F_{j+1}\}$

Repeat until no nachos left. a(n) = number of stages.

3 stages, so a(23) = 3

Fibonachos, cont.

A280523: When do we first see n? 1, 3, 10, 30, 84, 227, 603, 1589, 4172, 10936, ..., 20365011049 (25 terms known)

Conjecture: This is bisection of A215004:

$$c(0)=c(1)=1;$$

 $c(n) = c(n-1) + c(n-2) + floor(n/2).$
Why?

Postscript: Only a few hours after I gave this talk, Nathan Fox pointed out that the Reddit web page also mentioned a simpler formula, namely

c(n) = Fib(n+3) - floor((n+3)/2).

Using this Nathan was able to prove the conjecture. See A280523 for details.

Fibonachos, cont.

Generalize: Nachos based on S, where S = 1,... is a sequence of positive numbers.

S	a(n)	records at			
Fib.	A280521	A280523			
n	A057945	A006893			
n(n+1)/2	A104246	<=5 (conj.)			
2^n	A100661	A000325			
n^2	A280053	A280054			

No. of triang. nos. needed to represent n by greedy alg.

2^n-n

New

A28053 and A280054

Nachos based on Squares

```
n=36 subtract leaves
```

```
I 35
4 31
9 22
I6 6
I 5
4 I
I 0
```

3 stages, so a(36)=3

Smallest number with nachos value n

```
1 1
2 2
3 3
4 4
5 9
6 23
7 53
8 193
9 1012
10 11428
11 414069
12 89236803
13 281079668014
14 49673575524946259
15 3690344289594918623401179
16 2363083530686659576336864121757607550
17 1210869542685904980187672572977511794639836071291151196
18 444145001054590209463353573888030904503184365398155859130743499369619675545966466
```

24 terms from Lars Blomberg

What are these numbers?

Digital sums of Fibonacci numbers

A67182

Smallest Fibonacci number with digital sum n

Dec 26 2016: A067182 was in a deplorable state:

a(n) = smallest Fib. no. with digital sum n, or -1 if none exists

0, 1, 2, 3, 13, 5, -1, 34, 8, 144, 55, -1, -1, -1, 4181, -1, -1, 89, ...

All were conjectures!

Me to Seq. Fans.: No progress since 2002!

First reply: Not gonna happen!

Me: F_n mod 100 has period 300, might tell us something

Joseph Myers, Don Reble (indep.): You were close! Look at F_n mod 9999, period 600, no value is 6 mod 9999, so a(6) = -1 is true.

But all other—I entries are still conjectures.

Even in base 2 this is hard - see next slide

Digital sums in base 2

Row n: All Fibonacci numbers with Hamming weight n:

Charles Greathouse (Q) and Noam D. Elkies (replies) on MathOverflow, 2014:

The Hamming weight w(n) is the number of 1s in n when written in binary. Is there some effective bound on Fibonacci numbers F_n with $w(F_n) \le x$ for a given x?

Since you specify "effective" in the question I guess you know this already, but just in case: there are only finitely many such n, because $2^{e_1} + \cdots + 2^{e_x} = (\varphi^n - \varphi^{-n})/\sqrt{5}$ is an S-unit equation in x + 2 variables over $\mathbb{Q}(\sqrt{5})$; but in general no effective proof is known for such a result (though the *number* of solutions of $w(F_n) \leq x$ may be effectively bounded). – Noam D. Elkies Mar 2 '14 at 6:28

A222296

Theorem:

Noam D. Elkies, Mar 2 2014

3, 5, 34, 144 are the only Fib. nos. with wt 2.

The case x=2 is still tractable. If $F_n=2^e+2^f$ with e < f then e < 5, else $F_n\equiv 0 \mod 2^5$, which happens iff $n\equiv 0 \mod 24$, and then $7\mid 21=F_8\mid F_{24}\mid F_n$, which is impossible because 2^e+2^f is never a multiple of 7. So we have only a few candidates for e, and we can deal with each of them separately, possibly even by elementary means, to show that (n,e,f)=(12,4,7) is the last solution.

 \langle **EDIT** \rangle Here's such an elementary proof. For each e (other than the trivial e=2), we choose some $f_0 > e$, try each f with $e < f_0 < f$, and then once $f \ge f_0$ we use the condition $F_n = 2^e + 2^f \equiv 2^e \mod 2^f$ to get a congruence condition on n, and then reach a contradiction by considering F_n modulo some odd prime (usually 3, but with one much larger exception).

e=0: We take $f_0=4$. Trying f=1 and f=2 yields the Fibonacci numbers $F_4=3$ and $F_5=5$, and f=3 yields the non-Fibonacci number 9. Once $f\geq 4$ we have $F_n\equiv 1 \mod 16$. But $F_n\mod 16$ is periodic with period 24, and it turns out that the remainder is 1 only for $n\equiv 1,2,23\mod 24$. But $F_n\mod 3$ has period 8, which is a factor of 24; and $F_1=F_2=F_{-1}=1$. We deduce $F_n\equiv 1\mod 3$. Hence $2^f\equiv 0\mod 3$, which is impossible.

e=1: The Fibonacci numbers F_n congruent to $2 \mod 4$ are those with $n\equiv 3 \mod 6$, and these always turn out to be $2 \mod 32$. Thus $f\geq 5$, and f=5 yields the Fibonacci number $34=F_9$. We claim that this is the only possibility, using $f_0=6$. Once $f\geq 6$ we have $F_n\equiv 2 \mod 64$, and then $n\equiv \pm 3 \mod 24$. But (again thanks to 8-periodicity mod 3) this implies $F_n\equiv 2 \mod 3$, so once more we reach a contradiction from the congruence $2^f\equiv 0 \mod 3$.

e=2: impossible because F_n is never 2 mod 4.

e=3: We take $f_0=5$. Since $2^3+2^4=24$ is not a Fibonacci number, we may assume $f\geq 5$, and then $F_n\equiv 8 \mod 32$. This is equivalent to $n\equiv 6 \mod 24$, which again yields a contradiction mod 3 since $2^f=F_n-2^e$ would have to be a multiple of 3.

e=4: This is the hardest case: because f=7 yields $144=F_{12}$, it is not enough to use congruences that can be deduced from $F_n\equiv 2 \mod 2^7$, and we must take $f_0>7$. It turns out that $f_0=9$ works. Then f=5,6,8 yield the non-Fibonacci 48,80,272. Once $f\geq 9$ we must have $F_n\equiv 16 \mod 2^9$. Now $F_n\mod 2^9$ has period 768, but the condition $F_n\equiv 16 \mod 2^9$ determines $n\mod 384$ (half of 768), and we compute $n\equiv -84 \mod 384$. Now $n\mod 384$ determines $F_n\mod 384$ (the period is 128), and we find $F_n\equiv 2284 \mod 4481$, whence $2^f=F_n-2^e\equiv 2284-16=2268 \mod 4481$. But this is impossible because 2 is a fourth power (even an 8th power) mod 4481, and 2268 is not.

(/EDIT)

But I doubt that one can prove that such a technique can work for all x...

Carryless Stuff

(No caries)

Recall!

Carryless Arithmetic

Dedicated to Martin Gardner

No carries in the Carryless Islands!

(former penal colony - prisons have excellent dental care)

$$6 + 7 = 3$$

$$6 \times 7 = 2$$

785 +376

643 \times 59

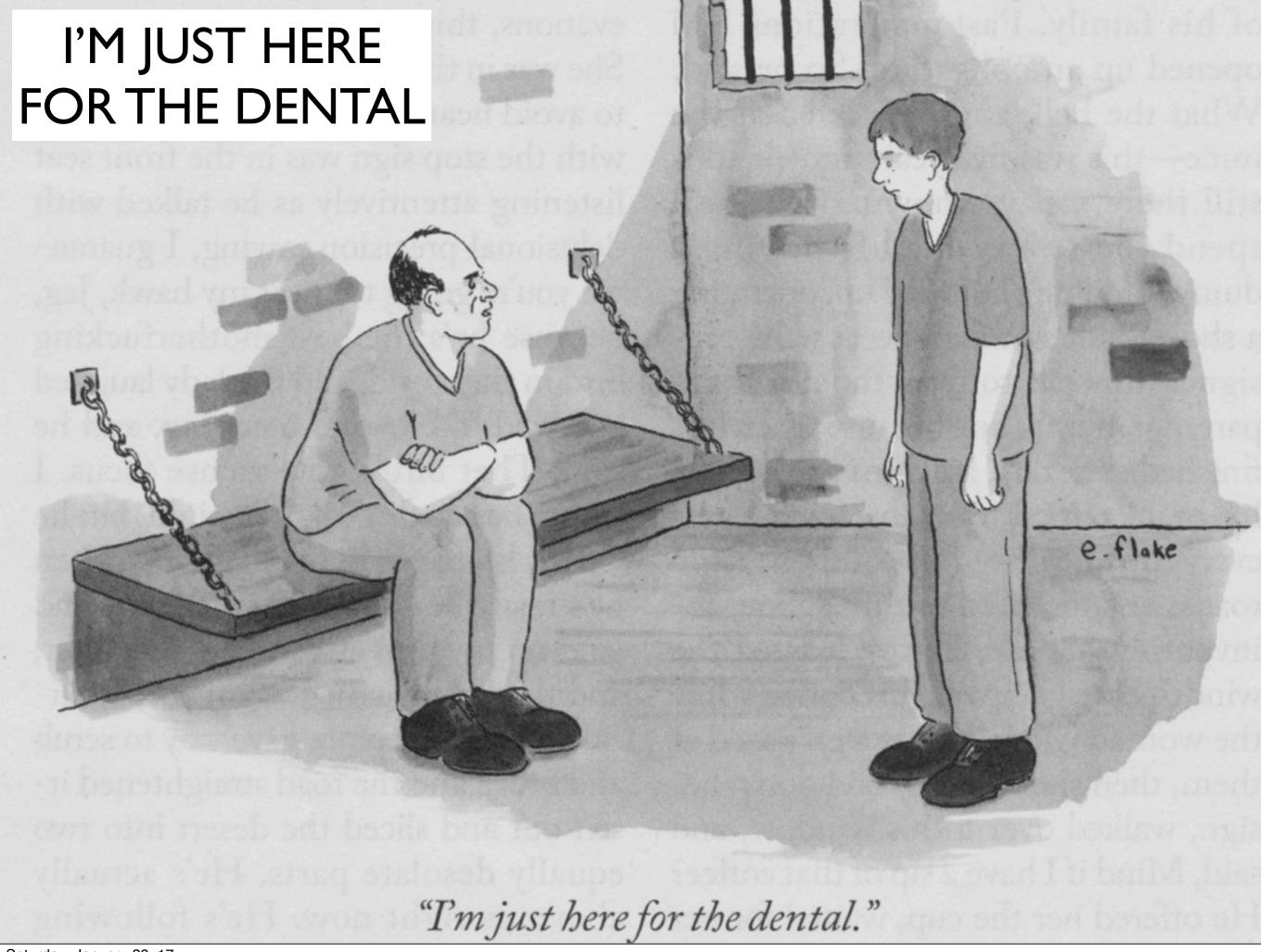
= 051

467

0050

= 417

Applegate, LeBrun, Sloane College Math. J. 2012 George Polya Prize



Recall!

What are the carryless primes?

First try fails!

Any number is divisible by 9, e.g. $9 \times 99 = 11$, so no primes exist

Better: Note that $3 \times 7 = 1$, $9 \times 9 = 1$

So 1, 3, 7, 9 are UNITS, and don't count.

p is prime if only factorization is $p = u \times p'$, where u is 1, 3, 7, 9

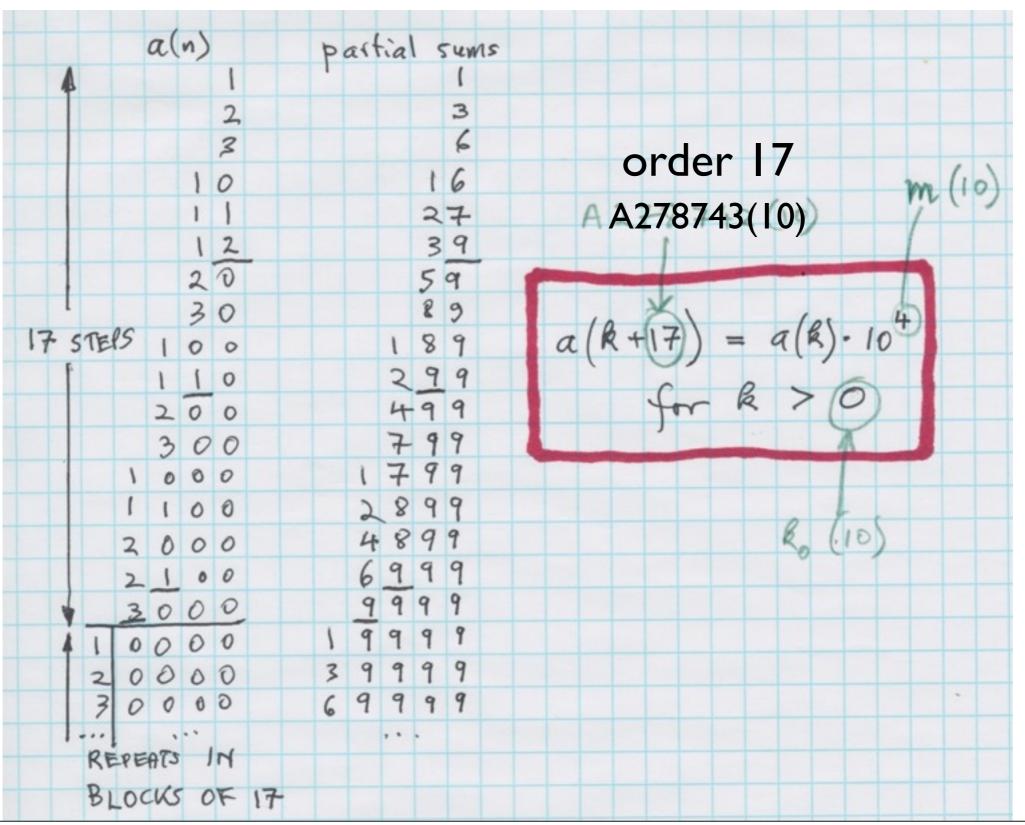
Carryless primes are 21, 23, 25, 27, 29, 41, 43, 45, ...

Sequence A169887 in OEIS

(Be careful: $2 = 4 \times 5005555503!$)

New

a(n) = smallest s.t. a(1) + ... + a(n) has no carries.A278742 Rémy Sigrist Nov 27 2016



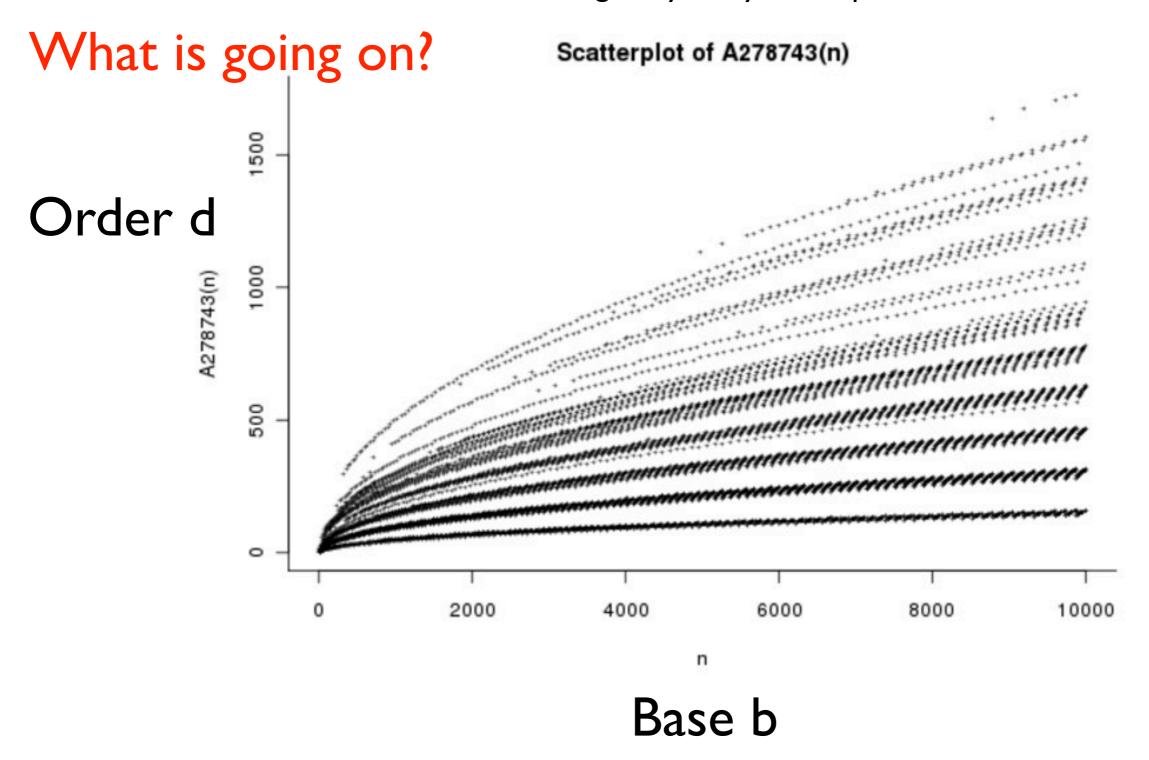
B	ASE 9		
	base 9	base 10	A281366, A280731
1	1	1	
2	2	2	
3	3	3	
4	10	9	A278743(9) m (9)
5	11	10	4.
6	20	18	a(R+4) = a(R) - 9
7	21	19	
8	100	81	-for k > (3)
9	110	90	R (9)
61	200	162	
	210	171	
[2	1000	729	
13	1100	810	
14	2000	1458	
15	2100	1539	
16	10000	6561	

Signist's Conjecture A278743, A280051, A280052 For any base b, I d, Ro, m such that $a(R+d) = a(R) \cdot b^{m}$ for $R > R_{o}$ A278743: d 5 5 0 2 A280051 : Ro 6 9 0 3 A280052 : m 2 12

A278743

From Sigrist's conjecture:

Order of recurrence for greedy carryless sequence in base b



The Tisdale Sieve

A141436

The Tisdale Sieve A141436

Dec 25 2016: Editor J.E.S. said: A141436 is a mess! Me: I will edit it! And discovered a diamond.....

Let
$$P = primes 2,3,5,7,11,13,...$$
 $N = nonprimes 1,4,6,8,9,10,12,...$

Define ∞ set of sequences S_1 , S_2 , S_3 ,... by

 $S_i(1) = simallest$ number not yet used

 $S_i(j+1) = either P(S_i(j))$ or $N(S_i(j))$ so that primes and nongrimes alternate in S_i .

 $S_i = 1,2,4,7,12$
 $S_2 = 3,6,13,21,12$
 $S_3 = 5,9,23,...$
 $S_1 = 1,3,5,8,10,11,...$

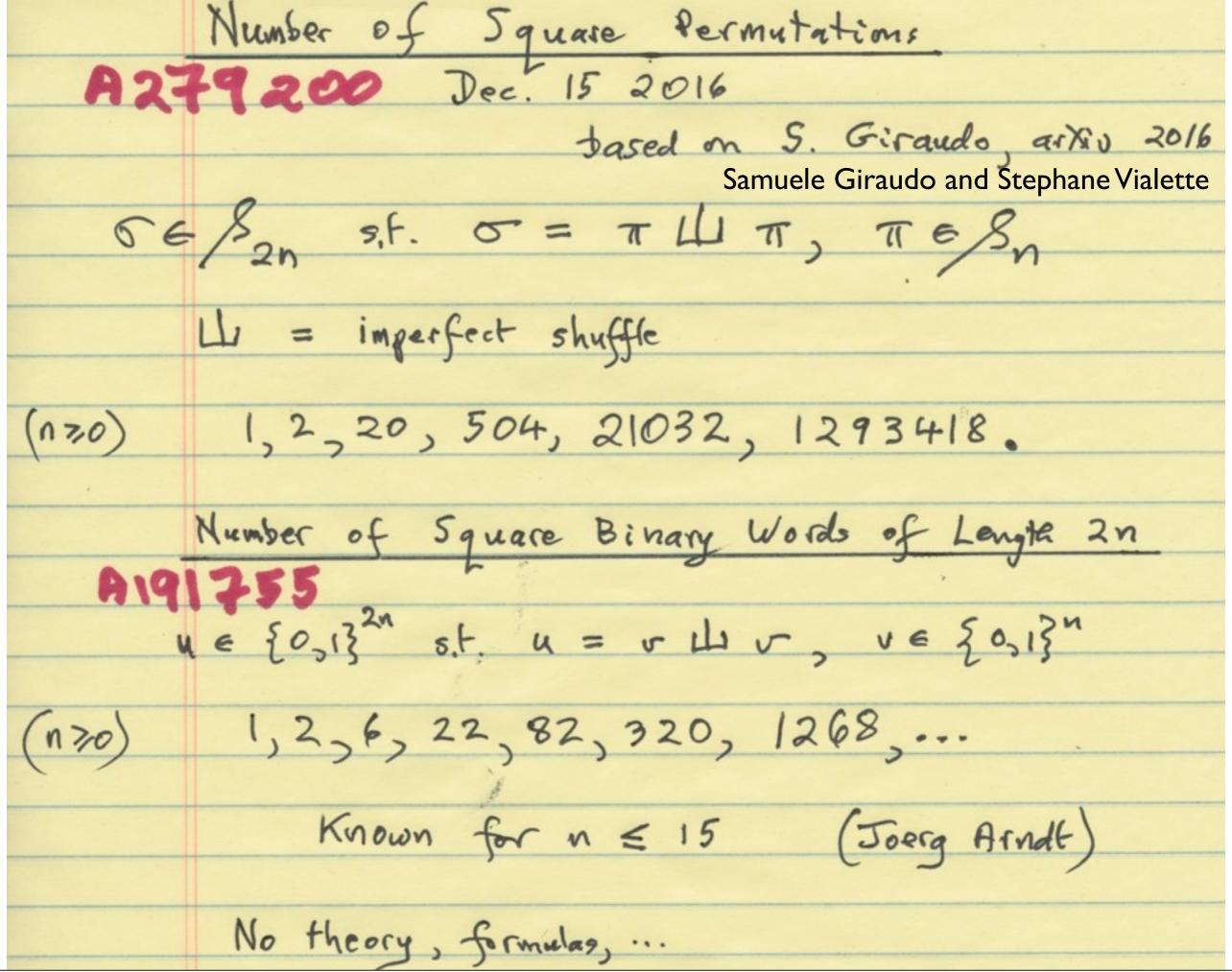
Conjecture $(R,J,Mathar)$: This is union of $A_0 = A_0 = A$

troof of Mathais conjecture by David Applegate Lemma Given $f: \mathbb{Z}^+ \to \mathbb{Z}^+$ with f(R) > R for all R.

Define $S_1, S_2, S_3 S_1 = f(R)$ Si(i) = smallest number not yet used si(j+i) = f(si(j))Then 5,(11, 52(1), 53(1), --- = R = Im(f). Proof If R& In(f) then R = 5i(j), j > 1i. R = 5i(1) for some i Conversely: If Rie In (4), then I m with f(m) = R. - if $m \in Im(f)$ then by IH $m = S_i(j), j > 1$ so $k = S_i(j+i)$ - if m & Im (5) then m = Si(1), R = Si(2)-Apply Lemma with f(R) = P(R) if REN, N(R) if REP.

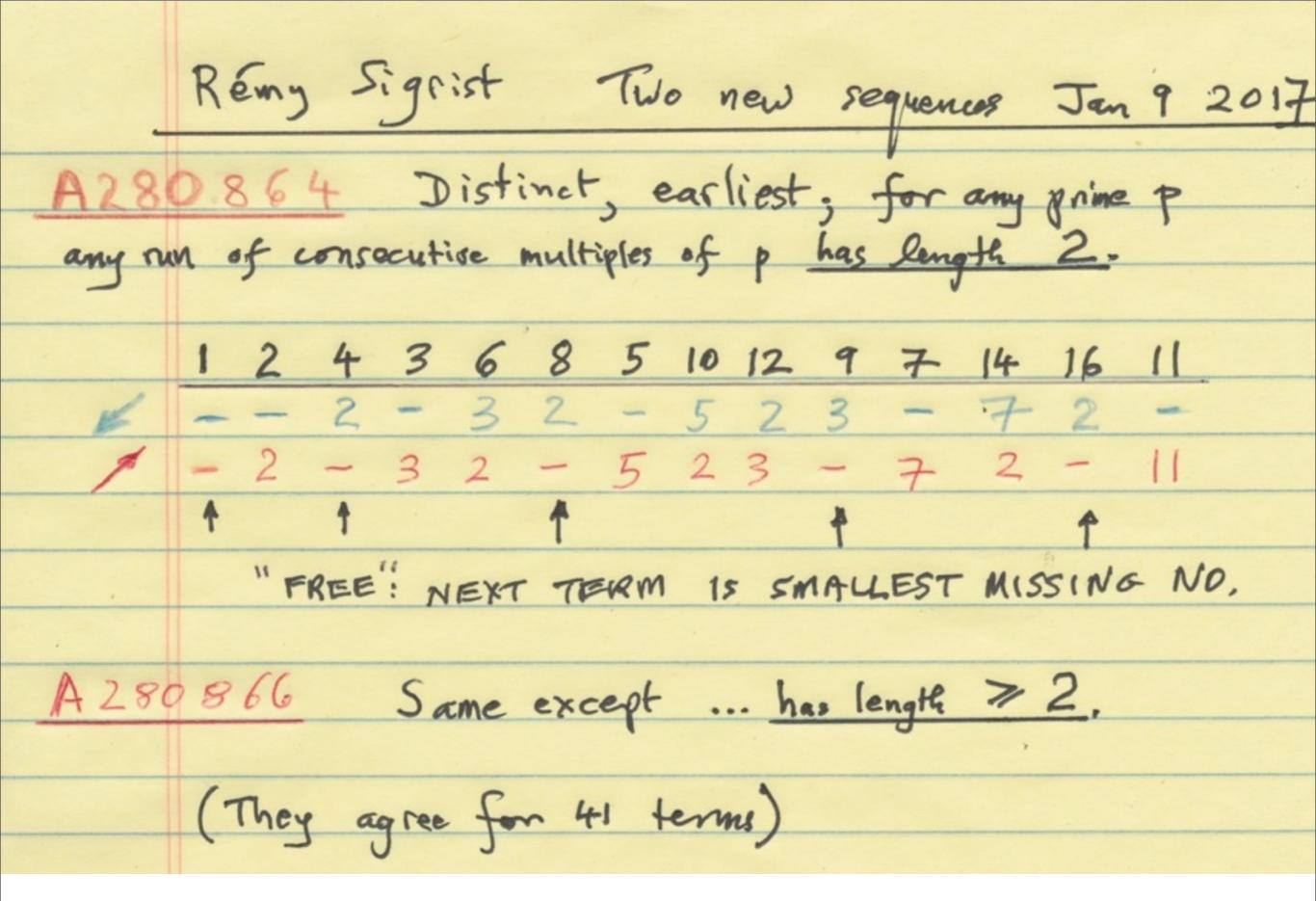
Saturday, January 28, 17

Square Permutations and Square Binary Words



Remy Sigrist's New Sequences

A280864, A280866



A 280866 This is a perw of natural number troof 1. clearly infinite 2. Any m is either in sequence, or I no 8t. n>n0 => a(n)> m 3. For any prime p, 3 term divisible by p (If to never appears, then no prime > p can appear. : all terms are mults products of 235 ... p-1 Go out past p!. Then cambidates for next term are p and p. { any product of district primes < p} as there are < p! so will appear next) 4. The For a prime p, let a (n) be first multiple of p. Exter a (n) = p, & a (n-i) was free, or a(4) = kp, a(n+1) = p and is free. . . 00 many fee term · . · · crey minder appear

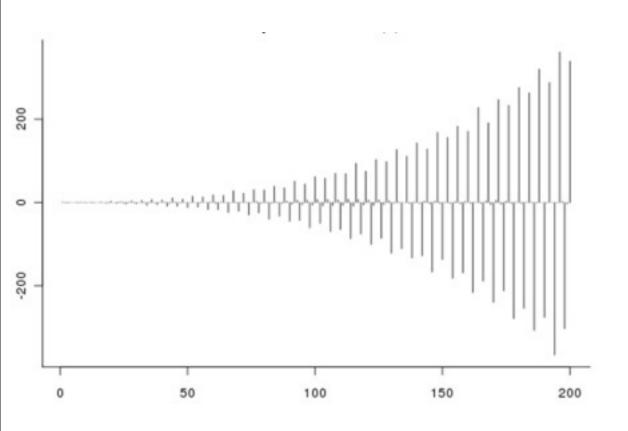
New A281488
with key-words
"look" and "hear"

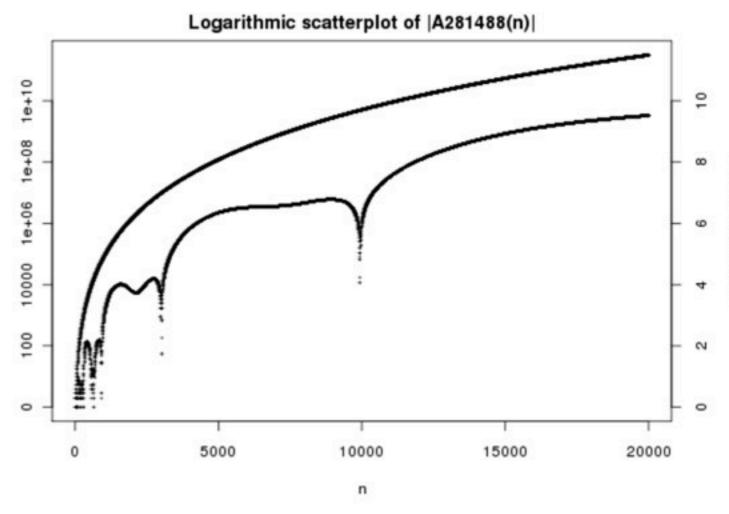
A281488 from Andrey Zabolotskiy January 22 2017

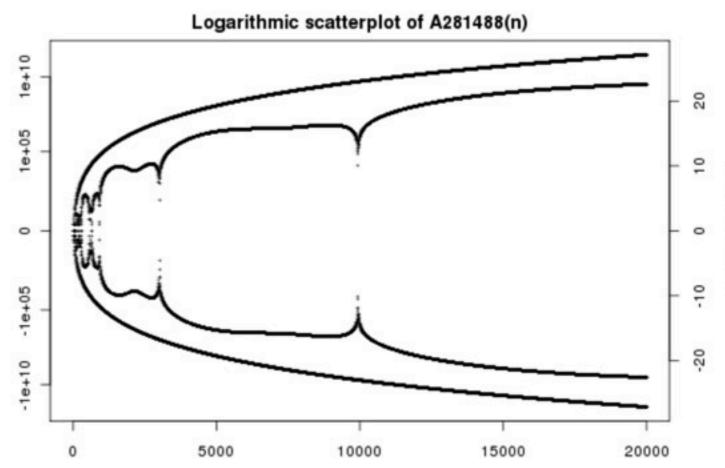
$$a(n) = -\sum_{\substack{d \mid (n-2) \\ 1 \le d \le n-1}} a(d)$$

1, -1, -1, 0, 0, 0, -1, 1, 0, -1, 0, ...









Two compositions from Michael Nyvang (Copenhagen) based on OEIS sequences

- surreal-cantata--final.mp3
- A276207-and-neighbors-music-forNJ.mp3

Like these problems?

Become a volunteer OEIS editor!

Contact Neil Sloane, njasloane@gmail.com