Homework 2

COMP 590-144
Program Verification and Synthesis

Due on: 7/28/2020 *

Note: Please read the instructions for submitting homework and follow the homework policy given on Sakai; submissions not following these guidelines will not be graded.

Problem 1 [Silly Puzzles] Encode the following problems as SAT problems and use your favorite SAT solver (Z3 or Minisat) to solve these problems.

1. There is a basket containing an apple , a banana , a cherry and a date. Four children named Erica , Frank , Greg and Hank are each to be given a piece of the fruit.

   (a) Erica likes cherries and dates.

   (b) Frank likes apples and cherries.

   (c) Greg likes bananas and cherries.

   (d) Hank likes apples , bananas , and dates.

   The problem is to give each child a piece of fruit that he or she likes. **[7.5 points]**

2. Three fellows accused of stealing CDs make the following statements:

   - Ed: "Fred did it, and Ted is innocent".
   - Fred: "If Ed is guilty , then so is Ted".
   - Ted: "'Im innocent , but at least one of the others is guilty".

   If the innocent told the truth and the guilty lied, who is guilty? **[7.5 points]**

Problem 2 [Graph Coloring] Attached with this homework are four files with description of graphs. The first line in the file gives the list of vertices (from 0 to $n-1$). The second line contains a sequence of pairs that represents the edges in the graphs. Use SAT solver to encode the coloring constraints and find out the colorability of the graphs.

- Graph of 6 nodes. **[5 points]**
- Graph of 20 nodes. **[5 points]**
- Graph of 50 nodes. **[10 points]**
- Graph of 100 nodes. **[10 points]**

[0]The deadline is flexible when the graduate students have special requests such as traveling for conference, paper deadlines, or unfamiliar background.

Problem 3 [Solving Sudoku Using SAT Solvers] Sudoku is a popular number-placement puzzle that originated in France in the end of the 19th century. Modern Sudoku was likely invented by Howard Garns from Connersville, Indiana and was first published in 1979 under the name "Number Place". The objective of the puzzle is to place numbers 1 - 9 on a $9 times 9$ grid, such that each number occurs only once in every row, every column, and every of the nine 3×3 sub-grids that compose the main grid. Sudoku puzzles are grids that have been partially occupied with numbers. The task is then to occupy the remaining fields in such a way that the constraints on rows, columns, and sub-grids are satisfied. A sample Sudoku problem and its solution are given in Figure 1. For more information about Sudoku refer to its Wikipedia page at `http://en.wikipedia.org/wiki/Sudoku`.

This problem has two parts. In the first part, you will write the boolean constraints in mathematical notation for solving a Sudoku puzzle. In the second part, you will write code and invoke a SAT solver to solve the Sudoku instance.

Part 1:

1. Write the boolean formula for the constraints that each number can occur at most once in every row. **[5 points]**

2. Write the boolean formula for the constraints that each number can occur at most once in every column. **[5 points]**

3. Write the boolean formula for the constraints that each number can occur at most once in every 3×3 sub-grid. **[5 points]**

Part 2: Encode the above constraints in a SAT solver and solve the Sudoku instance given in Figure 1. One of the widely used techniques is to encode these constraints using the API of the SAT solver. Two popular options are as follows; 1) Using Z3 (downloadable at `http://z3.codeplex.com/`) and use Python API and 2) Use MiniSAT (downloadable at `http://minisat.se/`) and use C++ API. Alternatively, you can write code to generate constraints in SMT-2 format (described at `http://smtlib.cs.uiowa.edu`) and give the SMT-2 file as an input to the binary of SAT solver. **[25 points]**

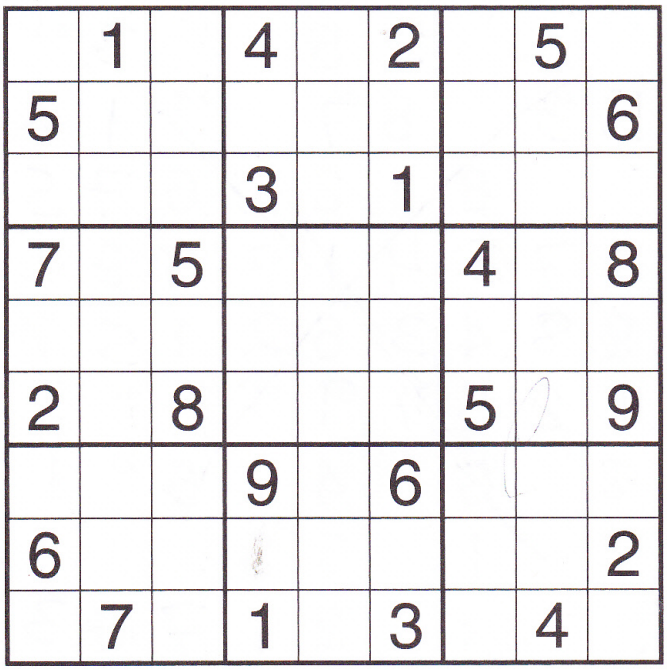

Figure 1: Sudoku puzzle for Problem 3.