Quantum-inspired algorithms combined with machine learning methods.

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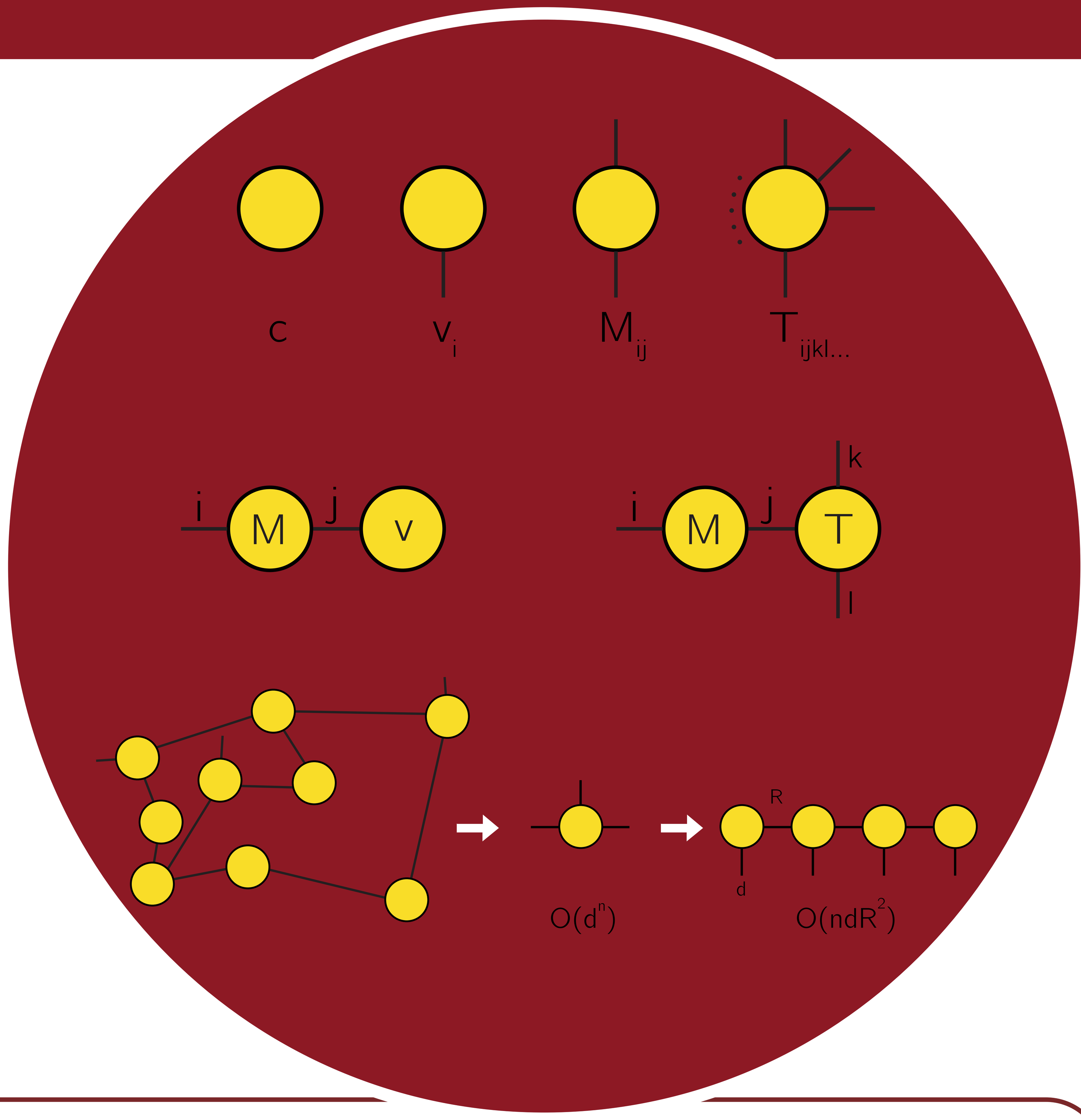


1 Introduction

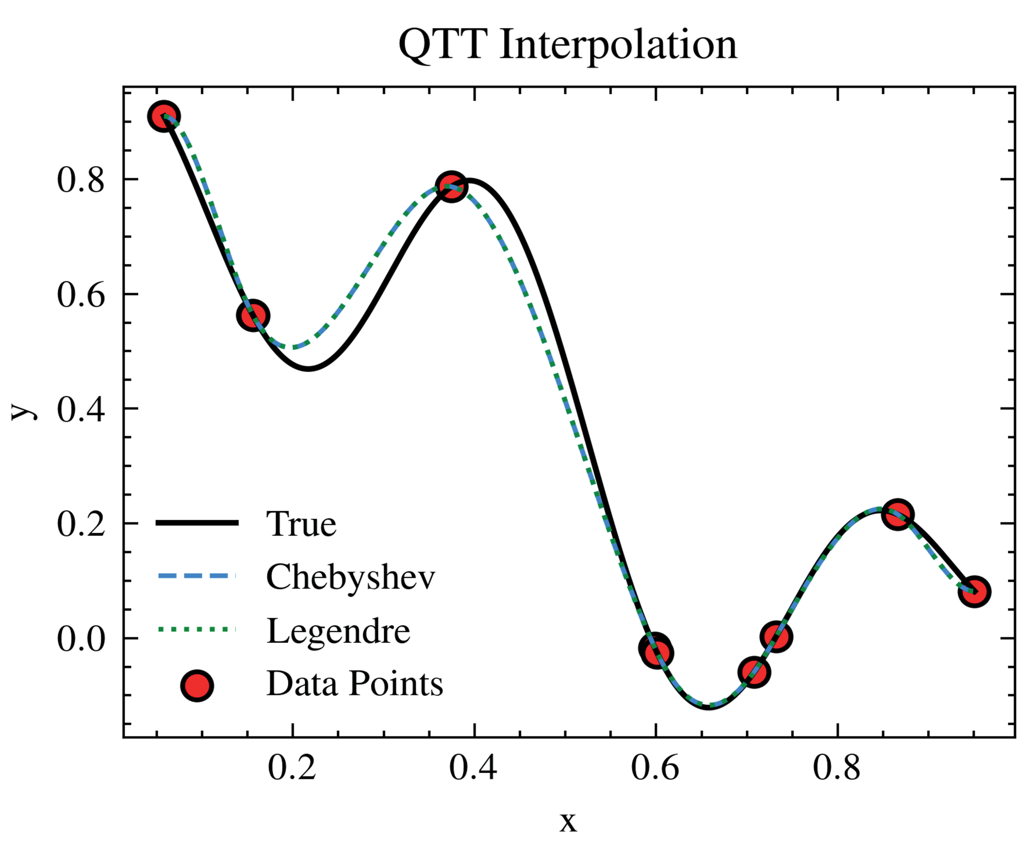
Solving high-dimensional partial differential equations (PDEs) is often constrained by the curse of dimensionality. To address this, we utilize tensor network methods that compress high-dimensional data and reduce computational complexity.

This work focuses on integrating data-driven approaches, such as physics-informed neural networks (PINNs), into the quantized tensor train (QTT) framework by combining compact tensor representations and data interpolation techniques. This would establish a bridge between numerical methods and machine learning. This integration enhances data learning capabilities while enabling efficient solutions for PDEs with complex boundary conditions and source terms.

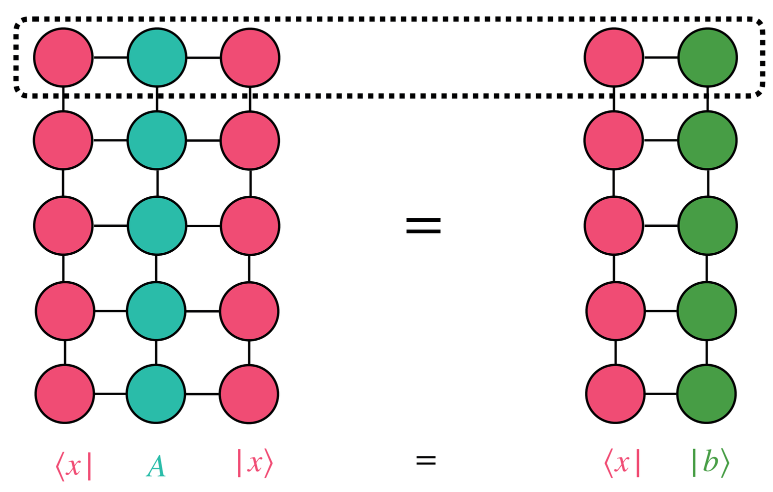
2 Algorithm

To integrate data points into the QTT framework, we follow three key steps:

- **Interpolation Function**: Construct a smooth spline $s(x)$ from data points $\{x_i, y_i\}$ using cubic or B-splines.
- **Interpolation Nodes**: Select interpolation nodes $\{x_j\}$ as evaluation points, commonly using Chebyshev or Legendre nodes.
- **Hyperparameter**: Define the number of interpolation nodes, which determines the bond dimension in the QTT representation.



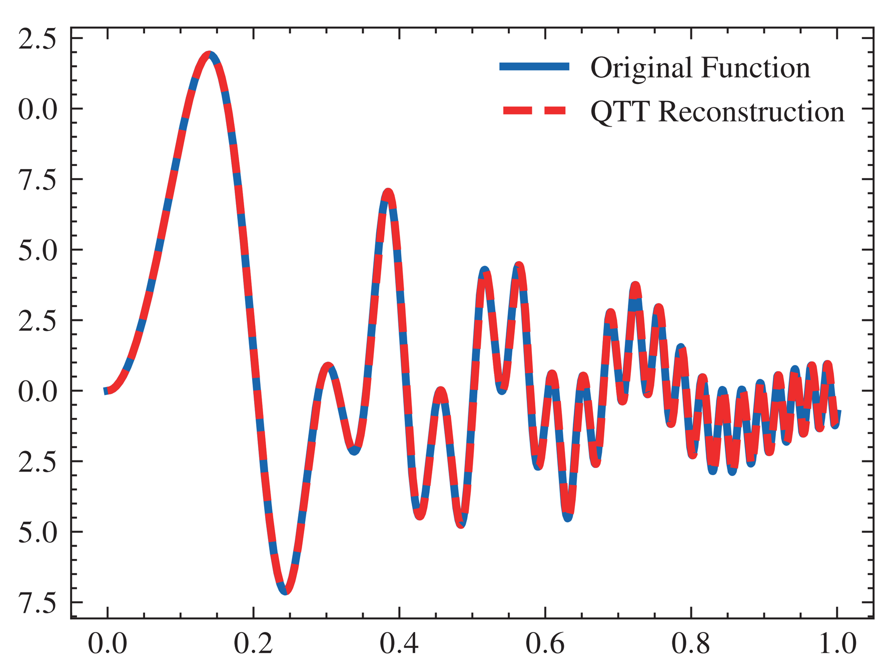
This approach provides a QTT representation of the function on the boundary and can be combined with standard quantum-inspired algorithms.

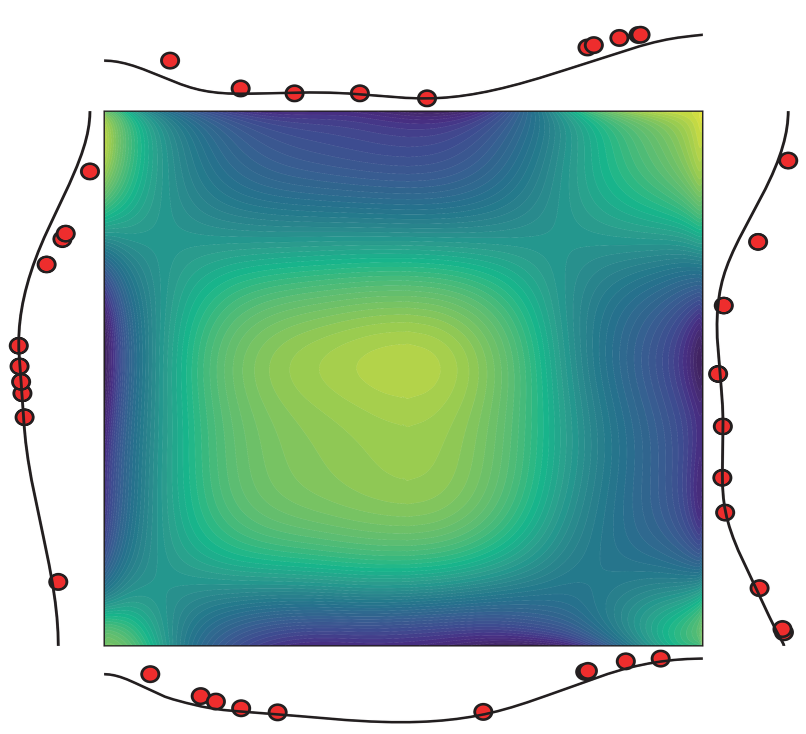


3 Results

This approach enables the representation of highly oscillatory functions within the QTT framework.

This approach can be combined with other QTT methods such as solving the 2D Laplace equation, $\Delta u = 0$ with random samples on the boundaries.





4 Conclusion

This approach integrates the efficiency of quantum-inspired algorithms with the flexibility of a data-driven framework.

Our approach ensures:

- The bond dimension can be increased to capture highly oscillatory behaviors.
- It can be easily extended to higher dimensions.
- A similar methodology can be applied to images instead of functions.