Fourth compulsory exercise

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This problem is concerned with the excited states in ^{20}Ne that decay into the α+^{16}O channel. The first questions relate to a beta-decay experiment.

> a) Find the experimental spin and parity of the nucleus ^{20}Na that beta-decays into the excited states in ^{20}Ne. We only consider the allowed beta-decays. Several of the states fed in beta-decay will subsequently deexcite by emitting an alpha-particle, find the experimental threshold energy in ^{20}Ne for alpha emission to the ground state in ^{16}O. What will the spin and parity be of the levels that enter in such beta-delayed alpha decays?

Since we are only considering allowed beta-decays know the following from figure 24.4

$$\text{allowed}: \begin{cases} \text{F+GT}; & \Delta J = 0 \\ \text{GT}; & \Delta J = 1, \end{cases} \tag{1}$$

and there is no change in the parity. We only know that the ground state in ^{16}O is even-even which means $J^\pi = 0^+$. In order for the α-decay to preserve the parity we require that $J^\pi = 2^+$ for some of the excited states in ^{20}Ne. Since the beta-decay is an allowed decay $\Delta J = 0$. It cannot be $\Delta J = 1$ since this would change the parity. Figure 1 shows the energy diagram.

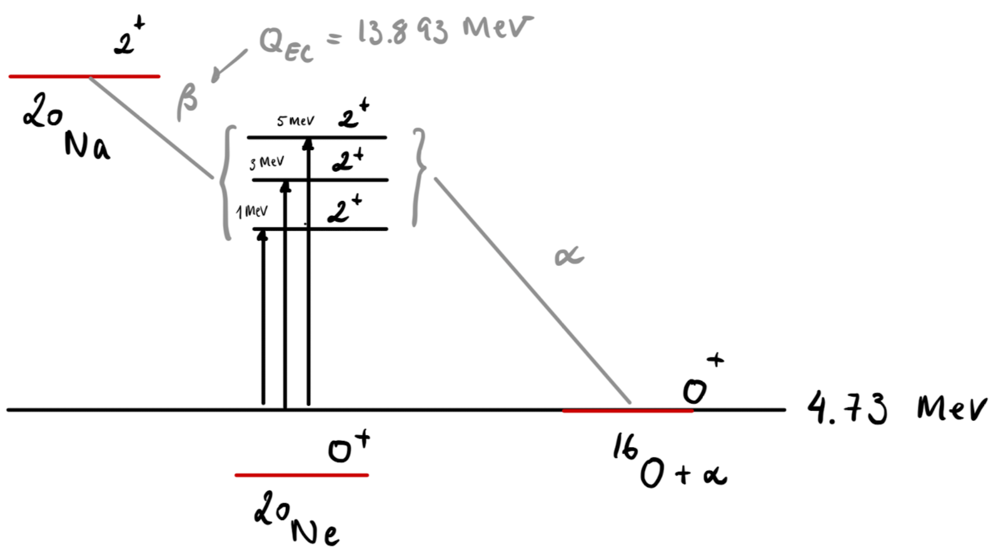

Figure 1: Energy diagram with J^π. Also includes three excited states with energies 1,3,5 MeV above the 4.73 MeV line.

To find the experimental threshold energy in ^{20}Ne for alpha emission I utilize that the Q-value for a decay is the energy (positive or negative) released in the decay. I use the definition of the mass excess which yields

$$\begin{aligned} Q_\alpha &= (M_P - M_D - m_\alpha)c^2 \\ &= -7041.931\,\text{keV} + 4737.001\,\text{keV} - 2.425\,\text{MeV} \\ &= -4.730\,\text{MeV} \end{aligned}$$

This value is also drawn on figure 1.

> b) Imagine that three levels in ^{20}Ne are placed so that they all can participate in beta-delayed alpha decay with total centre-of-mass energies of 1, 3 and 5 MeV, respectively, in the α-^{16}O system. Estimate the relevant α penetrability and the beta-decay f-factor for each level.

The α penetration factor can be expressed as

$$\begin{aligned} P_\alpha^{\ell=2} &= \exp\left(-\Delta\gamma\right)P_0 && (2) \\ &= \exp\left(-\ell(\ell+1)\sqrt{\frac{2}{\eta kR}}\right)\exp\left(-2\pi\eta - \sqrt{32\eta kR}\right), \quad k = \frac{\sqrt{2\mu E}}{\hbar}, && (3) \end{aligned}$$

where there are several things to note. First of all, equation (2) is the penetration factor which must be multiplied with kR to get the penetrability. Secondly, the mean square radius of the nuclear density is given by

$$R = r_0 A^{1/3}, \tag{4}$$

where r_0 can be between 1.2 fm for the charge radius and 1.4 fm for the full potential. The radius also depends on the composite system of the nucleus and the alpha particle. This means the radius can be expressed as $r_0(A_1^{1/3} + A_2^{1/3})$. Changing this parameter will affect the penetrability be several orders of magnitude. The values I have chosen can be seen in the Python code at the end of the document.

Equation (2) is not exact and uses a Taylor expansion of $\exp\left(-\gamma\right)$ which assumes small values compared to the height of the Coulomb barrier. This is not true and in particular the penetrability for $E = 5$ MeV is not correct. Using the values in the code I got the following values for the penetrability

$$P_\alpha^{\ell=2} = \begin{cases} 3.027\cdot 10^{-8}, & E = 1\,\text{MeV} \\ 8.421\cdot 10^{-3}, & E = 3\,\text{MeV} \\ 0.435, & E = 5\,\text{MeV}. \end{cases}$$

To calculate the beta-decay f-factor I have to account for the fact that the final state is an excited state and also include the correct relativistic expression.

$$\begin{aligned} E &= T + m_e c^2 \\ &= Q_\beta^* + m_e c^2 \\ &= Q_\beta - E^* \\ &= Q_{EC} - 2m_e c^2 - E^* + m_e c^2 \\ &= Q_{EC} - 2m_e c^2 - (B_E + E_{1,3,5}) + m_e c^2 \\ &= Q_{EC} - m_e c^2 - (B_E + E_{1,3,5}), \end{aligned}$$

where E_i denotes the energy above the ^{16}O+α system in MeV. The f-factor is given by

$$f = \left(\frac{E}{m_e c^2}\right)^5 \frac{1}{30}. \tag{5}$$

Using the calculations above yields

$$f = \begin{cases} 25098, & E = 1\,\mathrm{MeV} \\ 5518, & E = 3\,\mathrm{MeV} \\ 621, & E = 5\,\mathrm{MeV}. \end{cases}$$

Comparing equation (2) and equation (6) shows that the penetrability increases for increasing energy while the f-factor decreases for increasing energy. However, the α penetrability results also get more incorrect for increasing energy. This can be justified by considering the height of the Coulomb barrier given by

$$\begin{aligned} V_C &= \frac{Zz\alpha\hbar c}{R} \qquad (6) \\ &= \frac{Zz\alpha\hbar c}{r_0 A^{1/3}} \qquad (7) \\ &= 6.537\,\mathrm{MeV}, \qquad (8) \end{aligned}$$

which means the highest state we are considering is just below the maximum of the Coulomb barrier.

c) The experimental spectrum can be found in the file **na20 spectra.pdf**. Apart from peaks that may be fitted with Breit-Wigner type expression like in equation (20.77) there is an interesting minimum close to 3 MeV alpha particle energy. One possible interpretation is that it is due to interference between the strong peak (named 5) just below 2.2 MeV and a quite broad level (named 7) somewhat above 3 MeV. Can you devise an expression that could describe this?

To describe the behaviour of the interference I would make use equation (2.76) which is the Breit-Wigner for an isolated resonance, $a \longrightarrow b$

$$\sigma_r^{ba} = \frac{\pi}{k_a^2}\left|\frac{A^b A^{A^{a*}}}{E - E_r + \frac{i}{2}\gamma}\right| \tag{9}$$

This equation can also be used to describe the interference mentioned in the problem. As indicated on figure 1 the excited states in Ne have the same spin and parity which means one cannot distinguish the levels from each other if they interfere[1]. This means the behavior can be described

$$\sigma_r^{ba} = \frac{\pi}{k_a^2}\left|\frac{A_5}{E - E_r + \frac{i}{2}\Gamma_5} - \frac{A_7}{E - E_r + \frac{i}{2}\Gamma_7}\right|^2, \tag{10}$$

where you would have to make an educated guess for E_r. The subscript refer to the peak (5) and the broad level (7) this equation can be adjusted to look like the experimental spectrum. For appropiate values equation (10) can illustrate the interference–this is sketched on figure 2.

[1] They could in principle also interfere with all other levels.

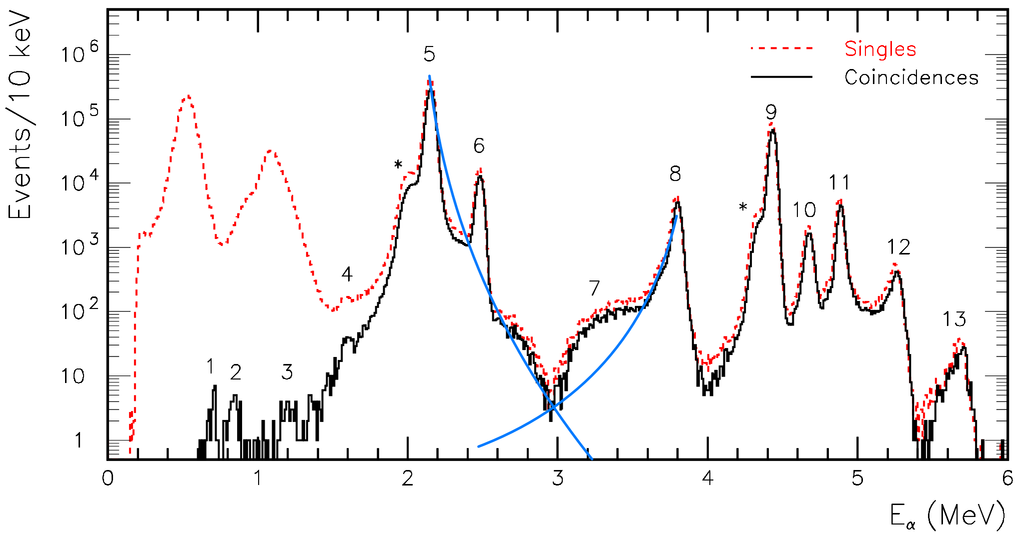

Figure 2: The experimental spectrum for ^{20}Na. I have tried to illustrate the interference between the strong peak (5) and the broader level (7) as a blue line.

> d) The resonances in ^{20}Ne may contribute to the Helium burning in red giant stars through the ^{4}He+16O reaction. Calculate the position and width of the Gamow peak for typical temperatures. Compare with the compilation of adopted 20Ne levels given at TUNL [**https://nucldata.tunl.duke.edu/chain/20.shtml**]. Do you expect resonance capture to be important?

To calculate the position of the Gamow peak I use equation (11) in the notes given by

$$E_0 = \left(\frac{E_G K_B^2 T^2}{4}\right)^{1/3} = 1.220\,\text{keV}(z^2 Z^2 \mu T_6^2)^{1/3}, \tag{11}$$

where T_6 is the temperature in units of 10^6 K. An appropiate value is $T \simeq 10^8$ K, hot enough to begin fusing helium to carbon via the triple-alpha process. This leads to a Gamow peak located at

$$E_0 \simeq 246\,\text{keV} \tag{12}$$

The width of the Gamow peak is given by

$$\Delta = \frac{4}{\sqrt{3}}\sqrt{E_0 K_B T} \tag{13}$$

$$= 0.749\,\text{keV}(z^2 Z^2 \mu T_6^5)^{1/6} \tag{14}$$

$$\simeq 106\,\text{keV} \tag{15}$$

Code

```
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
sns.set_theme(style="darkgrid")

```

```
#Alpha penetrability, height of Coulomb barrier and Gamow peak.

l=2                              #angular momenta
r0 = 1.4*10**(-15)               #meters
alpha = 1/137                    #fine structure constant
hbar = 1.054571817*10**(-34)     #J/s
c = 3*10**8                      #m/s
E = 8.011*10**(-13)              #Joules
A = 16                           #atomic number
Z1 = 8                           #charge 1
Z2 = 2                           #charge 2
m1 = 2.6560181*10**(-26)         #mass 1 in kilograms
m2 = 6.64648*10**(-27)           #mass 2 in kilograms
mu = m1*m2/(m1+m2)               #reduces mass of the two particles
beta = np.sqrt((2*E)/(mu*c**2)) #velocity in units of the speed of light
eta = Z1*Z2*alpha/beta           #Sommerfeld parameter
k = np.sqrt(2*mu*E)/(hbar)
R = r0*A**(1/3)                  #radius

gamma = 2*np.pi*eta-np.sqrt(32*eta*k*R)
deltagamma = l*(l+1)*np.sqrt(2/(eta*k*R))

Palpha = np.exp(-deltagamma)*np.exp(-gamma)*k*R #the penetrability times the penetration factor

h = (Z1*Z2*alpha*hbar*c)/(R*1.602*10**(-13)) #height of the Coulomb barrier
#print(h)

E0 = 1.220*(Z1**2*Z2**2*mu*(1*10**6)**2)**(1/3)   #position of the Gamow peak
Delta = 0.749*(Z1**2*Z2**2*mu*(1*10**6)**5)**(1/6) #width of the Gamow peak
#print(Delta)
```