Week 3

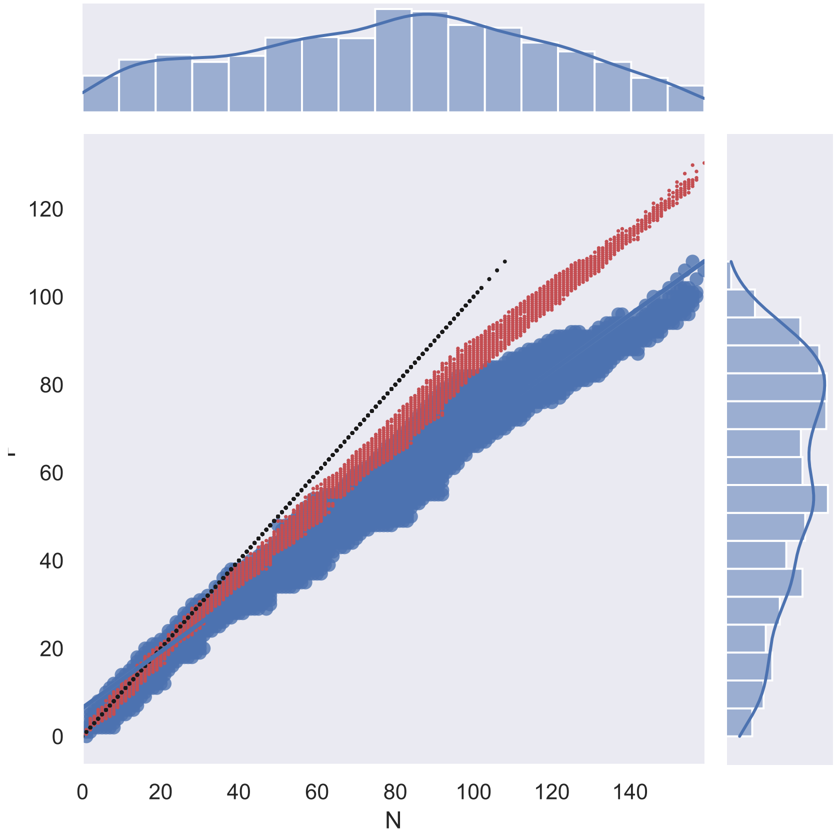
1. Q — i) Calculate the neutron separation energy of a nucleus (equation (8.2)), when will ground state neutron emission be possible in the heavy Na isotopes? ii) Find the Q-value for alpha emission, when will this become positive for nuclei in the valley of beta-stability

A — The seperation energy required to remove a neutron from the nucleus is

$$E_n = [M(Z,A-1) + M_n - M(Z,A)]c^2 = B(Z,A) - B(Z,A-1) \tag{1}$$

$$\begin{aligned} Q_\alpha &= (M_P - M_D - m_\alpha)c^2 = E_D + E_\alpha \qquad (2)\\ &= \frac{T_\alpha(M_D + m_\alpha)}{M_D} \qquad (3)\\ &\approx \frac{T_\alpha A}{A-4} \qquad (4) \end{aligned}$$

2. Q — Plot the nuclear chart with the line of beta-stability and the r-process region for Z in the range 30 to 100.



3. Q — Neutron star as a liquid drop model?

A — Staring From

$$B(A,Z) = b_{\mathrm{v}}A - b_{\mathrm{s}}A^{2/3} - b_{\mathrm{symm}}\frac{(N-Z)^2}{2A} - b_{\mathrm{c}}\frac{Z^2}{A^{1/3}} + b_{\mathrm{p}}\left[(-1)^N + (-1)^Z\right]\frac{1}{\sqrt{A}} \tag{5}$$

Assume the neutron star consists only of neutrons, $A = N$

$$B(N) = \left(b_{\mathrm{v}} - \frac{b_{\mathrm{sym}}}{2}\right)N - b_{\mathrm{s}}N^{2/3} + b_{\mathrm{p}}\frac{(-1)^N}{\sqrt{N}} \tag{6}$$

For very large N the surface term and the pairing term can be ignored since they scale as $N^{2/3}$ and $N^{-1/2}$ respectively. I add a Newtonian energy term with the geometic factor in front (gravitational binding energy for a spherical body of uniform density)

$$B(N) = \left(b_{\mathrm{v}} - \frac{b_{\mathrm{sym}}}{2}\right)N - \frac{3GM^2}{5R}, \tag{7}$$

where $M = Nm_n$ and $R = r_0A^{1/3}$. Inserting this and using $r_0 = 1.2$ fm leads to

$$\begin{aligned} B(N) &= \left(b_{\mathrm{v}} - \frac{b_{\mathrm{sym}}}{2}\right)N - \frac{3Gm_n^2N^2}{5r_0N^{1/3}} \\ &= \left(b_{\mathrm{v}} - \frac{b_{\mathrm{sym}}}{2}\right)N - \frac{3Gm_n^2}{5r_0}N^{5/3} \end{aligned}$$

This leads to a bound system since $B(N) < 0$. You can also solve for N

$$N(0) = \left(\frac{5r_0 \left(b_\mathrm{v} - \frac{b_\mathrm{sym}}{2} \right)}{3Gm_n^2} \right)^{3/2} = 6.2 \cdot 10^{55} \tag{8}$$

Comparing this to the Tolman–Oppenheimer–Volkoff limit for neutron stars

$$\frac{1.5 \cdot M_\odot}{m_n} = 1.78 \cdot 10^{57} \tag{9}$$

4. Q — The mirror nuclei ^{17}N and 17Ne both have ground states with spin-parity 1/2−. What is their most likely shell-model configuration?

A — Using the shell model (Woods-Saxon, figure 8.4) and that the maximum ocupancy for each subshell is $2j+1$.
For ^{17}N

$$\begin{aligned} &(0s_{1/2})^2(0p_{3/2})^4(0p_{1/2})^1, && \text{7 protons} \\ &(0s_{1/2})^2(0p_{3/2})^4(0p_{1/2})^2(0d_{5/2})^2, && \text{10 neutrons} \end{aligned}$$

For ^{17}Ne

$$\begin{aligned} &(0s_{1/2})^2(0p_{3/2})^4(0p_{1/2})^2(0d_{5/2})^2, && \text{10 protons} \\ &(0s_{1/2})^2(0p_{3/2})^4(0p_{1/2})^1, && \text{7 neutrons} \end{aligned}$$