Week 11

1. Q — **Problem 26**. Draw an energy diagram with all levels. Suggest spins and parities for all mentioned levels

A — Drawing the energy diagram

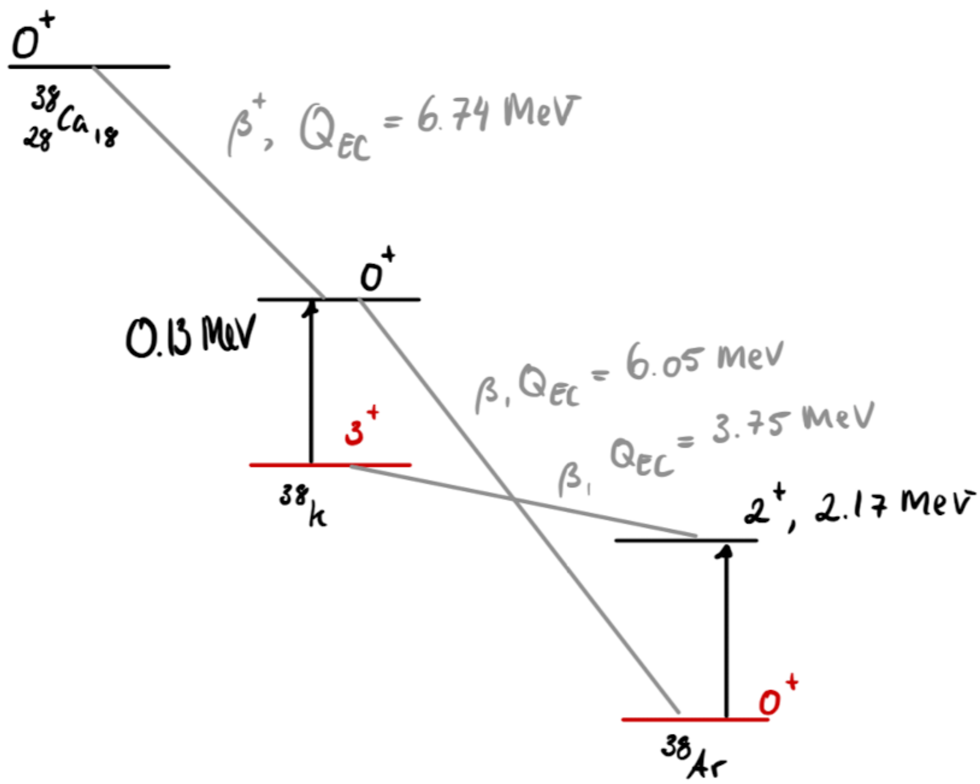

Figure 1: Energy levels

Using the following equations

$$\mathbf{J_i} = \mathbf{J_f} + \ell + \mathbf{S} \tag{1}$$

$$P_i = P_f(-1)^{\ell} \tag{2}$$

As stated explicit in the problem, the transition from the Ca ground state to the K excited state is a superallowed transition. This means $\Delta J = 0$ and $\Delta\pi = 0$. This means the excited state must have $J^{\pi} = 0^{+}$. The next transition is to the ground state in Ar which is an even-even so 0^{+} and the transition is again superallowed. The excited state in Ar has $J^{\pi} = 2^{+}$ and from the following equation we can conclude that the ground state in K must have either $J = 1, 3$. The ground state in K must have $J^\pi = 3^+$ since otherwise there would be Gamow-Teller transitions to the ground state in Ar. Also, note that the two superallowed transitions have similar Q-values and halflife–this indicates they could be the same type. The excited state in K does not decay by a γ transition since this is an octupole transition (M3) which is suppressed.

$$Q^*_{EC} = Q_{EC} - E^* \tag{3}$$

The spin and parity of K can be calculated

$$^{38}\mathrm{K}(?) + e^- \to^{38} \mathrm{Ar}^*(2^+) + \nu_e \tag{4}$$

$$^{38}_{28}Ca_{18} + e^- \longrightarrow {}^{38}_{19}K^*_{19} + \nu_e$$

$$Q^*_{EC} = Q_{EC} - E^* = 6.74\ \mathrm{MeV} - 0.13\ \mathrm{MeV} = 5.44\ \mathrm{MeV}$$

$$\Delta J = \pm 1, 0 \quad : \quad s_e = 1/2 \ , \ s_{\nu_e} = 1/2$$

Superallowed : $\Delta J = 0 \ , \ \Delta\pi = 0$

$$^{38}K^*(0^+) + e^- \longrightarrow {}^{38}Ar(0^+) + \nu_e$$

$$^{38}K(?) + e^- \longrightarrow {}^{38}Ar^*(2^+) + \nu_e \quad \Big\} \quad \vec{J}_i = \vec{J}_f + l + s = 2 + 0 + 1 = 3$$

$$P_i = P_f(-1)^l = P_f$$

Figure 2: Parity

2. Q — **Problem 8**. Find the total isospin and its projection for the ground states of the two pair of nuclei ^{26}Mg, ^{26}Al and ^{51}Ti, ^{51}V. Find their masses in the mass tables in mass16round.txt and argue which of the nuclei in each pair can beta decay. Is Fermi decay possible in any of the cases?

A — Using the equations for total spin and spin projecions

$$T_3 = \frac{A}{2} - Z \tag{5}$$

$$\frac{1}{2}|N - Z| \leq T \leq \frac{1}{2}(N + Z) \tag{6}$$

Using these equations the total isospin and its projection can be calculated.

For ^{26}Mg, $M = 25.982593$ u

$$T_3 = 1 \tag{7}$$
$$1 \leq T \leq 13 \tag{8}$$

For ^{26}Al, $M = 25.98689$ u

$$T_3 = 0 \tag{9}$$
$$0 \leq T \leq 13 \tag{10}$$

For ^{51}Ti, $M = 50.94660$ u

$$T_3 = 7/2 \tag{11}$$
$$7/2 \leq T \leq 51/2 \tag{12}$$

For ^{51}V, $M = 50.94395$ u

$$T_3 = 5/2 \tag{13}$$
$$5/2 \leq T \leq 51/2 \tag{14}$$

We see that $M(^{26}\mathrm{Mg}) < M(^{26}\mathrm{Al})$ which means that Al can β-decay. The same thing applies for $M(^{51}\mathrm{V}) < M(^{51}\mathrm{Ti})$ which means that Ti can β-decay. Looking at the isospin projection, T_3 where is at least a $\Delta T_3 = 1$ which means no Fermi decays are possible.

3. Q — The first paragraph in chapter 17.1 introduces a "quantality" parameter $\Lambda = \hbar^2/(Ma_2V_0)$ (used several times by Ben Mottelson) for a many-body system composed of particles with mass M and an interaction that has a minimum at a relative distance a and a strength V_0. As mentioned this is an estimate of the ratio between the "localization energy" and the potential energy. It can be used for different systems at (close to) zero temperature. Find parameter values for the nucleon-nucleon interaction and insert. Compare this to condensed systems of noble gases: the distance a is around 3 Å, and the potential strength is for Ne (and for H2 molecules) about 3 meV but goes down to 1 meV for He. Do you get any insight from the Λ values?

A — For the nucleon-nucleon interactions we are dealing with orders of magnitude similar to the following

$$a \simeq fm, \quad m \simeq 1000 MeV/c^2, \quad V_0 \simeq 1 - 100 MeV \tag{15}$$

This leads to the following parameter for the nucleon-nucleon interaction

$$\Lambda_{nn} = \frac{\hbar^2}{Ma^2V_0} \simeq 0.3894 \tag{16}$$

Comparing this is a condense system of noble gasses

$$\Lambda_{\mathrm{ng}} = \begin{cases} 0.008, & Ne \\ 0.116, & He \end{cases} \tag{17}$$

So Λ_{nn} and Λ_{He} are within the same order of magnitude and Λ_{Ne} is approximately a factor of 100 smaller. This depends on the chosen value for V_0 for nn. The interpretation of this is as follows. The parameter Λ localization energy and is a measure of how localised the system is in space. Due to the uncertainty relation, this means a higher spread in momentum. This is related to superfluidity since ^{3}He and ^{4}He can form a superfluid and similar physics can be expected from the nucleon-nucleon interaction from the localization energy alone. The localization energy can also be interpreted as a measure of much energy you need to keep the system in place. The value for Ne is much lower and this is related to how Ne crystalizes as $T \to 0$. As mentioned, something different happens to He and the nucleon-nucleon interaction and we have to deal with a Fermi liquid.