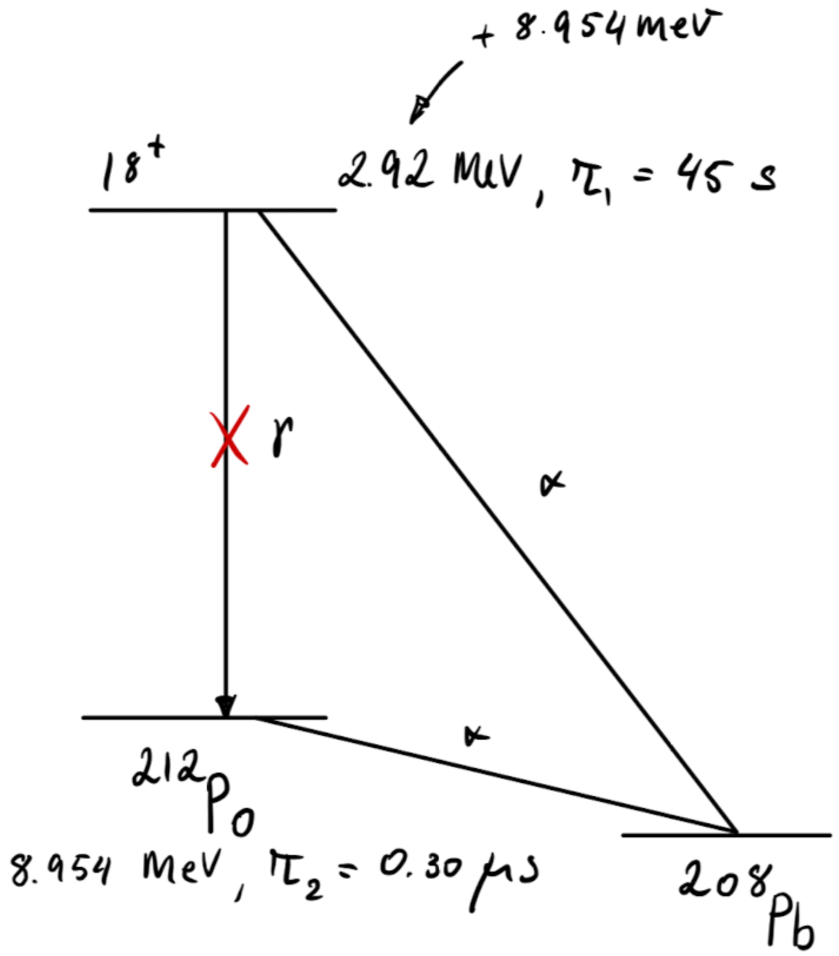
Week 10

1. Q — **Problem 24 – alpha vs. gamma decay**. Discuss the fact that the 2.92 MeV state decays by alpha rather than gamma emission, and the fact that the state lives longer than the ^{212}Po ground state.

A — First of all ^{208}Po is a double magic state so it's very favorable. The energy levels and transitions are shown in the figure.



The 2.92 MeV state decays by alpha emission due to the high J in the state. This means a lot of angular momentum is stored in this state. The high angular momentum and the energy scale ($\simeq$ 10 MeV) means the angular momentum barrier (centrifugal term) is non-negligible. This term is given by $\ell(\ell+1)\hbar^2/(2mr^2)$. The heavier mass of the α particle compared to the photon means α

emission is not punished as much as the gamma transition. Loosely speaking, one can pack more angular momentum into the heavier particle.
The 2.92 MeV states lives longer since the energy is smaller. This can be seen from the following equation

$$\ln \tau_{1/2} = A - \frac{BZ}{\sqrt{E}}, \tag{1}$$

where E is the energy is of the alpha-decay, and b is between 1.4-1.5.

2. Q — Check first the empirical formula for alpha-decay halflives given at the end of the note on alpha decay. Use the semiempirical mass formula first to estimate where in the nuclear chart alpha decay is energetically allowed

A — The empirical formula for alpha-decay halflives is given by 10^{λ}, where λ is

$$\lambda = -25.752 + 1.5913\frac{Z}{\sqrt{q}} - 1.15055\frac{A^{1/6}}{\sqrt{Z}} \tag{2}$$

To answer the second part of the question consider the following

$$Q_\alpha = M(Z,A)c^2 - M(Z-2,A-4)c^2 - m_\alpha c^2 \tag{3}$$

$$' = -B(A,Z) + B(Z-2,A-4) + B_\alpha, \tag{4}$$

where $B_\alpha = 28.296$ MeV and the other terms can be calculated using the semi-empirical mass formula. Plotting this for $40 \leq N,Z \leq 100$ which is approximately a straight line, where under the line $Q_\alpha < 0$ and α decay is forbidden. The opposite is true above the line. One can also make considerations about the binding energy per nucleon number for the alpha particle. The two half-lives can also be represented as two straight lines with a steeper slope.

3. Q — The first paragraph in chapter 17.1 introduces a "quantality" parameter $\Lambda = \hbar^2/(Ma_2V_0)$ (used several times by Ben Mottelson) for a many-body system composed of particles with mass M and an interaction that has a minimum at a relative distance a and a strength V_0. As mentioned this is an estimate of the ratio between the "localization energy" and the potential energy. It can be used for different systems at (close to) zero temperature. Find parameter values for the nucleon-nucleon interaction and insert. Compare this to condensed systems of noble gases: the distance a is around 3 Å, and the potential strength is for Ne (and for H2 molecules) about 3 meV but goes down to 1 meV for He. Do you get any insight from the Λ values?

A — For the nucleon-nucleon interactions we are dealing with orders of magnitude similar to the following

$$a \simeq \text{fm}, \quad m \simeq 1000\,\text{MeV/c}^2, \quad V_0 \simeq 1-100\,\text{MeV} \tag{5}$$

This leads to the following parameter for the nucleon-nucleon interaction

$$\Lambda_{\text{nn}} = \frac{\hbar^2}{Ma^2V_0} \simeq 0.3894 \tag{6}$$

Comparing this is a condense system of noble gasses

$$\Lambda_{\text{ng}} = \begin{cases} 0.008, & \text{Ne} \\ 0.116, & \text{He} \end{cases} \tag{7}$$

So Λ_{nn} and Λ_{He} are within the same order of magnitude and Λ_{Ne} is approximately a factor of 100 smaller. This depends on the chosen value for V_0 for nn. The interpretation of this is as follows. The parameter Λ localization energy and is a measure of how localised the system is in space. Due to the uncertainty relation, this means a higher spread in momentum. This is related to superfluidity since ^{3}He and ^{4}He can form a superfluid and similar physics can be expected from the nucleon-nucleon interaction from the localization energy alone. The localization energy can also be interpreted as a measure of much energy you need to keep the system in place. The value for Ne is much lower and this is related to how Ne crystalizes as $T \to 0$. As mentioned, something different happens to He and the nucleon-nucleon interaction and we have to deal with a Fermi liquid.