Lecture Notes

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Course overview

February: The connections to the underlying particle physics. Isospin, the mass formula, Fermi gas model and simple shell model.

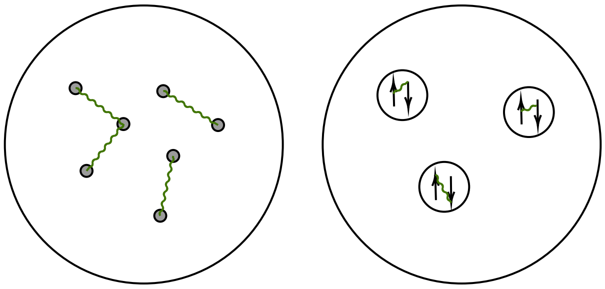
March: Nuclear structure models. Collective vibrations. Then elaborations on the shell-model, including an update on how nuclear shells may change. Also deformations and the theory of pairing.

April: Nuclear decays. Gamma decay, fission, alpha decay and beta decay. Also collective modes, in particular at rotations.

May: Nuclear astrophysics and nuclear reactions

1 Lecture 1: Free quarks and the strong nuclear force

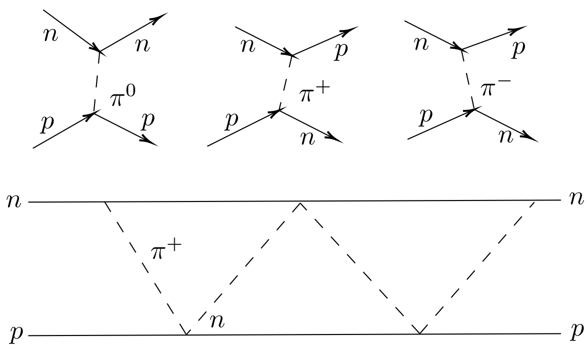
Nuclei consist of protons and neutrons, but why is this configuration energetically favored over free quarks. The nuclear forces that keep the nucleus together are induced through the exchange by mediating quanta – mesons. Considering two quarks with spin. The spin configurations can be aligned or anti-aligned where the energy is highest for the aligned state. This leads to an energetically favored state when the spins are anti-aligned. This can explain why the nuclei consists of protons and neutrons and not of free quarks since this would add more degrees of freedom. When the quarks are confined in protons and neutrons the gluons can only interact within the proton or neuton instead of all the free quarks in the nuclei.
At higher energies the bound system of protons and neutrons break down since the energy is now minimal for a free soup of quarks. This is the quark-gluon plasma.



Nucleons take part in all known interactions. The main feature of the nuclear world are sharped by the strong force – particles participating in strong interactions are called hadrons. Nucleons are the lightest fermions among the hadrons. The bosons among the hadrons are called mesons. The range of the strong force can be estimated by the uncertainty relation and the mass of π^0

$$\Delta R \sim c\Delta t \sim \frac{\hbar}{mc} = 1.46\,\mathrm{fm} \tag{1}$$

Inside the nucleus one cannot distinguish between protons and neutrons since they interchange through the pion. This means one can only distintguish a proton from a neutron when the particle decays. This means one consideres nucleons because it is an interacting system. This motivates the isospin formalism since this shows the net number of protons or neutrons.



2 Lecture 2: Isospin

Since one cannot distinguish the proton and the neutron inside the nucleous we can consider the two particles as two different states of the same strongly interacting object. This leads to a two-level system: the nucleon.

$$|p\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad |n\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2}$$

This leads to three different objects to look at, nn, np, pp. Also, the wavefunction consists of three parts: the radial term, the angular part and the spin part. The symmetry with respect to the exchange of spin varialbes is tested by the Bartlett operator. The triplet states are symmetric and the singlet states is anti-symmetric under spin exchange.

$$\mathcal{P}^{\sigma} = (1-)^{S+1} \tag{3}$$

The space inversion chances the sign of the relative coordinate

$$\mathcal{P}^{r} = (-1)^{l} \tag{4}$$

Both spin and orbital is given by the Heisenberg exchange operator

$$\mathcal{P} = (-1)^{l+S+1} \tag{5}$$

An example of symmetric and anti-symmetric

$$\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}, \quad \mathbf{S_1} = \frac{1}{2}, \quad \mathbf{S_2} = \frac{1}{2} \tag{6}$$

To get the expectation value of the spin product: square both sides. This leads to

$$\langle \mathbf{S_1} \cdot \mathbf{S_2} \rangle = \begin{cases} -\frac{3}{4}, & \text{singlet, anti-symmetric} \\ \frac{1}{4}, & \text{triplet, symmetric} \end{cases} \tag{7}$$

In gerneral the relative angular momentum, l, and the spin $\mathbf{S}$ is not conserved, but the rotational invariance makes the total angular momentum conserved. Another argument is that this operator commutes with the Hamiltonian.

$$\mathbf{J} = \mathbf{l} + \mathbf{S} \tag{8}$$

So the conserved quantum number is J^{π}. For a two-nucleon state:

$$^{2S+1}l_J \tag{9}$$

To introduce isospin one can consider Noether's theorem. This also means that an approximate symmetry yields some perfered quantites, i.e selection rules. The strong interaction cannot distinguish the proton from the neutron but the Coulomb interaction can. This leads to an approximate symmetry which leads to the isospin formalism.

Again considering the nucleon from equation (2) the isospin acts on the basis with a certain charge given by

$$Q = \frac{1}{2} - t_3 \tag{10}$$

Here t_i is analogous to the Pauli matrices for regular spin given. This also means there are three different operator so isospin is a vector with three components.

$$t_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad t_2 = \begin{pmatrix} 0 & -1 \\ i & 0 \end{pmatrix}, \quad t_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{11}$$

This also leads to lowering and raising operators in the same way one cna lowe and raise components of angular momentum $J_{\pm} = J_x \pm iJ_y$. For isospin the relation is given by

$$\tau_{\pm} = \tau_1 \pm i\tau_2 \tag{12}$$

The strong force cannot distinguish the proton from the neuton so isospin is invariant. This means the totalt isospin is a conserved quantum number

$$T^2 = \sum_a \mathbf{t_a} \tag{13}$$

The eigenvalue of the isospin "length" is $\mathbf{T^2} = T(T+1)$. One can expand the wavefunction by A factors for each nucleus and this is still an invariant

$$[\hat{\mathbf{T}}, \hat{H}_{\text{strong}}] \tag{14}$$

One can also consider the minimum and maximum of the isospin projection, T_3. This quantity is related to electrical charge

$$Q = \sum_a \left(\frac{1}{2} - t_{3a}\right) = \frac{A}{2} - T_3 \tag{15}$$

All states in a given nucleus (vertical scale) have the same projection

$$T_3 = \frac{1}{2}(N - Z) = \frac{A}{2} - Z \tag{16}$$

And belong to the horizontal scale. This is illustrated in figure 2.1 in the book. To conclude, the total wavefunction for the nucleus must also include Isospin

$$\Psi = RY_{lm}\chi\Omega. \tag{17}$$

3 Lecture 3: Selection rules and liquid drop model

Selection rules

Consider an arbitrary one-body operator, $\hat{O}$ which is a sum of single particle operators

$$Q = \sum_1^A q_a \tag{18}$$

This can be split into different terms if the operator can distinguish neutrons and protons

$$\begin{aligned} Q &= \sum_n q_n + \sum_p q_p \\ &= \sum_a \left(q_n \frac{1+\tau_{3a}}{2} + q_p \frac{1-\tau_{3a}}{2}\right) \\ &= \sum_a \frac{q_n + q_p}{2} + \sum_a \frac{q_n - q_p}{2}\tau_{3a} \end{aligned}$$

The first equality is on a neutron, proton level but using the isospin formalism we can move to a nucleon level since the strong force cannot distinguish protons from neutrons. This leads to the second equality where denominator is 1 if a neutron and 0 if a proton. The other way around for the next term. This leads to the third equality where the terms are called the isoscalar ($\tau = 0$) and the isovector $\tau = 1$ respectively. The physical interpretation of this is that for the isoscalar the neutrons and protons oscilllate in phase and out of phase for the isovector. This also leads to $\hat{\tau}_{\pm}$ being interpreted as a β-decay operator, look at equation (12).

Liquid drop model

The binding energy is traditionally given by

$$B(A,Z) = (ZM_p + NM_n)c^2 - E_{\text{tot}}(A,Z) \tag{19}$$

But in practice it is usefull to include the mass of the electrons

$$B(A,Z) = (ZM_p + NM_n)c^2 - E_{\text{tot}}(A,Z). \tag{20}$$

The dynamical features requires a reasonable chouse of dynamic variables. We consider a continous medium where excitations should resemble propagating waves. The lowest limit for the wavelength us of order r_0 (mean interparticle distance) which is close to the range of the nuclear force.

$$\frac{1}{k} > r_0 \simeq \frac{R}{A^{1/3}}. \tag{21}$$

This is rewriting by multiplaying $\hbar$

$$\frac{\hbar k R}{\hbar} < A^{1/3}, \tag{22}$$

and since $\hbar k$ is momentum, $\hbar k R$ is angular momentum. This motivates the use of spherical harmonics, $\alpha\lambda\mu Y_{\lambda\mu}(\hat{n})$. This means in a collective wave of deformation the radius R can be representated by a superposition of the spherical harmonics

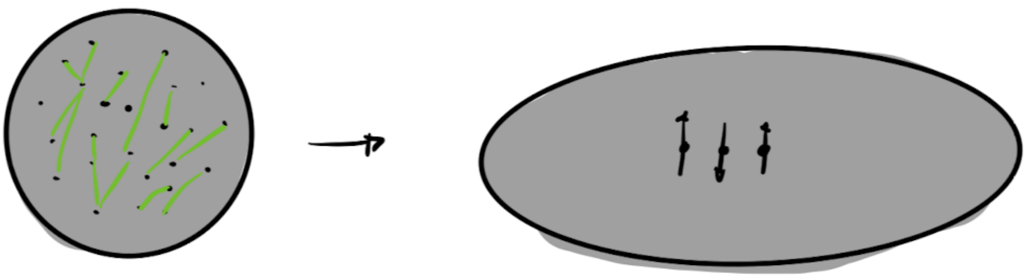
$$R(\mathbf{n}) = R_0 \left[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\mathbf{n})\right]. \tag{23}$$

4 Lecture 4: Fermi gas model

Introduce mean-field approximation and split the Hamiltonian into a Hamiltonian for each particle and a Hamiltonian for the interaction.

$$\hat{H} = \hat{H}_{\text{each particle}} + \hat{H}_{\text{interaction}} \longrightarrow \sum_{ij} v_{ij} + \sum_{ijk} v_{ijk} = \sum \left(\hat{H}_{\text{each particle}} + \hat{H}_{\text{interaction}}\right) - \sum \hat{H}_{\text{residual}} \tag{24}$$

Also introduce another quark model structure



In this section we considered a cubic box of size $L = V^{1/3}$. Each single-particle orbit in the Fermi gas model is characterized by the momentum $\mathbf{p}$ and spin-isospin quantum numbers – also normalized by the volume, V

$$\psi_{\mathbf{p}\sigma\tau} = \frac{1}{\sqrt{V}} e^{\frac{i}{\hbar}(\mathbf{p}\cdot\mathbf{r})} \chi_\sigma \Omega_\tau \tag{25}$$

So we assumed plane wave solution and isotropic in momentum-space.

$$\epsilon(p) = \frac{p^2}{2m^*} \tag{26}$$

This leads to the single-particle level density

$$\nu(\varepsilon) = \int \frac{\mathrm{d}^3p\mathrm{d}^3r}{(2\pi\hbar)^3}\delta(\epsilon - \epsilon(p)) = \frac{Vg}{2\pi^2\hbar^3}p^2\frac{\mathrm{d}p}{\mathrm{d}\epsilon}, \tag{27}$$

where g is the degeneracy factor and $(2\pi\hbar)^3$ is the volume the particle occupies. Now move from first quantization to second quantization and express the wavefunction in terms of the occupation number.

$$Z = \sum_{\mathbf{p}\sigma} n_{\mathbf{p}\sigma-1/2}, \quad N = \sum_{\mathbf{p}\sigma} n_{\mathbf{p}\sigma 1/2} \tag{28}$$

Introduce the Fermi energy diagram and express A in terms of p_F. This leads to the final expression

$$p_F = \hbar k_F = \hbar\left(6\pi^2\frac{n}{g}\right)^{1/3}, \tag{29}$$

where the fraction is the number density per degree of freedom. This means that all you need is the number density to get the Fermi momentum. Also that the Fermi gas is highy degenerate. From the Fermi energy you can get the seperation energy which leads to the potential, $\mathcal{U}_0 = 40 - 50$ MeV depending on m^*.

5 Lecture 5: Spherical mean field

We now examine the clustering of single-particle level called shell structure. This is illustrated in figure 5.1. The shell structure also implies dynamical properties such as excitations, capture etc. In nuclear physics there are some numbers for protons and neutrons that lead to especially stable nuclear systems – analogous to the noble gases in atomic physics. Those magic numbers for nuclei are placed along the valley of stabily and are given by

$$2, \quad 8, \quad 20, \quad 50. \quad 82, \quad 126. \tag{30}$$

Since the shape of the mean field is nearly the same for protons and neutrons the magic numbers turn out to be the same as well. The most stable nucleai are of course the double magic numbered nuclei. Keep in mind the magic numbers are not invariant since the potential changes when you add more protons or neutrons.

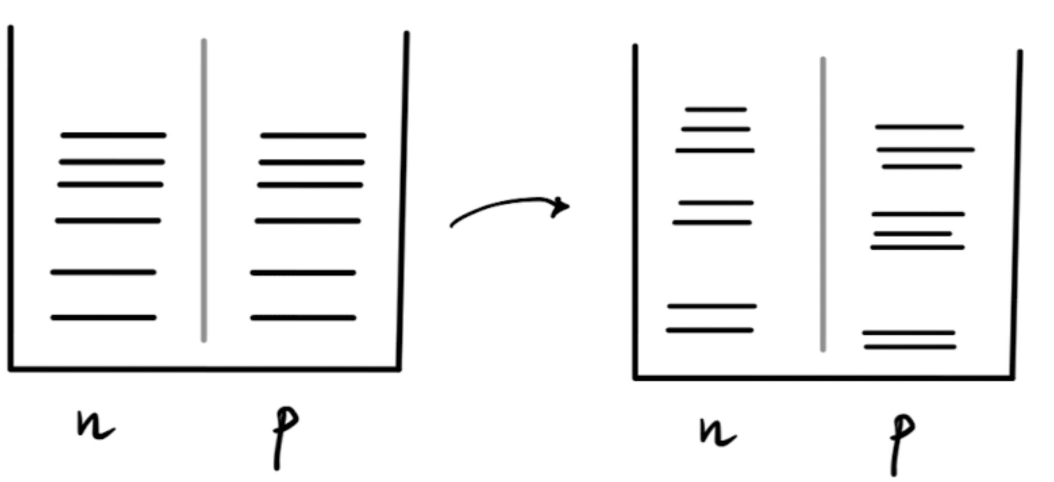


Figure 5.1: Energy gaps

Nuclei with one nucleon on top of a filled major shell have diminished nucleon seperation energy given by

$$S_n(A,Z) = B(A,Z) - B(A-1,Z)$$
$$S_p(A,Z) = B(A,Z) - B(A-1,Z-1)$$

Considering the shell structure the simplest approach to solve the model of mean field is the isopropic harmonic oscillator. This is easily done analytically but the drawbacks are in its unrealistic features that include excessive symmtry. This can be seen from the Hamiltonian

$$\hat{H}_{\text{HO}} = \frac{\mathbf{p}^2}{2M} + \frac{1}{2}M\omega_0^2\mathbf{r}^2, \quad \mathbf{r}^2 = (x+y+z)^2 \tag{31}$$

This means symmetric excitations in $N = n_x, n_y, n_z$. The energi levels ar

$$E_n = \hbar\omega_0 \left(N + \frac{3}{2}\right) \tag{32}$$

And the parity of the three-dimensional states is given by

$$\Pi = (-1)^N \tag{33}$$

It is important that all states in a major shell have the same parity. Therefore, the linear combinations of the major shell orbits with proper rotational symmetry should have only even (odd) l for even even (odd) N.

A more realistic potential is the Woods-Saxon potential which only consideres the nearest interaction and spin-orbit coupling. This means that the potential is an improvement to the harmonic oscillator potential since it includes surface effects. Another arguement for the surface effects is that angular momentum is with respect to a point. This is seen on figure 8.4. Also consider how degeneracy leads to shell structures from an energy perspective. The distance from $s \to p$ decreases as the nucleus becomes larger because the wavelength becomes larger and the kinetic energy decreases. The energy as a function of major shell number and angular momentum is given by

$$\begin{aligned} E(n,l) &= E(n_0,l_0) + (n-n')\frac{\mathrm{d}E}{\mathrm{d}n} + (l-l_0)\frac{\mathrm{d}E}{\mathrm{d}l} \\ &\simeq b\frac{\mathrm{d}E}{\mathrm{d}n} + a\frac{\mathrm{d}E}{\mathrm{d}l}, \end{aligned}$$

where the second term is the energy to add for a radial excitation and the third term is the energy you must add for an angular excitation. Also, a, b are small. This leads to degeneracy which leads to shell stuctures.

6 Lecture 6: Two-body dynamics

Low-energy nuclear forces

In the low-energy domain we can almost always limit ourselves to considerations of nucleon degrees of freedom. This means we only need a Hamiltonian expressed in terms of the nucleon variables: the coordinates $\mathbf{r}$, momenta $\mathbf{p}$ and spins $\mathbf{s}$. To construct a spin dependent Hamiltonian we need a combinations of spins and does not change sign under spatial inversion or time reversal. Another argument is that $\mathbf{r} \times \mathbf{p}$ does not change under parity. This means we are left with

$$\sigma_{\mathbf{1}}\sigma_{\mathbf{2}}(\sigma_{\mathbf{1}} \cdot \mathbf{n_1})(\sigma_{\mathbf{2}} \cdot \mathbf{n_2}), \tag{34}$$

where the first product is scalar and does not depend on the orientation of $\mathbf{n}$ and can therefore be ignored when we average over all angles. The rest are tensor forces. The angular average is

$$\overline{n_k n_l} = \frac{1}{4\pi} \int \mathrm{d}o\; n_k n_l = \frac{1}{3}\delta_{kl}. \tag{35}$$

This means the average part of the operator is $1/3(\sigma_{\mathbf{1}} \cdot \sigma_{\mathbf{2}})$ and belongs to spin-spin forces. We can now define a pure tensor operator

$$S_{12}(\mathbf{n}) = 3(\sigma_{\mathbf{1}} \cdot \mathbf{n})(\sigma_{\mathbf{2}} \cdot \mathbf{n}) - (\sigma_{\mathbf{1}} \cdot \sigma_{\mathbf{2}}) = 2[3(\mathbf{S} \cdot \mathbf{n})^2 - \mathbf{S}^2] \tag{36}$$

And this operator has angular average of 0. This pure tensor operator is responsible for any noncentral forces. The general momentum-independant interaction of two spin-$1/2$ particles may only contain three types of forces each with their own radial depenence: central, spin and tensor. The Hamiltonian for the momentum-independent interaction is given by

$$H_s(\mathbf{r}, \sigma_{\mathbf{1}}\sigma_{\mathbf{2}}) = U_c(r) + U_\sigma(r)(\sigma_{\mathbf{1}} \cdot \sigma_{\mathbf{2}}) + U_t(r)S_{12} \tag{37}$$

And this Hamiltonian is symmetric under spin exchange

$$[\mathcal{P}^\sigma, H_s] = 0 \tag{38}$$

We also need to add an isospin term to this Hamiltonian. This leads to a new Hamiltonian which can be expressed in terms of four forces: a central Wigner, Majorana ($\propto \mathcal{P}^r$), Bartlett ($\propto \mathcal{P}^\sigma$)m and Heisenberg ($\propto \mathcal{P}^r\mathcal{P}^\sigma$)

$$H_s' = U_W(r) + U_M(r)\mathcal{P}^r + U_B(r)\mathcal{P}^\sigma + U_H(r)\mathcal{P}^r\mathcal{P}^\sigma \tag{39}$$

Meson exchange

Mesons act as mediators of strong forces. The free meson field $\phi^\alpha(\mathbf{r}, t)$, where α is the isospin (charge) characteristic $t_3 = -Q$ of the meson that satisfies the Klein-Gordon equation

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \mu^2\right)\phi^\alpha = 0, \quad \mu = \frac{mc}{\hbar} \tag{40}$$

We now consider the Coulomb potential mediated by the photon with mass equal to zero and solve the Poisson equation. We do this since we are doing a similar approach for the Klein-Gorden equation. The Poisson equation is

$$\nabla^2\phi = -4\pi\delta(\mathbf{r}) \tag{41}$$

With the solution

$$\phi(\mathbf{r}) = \frac{e}{|\mathbf{r}-\mathbf{r_1}|} \tag{42}$$

For the Klein-Gorden equation we obtain

$$\left(\nabla^2 - \mu^2\right)\phi^\alpha(\mathbf{r}) = -\frac{g}{\mu}\tau^{alpha}(\sigma\cdot\nabla)\delta(\mathbf{r}-\mathbf{r_1}) \tag{43}$$

With solution

$$\phi^\alpha = \frac{g}{4\pi\mu}\tau^\alpha(\sigma\cdot\nabla)\frac{\mathrm{e}^{\mu|\mathbf{r}-\mathbf{r_1}|}}{|\mathbf{r}-\mathbf{r_1}|} \tag{44}$$

The final result is the pion-exchange potential that is symmetric with respect to nucleons

$$U_\pi = \frac{g^2}{4\pi\mu^2}(\tau_\mathbf{1}\cdot\tau_\mathbf{2})(\sigma_\mathbf{1}\cdot\nabla)(\sigma_\mathbf{2}\cdot\nabla)\frac{\mathrm{e}^{-\mu r}}{r} \tag{45}$$

Tensor forces and d-wave

Now also taking the tensor terms into account. The two tensor terms arises from the Majorana operator

$$H_s'' = [\mathcal{U}_{tW}(r) + \mathcal{U}_{tM}(r)\mathcal{P}^r]S_{12} \tag{46}$$

This yields the noncentral spin-dependent potential

$$\mathcal{U}(r) = \mathcal{U}_0 + \mathcal{U}_t(r)S_{12}, \quad \mathcal{U}_t(r) = \mathcal{U}_{tW}(r) + \mathcal{U}_{tM}(r) \tag{47}$$

The complete deuteron wavefunction now contains two radial parts. Factoring out the wave factor, $1/r$ yields

$$\Psi_M = \frac{1}{\sqrt{4\pi}}\frac{1}{r}\left(u_0(r) + \frac{1}{\sqrt{8}}u_2(r)S_{12}\right)\chi_{1M} \tag{48}$$

This equation was rewritten using the convient coupling represented by

$$\Theta_M = \frac{1}{\sqrt{32\pi}}S_{12}\chi_{1M} \tag{49}$$

Plugging this into the Schrödinger equation

$$u_0'' - [\kappa^2 + \mathcal{U}_0(r)]u_0 - \sqrt{8}\tilde{\mathcal{U}}_t(r)u_2 = 0 \tag{50}$$

$$u_2'' - [\kappa^2 + \frac{6}{r^2} + \tilde{\mathcal{U}}_0(r) - 2\tilde{\mathcal{U}}_t(r)]u_2 - \sqrt{8}\tilde{\mathcal{U}}_t u_0 = 0, \tag{51}$$

where the tilded potentials include the factor $2m/\hbar^2$. Now these are almost the same but what is the coupling terms? $S_{12}\frac{u_0}{r}Y_{02}\chi_{1M}$ is the s-wave but transforms into a d-wave contribution. This can be seen if one compares equation (48) to the last term in equation (50). Do the same consideration for the other terms.

$$r_{12}\frac{u_2}{r}\frac{1}{\sqrt{8}}S_{12}Y_{00}Y_{1M} = \frac{u_2}{r}\frac{8}{\sqrt{8}}Y_{00}\chi_{1M} - \frac{u_0}{r}\frac{2}{\sqrt{8}}S_{12}Y_{00}\chi_{1M} \tag{52}$$

Plug in potential and you get figure 3.5 in the book.

7 Lecture 7: Two-body scattering

Scattering theory

Consider elastic scattering in the center-of-mass frame, where $\mathbf{r}$ is the relative distance between the particles. We neglect interaction forces and consider free motion. This means the asympotic form of the wave function of the relative motion can be written as a combination of the incident plane wacve and the outgoing spherical wave

$$\psi(r) \simeq e^{i(\mathbf{k}\cdot\mathbf{r})} + f(\mathbf{k}', \mathbf{k})\frac{e^{ikr}}{r}, \tag{53}$$

where $k = k' = \sqrt{2mE/\hbar^2}$ is the wave vector, m is the reduced mass and $f(\mathbf{k}', \mathbf{k})$ is the scattering amplitude of dimension length. This is related to the differential cross section of scattering given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}o} = \left|f(\mathbf{k}', \mathbf{k}\right|^2. \tag{54}$$

Also, θ is the angle between $\mathbf{k}'$ and $\mathbf{k}$ For low k one can use partial wave expansion which means to new expressions for $e^{i\mathbf{k}\cdot\mathbf{r}}$ and $f(\theta)$

$$e^{i(\mathbf{k}\cdot\mathbf{r})} = e^{ikr\cos(\theta)} = \sum_{\ell=0}^{\infty}(2\ell+1)i^\ell j_\ell(kr)P_\ell(\cos(\theta)) \tag{55}$$

$$f(\theta) = \sum_\ell (2\ell+1)P_\ell(\cos(\theta))f_\ell, \quad f_\ell \in \mathbb{C} \tag{56}$$

Plugging this in

$$\psi(\mathbf{r}) \simeq \frac{i}{2kr}\sum_\ell (2\ell+1)P_\ell(\cos(\theta))[(-1)^\ell e^{-ikr} - (1+2ikf_\ell)e^{ikr}] \tag{57}$$

Now, the outcomming wave is distorted – its amplitude is not equal to 1. This is the term $\propto e^{ikr}$

$$S_\ell = 1 + 2ikf_\ell = e^{2i\delta_\ell}, \tag{58}$$

where δ_ℓ is the phase shift.

Lecture 8: Giant Resonances

Giant resonance is a high-frequency collective excitation of atomic nuclei, as a property of many-body quantum systems. In the macroscopic interpretation of such an excitation in terms of an oscillation, the most prominent giant resonance is a collective oscillation of all protons against all neutrons in a nucleus.

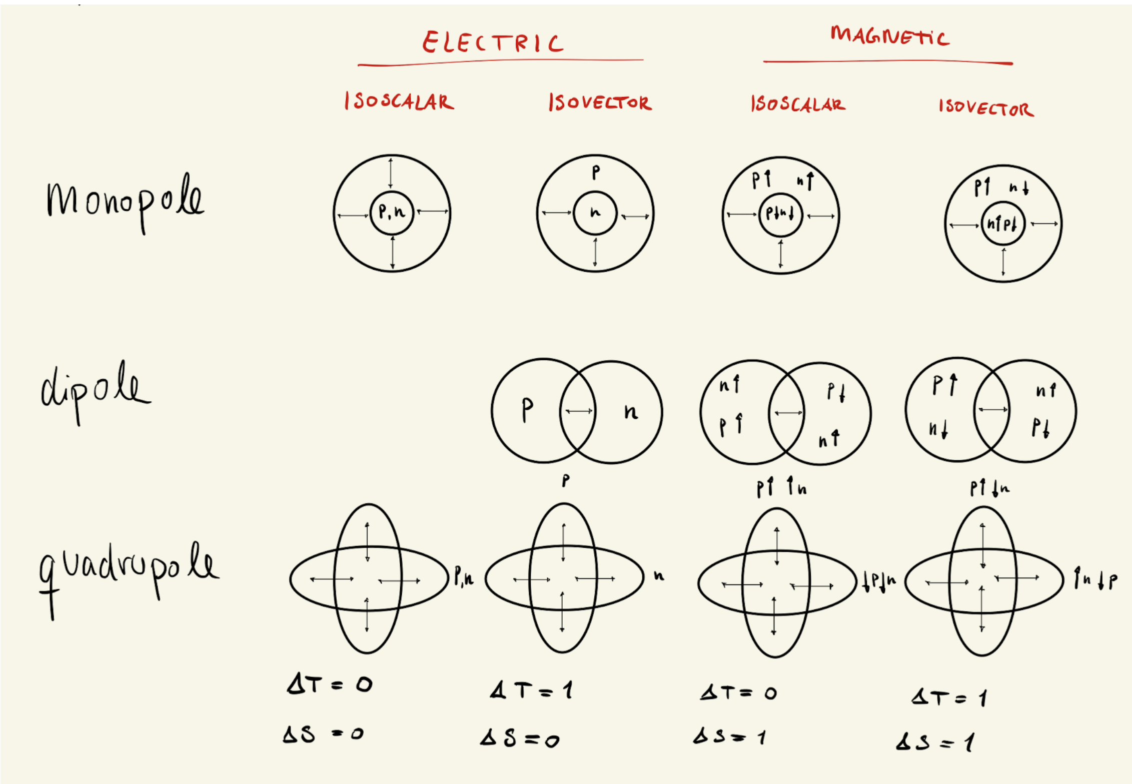


Figure 7.1:

The giant resonances are universal and only weakly correlate with the peculiarities of the individual nuclei. The most striking discrepancy between the liquid drop model and reality is revealed in the low-lying "surface" modes. which are very sensitive to the details of the ground state. This can be seen on figure 6.3 which shows these surface effects (vibrational states).

Lecture 9: Independant Particle Shell Model

Introducing second quantization. The single-particle orbits are given by

$$\lambda = n, \ell, j, m, \tau, \tag{59}$$

where τ is the isospin projection. The simplest many-body state allowed by Fermi-statistics is the Slater determinant given by

$$\Psi = \frac{1}{\sqrt{N}} \begin{vmatrix} \psi_{\lambda_1}(r_1) & \psi_{\lambda_2}(r_1) & \psi_{\lambda_3}(r_1) \\ \psi_{\lambda_1}(r_2) & \psi_{\lambda_2}(r_2) & \psi_{\lambda_3}(r_2) \end{vmatrix}, \tag{60}$$

which of course should be expanded to N-dimensions. For a 2 particles this is exactly the two-body wave function. In general, if you swap to columns you change a sign in the total wave function. The Slater determinant is defined by the set n_λ of occupation numbers $(= 0, 1)$ which fulfils

$$\sum_\lambda n_\lambda = A, \quad E(n_\lambda) = \sum_\lambda \epsilon_\lambda n_\lambda. \tag{61}$$

Section 11.1 introduces the second quantization notation/the occupation number representation. Note the difference between single-particle and many-body wavefunctions (round and angular brackets). The main result is equation (11.13) that gives the expression for a wavefunction of A fermions, and is the second quantization way of writing a Slater determinant.

$$|\Phi\rangle = \prod_{\lambda(n_\lambda=1)} a_\lambda^\dagger |0\rangle \tag{62}$$

It makes use of creation and annihilation operators defined at the bottom of page 204.

$$a_\lambda |0\rangle = 0, \quad \forall\lambda \tag{63}$$

They may seems frightingly deep and obscure if you have never encountered them before - they are not, as you will find out when you have gotten used to them, so think about them as smart ways of changing/probing occupation numbers. The important technical feature is the anticommutation relations in equation (11.8)

$$[a_\lambda, a_{\lambda'}]_+ = 0, \quad [a_\lambda^\dagger, a_{\lambda'}^\dagger]_+ = 0, \quad [a_\lambda, a_{\lambda'}^\dagger]_+ = \delta_{\lambda\lambda'} \tag{64}$$

Check that you understand them by verifying equation (11.23) (and why not also (11.24) that introduces pairwise contractions)! The basis transformations that are discussed around equation (11.5) may seem trivial or unimportant at first.

$$|\lambda) = |\nu) = \sum_\lambda |\lambda)(\lambda|\nu) \tag{65}$$

However, we need them to rewrite the expression for a general one-body operator in section 11.2, i.e. to go from equation (11.30) (make sure you are comfortable with that expression) to equation (11.33). It is not the most important derivation in the course, but at least the theoretically minded should try to follow it. The rest of section 11.2 is examples that are not very essential. Section 11.3 gives the corresponding general expression for a two-body operator in equation (11.39). We skip the derivation of that, but take a moment to ensure that you can see what elements are in (11.39).

$$F = \sum_{1234} (12|f|34) a_1^\dagger a_2^\dagger a_4 a_3 \tag{66}$$

Finally, section 11.4 employs the formalism to write down an expression for the two-body interactions in nuclei. The important results are equations (11.46) and (11.50), so check that you understand that they are equal (in (11.49) we only need the negative sign as we deal with fermions).

Lecture 10: Nuclear deformation

Nuclear deformation and collective model

The independent particle shell-model cannot be expected to work for nuclei in general and is seen experimentally to give a good description only for magic and near-magic nuclei in the ground state or in (some) low-lying excited states. (We have seen from the Island of Inversion that it may even fail for some nuclei with magic numbers such as $N = 20$.)

The two most important effects from the residual interaction are the pairing that we shall look at in chapter 13, and the deformation that is the topic of this chapter. We did not look into the detailed dynamical description of nuclear shapes in the second half of chapter 5 and will therefore only look at selected parts of chapter 12. There is a subtle point discussed on page 224 that is conceptually important, but in my experience may take some time to get used to: a deformed nuclear state can have $J = 0$ if the total wavefunction of the system has equal probability for all orientations of the deformed state. (So we think in terms of an "intrinsic frame" where the nucleus is deformed and the "exterior frame" - the one the atomic electrons see - where $J = 0$ and we average over all orientations. If you have difficulties in making sense out of the intrinsic frame, you can think of the deformed state as one that has strong spatial correlations between nucleons leading to the total density being non-spherical.) There are no static properties of a state with $J = 0$ that can tell us that it is deformed, so this information must be deduced from transition matrix elements to other states or from the general pattern of excited states – such as the rotation spectra we shall look at in chapter 16.

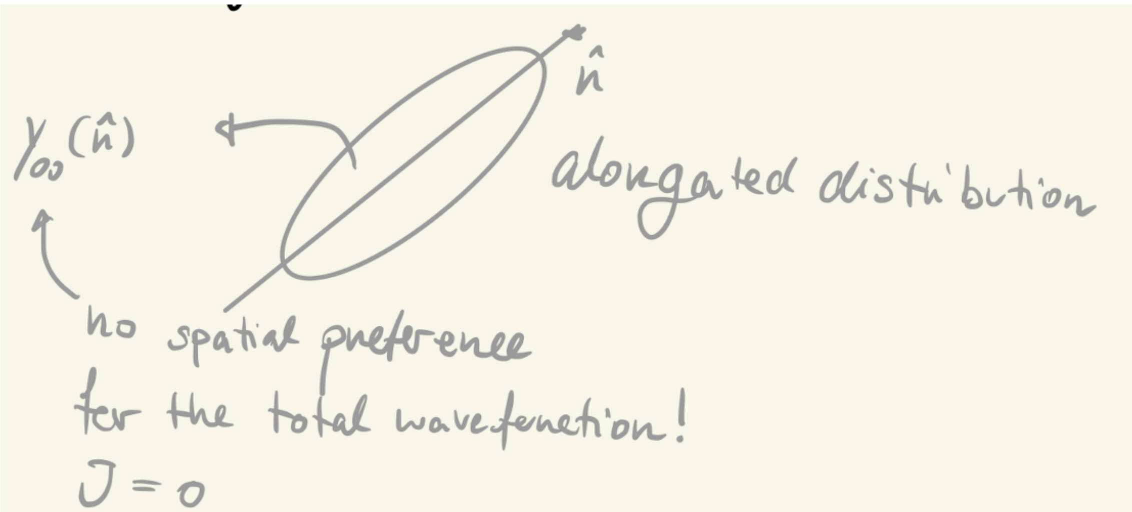


Figure 7.2:

The last paragraph of chapter 12.1 mentions the crucial observation that the system of a single particle outside a spherical core may gain energy if the core is slightly deformed (that would increase the spatial overlap of the single particle with the core which could gain energy as the nucleon-nucleon force is attractive) - when sufficiently many particles (more than one, but a handful or two is often enough) act together this will give permanent deformation. Chapter 12.2 gives a mathematical description of how such a deformation may be expressed. The mean field potential will now no longer be spherical and page 225 derives how one can rewrite. We will not need to use the details of this, but you should note that this (at least for small deformations) is mainly a surface effect - as illustrated in figure 12.1. Equation (12.7) could be the starting point of the classical (Bohr-Mottelson) treatment of deformation, it combines the shape degrees of freedom (from second half of chapter 5, we did not look at them) with a single-particle Hamiltonian and the derived coupling term. The following sections in the chapter explores this in more detail – we shall skip that except for three observations (without proof):
1. The expectation value of the coupling term is given in equation (12.20) for the most important case of a Y_{20} deformation term, note that there is a splitting in m, but that the $+m$ and $-m$ terms are degenerate.

$$\Delta\epsilon_{nljm} = \kappa_{nl}\sqrt{\frac{5}{64\pi}}\frac{3m^2 - j(j+1)}{j(j+1)}\alpha_{20}. \tag{67}$$

2. There are several ways of parametrising deformation, we shall use mainly β or δ, β is defined in equation (12.29)

$$\alpha_0 = \beta\cos(\gamma), \quad a_2 = a_{-2} = \frac{1}{\sqrt{2}}\beta\sin(\gamma) \tag{68}$$

(here we only consider Y_{2m} deformation) with equation (12.48) giving a more practical relation between the quadrupole moment Q and β (so β is more "experimental"). δ is almost the same, cf. equation (12.49), but is more convenient in some theoretical derivations.

3. Figure 12.3 shows how widespread deformation is among nuclei.

Single-particle quantum numbers and anisotropic harmonic oscillator

We consider in chapter 12.9 how (single) particles move in a deformed potential. We take from equation (12.20) that a spherical j-level will split up (expected, as the spherical symmetry now is broken and L no longer is a good quantum number). The coupling term, equation (12.59), is proportional to Y20 and is therefore able to mix states with different j but not to change parity - note the selection rules in the text.

$$H_{\text{coupl}} = -\kappa(r)\sqrt{\frac{5}{4\pi}}\beta P_2(\cos(\theta)), \quad \kappa(r) = r\frac{\mathrm{d}U}{\mathrm{d}r} \tag{69}$$

For the case of an axially symmetric (quadrupole) deformation the surviving quantum numbers are parity and m, the angular momentum projection on the symmetry axis. The book outlines how one would still expect simple rules for ground state spin-parity of even-even nuclei and odd-A nuclei, since rotation of the intrinsic frame will give states of higher energy.
Chapter 12.10 is technical and describes how the harmonic oscillator will change when deformed - it gives simply an anisotropic harmonic oscillator - and how the oscillator frequencies and energies will depend on the deformation, here expressed in terms of the delta parameter. The important outcome is figure 12.5 and the fact that there will be "new" magic numbers for certain deformations. For those who wish to follow the technical derivations: show and use that $r^2P_2 = (2z^2 - x^2 - y^2)/2$ to derive (12.62); to show (12.66) start reversely by inserting N, $\overline{\omega}$, δ_{osc} etc in it and show that it reduces to (12.65).
Chapter 12.11 continues the discussion of asymptotic quantum numbers from chapter 12.9, but adds spin-orbit and the fact that real nuclear potentials are flatter than the harmonic oscillator. It is a "discussion in principle" - try to get the gist of it, but we shall not employ these quantum numbers a lot later, so do not overspend time on it. The examples given in the rest of the chapter are more important. Chapter 12.12 outlines the (historically very) important Nilsson model - all the relevant single-particle neutron levels are shown in figure 12.6. You could compare it to figure 8.5 (for a fixed large A) and it is clear that deformation removes a lot of degeneracy of levels. Note how far down in energy some of the levels from the spherical high-j orbits in the upper shells can come at a deformation of 3:2. There are now less clear signatures for "deformed shells", but still a non-uniform level density. Chapter 12.13 discussed this and points to another prominent feature: that of avoided level crossing. Once you realise it is there you will find it is pervasive. We shall touch on avoided level crossing in extra problem 16, but to "tune your eye to it" try to look in figure 12.7 (that gives results starting from a Woods-Saxon potential) and compare the levels emerging from the spherical $g_{9/2}$: the $m = 9/2$ is essentially undisturbed, whereas the lower m values are much neater on the positive delta side than for negative delta where they tend to be bent downwards by similar-m-orbits from $g_{7/2}$ – or by even high-lying levels: all $m = 1/2$ levels in the figure (except for the very lowest one) are clearly perturbed by each other. Figure 12.8 adds octupole deformation and has even more mixing, to the point that wavefunctions at large deformations here are essentially chaotic.
Rather than getting lost in the quantum chaos (a topic the authors like a lot) you should remember that the spherical midpoint and the extreme large-deformation cases both have good (but different) quantum numbers. The transition region these two limits is where the deformed nuclei sit – and according to figure 12.3 this is a majority of nuclei.

Lecture 11: Pairing

Pairing is a tough subject to get into. In chapter 13.1 we look at one pair of nucleons. Chapter 13.2 gives the technical description of pair wavefunctions, also with more than one pair. We skip chapter 13.3. Chapter 13.4 presents a simplified model of pairing, and the rest of the chapter discusses different aspects of the BCS model of pairing. We shall only look at some of the main features of the BCS model, so will browse through selected parts of chapter 13.5-9 and skip chapter 13.10-11 completely. There are two reasons for this: the first is that it would be a lot of technical material with rather little physics behind; the second is that BCS is actually an approximate theory of nuclear pairing – it naturally leads to some of the main features of pairing (and focusses attention on them), but from e.g. a shell-model point of view, pairing is "just" one part of the residual interactions so a proper shell-model calculation will give the correct answers. It actually does, but seeing simple models - like the one in chapter 13.4 - hopefully will give you a better intuition for pairing.

Chapter 13.1 starts by reminding that pairing is not just the pairwise filling of levels in a Fermi gas picture (and mentions that $0+$ for even-even nuclei actually comes out of most models with randomly chosen interactions...). A crucial feature of pairing is the rather large pairing energy associated with it, this is a strong hint that it is a coherent effect. It is of course due to the attractive residual interaction between nucleons. The book makes the observation page 252 that the residual interactions can be expanded in L and gives the explicit formula for a delta-function interaction (it also points out that high L corresponds to small particle distance). More important for the following is the argument on the top of page 253 that two identical nucleons in the same j-shell will couple to $J = 0, 2, ...2j - 1$, i.e. all the odd values of J are excluded. The technical argument is given from the symmetry of Clebsch-Gordan coefficients and hinges partly on the parts of chapter 11 that we skipped (if you have plenty of time, you may look into it, otherwise just accept the result - note the reference to (12.83) on the top of page 253 should be to (11.86)). Let me remind that you have seen this general result already for the two-nucleon system, where nn and pp only existed for $J = S = 0$, not $J = 1$.

$$|JM\rangle = \frac{1}{\sqrt{2}} \sum_{mm'} C^{JM}_{jmjm'} a^\dagger_m a^\dagger_{m'} |0\rangle \tag{70}$$

We will use the result of problem 13.1 - but not do the calculations (perhaps as an exercise) - that using a delta-function interaction the $J = 0$ state will be shifted down in energy with respect to the others, see figure 13.2 that also show a typical experimental example close to a double-magic nucleus. Note that this state appears, cf equation (13.10), by putting the two nucleons in orbits with $+m$ and $-m$, i.e. orbits that maximise their spatial overlap. It is the "evenly distribution over all m-values" that makes the state coherent and gives the large energy gain. Pairing features are seen in many even-even nuclei, clearly close to magic numbers as illustrated in figure 13.4, but also - as shown in figure 13.3 - in nuclei that also have collective states (vibrational).

Chapter 13.4 presents a very schematic model for pairing. It is useful as a way of illustrating some very general features of pairing, but is too schematic to be used in more realistic calculations. The assumption is that Ω (an even number) orbits close to the Fermi surface participate in the pairing process, all the remaining orbits are decoupled. Pairs form in time-conjugate orbits and can scatter from one pair of orbits to another, there is a positive interaction energy connected to that. We assume furthermore that all the Omega orbits have the same energy ϵ and that all the matrix elements between pairs (notice: also from one pair to itself, a "self-energy") have the same value $-G$ where G is positive. The Hamiltonian is therefore (13.36):

$$H = \epsilon \hat{N} - G P^\dagger P \tag{71}$$

the first term is the one we have seen in (9.2) and (11.10), the second contains - once you expand the pair creation and annihilation operators - all the interaction terms mentioned above. The rest of the chapter now solves this Hamiltonian.
Intermezzo: the case of one pair of particles. This can be solved easily (I got the solution from the textbook of my older colleague, Aksel Jensen) - I present it for your information, you may skip it, it will not be part of the curriculum. Let $x_1, x_2, ...x_M$ be the amplitudes for the $M = \Omega/2$ pairs. Our wavefunction Ψ is then the vector of all x_i and from the interaction part of the Schrödinger equation we then get $-GV\psi = E\psi$, where V is a matrix with 1 on all entries (all orbits interact with each other orbit). Writing this out gives the equations $-G\sum x_i = Ex_1 = Ex_2 = Ex_M$. There are M solutions to this, one has $E = -GM$ and an eigenvector with the same amplitude for all pairs (i.e. a coherent state, the same as in (13.10)), all the others have $E = 0$. If you compare this to figure 13.2 you will see the same overall pattern - but the current model is indeed even more schematic.
The Hamiltonian can be solved exactly, but that is technically a bit intricate and requires the introduction of quasispin. The "spin" in that name is again more likely to confuse than to help, it simply means we can make a $SU(2)$ model and get results that are similar to those from spin (or isospin). The step-down, step-up and z-component of the operator algebra is given in (13.38) - for those who do not remember the commutators of $SU(2)$, you may find the commutators for the x, y, z-components and use that step-up/down is $x \pm iy$ to show that $[\mathcal{L}_-, \mathcal{L}_+] = -2\mathcal{L}_z$, $[\mathcal{L}_-, \mathcal{L}_z] = \mathcal{L}_-$ and $[\mathcal{L}_+, \mathcal{L}_z] = -\mathcal{L}_+$, corresponding to (13.19) and (13.37) (Ω is a number and commutes with all operators). The reason for introducing quasispin is that we now can denote all states in terms of the quasispin quantum numbers, corresponding to $\mathcal{L}_z$ and $\mathcal{L}_2$, they are given just before and after the important equation (13.39). Take time to ensure that you understand the values these quantum numbers have, $\mathcal{L}$ of course increases in steps of 1 and can be integer or half-integer.
The solutions of the Hamiltonian are now straightforward to write down. The interaction term can be rewritten as $-G\mathcal{L}+\mathcal{L}-$ and its value can be found from (13.39). Check that this gives (13.41) given by

$$E(\mathcal{L}, N) = \epsilon N + \frac{G}{4}\left(N - \frac{\Omega}{2}\right)\left(N - \frac{\Omega}{2} - 2\right) - G - \mathcal{L}(\mathcal{L}+1) \tag{72}$$

We often wish to rewrite the result in terms of seniority. The relation (13.42) reflects that seniority s starts out being 0 in the ground state and increase in steps of 2 as pairs are broken, whereas L starts out being $\Omega/4$ and decrease in steps of 1. (If you want to check that (13.41) gives (13.43) I recommend to "start at both ends" and write out all terms to see that they agree.)
Page 263 essentially collects interesting special cases and comments upon them. The loss of energy when going from $s = 0$ to $s = 2$ should give the pair binding energy (13.45) and is used to define the energy gap Δ. The relations can also be used for odd N and one can derive (13.48) showing that Δ is also the amount an odd system is pushed up with respects to the neighbouring even ones, i.e. the odd-even effect. We skip the last parts of the chapter, starting at the bottom of page 263, since that use the single j-level model from chapter 11 that we did not look into.

Lecture 12+13: Nuclear Gamma-Transitions

We skip chapter 14 on gamma-radiation, it would be useful but is not essential for us and some of you have probably seen similar content in other courses. The key result we shall use from here (a result, I believe you have seen in the introductory course) is the multipole expansion in chapter 14.10. In case you have never seen a proper derivation: it starts from an expansion of a plane wave as in equation (4.3). The Bessel function $j_l(x)$ can for small x be approximated as $xl/(2l+1)!!$ (the latter !!-notation means: multiply all odd numbers 1, 3, 5... up to $2l + 1$). We shall postulate the

result in equation (14.99), note in particular that decay rates go as $E2L + 1$ (E is proportional to k for photons). The $B(EL)$ that appears in the equation is the transition matrix element squared and is defined in the previous equations.
Chapter 15.1 treats single-particle gamma transitions, the only ones we will have in the independent particle model. As mentioned several places in the chapter pairing correlations will modify the results, and collective phenomenon will appear as we move away from the magic numbers, so the detailed model results in this chapter are a guideline rather than reliable predictions. In the model the general selection rules that are useful for determining which multipole order a transition has (used already in the introductory course, repeated on the top of page 306) are supplemented by a pure ℓ selection rule, proven in problem 15.1. (As usually, we do not go through this problem.) It is costumary to measure the $B(EL)$ values in terms of the Weisskopf single-particle units that are a very simple order-of-magnitude estimate. There are two similar sets of units for the $B(ML)$ values. Single-particle transitions almost never reach a full Weisskopf strength, collective transitions often go above, so they can be used to get a quick idea about what type of transition one is dealing with.
Chapter 15.2 outlines why collective transitions are so strong, you should focus on the argumentation up to and including equation (15.23). The argument is rather general - we shall see a more elaborate variation of it in chapter 18.1 - and hinges on the fact that the square of a coherent superposition of amplitudes is (much) larger than the sum of the squares of the individual amplitudes. This is in a way trivial, but if you have never met the argument before you should take the time to make sure you follow it here; it is the basis of many collective phenomena in many parts of physics. I would like to single out one detail that the book presents very nicely: the structure of the collective state and the operator we look at have to "match" in order that we get the coherence that makes up the collective transition. (We discussed different giant modes in chapter 6.4, they all give collective states of different quantum numbers. A given collective state would only give rise to one specific collective transition.)
Chapter 15.3 discuss nuclear isomers, i.e. excited states that have unusually long lifetimes. (The book does not bother to say what is usual lifetime values - many normal transitions are in the range $10 - 15$ to $10 - 9$ s, one often calls a state an isomer when its lifetime is above a µs.) One obvious way of getting a long lifetime is to have a transition with large angular momentum difference L between the two states (the one that emits the gamma ray and the final state), to see this notice by combining (14.99) and (15.13) that transition rates has a factor $(kR)2L + 1$ and rewrite kR as $ER/(\hbar c)$, only few gamma transition reaches values above $1/40$. Make sure you follow the elegant argument why isomers "pile up" at the end of a shell: the flat bottom of the potential gives a level ordering within a shell with high ℓ orbits lowest and lowest ℓ orbits at the top and the spin-orbit then mixes in high-l orbit (with opposite parity) from the next shell at the top - see e.g. figures 8.4 and 8.5

Lecture 14: Collective modes

Nuclear structure theory combines the complexities of the strong interaction with the complexities coming from many-body quantum mechanics. The first four pages of chapter 18.1 outlines a schematic model that is very useful and often used to describe microscopically the collective modes of a system (such as the giant resonances of chapter 6). It could be worth also to recheck section 15.2 and the remark from there that collectivity is often measured with respect to a single operator and arises when the residual interactions "are aligned" to a specific multipole structure. The model is rather schematic, but at the same time generic - it can be useful in many situations.
Formally, we are in a perturbation situation in (18.1) with known eigenstates for the (mean field)

unperturbed Hamiltonian $H0$ and a perturbation H' whose matrix elements will be assumed to be non-zero between states that have the same quantum numbers. The eigenfunctions for the total H can be expanded as in (18.4) and by inserting that in the Schrödinger equation the formal solution in (18.7) is obtained. So far it is very general.
We now assume the specific structure (18.9) for H'. It turns out to be applicable for many cases where the model is relevant, one example discussed explicitly in the book is one-particle one-hole excitations (e.g. of $E1$ type). With this structure one rewrites quickly (18.7) to the key equation (18.12). There are here trivial solutions with no change to the system - but the more interesting case is the one of (18.13) that is illustrated in figure 18.1: there will be $N - 1$ solutions close to the unperturbed energies, and one collective solution positioned above/below the other solutions for a repulsive/attractive H' (cf the sign of kappa). (If the strength of H' is weak, the "collective" solution will look quite a lot like the others.)
For a strong residual interaction one gets the properties in (18.15)-(18.17), note that when matrix elements are similar one gets the "equal contribution of each unperturbed state" that was used in section 15.2. The book goes on to show that the collective state can exhaust the sum rule for the strength, but this is beyond what I would expect you to have time to look into.

Nuclear Rotation

You may have met rotation several places in physics - some may have heard about the (Noether) argument that symmetry gives conservation laws (translation/rotation giving conservation of momentum/angular momentum). Chapter 16.1 starts with a "warning", namely that rotational behaviour is more complicated than translational behaviour - this is mainly for the theoretically minded among you.
We have rotation in nuclei (and in molecules and particles, not in atoms), this is seen via rotational bands as in figure 16.1. A loose "definition" of a band is that all levels have the same (deformed) internal structure that is rotating more and more rapidly, this automatically gives strong overlap of wavefunctions and therefore fast gamma-transitions within a band and much slower transitions to other states. The energy is naturally given by the Hamiltonian in equation (16.4) (since J is a vector in that equation, the $\hbar^2$ should be removed), corresponding to energies given by (16.3). Note the concept of a bandhead state, the lowest level that sets the starting energy and spin for the band. There can be many different bands in a given nucleus, and we should use the shell-model or other mean-field models to describe the difference in structure between the different bandheads - note in particular in figure 16.1 that all excited bands start after an energy gap 1.7 MeV, this is a pairing energy gap.
We shall not look into the theoretical treatment of the moment of inertia for a band, equation (16.5) - as indicated in figure 16.3 the values for nuclei lie between the two limits of rigid motion (solid) and irrotational motion (liquid), so it is complex to describe. The book mentions two ways of estimating the moment of inertia experimentally, they are inspired by the first and second derivate of $E(J)$, are called "static and dynamic" and correspond to global and local estimates... The values we get for ground state bands are as in figure 16.3, the pairing tends to lower the moment of inertia. For excited bandheads one can get significantly higher moments of inertia - one example is the superdeformed bands mentioned page 340 (remember here the discussions in chapter 12 as well as the shape/fission isomers in chapter 21).
There are other ways of getting high angular momentum in a nucleus than collective rotation, one can also build it up via coupling of the nucleons - and in spherical nuclei one can get a very different type of level scheme. Compare figures 16.1 and 16.4 (and note that this is also done for a selected

set of states in figure 16.5) - whenever nuclear rotation occurs we get an enormous simplification of description of the structure.
The reason we can access the high-angular-momentum states at all experimentally is that thay are likely to be produced in heavy ion reactions where impact parameters around the nuclear radius will be collisions with very high angular momentum. The compound nucleus gets rid of energy via emission of neutrons and gamma rays, but this does not reduce angular momentum very much. We therefore in many different collisions end up in the states with lowest energy for a given spin - called the yrast states (from Swedish "yr", meaning "dizzy" - it corresponds to "ør" in Danish). Figure 16.5 is one way of highlighting the yrast line.

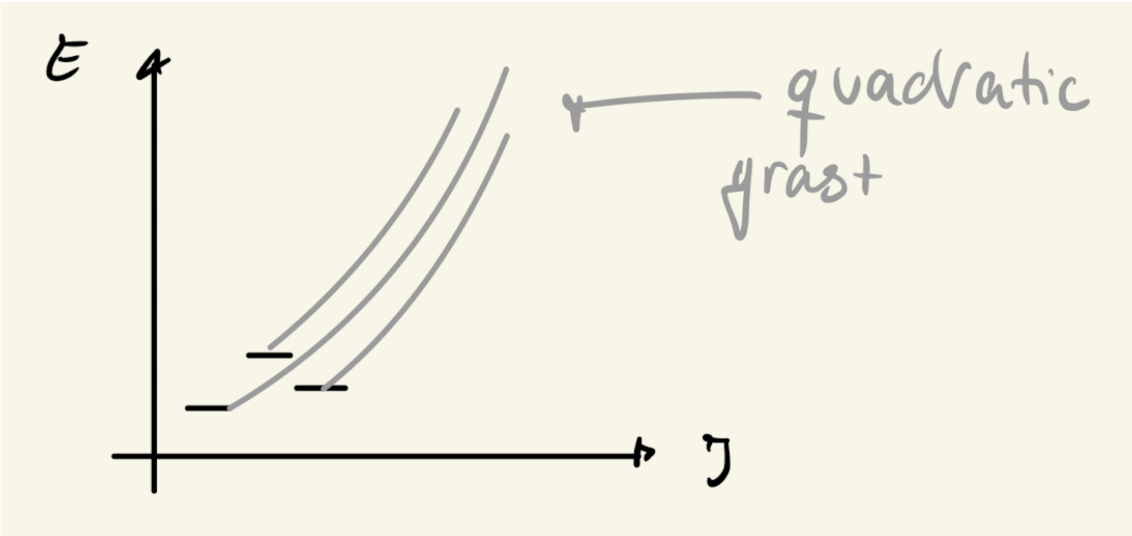


Figure 7.3: Yrast band

The first two pages in section 16.23 discusses band crossing that will appear because higher bandhead states can have higher moments of inertia and the bands built upon them therefore eventually will become the yrast band, cf. figure 16.10 (focus on the left hand side of this figure, the right hand is a standard way of plotting the same data in a transformed manner, but takes a bit longer to explain). Notice that the two bands normally will mix when they cross (remember our discussion of two-level mixing !) so the transition will be softened a bit.

Lecture 16: Nuclear Fission

Chapter 21.1 introduces some of the basic facts on fission. We already saw in chapter 5 (figure 5.1 and equation (5.96)) that fission is energetically favourable for heavy nuclei and that the critical value of the fission parameter $Z2/A$ is about 49. For values below the critical one spontaneous fission can still happen via tunneling (as in alpha-decay), but it is first in Cf and Fm ($Z = 98$ and 100) that one finds isotopes where spontaneous fission is the main decay channel. It has been measured as low as Th ($Z = 90$). Note the discussion around figure 21.1 on the fission barrier. The height varies with A and Z, it is around 6 MeV in U. Nuclei excited to high energy can fission, it has been seen even down to 208Pb - absorption of a low-energy neutron may in some cases lead to fission: it will give a final nucleus excited by the neutron separation energy that as an example in 235-239U is (in MeV) 5.3, 6.5, 5.1, 6.1, 4.8, note the even-odd staggering due to pairing. So low-energy neutrons can induce fission in 235U (final nucleus 236U), but not in 238U (final nucleus 239U) where a neutron kinetic energy above 1.1 MeV is needed to give fission. Note on page 501 that Problem 21.1 ends just below equation (21.2) - in the first print of the book the separation line is placed a paragraph too low.

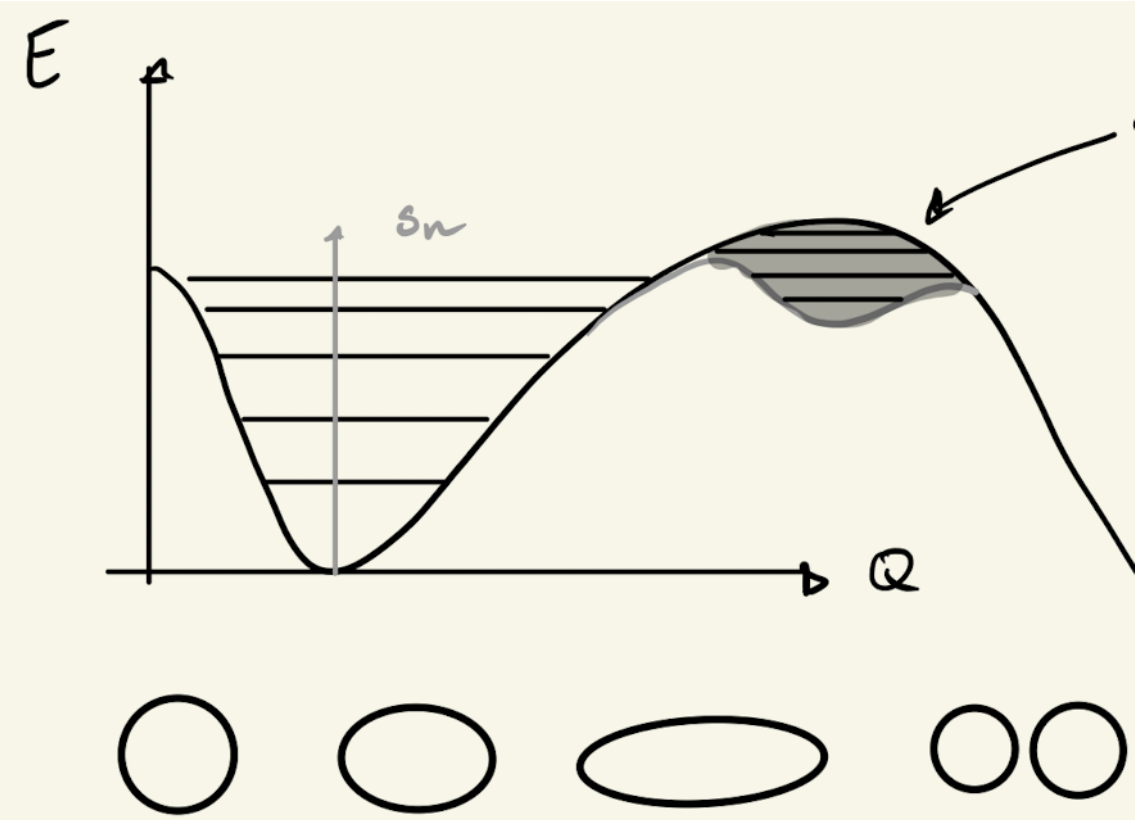


Figure 7.4: Fission barrier. Second minimum due to shell model

As an intermezzo, I recommend looking at the two paragraphs on page 507 (most of the page) that describes neutrons in different energy ranges, and what they may be used for. You may have heard about the ESS (European Spallation Source) that is being built at the moment close to Lund - it produces neutrons by spallation (high-energy reactions of protons on W targets) for use mainly in condensed matter physics and biology. Other similar neutron facilities, such as the $n-TOF$ at CERN, also have a strong physics programme; several research nuclear reactors also provide neutrons for many different applications. (They are sensitive to magnetism and are better than synchroton radiation in distinguishing light elements, such as those of relevance in biology.)
Back in chapter 21.1: heavy nuclei are neutron rich and most fission fragments therefore lie below the line of beta stability and are radioactive, some emitting beta-delayed neutrons during their decays. These are very important for nuclear reactors. If all neutrons liberated in fission were prompt, the timescale for the chain reaction in nuclear reactors would be very fast, so if slightly more than one neutron is produced per fission the energy production would increase very rapidly and therefore easily get out of control and lead to an explosion - with a few percent of the neutrons coming delayed by around a second the overall timescale is sufficiently longer to make it possible to control a nuclear reactor...
Chapter 21.2 discusses alpha-decay and the estimates of their halflife due to penetration of the Coulomb barrier. This is all material you should have seen in the introductory course, one exception being the discussion on page 504 on the pre-exponential factor that contains the matrix element between initial and final nucleus and is expected to be strong just above double magic shells. Note

that this formalism of course also could be used for proton radioactivity (nuclei just beyond the proton dripline may live up to a second before decaying by proton emission) or for emission of heavier nuclei - among the ones being emitted are 14C and 22Ne, but these latter processes are rather rare. Finally, alpha-decays are an essential ingredient of the decay-chains that start in the longlived naturally abundant U/Th isotopes and go all the way down to Pb/Bi. They probably provide a substantial part of the heat in the interior of the Earth, the decays can be used to estimate the age of the solar system, and these decay give an important part of the natural background radiation on the surface of the Earth - most famous is the Radon background that is due to 222Rn (produced by alpha-decay of 226Ra, occuring in the 238U decay chain).
Chapter 21.7 is not as overall essential knowledge as the other sections, but it ties together several subjects in our curriculum in a way that you hopefully will find satisfactory rather than confusing. We have looked at the transition from spherical to deformed nuclei in the liquid drop model (Chapter 5) and in the shell model (Chapter 12), the former lacked shell effects, but these may be restored by shell corrections - those are the topic of the last half of this section. As seen in figures 12.5 and 12.6 we may for certain masses get new shell structure (and therefore extra stability) for axis ratios e.g. $2:1$ and $3:1$ (superdeformation / hyperdeformation). The very smooth dependence of energy on deformation from chapter 5 will therefore be modulated as shown schematically in figure 21.6 - we can have second (and even third) minima at larger deformation. Low-lying states in these mimima tend to be longlived (states above the barriers are of course not) and are therefore isomers, often called shape isomers. Shape isomers will happen over much of the nuclear chart - some examples are given at the end of the section - but they may play a special role in fissioning nuclei where they are often called fission isomers. States in the "normal" deformed shell may (e.g. via reactions) go into states in the "second minimum", these can then decay back into the normal minimum or continue outwards and lead to fission - the states in the "second minimum" in certain cases function as "stepping stones" to fission. Try to go through the main principle in the derivation of the shell correction term equation (21.45) on page 517-8: the basic idea is to take a shell model energy level spectrum as a function of the deformation parameters Q, their level density - a set of δ-functions in (21.41) - is smeared out first by 1-2 MeV to give a continuous function with all major structure still present, then smeared out much more to get a structureless density. The latter would correspond to "no microscopic features" as we have in the liquid drop model. The difference in total energy (for a given number of nucleons) for these two smeared level densities is the shell correction. (And why do we have to use this complicated procedure? - because absolute energies are given much more reliably by the liquid drop model than by the shell models, so we need to combine the two sets of models...)

Weak Interactions

This chapter gives selected examples of the interplay beteen weak interactions and nuclei (a warning: this is very much my research area, so I may put too much focus of the details). After the introduction in chapter 24.1, we shall focus on chapters 24.2-3 and 24.6 that outlines the standard theory of beta-decay and look quickly at chapter 24.13 that deals with double beta-decay. We skip the connections to particle physics and the standard model that is explained in chapters 24.4-5 and 24.10-11, as well as the neutrino oscilltion chaper 24.12 (other courses cover this material). I will assume that you have heard about parity violation (chapter 24.7) from the introductory course - that subject, as well as the electric dipole moment covered in chapters 24.8-9, is a good reminder that much of the early characterization of the weak interactions was done in nuclear physics. There is still a substantial part of current work in nuclear physics on the "fundamental" aspects of weak interactions

(precision low-energy experiments can compete with higher-energy experiments), but of course also many places where weak interactions can be used to study nuclear behaviour as well as several cases where weak interactions are a key in application of nuclear physics (e.g. supernovae dynamics).
Chapter 24.1 reminds at the end that the reason the weak interactions are weak, is the large rest mass of the W and Z bosons. The weakness implies that timescales for weak decays or reactions are long on the typical nuclear scales, but weak processes are nevertheless important in several cases: (1) nuclear beta decays (β-, $\beta+$ and electron capture) - the energy scales are here so low that outgoing lepton wavefunctions can be replaced by plane waves, i.e. we get a multipole decomposition as for gamma decays (see sections 15 and/or 14.10, the mathematical expansion was given in equation (4.3)); (2) a variation of the process covered in the book is muon capture (as (24.3) but with an initial muon) that happens with stopped cosmic negative muons, here the energy is too large for the expansion to be useful; (3) another variation, discussed around (24.4), is neutrino detection; (4) single beta-decay can be energetically forbidden, but double beta-decay allowed - however, the timescales are very long; (5) weak reactions are normally far too weak to be of importance, but (24.5) is an exception and is a key reaction for hydrogen burning in stars; (6) tests of fundamental symmetries often involve weak interactions, this is mainly a subject in between nuclear and particle physics, but in a few cases also relevant for nuclear structure.
The decay rate for beta-decay is given by time-dependent pertubation theory, equation (24.6). I shall go through this in a different order than the book. We look first at the matrix-element. The operators for Fermi and Gamow-Teller decays were given in (6.53) and (6.54) - they change a proton into a neutron (or opposite) and also can do a spinflip for Gamow-Teller, but they do not change the spatial wavefunction! The corresponding sum rules were outlined in problem 6.2. The coupling constant is measured from well-understood Fermi decays (also used to establish the quark mixing, as you may have heard in the introductory course) and is given in (24.25) - the coupling constant in principle differs from Fermi and Gamow-Teller decays, as mentioned on page 595.
We next look at the wavefunctions. We shall not write down explicit expressions for the nuclear wavefunctions, but can use that Fermi decays goes (to a high precision) within isospin multiplets so one can write down general expressions for the strength (the reduced matrix element squared) as done in problem 24.1. Then the lepton wavefunctions: the wavenumbers are small (momenta typically less than 5 MeV/c) so kr is much less than 1. It is then a good approximation to do a plane wave approximation and (as for gamma rays) do an angular momentum expansion (the equation (4.3)). This is in principle all for the neutrino, but the electron/positron will be affected by the nuclear Coulomb field and the wavefunction enhanced/suppressed, in particular at lower energies. This effect is collected in the Fermi function that to lowest order is given by (24.16). Now, the lowest order in the expansion, $\ell = 0$, gives the allowed transitions - that is what chapters 24.2-3 focus on. The corresponding selection rules are given page 591-592. If allowed transitions cannot occur we must go to higher orders in the explansion, this is the subject of chapter 24.6 - in order ℓ there is an extra factor $(kr)^{\ell}$ in the matrix element, which (as the weak interaction is a point interaction on the nuclear scale) gives a factor rll in the nuclear matrix element. There are other smaller corrections that all in all imply that you should stay away from these higher orders - the forbidden transitions - if at all possible... As an order of magnitude the beta strengths gets extra factors of order $(kR)^{2\ell}$, i.e. a suppression of 3-4 orders of magnitude (depending on the Q-values of the transition). This is the major reason behind the pattern in figure 24.4 - the nuclear matrix elements will of course give variations as well, but typically only vary a few orders of magnitude. Chapter 24.6 has at the end some more examples (e.g. of very hindered transitions) and comments, note that most beta transitions you will see are Gamow-Teller.
The last step is to look at the phase space factors, and we will (as is customary) only do this

for allowed transitions (to avoid the k^ℓ factors in higher orders). The conservation laws simplify greatly since the energy given to the final state nucleus can be neglected (it rarely reaches the keV range), so the energy conservation gives $E = E_e + E_\nu$ whereas the momentum conservation gives $0 = p_{\text{nuc}} + p_e + p_\nu$, so we can take the two lepton momenta as independent, this is the reasoning behind (24.7). Note here that we treat the process in a (large, but) finite quantization volume V - this is also the reason for the factor $V^{-1/2}$ in the lepton wavefunctions, the volume cancels out and does not enter in the final expressions. Note also that the lepton energies are the relativistic ones, i.e. includes the rest mass. It is now a simple exercise to rewrite the phase space factor - first in chapter 24.2 as a differential (a function of the electron energy) - then in chapter 24.3 after integration over electron energy (or momentum, as the book chooses to do). A longer digression on page 588-589 considers how a neutrino mass term would enter (or mass terms, cf (24.12)) - this is important as it is the only model independent test of a neutrino mass; the current limit (2019, from the KATRIN collaboration) on the effective neutrino mass is 1 ev. The final result in (24.18) is now obtained with the definition of the f-factor in (24.17), note the useful - and very often used - rewriting into dimensionless quantities (electron energies in units of $m_e c_2$, momenta in units of $m_e c$).
Digression/exercise: the approximation of f given in extra problem 27b is obtained by neglecting both the neutrino and electron rest masses, use then (24.10) and integrate over (reduced) electron energy to get the result. (In applications, do remember that E is the relativistic energy!)
All of the above was for β-decays. In electron capture the energy conservation is given by (24.14) where the nuclear recoil energy again can be neglected, the nuclear recoil momentum is simply equal to the neutrino momentum, so the phase space factor is simply given by the neutrino factor at the end of (24.17) since there is a two-body final state. The total phase space is here proportional to $p_n u^2$ and $p_n u$ is (apart from a factor c) for essentially all decays equal to $E_n u$ in (24.14).
The last part of chapter 24.3 (after equation (24.20)) gives, as the last part of chapter 24.6, examples that may be useful for getting a feeling for how beta decays are happening in practice. For a practitioner, equation (24.33) is particularly useful - and do not forget figure 24.4.
The final chapter 24.13 tells about double beta-decay, figure 24.7 is here the essential one. Skim through the text to get a quick overview of what concepts are at play here - for those who have had more particle physics, the discussion on possible Majorana type neutrinos and the (so far unsuccessful) search for neutrinoless double beta-decay may be illuminating.

Lecture 18+19+20+21: Nuclear Astrophysics

See note.

Lecute 22+23: Compound Nucleus Resonances

We typically divide reactions into direct reactions and compound reactions (one can look at intermediate situations, but these two limits very often suffice). The first paragraphs on page 5 and figure 1.1 outlines the difference. We shall look at direct reactions in these notes and at compound reactions in chapter 20.9 in the book.
Section 1.2 summarizes the notation that is rather detailed (since we have to cover several different cases), but in principle is straightforward to decipher. Equations (1.4) and (1.5) describe the relative motion of two nuclei, the internal Hamiltonian of each of the two nuclei and the interaction term between the two nuclei. The new ingredient is the interaction term - we look at it in section 2. Note that we need an extra index, α for the incoming channel, α' for inelastic scattering, β for rearrangement channels, to keep track of what in- and out-going nuclear systems we are describing.

We do the standard scattering theory assumption of incoming plane wave and outgoing spherical-type-waves (as in the book equation (4.1)) for the asymptotic wave functions. Equations (1.8) and (1.9) contains everything: incoming plane wave, elastic scattering, the many different possible inelastic scatterings and the many different rearrangements reactions. The way to get cross sections is generalized from (4.2) in the book to (1.10) in the notes. Reaction people often introduce the T-matrix (transition matrix) that is proportional to the scattering amplitude - we shall assume without any attempt of proof that equation (1.13) can be derived and shall only use it in a limited way in section 4. (Note that K as usual is a wave number, there is a $\hbar$ missing 3 lines below (1.13)...)
Section 2 deals with elastic scattering, we skip the complications coming from the Coulomb potential (sections 2.3 and 2.4) and the final sections 2.9 and 2.10 - and section 2.7 is empty. Page 13 tries to discuss how one may use an effective interaction potential - it is based partly on the more theoretical (projection operator) formalism in section 1.3 that we skip, so do not attempt to follow the arguments (they are anyway not that strict) - - we simply accept that one often can use an effective interaction potential that in principle can be complex (in the meaning: having real and imaginary parts), non-local (we neglect that) and energy-dependent. The rest of section 2 makes this more concrete.
Section 2.1 introduces the partial wave expansion and phase shifts that we have used in the book in chapter 4. Compare equations (2.4) and (2.6) in the notes with (4.13) and (4.3) in the book to see that the partial wave expansion is essentially the same. Moro's notes has a more general formalism (including the F, G and H functions), but leads to the same phase shift - compare equation (2.14) in the notes with equations (4.8) and (4.9) in the book. The main interesting point in this section is the four bullet points at the end - focus on understanding them, and note in particular that an imaginary part of the potential will lead to loss of flux, which is seen as a norm of the S-matrix element being less than 1. This is the only point where the note really goes beyond the book chapter 4.
The brief section 2.2 again agress with the book - compare equation (2.19) in the notes with equations (4.12) and (4.10) in the book.
It turns out that it is very hard to deduce the effective interaction potential from experiments. One therefore often chooses the form presented in section 2.5 that you will recognize as a Woods-Saxon potential including spin-orbit where the real part has to go smoothly into the potential given in the book chapter 8.9 when we go to bound states, and the main addition is imaginary terms (of the same shape) and the fact that the strength parameters of the potentials can be energy dependent. This is called the phenomenological optical potential - (optical because scattering of light including absorption can be treated with a similar imaginary term...). For scattering of a single nucleon against a target it would be natural to assume the Woods-Saxon type of potential shape - - for more complicated projectiles one could try to use the folding procedure outlined very briefly in section 2.6, but in practice it turns out that the standard form of the optical potential is working well in most applications.
Physically, at small impact parameters (when the nuclei overlap) the strong interactions will make it very unlikely that the nuclei emerge undistrubed, i.e. we have absorption rather than elastic scattering. This is mathematically taken care of by the imaginary term in the optical potential. Section 2.8 describes the strongly absorbing region in more detail - it resembles somewhat the description we shall look at later in the book chapter 22.2 (you can already now look at figure 22.2 for an illustration of the grazing angle). Notice the three different regimes, shown in the notes figure 2.3, of Rutherford, Fresnel and Fraunhofer scattering - clear absorption at small impact parameters/large angles if the energy is sufficient to make the nuclei touch.
We skip section 3 on inelastic scattering and look in section 4 briefly on transfer reactions. Section 4.1 repeats the information (and notation) from section 1.2 with a bit more detail - do check the

naming of the different components in figure 4.1 where a fragment x (e.g. a nucleon) is transferred from one nucleus to another.
Section 4.2 looks heavier than it really is. All the general relations are given first and are then gradually approximated to something more understandable and practically useful. We follow mainly the case of the (d,p) reaction, i.e. transfer of one neutron unto a target nucleus. All the different interaction terms turns out to reduce approximately to equation (4.9), the key here is that there is an overlap between the wavefunctions of the target A and final nucleus B (that can be in an excited state). The overlap is quantified as a spectroscopic factor, equation (4.12), and is very often written in terms of a set of single-particle wavefunctions - note that the square of the amplitude in (4.13) is the spectroscopic factor and that it may be interpreted as the occupation number of the orbital in question. As Moro writes this type of definition is not completely unique, but it is very practical as the two examples in figure 4.2 and 4.3 show. Make sure that you understand how each single-particle level gives rise to a peak in the transfer cross-section and how for each peak the associated angular momentum can be found from the angular dependence of the cross-section.

Chapter 20.9: Statistical Description of Resonances

Many different experiments show, as in figure 20.2 for low-energy neutrons, strong peaks in reaction cross-sections that are called resonances (the system "resonates" at given energies) and are interpreted as many-body states - - their width Γ is here still less than their average distance D. They are normally interpreted in the compound model (originating with Niels Bohr) where the long lifetime (small Γ) is assumed to give a loss of memory of how the state was populated, so formation and decay are independent, as quantified in equation (20.71).
We shall look at s-wave neutrons and use the relations for elastic scattering derived in chapter 4, namely equations (4.6) to (4.11) that corresponds to (20.72) to (20.73). We postulate (20.74) for reactions and get (20.75) as total cross section. We furthermore postulate the expressions (20.76) and (20.77) which is the Breit-Wigner approximation for the resonance shape (it is partially derived in exercise 31) - the spin weightings obeys the standard rules of averaging over initial states and summing over final states.
By now we can take two different approaches to a spectrum such as the one in figure 20.2: (i) describe neutron scattering in a mean field (the optical potential, which for such a low energy is close to the shell-model potential), this would give a few broad "single-particle" resonances; (ii) describe each of the many narrow experimental resonances with the Breit-Wigner expression. How can these two pictures be connected? Experimentally, we can smooth the narrow resonances (as on page 517) to give the broad overall strength that should correspond to (i). Theoretically, we also have to perform an averaging - I shall denote it here by av(..) and shall proceed in a slightly different way than the book. The S-matrix can be written as $S = \overline{S} + S_{fl}$, where $av(S)$ is given in (20.78). By definition $av(S_{fl} = 0$, and you can check that $av(S_{fl}^2)$ is given by (20.79) – inserting this in (20.74) gives directly (20.80), saying that the average reaction cross-section is less than that given by inserting av(S), i.e. the one the optical model would give (it would correspond to the average behaviour). You can also average the elastic cross-section in (20.73) and show that it would be the optical model elastic cross-section (the one in (20.83)) plus σ_{fl}. Here two comments are needed - first, note that the total average cross-section is the same in the optical model as in the detailed description - second, the difference between the two points of view is, as indicated on the top of page 495, that a resonance that decays back into the elastic channel is counted as "reaction" in the optical model, and "elastic" otherwise...
This can be done even more explicitly by performing the averaging (i.e. energy integral) over an

interval I that contains n resonances, i.e. $I = nD$. The average over a single resonance is given by (20.86) and gives the average over total reaction cross-section in (20.89). Defining the transmission coefficient as in (20.90) our formula (20.74) turns into (20.91) and we obtain the important result (20.92). It is important since it relates the right hand side that can be measured in experiments and the left hand side that can be calculated in the optical model - it is therefore the way to tune optical models to fit experimental data.
Can we proceed the other way ? I.e. if we know the optical model parameters (they wary smoothly over the nuclear chart, as the shell-model does), can we predict average cross sections for processes? The answer is yes, and the details are given in the Hauser-Feshbach approximation, equation (20.94), but we do not take the extra two hours it would take to look into this in detail - - however, note that to use this approximation (as done often for heavier nuclei, e.g. in nuclear astrophysics) you need to know the optical potentials in all open reaction channels.

Heavy Ion Collisions

Heavy-ion reactions includes many subtopics that we will not have the time to go into, one example is the investigation of superheavy nuclei, i.e. the heaviest part of the nuclear chart up to $Z = 118$ (the element Oganesson). We shall here aim for a quick overview of what type of reactions can proceed and start in section 22.1 by noting that the many degrees of freedom in the colliding system makes a statistical approach useful. It is customary to look at typical timescales (or equivalently: widths Γ) where the timescale for equilibration should be considerably shorter than the timescale for decay out of the compound system - otherwise the statistical approach is not meaningful and the compound picture not appropriate. The decay timescale should again be shorter than the Weisskopf/recurrence time.
The overview in section 22.2 focusses on energies around and somewhat above the Coulomb barrier (5-10 MeV per nucleon). Here the de Broglie wavelength of the relative motion is much smaller than the typical length scales (given either by equation (22.1) or (22.2) depending on whether we are below or above the Coulomb barrier) - and we can use a semiclassical approach and speak of trajectories, impact parameters and orbital momentum, equation (22.3). Figure 22.2 sketches how one can divide the reactions into groups according to decreasing impact parameter b, equivalent to more and more violent collision: at the largest b we have pure Rutherford scattering i.e. elastic scattering, we can then have a region with inelastic scattering - first Coulomb excitation and then contributions from strong interactions - this extends down to the grazing angle θ_g, then follows what the book calls quasielastic processes and Antonio Moro would call rearrangement collisions (the point being that we have a target-like and projectile-like nucleus coming out of the reaction), and finally the deep-inelastic region with considerable exchange of nucleons – complete fusion would be an extreme variant of this, but the total excitation energy may be too high for the compound system to survive a long time.
One could imagine the division in figure 22.2 to correspond to different radii R_i, that can be related to impact paraters bi in the following way: angular momentum conservation gives $L = bp_{far} = Rp_{close}$, the momenta are proportional to the square root of the kinetic energy that decreases from E to $E - V(R)$ where V is the total potential (mainly the Coulomb potential for this purpose). Combing this gives $b^2E = R^2(E - V(R))$ and therefore leads to equation (22.4). Note, what the book does not discuss, that a plot of $E\sigma R$ as a function of E would allow to extract R and $V(R)$ experimentally. Note also that we can change between using angular momentum ℓ and b as variables.
The book discusses several phenomena occuring in the more violent regions. Note in particular the description of "orbiting", so that projectiles can be bent in the same scattering angle both by

"near-side" and "far-side" processes.

Liquid-gas phase transition, chapter 22.8 and NuPECC LRP 4.2.3

When exciting a nucleus more and more ("heating it up") we will reach a point where it will disintegrate ("evaporate") into smaller pieces, eventually into a gas of free nucleons. This liquid-to-gas phase transition is a bit challenging to describe: it happens in a finite system that furthermore is a quantum system. We cannot expect to have clean, sharp phase transitions like in classical macroscopic system, but will anyway use the terminology from there. A key quantity in detailed descriptions (we only touch upon it briefly) is the Equation-of-State (EoS) of nuclear matter, where recent research has focussed in particular on the dependence on isospin - whether the EoS is soft or hard has implications e.g. for the maximum mass of neutron stars.
We probe the EoS mainly through nuclear reactions. The book focusses on the splitting up into smaller fragments, called multifragmentation, where a key quantity is the number of fragments (IMFs) larger than the alpha particle and smaller than the original nuclei. This will first increase with excitation energy of the system, then decrease as all fragments are dissociated into nucleons (and/or alpha particles). Take time to understand the scaling shown in figure 22.8 in the book. (You can skip the statistical estimates on page 549-550.)
The NuPECC Long Range Plan (LRP) lists more information, but does not put it nicely into a logical structure (partly because it was written for a specialized audience, partly because this is still a challenge for the field). You should focus on Box 3 and the "order parameter" suggested there that experimentally indicated we do deal with a phase transition, and also on the isospin fractionation (an extreme case is the crusts of neutron stars, where at a certain depth nuclei will exist embedded in a neutron gas).
Note that LRP section 4.2.1 also has interesting comments on the EoS, the liquid-gas phase transition and some references to where in the cosmos these quantities are of interest.

Quark-gluon plasma phase, NuPECC LRP 4.2.1, 4.2.2 and 4.2.5

Section 4.2.1 gives an overview of phase transitions in nuclear matter, experimentally probed via collisions of the heaviest nuclei (traditionally Pb nuclei at CERN and Au nuclei in the US). The focus in this section should be on understanding the phase diagram in Box 1. Note that the extention of the liquid-gas phase transition in the figure is a bit too large. The transition between hadron matter and "deconfined quarks" is actually two transitions: a quark confined-deconfined one and a chiral symmetry breaking-resoration one. The figure also indicates several trajectories that could be followed after high-energy nuclear collisions (experimentally, it appears that the systems equilibrate rather quickly so that it makes sense to draw them into a phase diagram) - it is interesting that one expects a first order transition at high baryon density (sometimes chemical potential is used instead of density), but that there is a critical point so that one has a smooth crossover at lower densities, e.g. in the early Universe.
Section 4.2.2 reports on theoretical calculations from lattice QCD with figure 1 giving energy density and pressure as function of temperature. There is clearly two different phases (with many more degrees-of-freedom at high energy, corresponding to quarks being deconfined - note that it is actually the colour charges that are being deconfined), but the calculations are done on small lattices so the finite size effects are large. Box 2 attempts to cover also the chiral symmetry breaking, but for those of you that have not done a lot of particle physics this may be a tad too heavy to be understandable. These two transitions are expected to occur close to each other, the most recent values I saw was a transition temperature around 160 MeV. You can skip the details in the last half of this section.

Section 4.2.5 gives an account of the experimental situation - and it is therefore the most historical one, but also in a sense the most interesting one: the investigation of the phase transition to "quark-gluon plasma" has to a large extent been driven by experiment. The overall picture may have originated in theory, but whenever one experimentally was able to go to higher energy collisions new and unexpected experimental dicoveries has overturned previous concepts and led to new modelling. Unfortunately, there is no good quick introduction as yet to this rapidly changing field. One key concept now is "flow" that is a collective phenomena illustrated in Box 4. The unexpect result from analysis of the data showed that the flow happens with very low viscosity, the matter here is a nearly perfect fluid (close to the expected lower bound for a quantum system). As mentioned in the right column on page 96 quark-antiquark bound states may be "screened away" by free colour charges - this has been followed now as a function of beam energy and is an interesting probe of the phase. A surprise from RHIC experiments (left column, page 96) was jet quenching: a collision that in vacuum would give two jets of particles back-to-back may in the "plasma" have a quark going through several fm of matter - it will loose energy on the way, so that one jet will be missing - and one has in this way been able to estimate the energy loss due to colour forces (much larger than the one from electromagnetic forces...). But there are many other aspects that people are working on collecting into a unified picture.