Week 9

1. Q — **Problem 23: Gamma decay**
a) The note on gamma decay also states the relations coming from conservation of angular momentum and parity. Check first how these conservation laws are used to determine what type of gamma-ray will dominate the transition.

The photon carries away a total angular momentum given by L. The minimum value is 1 – there are, however, two possible polarisations states, $L_z = \pm 1$. This means that in in the transition there must be a change in L_z of ± 1. The allowed values of L are restricted by

$$\mathbf{J}_i = \mathbf{J}_f + \mathbf{L} \tag{1}$$

The electric multipole radiation has parity $P_\gamma^E = (-1)^L$ and the magnetic multipole radiation has parity $P_\gamma^M = (-1)^{L+1}$. From parity conservation we have

$$P_i = P_f P_\gamma \tag{2}$$

From these equations we have some selection rules and this also means we can determine which type will dominate the transition.

Multipolarity	Dipole		Quadru		Octu		?	
Type	E1	M1	E2	M2	E3	M3	E4	M4
L	1	1	2	2	3	3	4	4
P_γ	-1	1	1	-1	-1	1	1	-1

Table 0.1: Selection rules

If you have a transition $J_i^{P_i} \to J_f^{P_f}$, you'd have to look at L and the change in parity from equation (2).

b) Plug in numbers for the transitions width for i) 10 MeV, $A = 17$, ii) 1 MeV, $A = 200$.
Use the following equations

$$\Gamma(E1) = 0.068 E^3 A^{2/3}, \quad \Gamma(M1) = 0.021 E^3 \tag{3}$$

$$\Gamma(E2) = 4.9 \cdot 10^{-8} E^5 A^{4/3}, \tag{4}$$

where Γ is measured in eV and E is measured in MeV. The equations relating the decay width and the mean life-time:

$$\Gamma = \frac{\hbar}{\tau} = \frac{\hbar \ln(2)}{t_{1/2}} \tag{5}$$

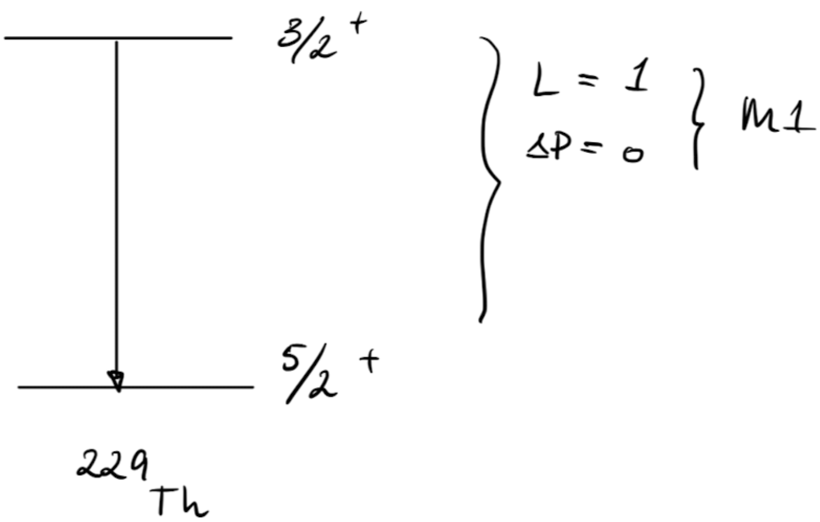
i) 10 MeV, $A = 17$

$$\Gamma(E1) = 450\,\text{eV}, \quad \tau = 1.46\,\text{as} \tag{6}$$
$$\Gamma(M1) = 21\,\text{eV}, \quad \tau = 31.34\,\text{as} \tag{7}$$

ii) 1 MeV, $A = 200$

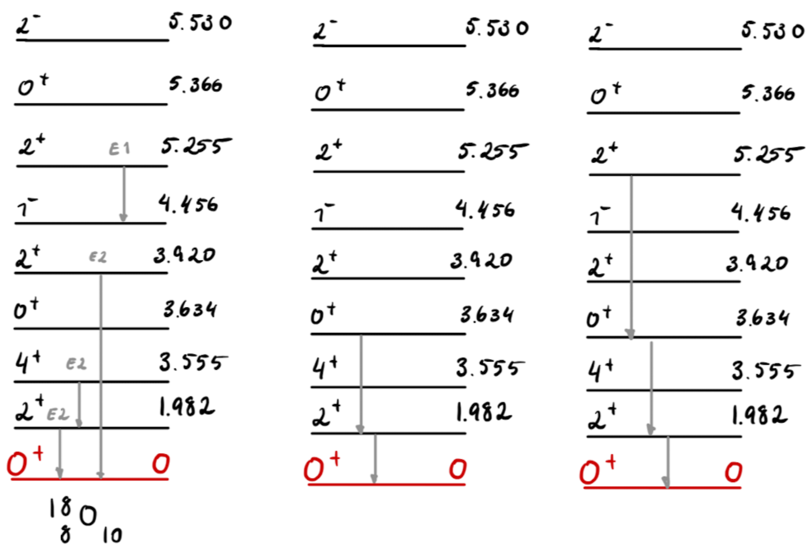
$$\Gamma(E1) = 2.32\,\text{eV}, \quad \tau = 283.7\,\text{as} \tag{8}$$
$$\Gamma(E2) = 5.73 \cdot 10^{-5}\,\text{eV}, \quad \tau = 11.49\,\text{ps} \tag{9}$$

c) ^{229}Th is famous due to having a 3/2+ excited state at an excitation energy of 7.8(5) eV above the 5/2+ groundstate. What type of gamma-ray would deexcite the state?



Calculating the mean-life time, $\tau = 64.7$ s.

d) Make first a quick sketch of the level scheme of ^{18}O. What gamma transitions do you expect to dominate?



Keep in mind that the $2^+ \to 2^+$ is possible. This is due to the fact that $J_i = J_f \neq 0$ is possible, compare to equation (1). In this case the $2^+ \to 1^-$ is faster due to J value.

e) Angular correlation in cascade of consequtive gamma rays

If you consider the $0^+ \to 2^+$ transition the $E2$ type would dominate. The initial state $J_1 = 0$ is fully symmetric and we can choose the quantization axis along the direction of the emitted photon. The intermediate state $J_2 \neq 0$ and there must be an angular dependency. The second photon originates from the same magnetic substate.

In the second case the initial state has $J = 2$ but we can choose the quantization axis to be along the direction of the emitted photon. From the 0^+ state the distribution is uniform since $J = 0$. Finally, from $2^+ \to 0^+$ there must be some angular denpendency.

2. Q — **Problem 14**: The shell-model configuration of the three states given for ^{17}O

A — For the ground state:

$$\begin{aligned} &(0s_{1/2})^2(0p_{3/2})^4(0p_{1/2})^2, \quad \text{protons} \\ &(0s_{1/2})^2(0p_{3/2})^4(0d_{5/2})^1, \quad \text{neutrons} \end{aligned}$$

For the first excited state

$$\begin{aligned} &(0s_{1/2})^2(0p_{3/2})^4(0p_{1/2})^2, \quad \text{protons} \\ &(0s_{1/2})^2(0p_{3/2})^4(1s_{1/2})^1, \quad \text{neutrons} \end{aligned}$$

For the second excited state

$$\begin{aligned} &(0s_{1/2})^2(0p_{3/2})^4(0p_{1/2})^2, \quad \text{protons} \\ &(0s_{1/2})^2(0p_{3/2})^4(0d_{3/2})^1, \quad \text{neutrons} \end{aligned}$$

The binding energy for the $d_{5/2}$ neutron in ^{17}O must be 4.2 MeV.

3. Q — **Problem 22** – Seniority