Week 2

1. Q — Explain the solution in problem 2.2

A — Determine the parity of π^- in the following

$$\pi^- + d \to n + n \tag{1}$$

The spin of π^- is zero and the spin of the deuteon is 1. In the final state both neutrons have $S = 1/2$. The pion is captured by the deuteron from a 1S state. This implies $l = 0$. This means the angular momentum in the initial state is $J_i = 1$. From conservation of angular momentum the final state must have $J_f = 1$.

$$J^\pi P_\pi P_d = (-1)^l P_\pi P_d = P_\pi, \quad \text{initial state} \tag{2}$$

$$P_n P_n (-1)^l = (-1)^l, \quad \text{final state} \tag{3}$$

The neutrons in the final state are two identical fermions and there is only one state with $J = 1$ which is the ${}^3P_1$ state. This implies $l = 1$ and this means

$$P_\pi = (-1)^l = -1. \tag{4}$$

2. Q — Make sketches similar to figure 2.1 for A=17 and A=32

A — All states in a given nucleus have the same projection

$$T_3 = \frac{1}{2}(N - Z) = \frac{A}{2} - Z \tag{5}$$

They belong to various isomultiplets

$$\frac{1}{2}|N - Z| \leq T \leq \frac{1}{2}(N + Z) \tag{6}$$

This means $T(T+1)$ new grounds states for each T due to isospin symmetry. The rest are excitations. We assume $[\hat{T}, \hat{H}_{\text{strong}}]$ even though this is only an approximation. The Coulomb force does not conserve isospin and this means the energy levels are not symmetric and for instance the protons are less tightly bound. The state with the lowest T is the ground state. For $A = 17$

$$T_3 = \frac{1}{2} \tag{7}$$

And

$$\frac{1}{2} \leq T \leq \frac{17}{2} \tag{8}$$

And the same thing for $A = 32$.

$$T_3 = 0 \tag{9}$$

And

$$0 \leq T \leq 16 \tag{10}$$

This is illustrated in the following figure

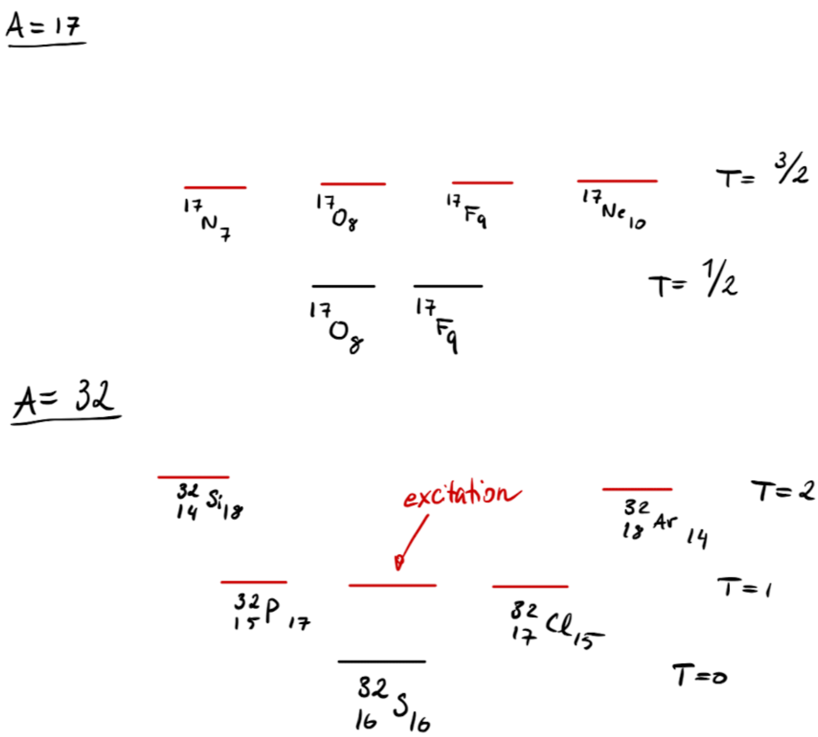

Figure 0.1: isomultiplets

3. Q — Describe how isospin conservation can explain the following two results from nuclear reactions i) $d + d \rightarrow^4$ He $+ \pi^0$ ii) $d + d \rightarrow^4$ He $+ \pi^0 + \pi^0$. Why is the first $T = 1$ state in ^{12}C strongly excited in inelastic proton scattering but weakly excited in inelastic alpha scattering?

A — Since the deuteron has isospin 0 the total isospin can couple to 0 and in order for the total isospin in the final state to couple to 0 there must be two pions.
The nuclear reactions are

$$p +^{12} C \longrightarrow p +^{12} C^* \tag{11}$$

and

$$\alpha +^{12} C \longrightarrow \alpha +^{12} C^* \tag{12}$$

The isopin of the proton is 1/2 and the carbon nucleus in the ground state 0. In equation (11) the total isospin in the final state can couple to either 1/2 or 3/2 and this is a strong interaction. For equation (12) the alpha particle has isospin equal to 0 and the total isospin can only couple to 1 and the reaction is electromagnetic. This equation (12) is isospin forbidden which yiels more observations of equation (11). Both reactions can occur due to isospin not being a perfect symmetry.

4. Q — Read the last 2-4 sentences of section 2.4. Discuss to what extent one can say that isospin measures the symmetry of a nuclear wavefunction.

A — Consider

$$\mathcal{P}^r \mathcal{P}^\sigma \mathcal{P}^\tau = (-1)^{l+S+T} = -1 \tag{13}$$

This implies a correspondance betwwen the isospin symmetry and the space-spin symmetry – for the two-nucleon the isospin symmetry is a consequence of the space-spin symmetry. For a higher T the less space-spin symmetry. Also, lowest T implies maximum space-spin symmetry. This leads to an interpretation of isospin as a measure of space-spin symmetry and the nuclear wavefunction.