Interacting Boson Model

Nuclear Physics II

7th of June 2021

Introduction

- Shell model
- Collective degrees of freedom
- Group theory and symmetry
- Phase transitions

Recap

Shell model

Even-even nuclei, valence nucleons, pairing (figure 13.4)

Collective behaviour

Shape variables, rotations, vibrations

$$R(\boldsymbol{n}) = R_0 \left(1 + \sum_{\lambda\mu} \alpha_{\lambda\mu}(t) Y_{\lambda\mu}(\boldsymbol{n}) \right)$$

Introducing bosons

Consider only s and d bosons

$$N = N_s + N_d = \frac{N_f}{2}$$

States generated by operators

$$\hat{s}^\dagger, \hat{d}^\dagger_m, \quad m = 0, \pm 1, \pm 2$$

Algebra of Boson Operators

Generators: $[d^\dagger s, s^\dagger s] = d^\dagger s$

Subgroups: subsets of generators that commute among themselves

Casimir operator $\rightarrow$ quantum numbers and degeneracy

Hamiltonian

$$H = H_s^\circ + H_s' + H_d^\circ + H_d' + H_{sd}',$$

where

$$H_s^\circ = \omega_s \left(s^\dagger s + \frac{1}{2} \right), H_s' = U(s^\dagger s^\dagger)(ss),$$

$$H_d^\circ = \omega_d \sum_m \left(d_m^\dagger d_m + \frac{1}{2} \right), \quad H_d' = \frac{1}{2} \sum_{L=0,2,4} V_L \sum_m (d^\dagger d^\dagger)_{LM} (dd)_{LM}$$

Coupling leads to

$$H = \epsilon N_d + \alpha C_{U(5)} + \beta C_{SU(3)} + \gamma C_{\mathcal{O}(6)} + \delta C_{\mathcal{O}(5)} + \eta C_{\mathcal{O}(3)} + \text{const.},$$

Chain algebra

$$\text{Vibational limit} : SU(6) \supset SU(5) \supset \mathcal{O}(5) \supset \mathcal{O}(3)$$

$$\gamma\text{-unstable limit} : SU(6) \supset \mathcal{O}(6) \supset \mathcal{O}(5) \supset \mathcal{O}(3)$$

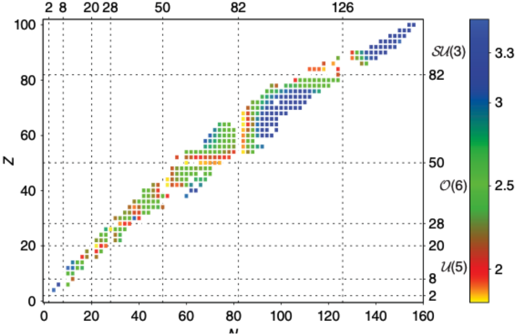
$$\text{Rotational limit} : SU(6) \supset SU(3) \supset \mathcal{O}(3)$$

Vibrational limit – $\mathcal{SU}(5)$

Limit of non-interacting d-bosons. Pure vibrational spectra

$$\frac{E(4_1^+)}{E(2_1^+)} \simeq 2$$

Compare figure 19.1 to figure 6.3 (cadmium isotopes)



Rotational limit – $\mathcal{SU}(3)$ limit

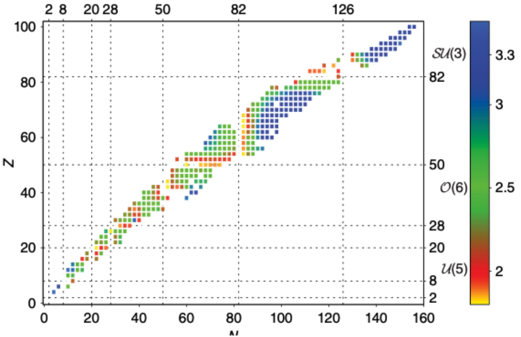
Prolate axial rotor – rotational-like spectra (figure 19.3)

$$\frac{E_4}{E_2} \simeq 3.3$$

(λ, μ) related to quadrupole deformation parameters

$$Q_0 = \frac{\hbar}{M\omega_0}(2\lambda + \mu), \quad Q_2 = \sqrt{\frac{3}{2}}\frac{\hbar}{M\omega_0}\mu$$

Related to β, γ. Figure 19.3

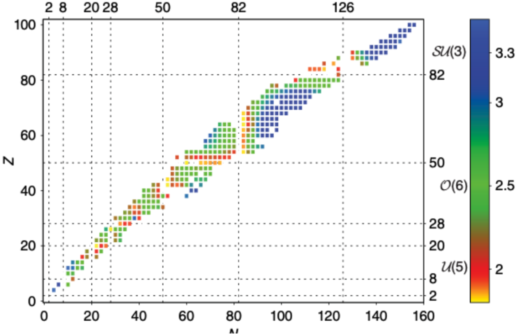


γ-unstable limit – $\mathcal{O}(6)$

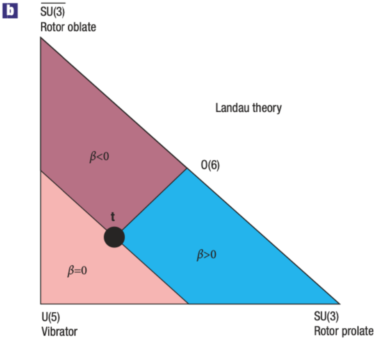
Figure 19.2

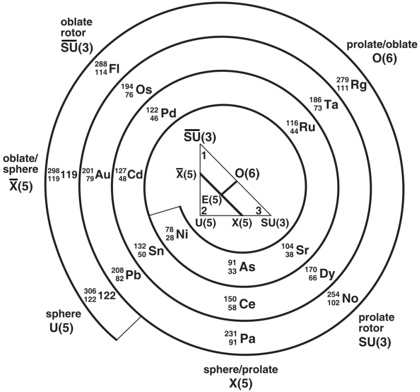
$$\frac{E_4}{E_2} \simeq 2.5$$

Broad region of $\mathcal{O}(6)$ nuclei near $Z = 54$ and $Z = 56$



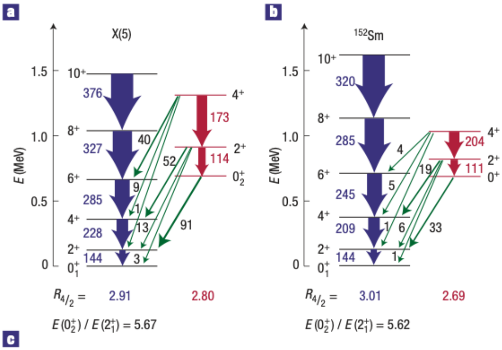
Shapes and Phase transitions

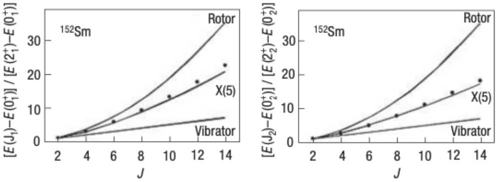




Compare to figure 12.2

Comparison


a
X(5)
E (MeV)
1.5
1.0
0.5
0
10+
8+
6+
4+
2+
0^+_1
376
327
285
228
144
40
52
9
1
13
3
91
173
114
4+
2+
0^+_2
$R_{4/2}$ = 2.91 2.80
$E(0^+_2) / E(2^+_1) = 5.67$
b
152Sm
10+
8+
6+
4+
2+
0^+_1
320
285
245
209
144
4
19
5
1
6
33
204
111
4+
2+
0+
$R_{4/2}$ = 3.01 2.69
$E(0^+_2) / E(2^+_1) = 5.62$
c


152Sm
Rotor
X(5)
Vibrator
$[E(J_1)-E(0^+_1)] / [E(2^+_1)-E(0^+_1)]$
$[E(J_2)-E(0^+_2)] / [E(2^+_2)-E(0^+_2)]$
J
0
10
20
30
2
4
6
8
10
12
14