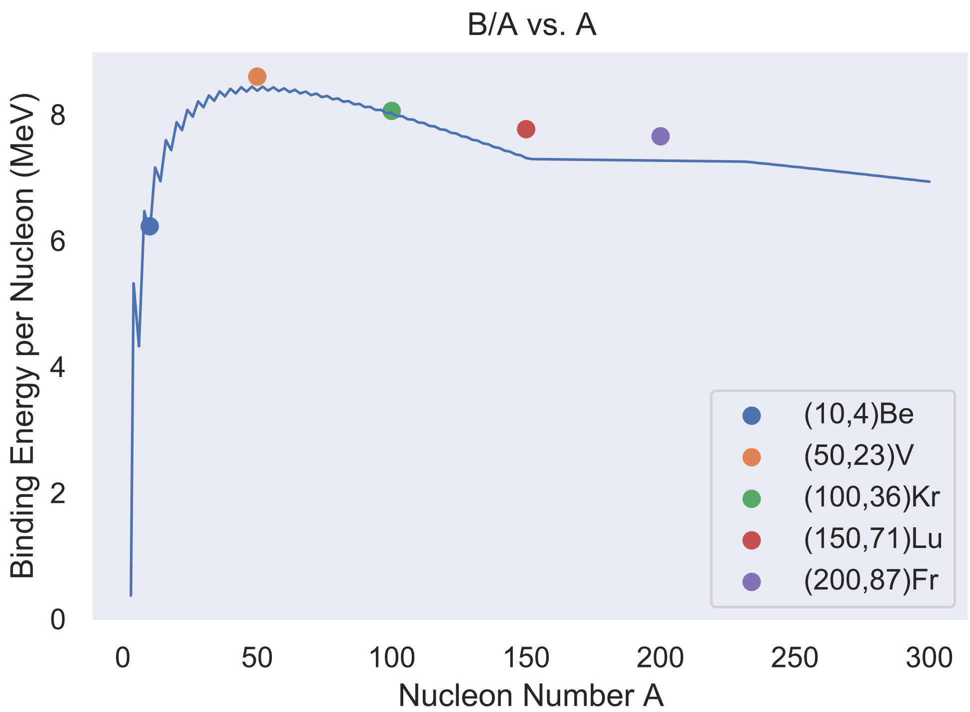
Week 1

1. Q — **Problem 6: The semi-emperical mass formula**. Make a script to calculate the binding energy B in MeV for a given mass number A, proton number Z (and N neutrons). Compare B/A to figure 5.1.

A — Plot



2. Q — **Problem 1: Isospin properties**. In a two-nucleon state with quantum numbrs T and T_3 for the total isospin and its third component, show that the expectation value of the vector product of t_1 and t_2 is $[2T(T+1)-3]/4$.

A — Starting from

$$\boldsymbol{T} = \boldsymbol{t}_1 + \boldsymbol{t}_2 \tag{1}$$

Taking the square to get an expression for the vector product

$$\left\langle \boldsymbol{T}^2 \right\rangle = \left\langle \boldsymbol{t}_1^2 \right\rangle + \left\langle \boldsymbol{t}_2^2 \right\rangle + 2\left\langle \boldsymbol{t}_1 \cdot \boldsymbol{t}_2 \right\rangle \tag{2}$$

And using the relations

$$\left\langle S^2 \right\rangle = S(S+1), \quad \left\langle S_i^2 \right\rangle = \frac{1}{2}\left(\frac{1}{2}+1\right) \tag{3}$$

This leads to

$$\begin{aligned}\langle \boldsymbol{t_1} \cdot \boldsymbol{t_2} \rangle &= \frac{T(T+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right)}{2} \\ &= \frac{2T(T+1)-3}{4}\end{aligned}$$

Which is the final expression.

3. Q — **Problem 5: The Deuteron and other s-waves in nuclei**. Consider only a central potential between the proton and the neutron. Use $U(r)$ and write down the Schrödinger equation in term of the radial deuteon wavefunction $u(r)/r$. Solve this either analytically or numerically and plot the wavefunction

A — Considering the potential between the proton and the neutron given by

$$U(r) = \begin{cases} -U_0, & r \leq R \\ 0 & r > R \end{cases}$$

The radial equation is given by

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2 u(r)}{\mathrm{d}r^2} + \left[U + \frac{\hbar^2 l(l+1)}{2mr^2}\right]u(r) = Eu(r) \tag{4}$$

This is identical to the one-dimensional Schrodinger equation with an effective potential, where the centrifugal term pushes the particle outward. To solve this analytically I rewrite the equation and consider the boundary conditions.

$$\frac{\mathrm{d}^2 u(r)}{\mathrm{d}r^2} = \left[-k^2 + \frac{l(l+1)}{r^2}\right]u(r), \quad \text{where} \quad k = \frac{\sqrt{2m(E+U_0)}}{\hbar} \tag{5}$$

For $l = 0$

$$\frac{\mathrm{d}^2 u(r)}{\mathrm{d}r^2} = -ku(r) \Rightarrow u(r) = A\sin(kr) + B\cos(kr) \tag{6}$$

Since $R(r) = u(r)/r$ and $\cos(kr)/r$ blows up as $r \to 0 \Rightarrow B = 0$ and the solution is

$$u(r) = A\sin(kr), \quad r \leq R \tag{7}$$

For $r > R$, $l = 0$ and $U(r) = 0$

$$\frac{\mathrm{d}^2 u(r)}{\mathrm{d}r^2} = -\frac{2m}{\hbar^2} = \kappa^2 u(r) \Rightarrow u(r) = Ce^{\kappa r} + De^{-\kappa r}, \quad \kappa = \frac{\sqrt{-2mE}}{\hbar} \tag{8}$$

Since $Ce^{\kappa r}$ blows up as $r \to \infty$. Now I can use the fact that the solutions must match at $r = R$. This must be true for both $u(r)$ and $u(r)'$. This leads to two equations for $r = R$

$$A\sin(kR) = De^{-\kappa R} \tag{9}$$

$$Ak\cos(kR) = -D\kappa e^{-\kappa R} \tag{10}$$

This leads to

$$-\cot(kR) = \frac{\kappa}{k} \tag{11}$$

Plugging in an appropiate value for $R = 2.127$ fm yields

$$\begin{aligned} U_0 &= \frac{\hbar^2\pi^2}{2mR^2} - E \\ &= \frac{\hbar^2\pi^2}{2mR^2} + B \\ &= 24.82\,\mathrm{MeV} \end{aligned}$$