Week 14

1. Q — **Problem 4**. We consider here the reaction ^{65}Cu+p going to ^{65}Zn*+n (Zn has 30 protons). With sufficiently high proton energy the final neutron spectrum will be a continuous distribution with one sharp peak superimposed, the peak corresponds to ^{65}Zn being left at an excitation energy of 7.3 MeV above the ground state. How may isospin be used to explain the presence of a strong sharp peak?

A — If you consider figure 2.1 in the book we have the same thing but for a much higher A. On figure 1 I have drawn the some of the isomultiplets. For the reaction mentioned above we effectively make a proton into a neutron. The total wave function must be a product of the coordinate, ψ, spin χ and the isospin, Ω. Since the spin and coordinate does not change the total wave functions must have the biggest overlap when the isospin part is the same. In this case it is the $T = 7/2$ state in Cu to the $T = 7/2$ state in Zn, which is an excited state. This argument follows from a two-nucleon interacting, but this can be generalized for multi-nucleon interactions.

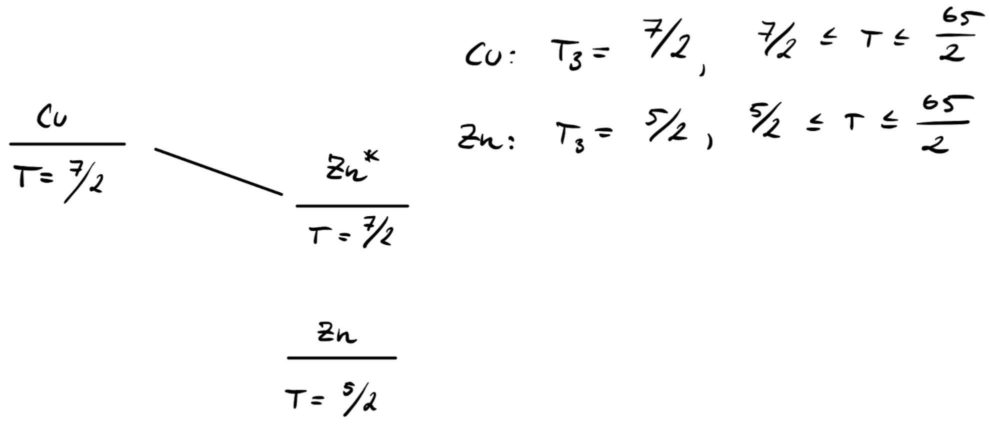

Figure 1: Isospin diagram

Another argument which is more clear is as follows. This is a strong interaction which means T_3 is a conserved quantity in when the Coulomb interaction is neglected. This means the strength of the reaction must be proportional to some matrix element which corresponds to the overlap between the wave functions – this is true for all the states within the isospin diagram. For the wave functions the only thing that changes is the isospin term, Ω_{T,T_3} and the overlap must be the largest when T_3 is the same for two states. This explains the sharp peak for $T_i = 7/2 \to T_f = 7/2$.

2. Q — **Problem 30**. The temperature dependence of the average reaction rate in equation (13) in note 4 is normally written as a power law T^ν. Determine an expression for the power ν by first

showing that it can be found as the derivate of the logarithm of the reaction rate with respect to $\ln(T)$. Then find the value of ν for the two reactions $p+p$ and $p+^{14}$N in the interior of the sun (temperature $1.5 \cdot 10^7$ K).

A — Equation (13) in the notes is given by

$$\langle \sigma v \rangle = \sqrt{\frac{2}{\mu (k_B T)^3}} \Delta S(E_0) \exp\left(-\frac{3E_0}{k_B T}\right), \quad \Delta = \frac{4}{\sqrt{3}} \sqrt{E_0 K_B T}, \quad E_0 \propto T^{2/3} \tag{1}$$

To find the temperature dependency I use the natural logarithm of the average reaction rate

$$\ln \langle \sigma v \rangle \propto \nu \ln(T), \tag{2}$$

where ν is the exponent of the temperature. Taking the the derivate of the logarithm of the reaction rate with respect to $\ln(T)$ and using the chain rule yields

$$\begin{aligned} \frac{\partial \langle \sigma v \rangle}{\partial \ln(T)} &= T \frac{\partial \langle \sigma v \rangle}{\partial T} \\ &= \frac{E_0}{k_B T} - \frac{2}{3} \\ &= \nu. \end{aligned}$$

Plugging in the appropriate numbers will give the temperature dependence. For $p+p$ at $T = 1.5 \cdot 10^7$ K

$$\nu_{p+p} = 3.9 \tag{3}$$

For CNO

$$\nu_{p+^{14}\mathrm{N}} = 20. \tag{4}$$

This shows the high dependency on the temperature and highlights the strong dependency for the CNO process.

3. Q — **Problem 16**. Shell-model predictions are expected to be most reliable very close to double-magic nuclei. Check this for the spin-parity values for the nuclei below close to ^{132}Sn and 208Pb. Does it matter whether you use figure 8.4 or 8.5?

A — Depends on if you have to add or subtract one proton / neutron from the double magic nuclei $^{132}_{50}\mathrm{Sn}_{82}$ or $^{208}_{82}\mathrm{Pb}_{126}$.

	$^{131}_{49}\mathrm{Th}_{78}$	$^{131}_{50}\mathrm{Sn}_{81}$	$^{133}_{50}\mathrm{Sn}_{83}$	$^{133}_{51}\mathrm{Sb}_{82}$	$^{207}_{81}\mathrm{Tl}_{126}$	$^{207}_{82}\mathrm{Pb}_{125}$	$^{209}_{82}\mathrm{Pb}_{127}$	$^{209}_{83}\mathrm{Bi}_{126}$
J^π	$9/2^+$	$3/2^+$	$7/2^-$	$7/2^+$	$1/2^+$	$1/2^-$	$9/2^+$	$9/2^-$
Fig. 8.4	$9/2$	$1/2$	$9/2$	$7/2$	$1/2$	$1/2$	$9/2$	$9/2$
Fig. 8.5	$9/2$	$1/2$	$7/2$	$7/2$	$1/2$	$1/2$	$9/2$	$9/2$

Figure 2:

The two figures yield different results with figure 8.4 having 6 correct and figure 8.5 having 7 correct. The parity also matches but I forgot to add these.