

Treatment of open quantum systems

Quantum Engineering II

3rd of June 2021

System-Reservoir Interaction

Atom, $|g\rangle, |e\rangle$

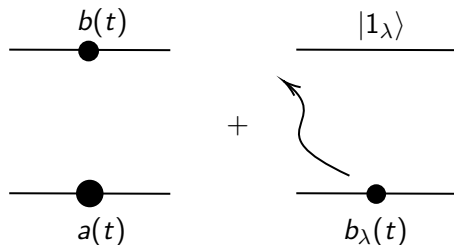
Quantized field, $|\{n_\lambda\}\rangle$

Initial state, $t = 0$, no photons

$$|\Psi_0\rangle = a_0|g\rangle + b_0|e\rangle$$

Evolves

$$|\Psi(t)\rangle = a_0|g\rangle \otimes |0\rangle + b(t)e^{-iE_A t/\hbar}|e\rangle \otimes |0\rangle + \sum_{\lambda} b_{\lambda}(t)e^{-iE_{\lambda} t/\hbar}|g\rangle \otimes |1_{\lambda}\rangle$$



Time A Evolution of Density Matrix

Two Level system coupled to a large reservoir.

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H, \rho] + \mathcal{L}_{\text{relax}}[\rho]$$

where

$$\mathcal{L}_{\text{relax}}[\rho] = -\frac{\Gamma}{2}(|e\rangle\langle e|\rho + \rho|e\rangle\langle e|) + \Gamma|g\rangle\langle e|\rho|e\rangle\langle g| \quad (1)$$

in the usual Lindblad form that ensures that $\text{Tr}(\rho(t)) = 1$ and positive expectation $\langle\psi|\rho|\psi\rangle > 0$.

Time A Evolution of Density Matrix

Notice we get this form ensures that we get the usual terms of the optical Bloch equation

$$\dot{\rho}_{ee} = \langle e | \frac{d}{dt} \rho | e \rangle = -\Gamma \rho_{ee}$$

$$\dot{\rho}_{gg} = \langle g | \frac{d}{dt} \rho | g \rangle = \Gamma \rho_{ee}$$

Bloch Sphere with Damping

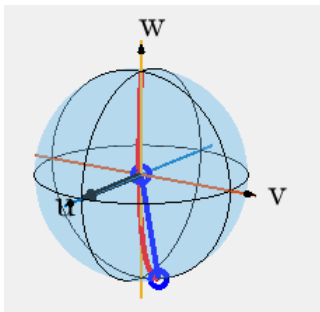


Figure: $\Omega \ll \Gamma \neq 0$, ($s \ll 1$)

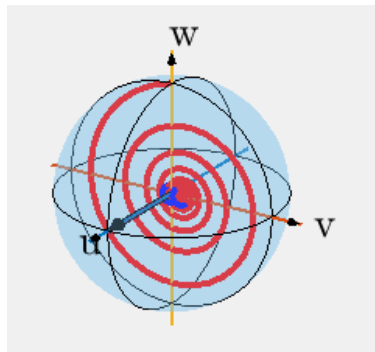


Figure: $\Omega \gg \Gamma \neq 0$, ($s \gg 1$)

Stochastic Wave functions

Two Level System

$$|\psi\rangle = \alpha(t)|g\rangle + \beta(t)|e\rangle$$

Quantum Jumps:

$$\begin{aligned}\Delta P &= \Gamma \langle \psi | \sigma_-^\dagger \sigma_- | \psi \rangle \Delta t, \quad \sigma_- = |g\rangle\langle e| \\ &= \Gamma |\beta|^2 \Delta t\end{aligned}$$

Obtain random number $r \in (0, 1)$.

$r < \Delta P$: Jump

$r > \Delta P$: No Jump

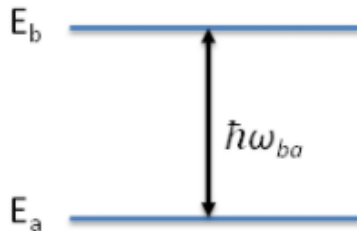


Figure: Single Two Level System

Stochastic Wave functions

Evolution

$$\text{Jump: } |\Psi\rangle \rightarrow \frac{\sigma_- |\Psi\rangle}{\sqrt{\langle \Psi | \sigma_-^\dagger \sigma_- | \Psi \rangle}} = |g\rangle$$

$$\text{No Jump: } |\Psi\rangle \rightarrow \frac{e^{-i\Delta t \hat{H}_{\text{eff}}}}{\sqrt{N}} |\Psi\rangle$$

$$\hat{H}_{\text{eff}} = \hat{H} - i\hbar(\gamma/2)\sigma_-^\dagger \sigma_- \quad (\text{non hermitian})$$

Repeat and Average over trajectories

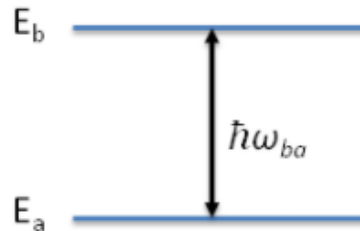


Figure: Single Two Level System

Stochastic Wavefunction

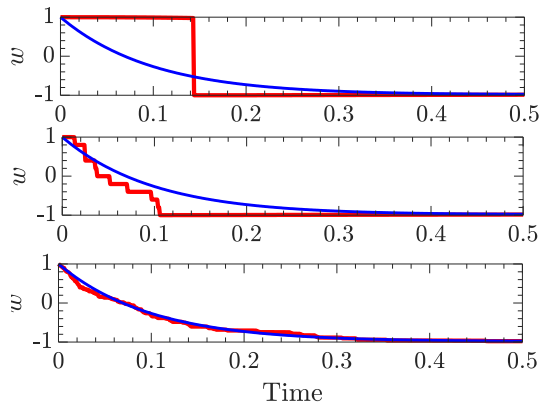


Figure: Average over $n = 1$, $n = 10$ and $n = 100$ and density matrix for $\Omega \ll \Gamma$

Stochastic Wavefunction

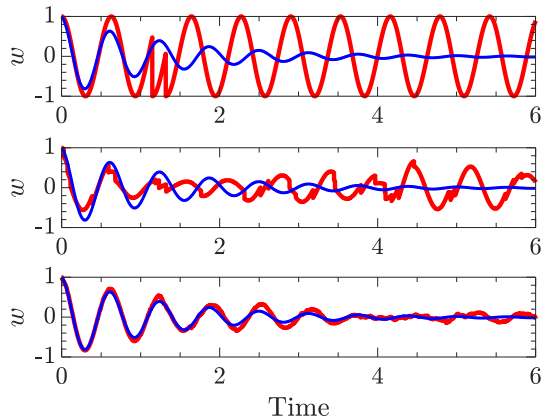


Figure: Average over $n = 1$, $n = 10$ and $n = 100$ and density matrix for $\Omega \gg \Gamma$