Discuss the theory of quantum measurements and its consequences for distinguishability of quantum states, quantum cryptography and sensing of physical parameters

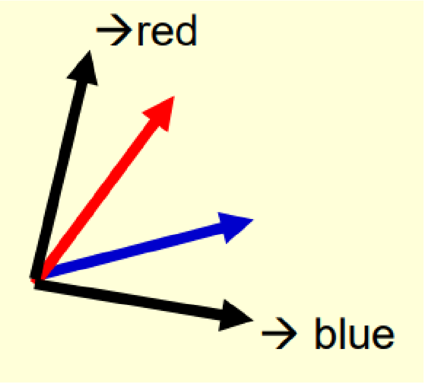
Quantum Engineering II

3rd of June 2021

Measurement

Problems

- Born's rule random outcomes
- Measurement back action



Standard Projective Measurement

Hermitian operator $\hat{H} = \sum_m \lambda_m |m\rangle\langle m| = \sum_j E_m \hat{P}_m$

$$|\psi\rangle = c_g|g\rangle + c_e|e\rangle,$$

A measurement of H yields one of the eigenvalues λ_m with prop. $|c_g|^2$

$$|\phi_g\rangle = \frac{\hat{P}_g|\psi\rangle}{\sqrt{\langle\psi|\hat{P}_g|\psi\rangle}} = \frac{c_g}{|c_g|}|g\rangle$$

Notice

$$\hat{P}_j\hat{P}_i = \hat{P}_i\delta_{i,j} \quad \langle\psi|\hat{P}_m|\psi\rangle > 0, \qquad \sum_m \hat{P}_m = 1.$$

Positive Operator Valued Measurement

A POVM is a set of operators $\{E_m\}$ such that

$$\langle\psi|E_m|\psi\rangle > 0, \qquad \sum_m E_m = 1.$$

Example

Alice and Bob

$$E_1 = \frac{\sqrt{2}}{1+\sqrt{2}} |1\rangle\langle 1|$$

$$E_2 = \frac{\sqrt{2}}{1+\sqrt{2}} |-\rangle\langle -|$$

$$E_3 = I - E_1 - E_2$$

Suppose we receive one of two states $|0\rangle$ or $|+\rangle$.

$$\hat{E}_1 \rightarrow |+\rangle$$

$$\hat{E}_2 \rightarrow |0\rangle$$

$$\hat{E}_3 \rightarrow \text{nothing to infer}$$

Quantum Cryptography

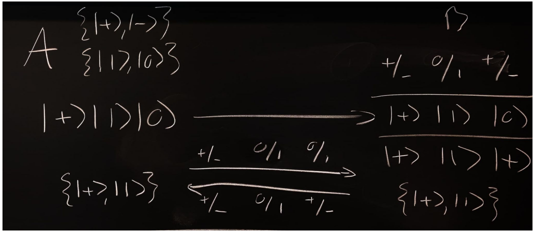
Quantum Key Distribution
– BB84 protocol.

$$|0\rangle$$

$$|1\rangle$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$



Bayes Counting

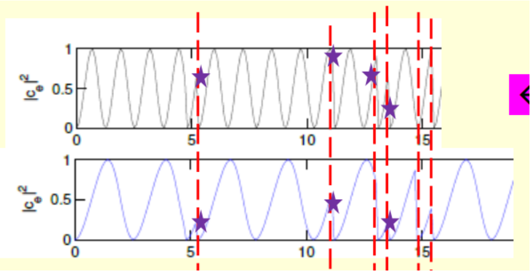


Figure: $P(\Theta_1|O) = P(O|\Theta_1)P(\Theta_1)$

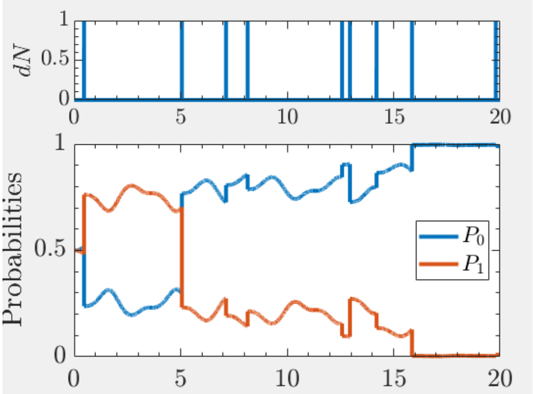


Figure: $\Omega_0 = 2$, $\Omega_1 = 4$