Basic idea of quantum computing and the operation of a specific quantum computing algorithm

Quantum Engineering II

4th of June 2021

Quantum Parallelism

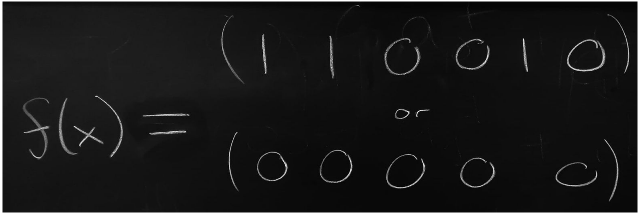
Quantum Parallelism: Parallel computation on superposition states

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}} = \hat{H}|0\rangle. \tag{1}$$

Interference can make calculation that depends on all results.

Deutsch-Jozsa algorithm

Determine is the function of 2^n entries even or odd?



Classical: Worst case $\frac{2^n}{2} + 1$ evaluations

Deutsch-Jozsa algorithm

Begin with a query and anwer registers

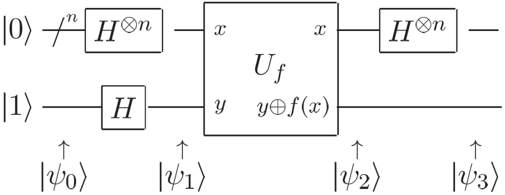
$$|\psi_0\rangle = |0\rangle^{\otimes n}|1\rangle$$

Put into superposition with Hadamard

$$|\psi_1\rangle = \sum_{x\in\{0,1\}^n} \frac{|x\rangle_n}{\sqrt{2^n}}|-\rangle$$

Calculate function $|-\rangle \rightarrow (-1)^{f(x)}|-\rangle$

$$|\psi_2\rangle = \sum_{x\in\{0,1\}^n} (-1)^{f(x)}\frac{|x\rangle_n}{\sqrt{2^n}}|-\rangle$$



Deutsch-Jozsa algorithm

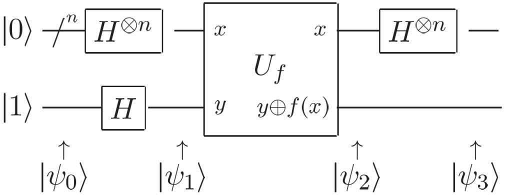
Use Hardamard on all again

$H^{\otimes n}|x\rangle_n = \sum_z (-1)^{\mathbf{x}\cdot\mathbf{z}}|z\rangle_n/\sqrt{2^n}$

$$|\psi_3\rangle = \sum_z \left(\sum_x \frac{(-1)^{\mathbf{x}\cdot\mathbf{z}+f(x)}}{2^n}\right)|z\rangle_n|-\rangle$$

$$c_z = \sum_x \frac{(-1)^{\mathbf{x}\cdot\mathbf{z}+f(x)}}{2^n}$$

$$c_0 = \sum_x \frac{(-1)^{f(x)}}{2^n} = \begin{cases} \pm 1 & \text{(constant)} \\ 0 & \text{(balanced)} \end{cases}$$



If projection onto $|0\rangle_n$ then (constant) if not then (balanced)

Pros and Cons

Pros

Only one iteration is needed compared $\frac{2^n}{2} + 1$

Cons

Classically you can be lucky after 2 iterations