

Discuss the theory of quantum measurements and its consequences for distinguishability of quantum states, quantum cryptography and sensing of physical parameters

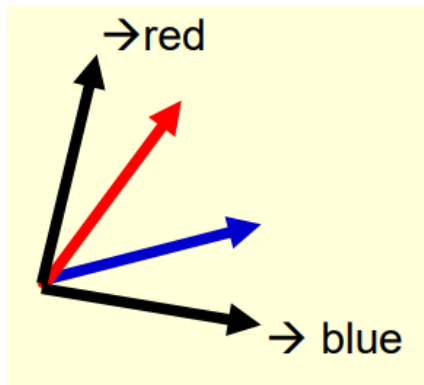
Quantum Engineering II

4th of June 2021

# Measurement

## Problems

- Born's rule random outcomes
- Measurement back action



# Standard Projective Measurement

Hermitian operator  $\hat{H} = \sum_m \lambda_m |m\rangle\langle m| = \sum_j \lambda_j \hat{P}_j$

$$|\psi\rangle = c_g |g\rangle + c_e |e\rangle,$$

A measurement of  $H$  yields one of the eigenvalues  $\lambda_m$  with prop.  $|c_g|^2$

$$|\phi_g\rangle = \frac{\hat{P}_g |\psi\rangle}{\sqrt{\langle \psi | \hat{P}_g | \psi \rangle}} = \frac{c_g}{|c_g|} |g\rangle$$

Notice

$$\hat{P}_j \hat{P}_i = \hat{P}_i \delta_{i,j} \quad \langle \psi | \hat{P}_m | \psi \rangle > 0, \quad \sum_m \hat{P}_m = 1.$$

# Positive Operator Valued Measurement

A POVM is a set of operators  $\{E_m\}$  such that

$$\langle \psi | E_m | \psi \rangle \geq 0, \quad \sum_m E_m = 1.$$

# Example

Alice and Bob

$$E_1 = \frac{\sqrt{2}}{1 + \sqrt{2}} |1\rangle\langle 1|$$

$$E_2 = \frac{\sqrt{2}}{1 + \sqrt{2}} |-\rangle\langle -|$$

$$E_3 = I - E_1 - E_2$$

Suppose we receive one of two states  $|0\rangle$  or  $|+\rangle$ .

$$\hat{E}_1 \rightarrow |+\rangle$$

$$\hat{E}_2 \rightarrow |0\rangle$$

$$\hat{E}_3 \rightarrow \text{nothing to infer}$$

# Quantum Cryptography

Quantum Key Distribution  
– BB84 protocol.

$$|0\rangle$$

$$|1\rangle$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Handwritten diagram illustrating the BB84 protocol:

- On the left, Alice's possible states are listed:  $\{ |+\rangle, |-\rangle \}$  and  $\{ |1\rangle, |0\rangle \}$ . A large 'A' is written next to them.
- Below these, the states  $|+\rangle|1\rangle|0\rangle$  and  $\{ |+\rangle, |1\rangle \}$  are written.
- Arrows indicate transformations:
  - A horizontal arrow from  $|+\rangle|1\rangle|0\rangle$  to  $|+\rangle|1\rangle|0\rangle$  with labels  $+/-$  above and  $0/1$  below.
  - A horizontal arrow from  $\{ |+\rangle, |1\rangle \}$  to  $\{ |+\rangle, |1\rangle \}$  with labels  $+/-$  above and  $0/1$  below.
  - A vertical arrow from  $\{ |+\rangle, |1\rangle \}$  to  $\{ |+\rangle, |1\rangle \}$  with labels  $+/-$  on the left and  $0/1$  on the right.
- On the right, the resulting states are listed:  $|+\rangle$ ,  $|+\rangle|1\rangle|0\rangle$ ,  $|+\rangle|1\rangle|+\rangle$ , and  $\{ |+\rangle, |1\rangle \}$ .

# Bayes Counting

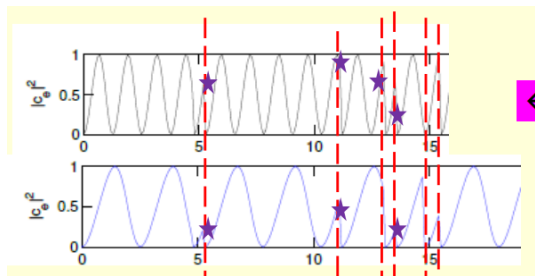


Figure:  $P(\theta_1|O) = P(O|\theta_1)P(\theta_1)$

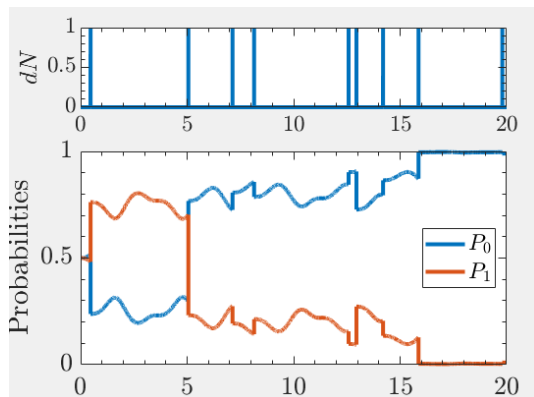


Figure:  $\Omega_0 = 2, \Omega_1 = 4$