Treatment of open quantum systems

Quantum Engineering II

4th of June 2021

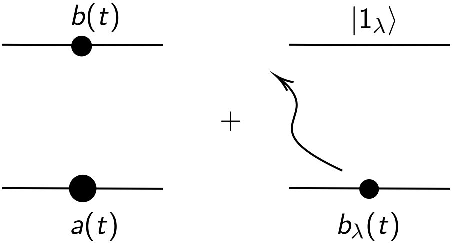
System-Reservoir Interaction

Atom, $|g\rangle$, $|e\rangle$

Quantized field, $|\{n_\lambda\}\rangle$

Inital state, $t = 0$, no photons

$$|\Psi_0\rangle = a_0|g\rangle + b_0|e\rangle$$



Evolves

$$|\Psi(t)\rangle = a_0|g\rangle \otimes |0\rangle + b(t)e^{-iE_A t/\hbar}|e\rangle \otimes |0\rangle + \sum_\lambda b_\lambda(t)e^{-iE_\lambda t/\hbar}|g\rangle \otimes |1_\lambda\rangle$$

Time A Evolution of Density Matrix

Two Level system coupled to a large reservoir.

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{1}{i\hbar}[H,\rho] + \mathcal{L}_{\text{relax}}[\rho]$$

where

$$\mathcal{L}_{\text{relax}}[\rho] = -\frac{\Gamma}{2}(|e\rangle\langle e|\rho + \rho|e\rangle\langle e|) + \Gamma|g\rangle\langle e|\rho|e\rangle\langle g| \tag{1}$$

in the usual Lindblad form that ensures that $\mathrm{Tr}(\rho(t)) = 1$ and postive expectation $\langle\psi|\rho|\psi\rangle > 0$.

Time A Evolution of Density Matrix

Notice we get this form ensures that we get the usual terms of the optical Bloch equation

$$\dot{\rho}_{\mathrm{ee}} = \langle e|\frac{d}{dt}\rho|e\rangle = -\Gamma\rho_{\mathrm{ee}}$$
$$\dot{\rho}_{\mathrm{gg}} = \langle g|\frac{d}{dt}\rho|g\rangle = \Gamma\rho_{\mathrm{ee}}$$

Bloch Sphere with Damping

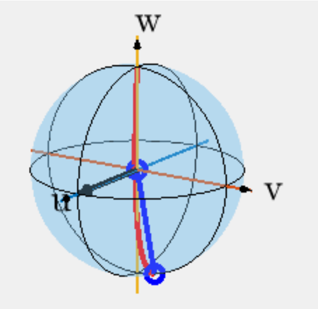


Figure: $\Omega \ll \Gamma \neq 0$, $(s \ll 1)$

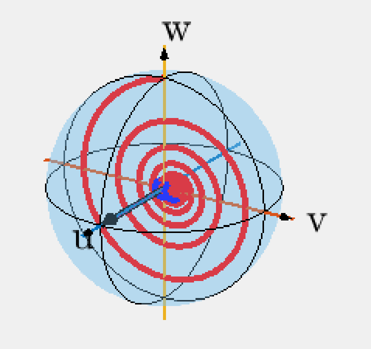


Figure: $\Omega \gg \Gamma \neq 0$, $(s \gg 1)$

Stochastic Wave functions

Two Level System

$$|\Psi\rangle = \alpha(t)|g\rangle + \beta(t)|e\rangle$$

Quantum Jumps:

$$\Delta P = \Gamma\langle\Psi|\sigma_-^\dagger\sigma_-|\Psi\rangle\Delta t, \quad \sigma_- = |g\rangle\langle e|$$
$$= \Gamma|\beta|^2\Delta t$$

Obtain random number $r \in (0,1)$.

$$r < \Delta P : \quad \text{Jump}$$
$$r > \Delta P : \quad \text{No Jump}$$

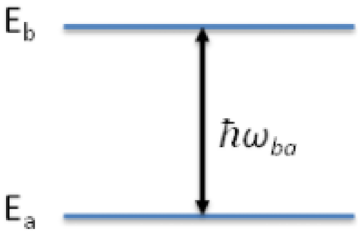


Figure: Single Two Level System

Stochastic Wave functions

Evolution

$$\text{Jump:} |\Psi\rangle \rightarrow \frac{\sigma_-|\Psi\rangle}{\sqrt{\langle\Psi|\sigma_-^\dagger\sigma_-|\Psi\rangle}} = |g\rangle$$

$$\text{No Jump:} |\Psi\rangle \rightarrow \frac{e^{-i\Delta t\hat{H}_{\text{eff}}}}{\sqrt{N}}|\Psi\rangle$$

$$\hat{H}_{\text{eff}} = \hat{H} - i\hbar(\gamma/2)\sigma_-^\dagger\sigma_- \quad \text{(non hermitian)}$$

Repeat and Average over trajectories

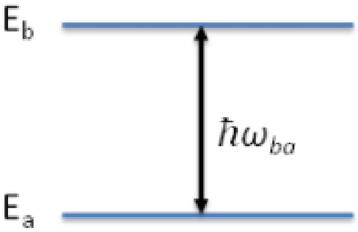


Figure: Single Two Level System

Stochastic Wavefunction

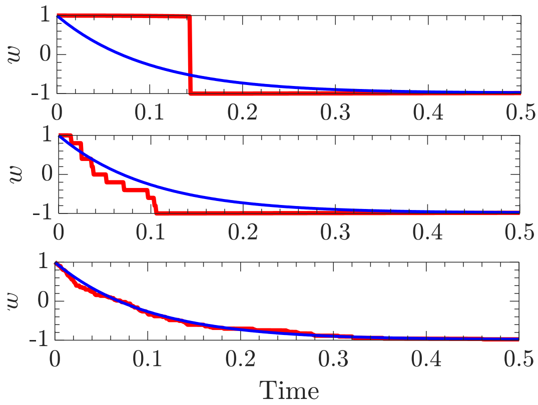


Figure: Average over $n = 1$, $n = 10$ and $n = 100$ and denisty matrix for $\Omega \ll \Gamma$

Stochastic Wavefunction

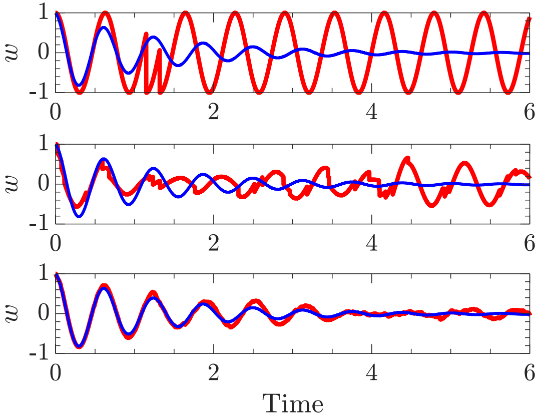


Figure: Average over $n = 1$, $n = 10$ and $n = 100$ and denisty matrix for $\Omega \gg \Gamma$