Quantum Engineering II

Squeezed States

3rd of June 2021

Quadrature Operators

The Quadrature Operators are defined by Coherent states,

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \tag{1}$$

Quadrature Operators

$$\hat{X}_1 = \frac{\hat{a} + \hat{a}^\dagger}{2} \tag{2}$$

$$\hat{X}_2 = \frac{\hat{a} - \hat{a}^\dagger}{2i} \tag{3}$$

Quadrature Squeezing

For a coherent state $|\alpha\rangle$ and the vacuum state $|0\rangle$.

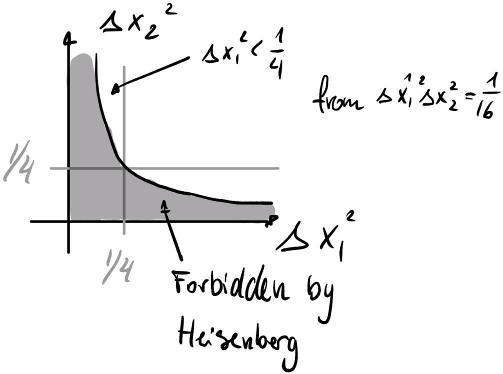
$$\langle(\Delta\hat{X}_1)^2\rangle = \langle(\Delta\hat{X}_2)^2\rangle = \frac{1}{4} \qquad (4)$$

Quadrature Squeezing exist whenever

$$\langle(\Delta\hat{X}_1)^2\rangle < \frac{1}{4} \quad \text{or} \quad \langle(\Delta\hat{X}_2)^2\rangle < \frac{1}{4} \qquad (5)$$

where Heisenberg is satisfied

$$\langle(\Delta\hat{X}_1)^2\rangle\langle(\Delta\hat{X}_2)^2\rangle \geq \frac{1}{16} \qquad (6)$$



Generation of squeezed states (mathematically)

Introduce "squeeze"operator

$$\hat{S}(\xi) = \exp\left[\frac{1}{2}\xi^* a^2 - \xi a^{\dagger 2}\right], \quad \xi = r \exp i\theta, \quad 0 \leq \theta \leq 2\pi \tag{7}$$

Two-photon generalization of the displacement operator, $\hat{D}(\alpha)$

Generation of squeezed states (mathematically)

On the vacuum state: $|\xi\rangle = \hat{S}(\xi)|0\rangle$ using Baker-Hausdorf

$$\langle\xi|\hat{X}_1|\xi\rangle = \langle\xi|\hat{X}_2|\xi\rangle = 0$$

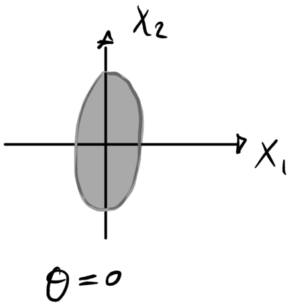
Furthermore for $\theta = 0$.

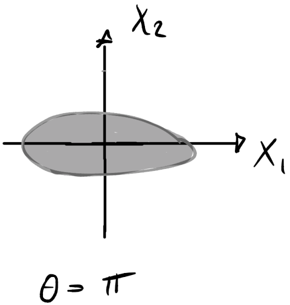
$$\left\langle(\Delta\hat{X}_1)^2\right\rangle = \frac{1}{4}e^{-2r}$$
$$\left\langle(\Delta\hat{X}_2)^2\right\rangle = \frac{1}{4}e^{2r}$$

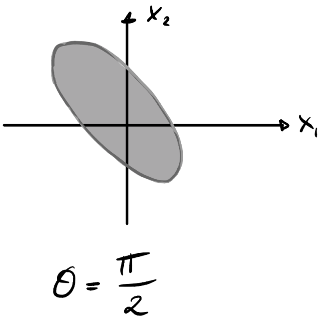
Notice

$$\left\langle(\Delta\hat{X}_1)^2\right\rangle\left\langle(\Delta\hat{X}_2)^2\right\rangle = \frac{1}{16} \tag{8}$$

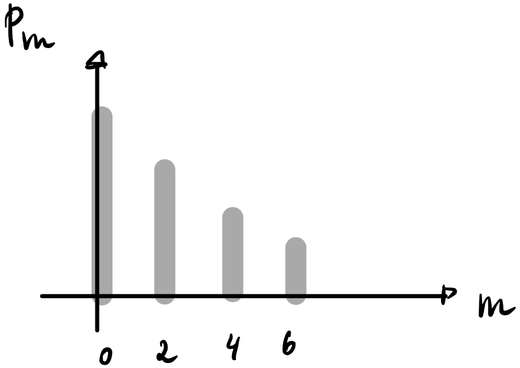
Generation of squeezed states (mathematically)







Photon probability Distribution for a Squeezed Vacuum



Displaced Squeezed State

Displacement operator

$$|\alpha\rangle = \hat{D}(\alpha)|0\rangle, \quad \text{where} \quad \hat{D}(\alpha) = \exp(\alpha \hat{a}^\dagger - \alpha^* \hat{a}) \tag{9}$$

Displace the squeezed state

$$|\alpha, \epsilon\rangle = \hat{D}(\alpha)\hat{S}(\xi)|0\rangle \tag{10}$$

The average number of photons

$$\bar{n} = \langle \alpha, \xi|\hat{n}|\alpha, \xi\rangle = |\alpha|^2 + \sinh^2(r) \tag{11}$$

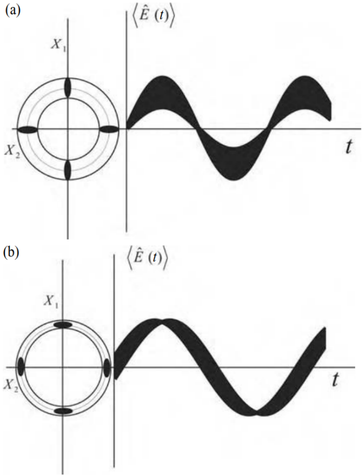
Amplitude and Phase Squeezing

(a) Squeezing initially in $\hat{X}_1$ (Phase Squeezed)

(b) Squeezing initially in $\hat{X}_2$ (Amplitude Squeezed)

where we have simple projected onto $\hat{X}_1$

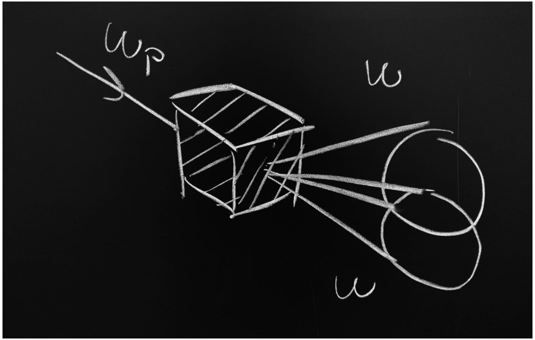
$$\hat{E}_x = \epsilon_0(\hat{a} + \hat{a}^\dagger)\sin(kz) \qquad (12)$$



Generation of Squeezed states

Degenerate parametric down-converter - nonlinear medium $\omega = \omega_P/2$

$$\hat{H} = \hbar\omega\hat{a}^{\dagger}\hat{a} + \hbar\omega_P\hat{b}^{\dagger}\hat{b} + i\hbar\chi^{(2)}(\hat{a}^2\hat{b}^{\dagger} - \hat{a}^{\dagger 2}\hat{b}).$$



Generation of Squeezed States (continued)

Parametric approximation $\hat{b} \to \beta e^{-i\omega_P t}$, $\hat{b}^\dagger \to \beta^* e^{i\omega_P t}$,

$$\hat{H} = \hbar\omega \hat{a}^\dagger \hat{a} + i\hbar(\eta^* e^{i\omega_P t} \hat{a}^2 - \eta e^{-i\omega_P t} \hat{a}^{\dagger 2}). \tag{13}$$

Transforming into the interaction picture

$$\hat{H}_I(t) = i\hbar(\eta^* e^{i(\omega_P - 2\omega)t} \hat{a}^2 - \eta e^{-i(\omega_P - 2\omega)t} \hat{a}^{\dagger 2}). \tag{14}$$

Now choosing $\omega_P = 2\omega$,

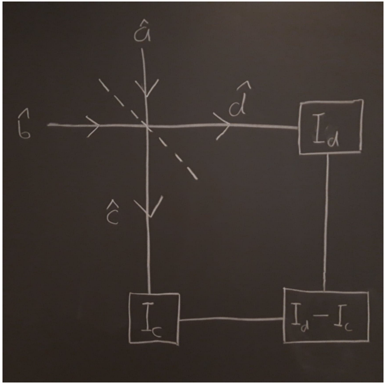
$$\hat{H}_I(t) = i\hbar(\eta^* \hat{a}^2 - \eta \hat{a}^{\dagger 2}). \tag{15}$$

Time evolution operator is now the squeeze operator, $\hat{U}_I(t, 0) = \hat{S}(\xi)$,

Measurement of Squeezed States

Mix squeezed state with strong coherent state.
Balanced homodyne detection.

$$\hat{c} = \frac{1}{\sqrt{2}}(\hat{a} + i\hat{b})$$
$$\hat{d} = \frac{1}{\sqrt{2}}(\hat{b} + i\hat{a})$$



Intensities

$$I_c - I_d = \langle \hat{c}^\dagger \hat{c} - \hat{d}^\dagger \hat{d} \rangle = i\langle \hat{a}^\dagger \hat{b} - \hat{a}\hat{b}^\dagger \rangle \tag{16}$$

Measurement of Squeezed States (continued)

As before, let $\hat{b} \to \beta e^{-i\omega t}$ with $\beta = |\beta| e^{i\psi}$,

$$I_c - I_d = |\beta| \langle \hat{a} e^{i\omega t} e^{-i\theta} + \hat{a}^\dagger e^{-i\omega t} e^{i\theta} \rangle,$$

where $\theta = \psi + \pi/2$. Assume now that the light from $\hat{a}$ also has frequency ω

$$I_c - I_d = 2|\beta| \langle \hat{X}(\theta) \rangle.$$

For strong coherent light

$$(I_c - I_d)^2 = 4|\beta|^2 \langle \Delta \hat{X}(\theta)^2 \rangle.$$