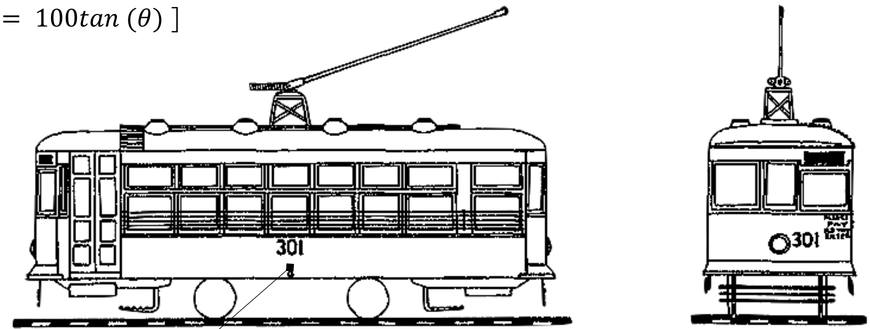
SPEED CONTROL OF A SF-DC MOTOR FOR TRACTION

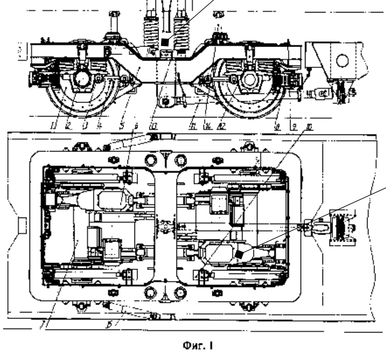
Design specifications:

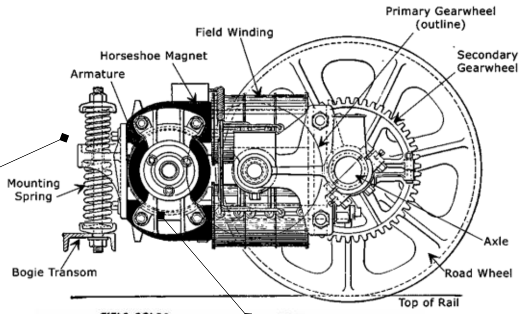
- Find the design parameters of the DC motor according to the data
- Design and simulate speed and current control in order to cover a 10km track considering the given Table characteristics. [The slope is $s\% = 100tan\,(\theta)$]

Data:

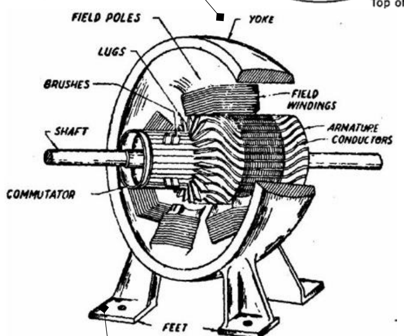


- **Line voltage:** $V_n = 600\ V$
- **Rated speed [rad/s]:** $\omega_n = 314\ \frac{rad}{s}$
- **Rated speed [km/h]:** $v_r = 60\ \frac{km}{h}$
- **Efficiency:** $\eta = 0.9$
- **Armature time constant:** $\tau_a = 10^{-3}\ s$
- **Excitation time constant:** $\tau_e = 1\ s$
- **Excitation rated voltage:** $V_e = 120\ V$
- **Excitation rated current:** $I_e = 1\ A$
- **Mass of std passenger:** $m_p = 80\ kg$
- **Number of passengers:** $n_p = 200$
- **Tramway mass:** $m_t = 10000\ kg$
- **Time of acceleration:** $t_a = 25\ s$

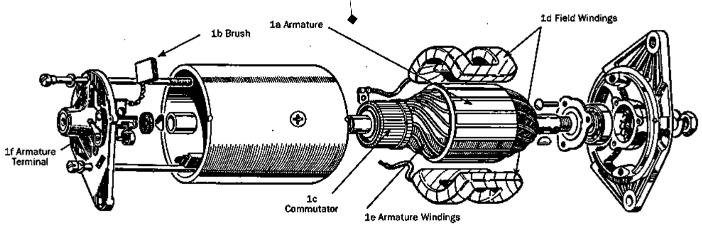


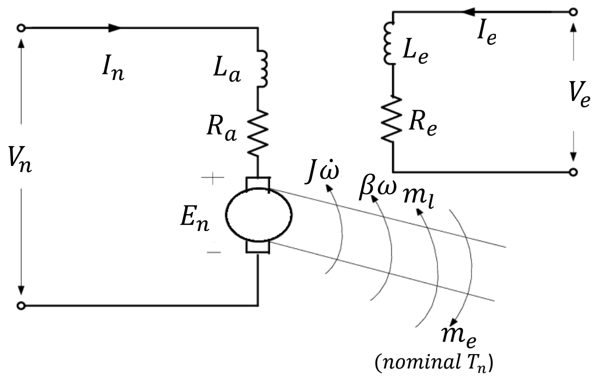


Model parameters identification



- ***Total mass:*** Overall mass of the vehicle loaded with all the passengers
  $M = m_t + m_p \cdot n_p = 26000\ kg$
- ***Rated speed [m/s]:*** Rated speed of the tram in m / s
  $v_{max} = v_r \frac{1000}{3600} \cong 16.667\ \frac{m}{s}$
- ***Average acceleration:*** Acceleration in the given time and space
  $a = \frac{v_{max}}{t_a} \cong 0.667\ \frac{m}{s^2}$
- ***Traction force***: Accelerating force
  $F_{trac} = M \cdot a \cong 17333\ N$
- ***Traction power***: Accelerating power
  $P_{trac} = F_{trac} \cdot v_{max} \cong 288.9\ kW$
- ***Rated power:*** Nominal total power
  $P_{tot} = P_{trac} + P_{fric} = P_{trac} + \frac{P_{trac}}{3} \cong 385.185\ kW$
- ***Electrical power absorbed*** by the DC motor:
  $P_{el} = \frac{P_{tot}}{\eta} \cong 427.983\ kW$
- ***Rated torque:*** Nominal torque
  $T_n = \frac{P_{tot}}{\omega_n} \cong 1227\ Nm$
- ***Rated current:*** Nominal armature current
  $I_n = \frac{P_{el}}{V_n} \cong 713.3\ Nm$
- ***Torque/e.m.f. coefficient :*** Torque constant of the motor
  $K = \frac{T_n}{I_n \cdot I_e} \cong 1.7197$





- ***Armature resistance :*** Taking into account the power joule losses

  $$P_{el} = V_n I_n = P_{tot} + R_a I_n^2 \rightarrow \quad R_a = \frac{P_{el} - P_{tot}}{I_n^2} \cong 0.0841 \ \Omega$$
- ***Armature inductance:***

  $$L_a = R_a \cdot \tau_a \cong 841.15 \ \mu H$$
- ***Rated e.m.f. :*** Considering the power relationships and the armature voltage expression

  $$V_n = R_a I_n + E_n \rightarrow \quad P_{el} = V_n I_n = R_a I_n^2 + E_n I_n = P_{tot} + R_a I_n^2 \rightarrow E_n I_n = P_{tot} = P_{el}\eta \rightarrow$$
  $$E_n = \eta \frac{P_{el}}{I_n} = \eta \cdot V_n = 540 \ V$$
- ***Excitation resistance:***

  $$R_e = \frac{V_e}{I_e} = 120 \ \Omega$$
- ***Excitation inductance:***

  $$L_e = R_e \cdot \tau_e = 120 \ H$$
- ***Equivalent inertia:*** From energy balance

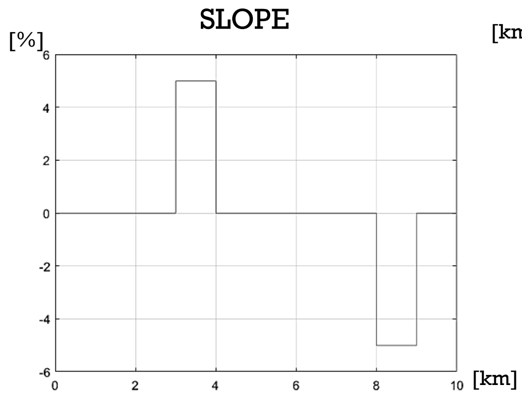
  $$\frac{1}{2} M \cdot v_{max}^2 = \frac{1}{2} J \cdot \omega_n^2 \rightarrow \ J = \frac{M \cdot v_{max}^2}{\omega_n^2} \cong 73.25 \ kgm^2$$
- ***Damping/friction factor:*** Assuming that the friction torque acting on whole the tram is

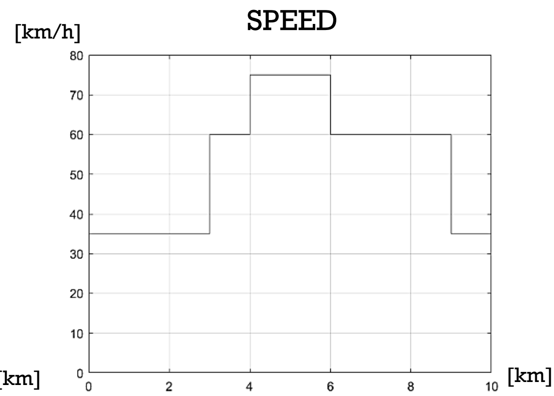
  $$T_{fric} = \frac{P_{fric}}{\omega_n} = \beta \cdot \omega_n \rightarrow \ \beta = \frac{P_{fric}}{\omega_n^2} = \frac{P_{trac}}{3 \cdot \omega_n^2} \cong 0.977 \ Nms$$

Operating regions

Before implementing any type of control, we must specify the desired operating conditions. In particular the track characteristics such as the slope (angle between the rising path and the ground) and the required speed.

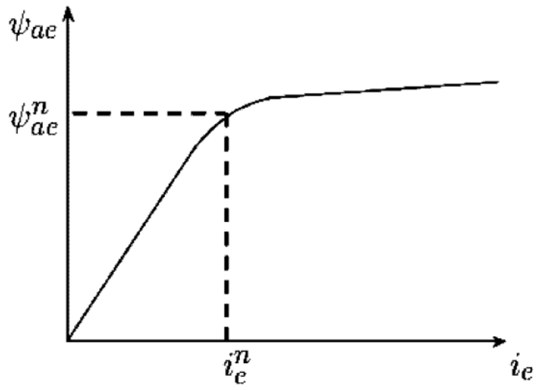
track	slope %	speed
$0-1\,km$	0	$35\ km/h$
$1-3\,km$	0	$60\ km/h$
$3-4\,km$	5%	$60\ km/h$
$4-6\,km$	0	$75\ km/h$
$6-8\,km$	0	$60\ km/h$
$8-9\,km$	−5%	$60\ km/h$
$9-10\,km$	0	$35\ km/h$





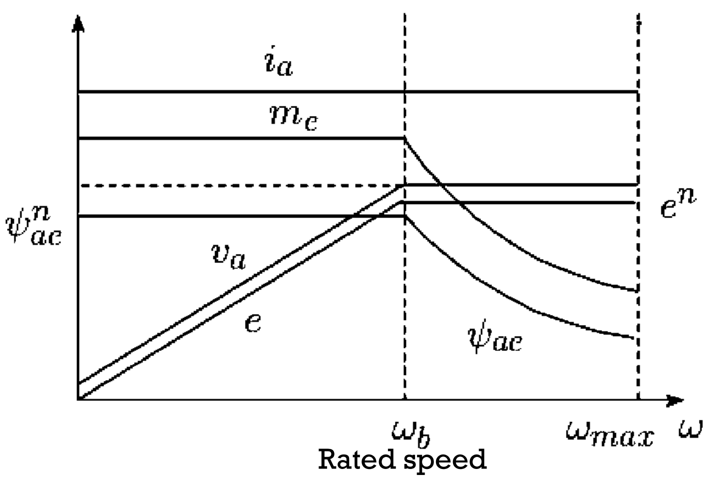
We can see that the tram is required to go above is rated speed 60 km/h (314 rad/s) and reach 75 km/h. To accomplish that, assuming a continuous use of the machine, we have to work in the flux weakening region, meaning that we have to control the linked flux ψ_{ae} . Knowing that $E = K \cdot \psi_{ae}(I_e) \cdot \ \omega$, for a constant E (with a maximum nominal value E_n) for an increasing speed (e.g. our case) we'll have to decrease the flux. The model variables' behavior is here represented:

Magnetizing curve: $\psi_{ae} \propto I_e$



(Proper use of the ferrogmatic material operating at the knee)

Operating regions:



Control scheme

The simplest linear model of a DC motor (permanent magnets case) is implemented in a cascade control architecture, which is based on two nested loops based on a speed and armature current controller. The goal of this scheme is to assure a reference speed tracking by manipulating the voltage (control variable) applied to the motor terminals through the power converter (actuator). In our case we're dealing with a separately excited DC motor, furthermore the previous analysis about the operating regions suggests the fact that to control the flux, in a speed range above the rated one, we must implement a control loop for the excitation as well as the canonic decoupled controllers for armature current and speed.

We shall then consider for our **separately excited DC motor** this model:

$$\boldsymbol{L_a}\frac{\boldsymbol{dI_a(t)}}{\boldsymbol{dt}} = V_a(t) - R_aI_a(t) - E = V_a(t) - R_aI_a(t) - K\psi_{ae}(I_e)\omega(t) = \boldsymbol{V_a(t) - R_aI_a(t) - KI_e(t)\omega(t)}$$

$$\boldsymbol{L_e}\frac{\boldsymbol{dI_e(t)}}{\boldsymbol{dt}} = \boldsymbol{V_e(t) - R_eI_e(t)}$$

$$\boldsymbol{J}\frac{\boldsymbol{d\omega(t)}}{\boldsymbol{dt}} = m_e(t) - m_l(t) - \beta\omega(t) = \boldsymbol{KI_e(t)I_a(t) - m_l(t) - \beta\omega(t)}$$

MIMO system definition:

$$\boldsymbol{x} = \begin{bmatrix}\omega\\ I_a\\ I_e\end{bmatrix} \quad \boldsymbol{u} = \begin{bmatrix}m_l\\ V_a\\ V_e\end{bmatrix} \quad \boldsymbol{y} = \begin{bmatrix}\omega\\ I_a\\ I_e\\ m_e\end{bmatrix}$$

Design characteristic and observations:

- The electrical dynamic and the mechanical dynamic are coupled through the back-emf E. Assuming to have a good speed estimator or a speed sensor we can add a **feedforward action providing a back-emf compensation $\hat{\boldsymbol{E}}$** (at steady-state)
- We have to **wait until the excitation circuit has reach the steady-state** condition before to apply a speed reference command to the motor. This can also be noticed by the fact that the time constant τ_e is larger than τ_a.
- From the **cascade loop theory**:
  The outer loop must be at least one decade slower than the inner loop in order to see a unitary closed loop transfer function for the inner one. This result in the proper choice of bandwidth for the controllers.

Considering the previous observations, we can then define the following dynamics for the 3 control loops:

Armature current control loop:
$$G_{i_a}(s) = \frac{1}{R_a + sL_a} = \frac{1}{0.08412 + 0.0008412\cdot s}$$

Excitation current control loop:
$$G_{i_e}(s) = \frac{1}{R_e + sL_e} = \frac{1}{120 + 120\cdot s}$$

Speed control loop:
$$G_m(s) = \frac{1}{\beta + s\cdot J} = \frac{1}{0.9767 + 73.25\cdot s}$$

Armature current controller:

To fulfill cascade constraints in terms of bandwidth between the nested loops, we can impose a cut-off frequency for the inner current controller at least ten times smaller than the outer speed one, allowing to approximate the inner closed-loop transfer function as a unitary gain.

So having:
- $\tau_{G_{i_a}} = \frac{L_a}{R_a} = 0.01\ s$
- $T_{a_{G_{i_a}}} = 5\cdot\tau_{G_{i_a}} = 0.05\ s$
- $T_{a_d} = \frac{T_{a_{G_{i_a}}}}{2} = 0.025\ s$ (e.g. choosing for the final settling time half of the previous one)

Requirements:

$\Rightarrow$ $\omega_{c_{i_a}} = \frac{5}{T_{a_d}} = 200\frac{rad}{s}$

$\varphi_m = 90°$

$\Rightarrow$ **From pidtool (or imposing pole/zero cancellation):**

$k_{p_{i_a}} = \omega_{c_{i_a}}\cdot L_a \cong 0.1682$

$k_{i_{i_a}} = \omega_{c_{i_a}}\cdot R_a \cong 16.8231$

$\Rightarrow$ **PI Controller:**

$$R_{i_a}(s) = k_{p_{i_a}} + \frac{k_{i_{i_a}}}{s} = \frac{0.1682\cdot s\ +\ 16.82}{s}$$

Speed controller:

Having:
- $\tau_{G_m} = \frac{J}{\beta} = 75\ s$
- $T_{a_{G_m}} = 5 \cdot \tau_{G_m} = 375\ s$

We respect the requirements by imposing for example:

Requirements:

$\Rightarrow \omega_{c_\omega} = \frac{\omega_{c_{i_a}}}{200} = 1\frac{rad}{s}$

$\varphi_m = 90°$

$\Rightarrow$ **From pidtool (or imposing pole/zero cancellation):**

$k_{p_\omega} = \omega_{c_\omega} \cdot J \cong 73.25$

$k_{i_\omega} = \omega_{c_\omega} \cdot \beta \cong 0.9767$

$\Rightarrow$ **PI Controller:**

$R_\omega(s) = k_{p_\omega} + \frac{k_{i_\omega}}{s} = \frac{73.25 \cdot s\ + 0.9767}{s}$

Excitation current controller:

Having:
- $\tau_{G_{i_e}} = \frac{L_e}{R_e} = 1\ s$
- $T_{a_{G_{i_e}}} = 5 \cdot \tau_{G_{i_e}} = 5\ s$

We implement the following characteristics:

Requirements:

$\Rightarrow \omega_{c_{i_e}} = \frac{\omega_{c_{i_a}}}{10} = 20\frac{rad}{s}$

$\varphi_m = 90°$

$\Rightarrow$ **From pidtool (or imposing pole/zero cancellation):**

$k_{p_{i_e}} = \omega_{c_{i_e}} \cdot L_e = 2400$

$k_{i_{i_e}} = \omega_{c_{i_e}} \cdot R_e = 2400$

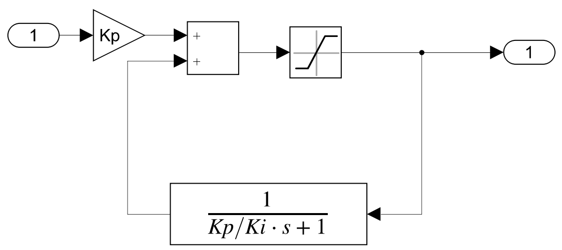
$\Rightarrow$ **PI Controller:**

$R_{i_e}(s) = k_{p_{i_e}} + \frac{k_{i_{I_e}}}{s} = \frac{2400 \cdot s\ + 2400}{s}$

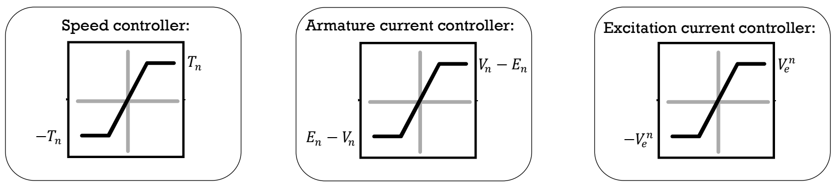
Anti-windup configuration:

Since the control variables actuators are subjected to saturation in terms of performances it's better if we implement for our controller an anti-windup configuration.

General scheme:



Saturation limits:



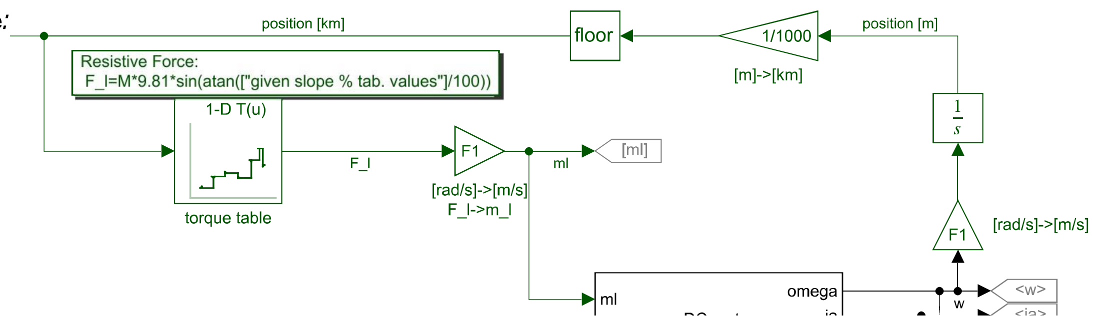
Simulink implementation

Load torque: We consider the torque related to the gravity force of the tramcar dependent on its slope $s\%$ wrt its position (extracted from the speed with an integrator).

$s\% = 100tan\ (\theta) \rightarrow F_l = M * 9.81 * sin(atan(["given\ slope\ \%\ tab.values"]/100))$

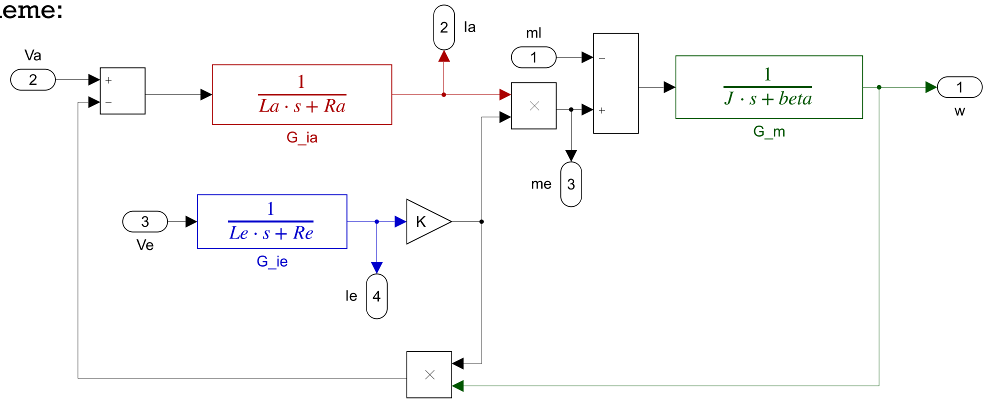
By multiplying for a proper dimensional conversion factor $F_1 = \frac{v_{max}}{\omega_n} \cong 0.0531\ \frac{m}{rad}$ ([rad/s]->[m/s]), we obtain the torque.

Blocks from the total scheme:



SE-DC motor transfer functions: In the Simulink model we can build a subsystem taking into account all the dynamics of the different loops specified before.

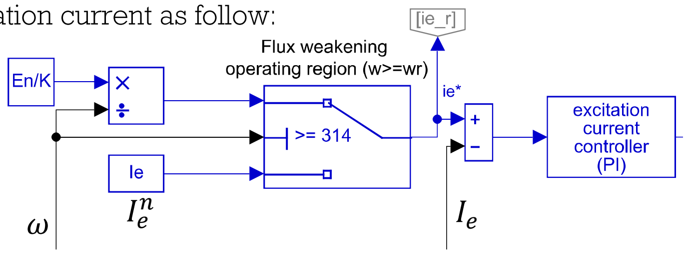
SE-DC motor scheme:



Excitation circuit behavior: To actualize the limits discussed previously in the operating regions paragraph, we can summarize the behavior of the excitation current as follow:

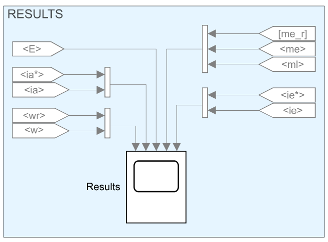
$$\begin{cases} I_e = I_e^n & \omega < \omega_n \text{ (rated speed)} \\ I_e = \dfrac{E_n}{K\omega} & \omega > \omega_n \end{cases}$$

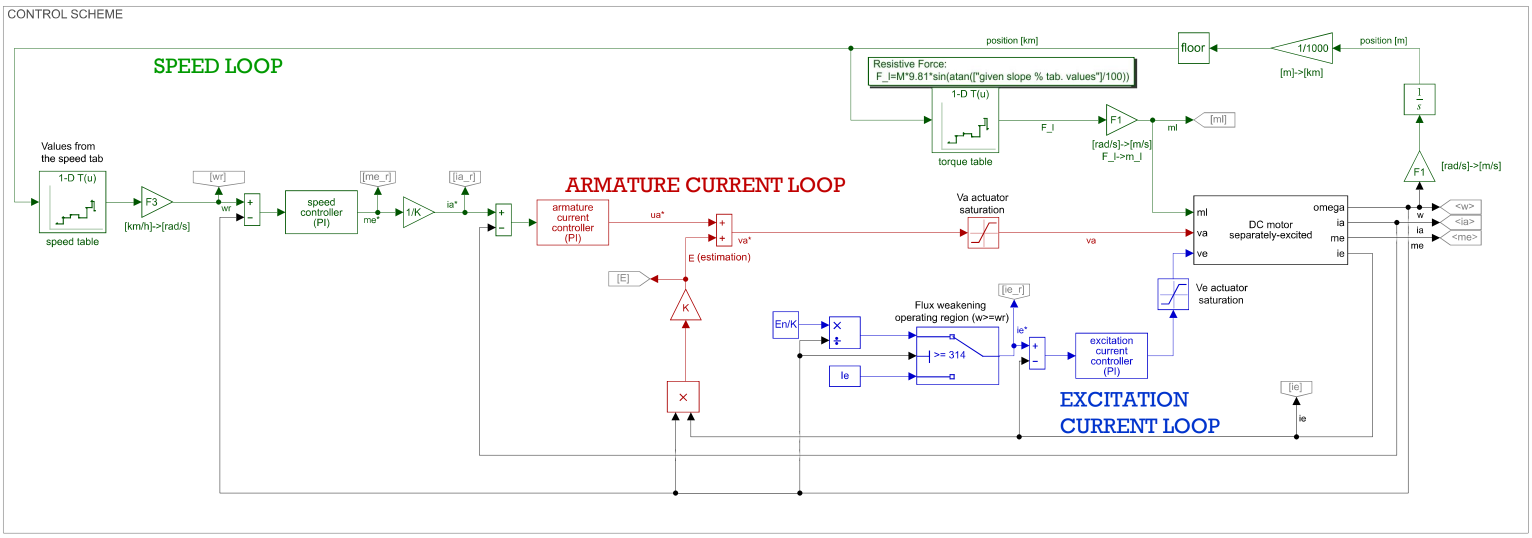
Flux weakening region switch implementation:



All considering the final simulink model is then:

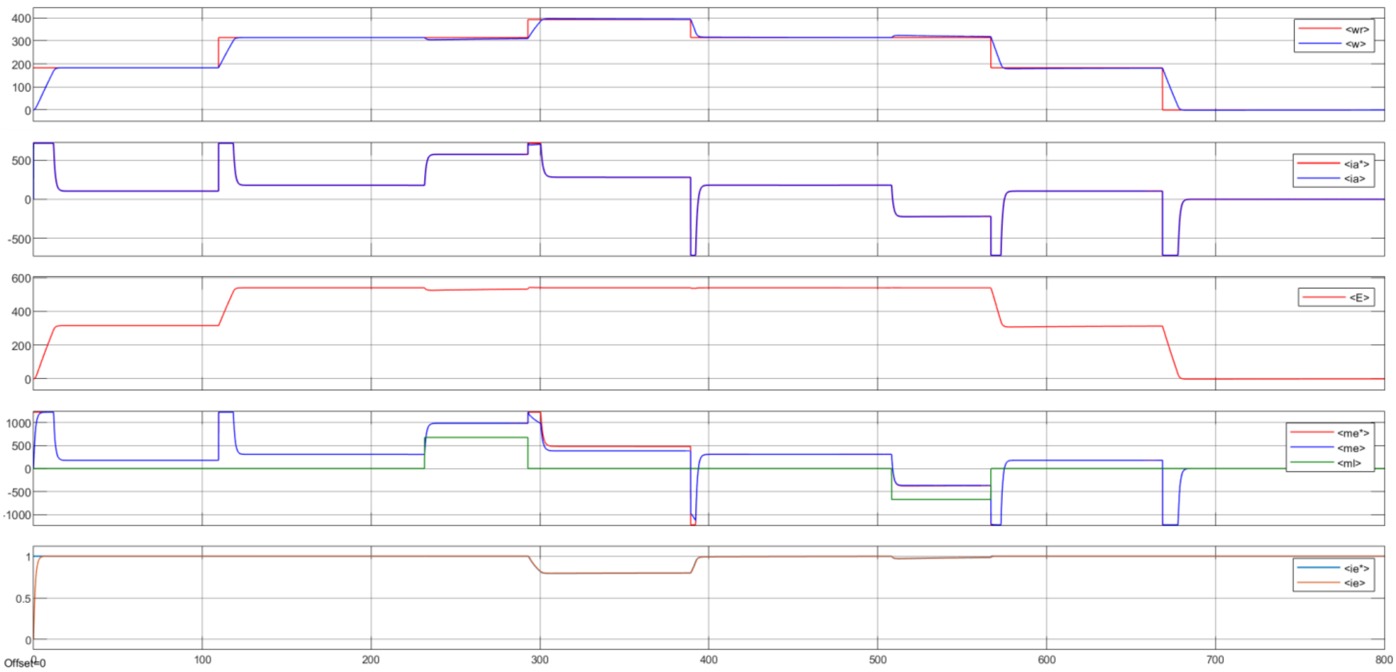
Total scheme



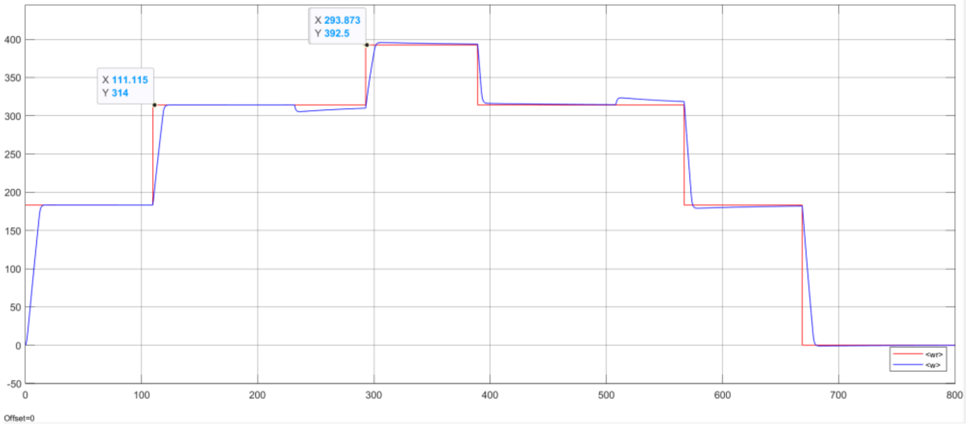


Results:

Running the Simulink model with the proper model parameters computed before, we obtain:



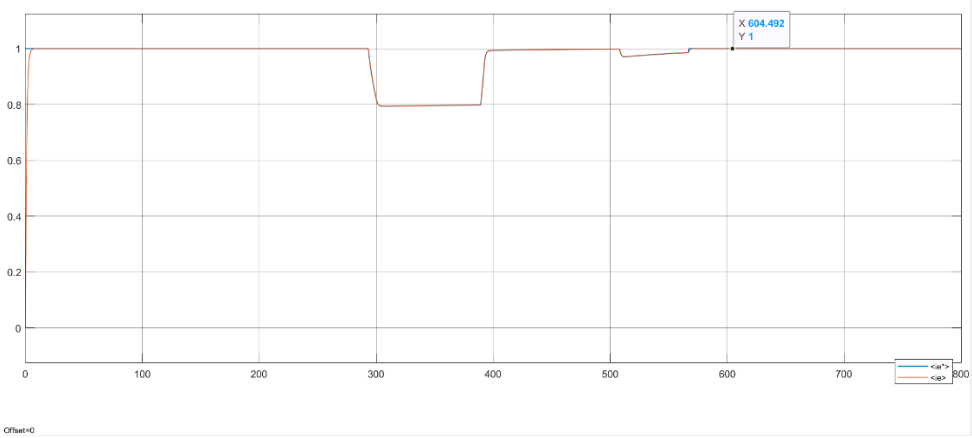
Speed:



Looking at the speed [rad/s] scope we can see that the controller designed allowed us to achieve a good tracking of the reference speed.

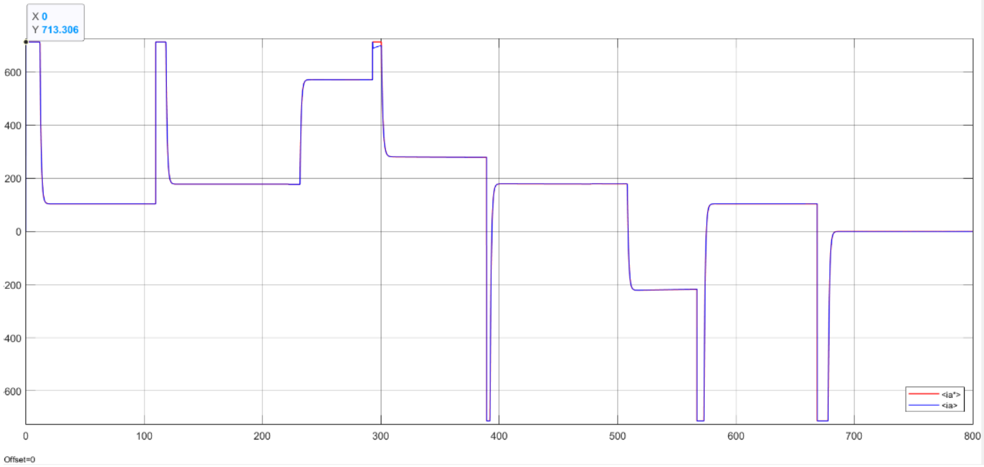
The small overshoots in the constant speed regions are caused by the changing slope of the track, that as seen before plays a role in the expression of the load torque.

Excitation current:



As expected, the excitation current change accordingly to the variation of the speed above the rated one, with a decrease that represents the flux adjustment in the so-called flux weakening region.

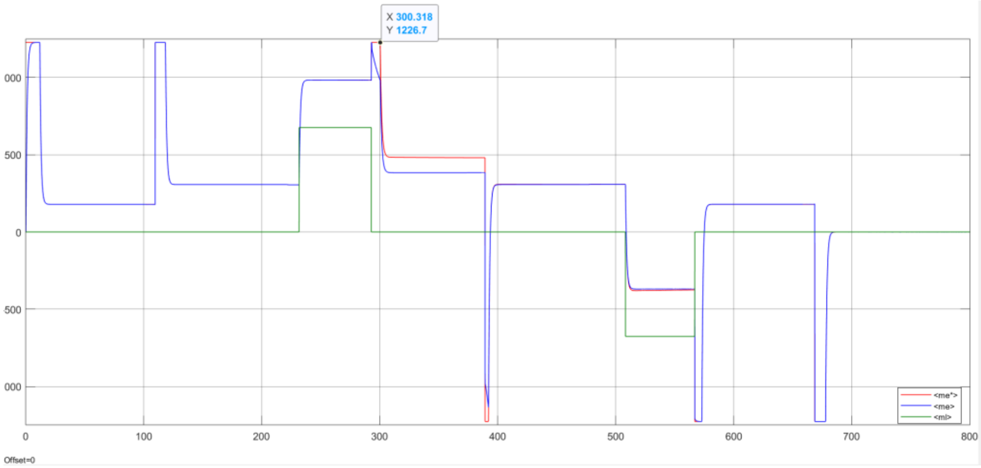
Armature current:



We can see how the peaks reach a maximum value equal to the nominal armature current, meaning that in that case we supply the max. current and have the max. acceleration to achieve more quickly the desired speed.

For example, after the first peak we can see that the current changes accordingly with the torque $m_e(I_a)$ necessary to balance at constant speed (so $\dot{\omega} = 0$) the friction forces. We can then see after the second peak that the speed has increased and so will do the friction force ($\propto \omega$) and then $m_e(I_a)$, hence the current.

Torque:

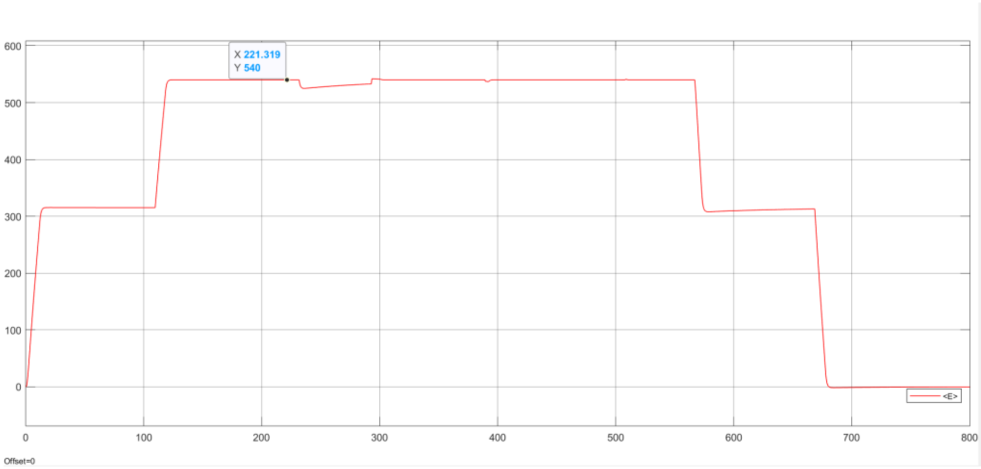


As stated before the behavior of the torque $m_e(I_a)$ is strictly dependent on the armature current.

From the scope we can also see that the resistive torque m_l is dependent from the slope.

Overall, we have good performance tracking.

E.m.f:



The small overshoots, in this case, are caused by the action of the speed controller (and maybe also by the excitation current controller not fast enough) in the flux weakening region where we're exceeding the rated speed and trying to keep the e.m.f. $E(\omega)$ constant.