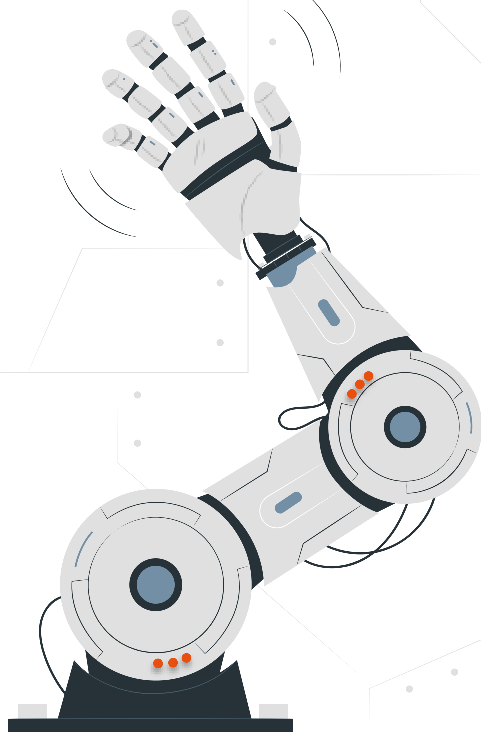
POLITECNICO
MILANO 1863



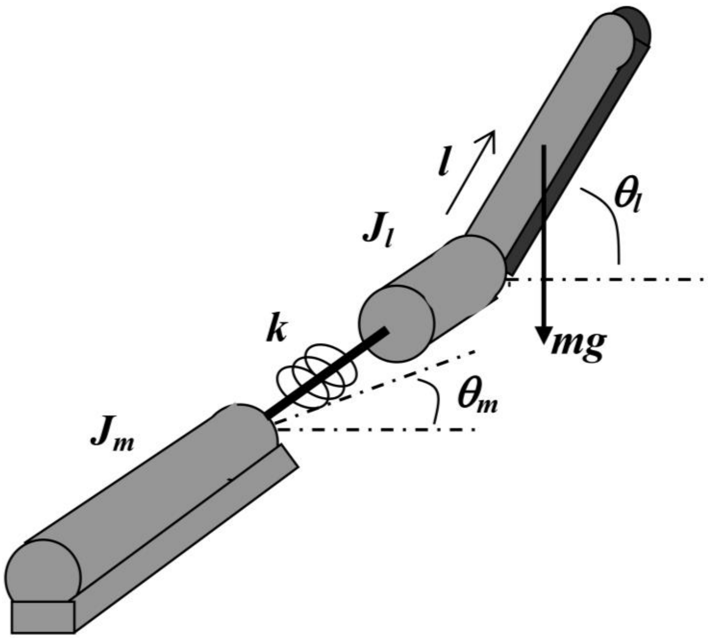
GRENOBLE INP UGA Ense3

Computer project on Nonlinear Systems

Robot arm control

"The Popov's Cops": R.Cazzato, C. Inviti, M. Molinaro, D. Ruiz

Robot arm with flexible joint



Dynamical equations:

$$J_l\ddot{\theta}_l + B_l\dot{\theta}_l + k(\theta_l - \theta_m) + mglcos(\theta_l) = 0$$

$$J_m\ddot{\theta}_m + B_m\dot{\theta}_m - k(\theta_l - \theta_m) = u$$

Data:

$$J_l = J_m = 4 * 10^{-4}\ Nms^2/rad$$

$$B_m = 0.015\ Nms/rad$$

$$B_l = 0.0\ Nms/rad$$

$$k = 0.8\ Nm/rad$$

$$m = 0.3\ kg$$

$$l = 0.3\ m$$

$$g = 9.8\ ms^{-2}$$

Non-linear system
State space representation

Dynamical equations:

$$J_l\ddot{\theta}_l + B_l\dot{\theta}_l + k(\theta_l - \theta_m) + mglcos(\theta_l) = 0$$

$$J_m\ddot{\theta}_m + B_m\dot{\theta}_m - k(\theta_l - \theta_m) = u$$

States:

$$x_1 = \theta_l$$

$$x_2 = \theta_m$$

$$x_3 = \dot{\theta}_l$$

$$x_4 = \dot{\theta}_m$$

System dynamic:

$$\dot{x_1} = x_3$$

$$\dot{x_2} = x_4$$

$$\dot{x_3} = -\frac{1}{J_l}[B_l x_3 + k(x_1 - x_2) + mglcos(x_1)]$$

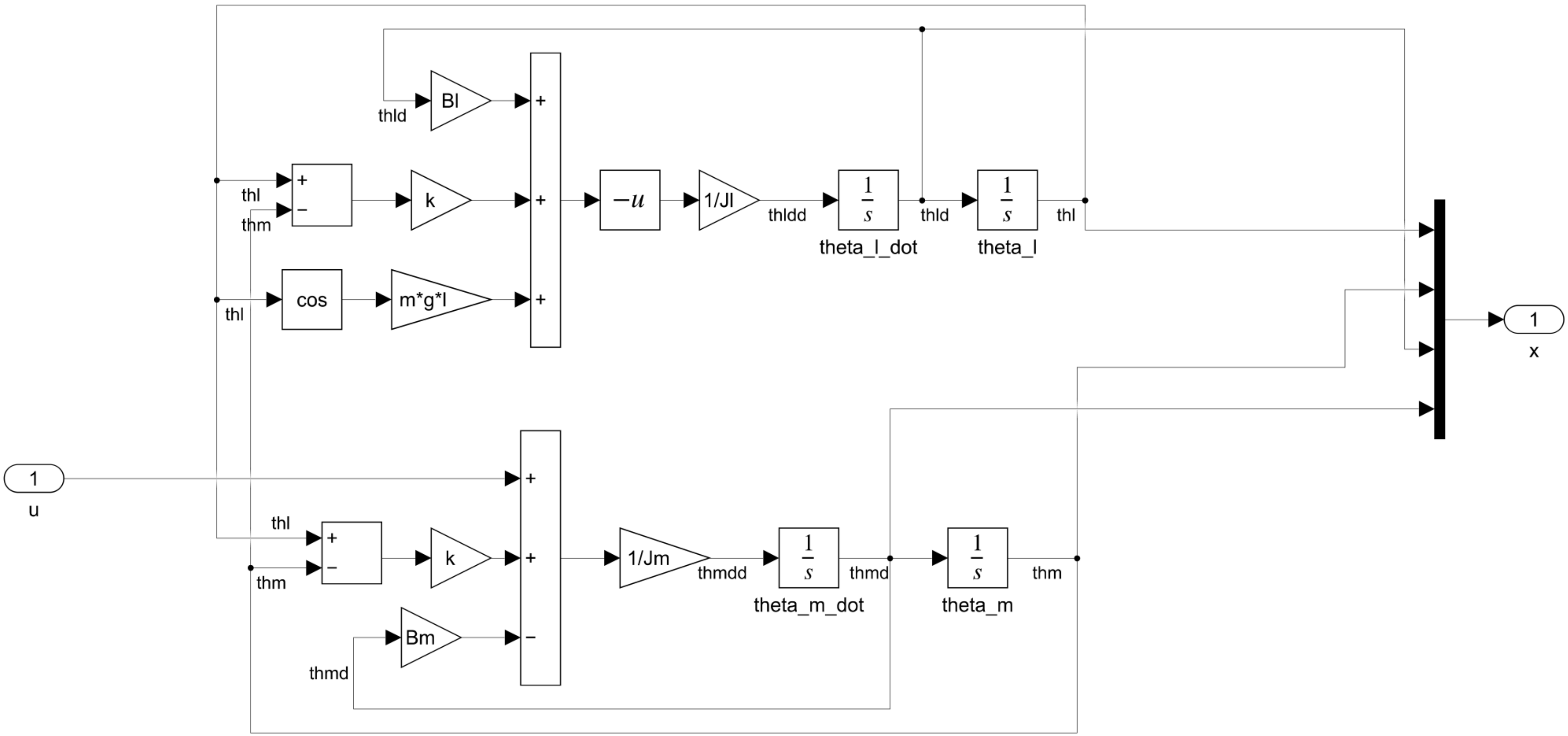
$$\dot{x_4} = \frac{1}{J_m}[-B_m x_4 + k(x_1 - x_2) + u]$$

System:

$$\begin{cases} \dot{x} = a(x) + b(x)u \\ y = c(x) = x_1 = \theta_l \end{cases}$$

$$a(x) = \begin{bmatrix} x_3 \\ x_4 \\ -\frac{1}{J_l}[B_l x_3 + k(x_1 - x_2) + mglcos(x_1)] \\ \frac{1}{J_m}[-B_m x_4 + k(x_1 - x_2)] \end{bmatrix} \qquad b(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_m} \end{bmatrix} \qquad c(x) = [x_1 0\ 0\ 0]$$

Non-linear system
Simulation scheme for the non-linear plant



Tangent linearization
Linear tangent approximation

Equilibrium:

$$\overline{x_1} = \bar{\theta}_l = \frac{\pi}{4}$$

$$\overline{x_2} = \overline{\theta_m} = \frac{mgl}{k} cos(\bar{\theta}_l) + \bar{\theta}_l$$

$$\overline{x_3} = \bar{\dot{\theta}}_l = 0$$

$$\overline{x_4} = \overline{\dot{\theta}_m} = 0$$

$$\bar{u} = k(\bar{\theta}_l - \overline{\theta_m})$$

$$\delta x = x - \bar{x}$$

Linear tangent approximation:

$$\delta \dot{x}_1 = \delta x_3$$

$$\delta \dot{x}_2 = \delta x_4$$

$$\delta \dot{x}_3 = -\frac{1}{J_l}[B_l \delta x_3 + k(\delta x_1 - \delta x_2) - mgl\ sin(\overline{x_1})\delta x_1]$$

$$\delta \dot{x}_4 = \frac{1}{J_m}[-B_m \delta x_4 + k(\delta x_1 - \delta x_2) + \delta u]$$

Linearized system:

$$\begin{cases} \delta \dot{x} = \bar{A}\delta x + \bar{B}\delta u \\ \delta y = \bar{C}\delta x \end{cases}$$

$$\bar{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{1}{J_l}[-k + mgl\ sin(\overline{x_1})] & \frac{k}{J_l} & -\frac{B_l}{J_l} & 0 \\ \frac{k}{J_m} & -\frac{k}{J_m} & 0 & -\frac{B_m}{J_m} \end{bmatrix} \quad \bar{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_m} \end{bmatrix} \quad \bar{C} = [1\ 0\ 0\ 0]$$

Stabilizing feedback for tangent-linearized system

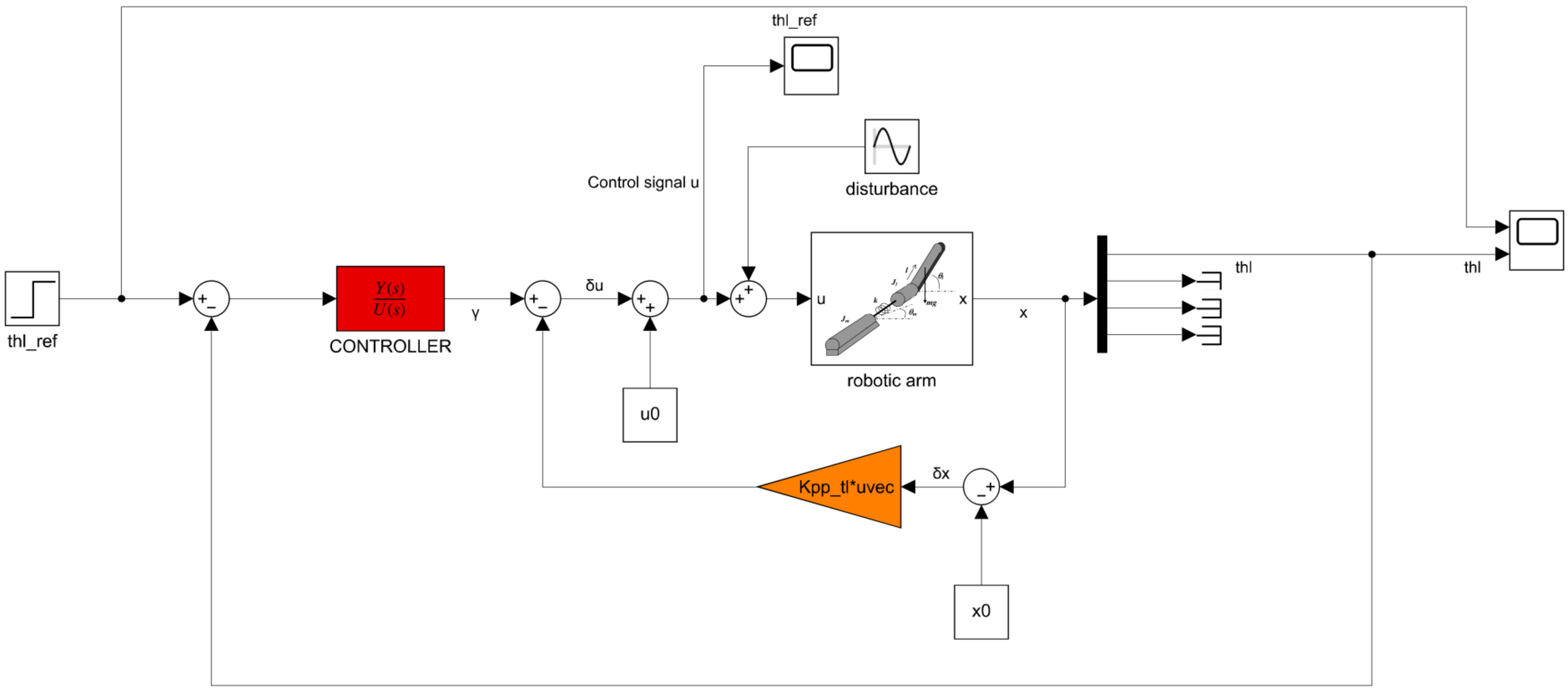
Stabilizing Strategy: Pole Placement

$$\delta u = -K\ \delta x + \delta v$$
$$\delta \dot{x} = \bar{A}\delta x + \bar{B}\delta u = \bar{A}\delta x - \bar{B}K\delta x + \bar{B}\delta v$$
$$\delta \dot{x} = (A - K)\delta x + \bar{B}\delta v$$

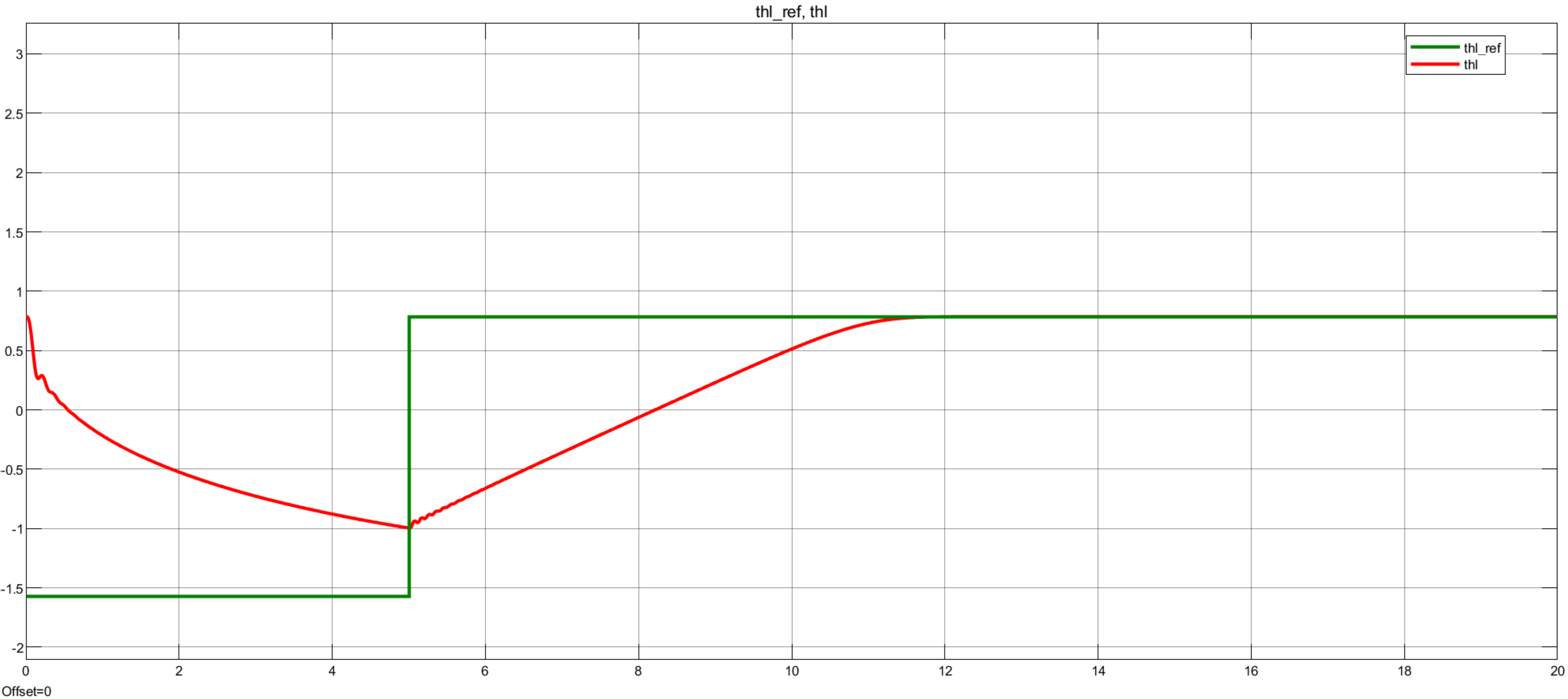
Control method for performance requirements:
Proportional Integral Controller

$$e = \delta y_0 - \delta y$$
$$\delta v = \left(K_p + K_i \frac{1}{s}\right) e$$

Tangent linearization
Simulation scheme for tangent-linearized system

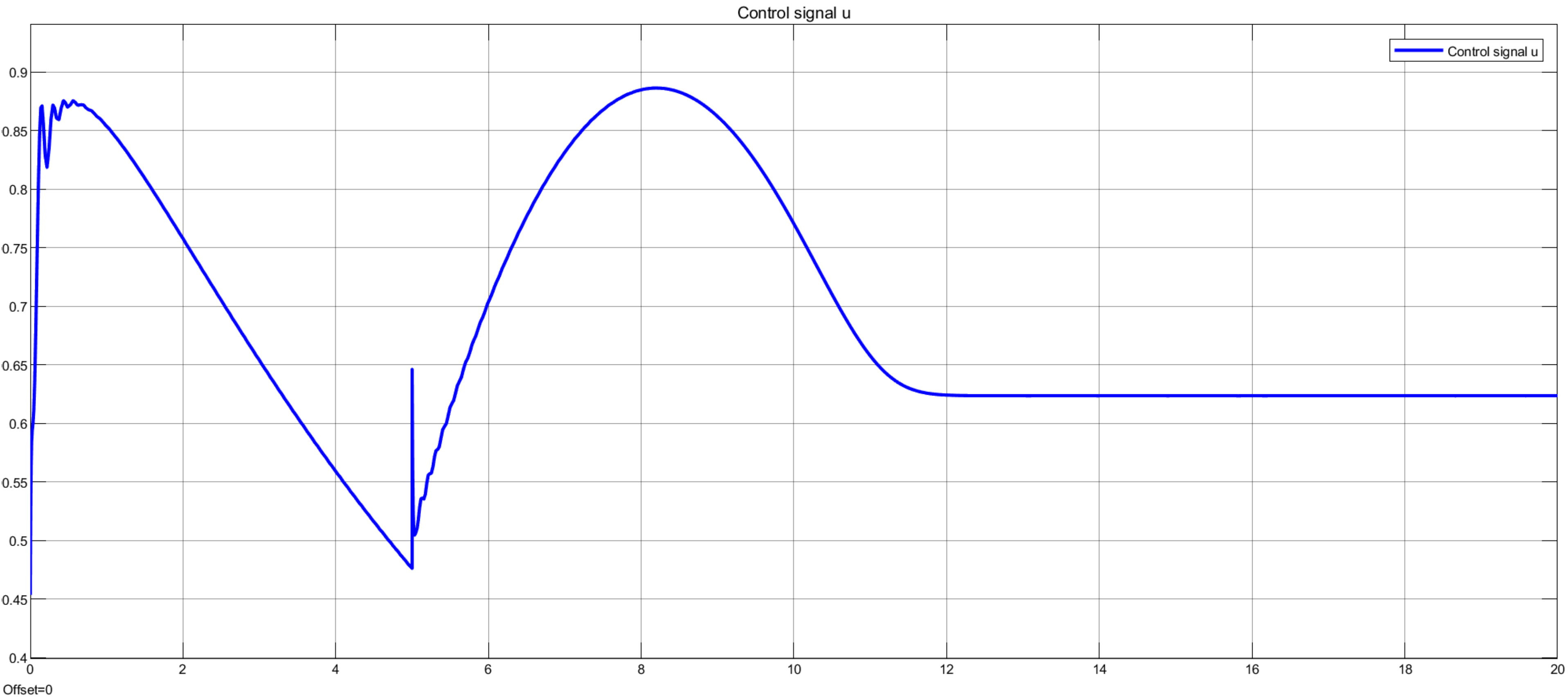


Tangent linearization
Simulation results for tangent-linearized system



Settling Time: 7s

Tangent linearization
Simulation results for tangent-linearized system

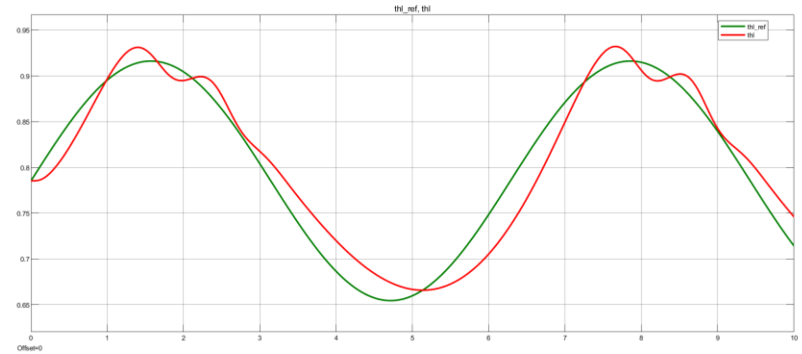


Tangent linearization

Simulation results for tangent-linearized system: NON LINEAR EFFECT

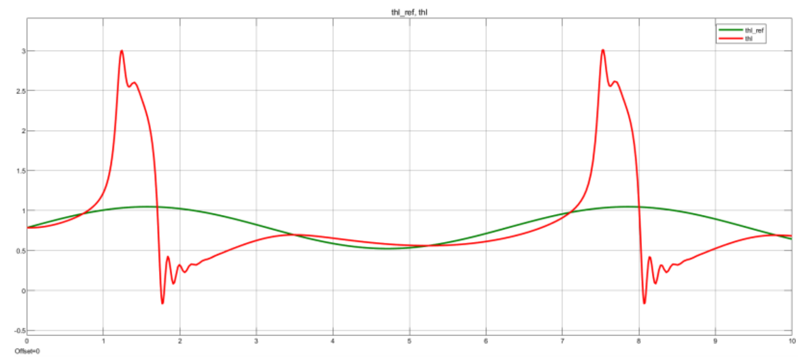
We also ran tests to check the non-linear effect on the tangent-linearized system. Applying a varying reference such as a sinusoidal signal around the equilibrium we discovered that:

- For small reference's amplitude ($\pm\pi/24$) the system remains stable



Sin: amplitude= $\pi/24$, bias=$\vartheta_{l0} = \pi/4$

- For larger values of the amplitude the system presents instability due to states approaching nearby equilibria.

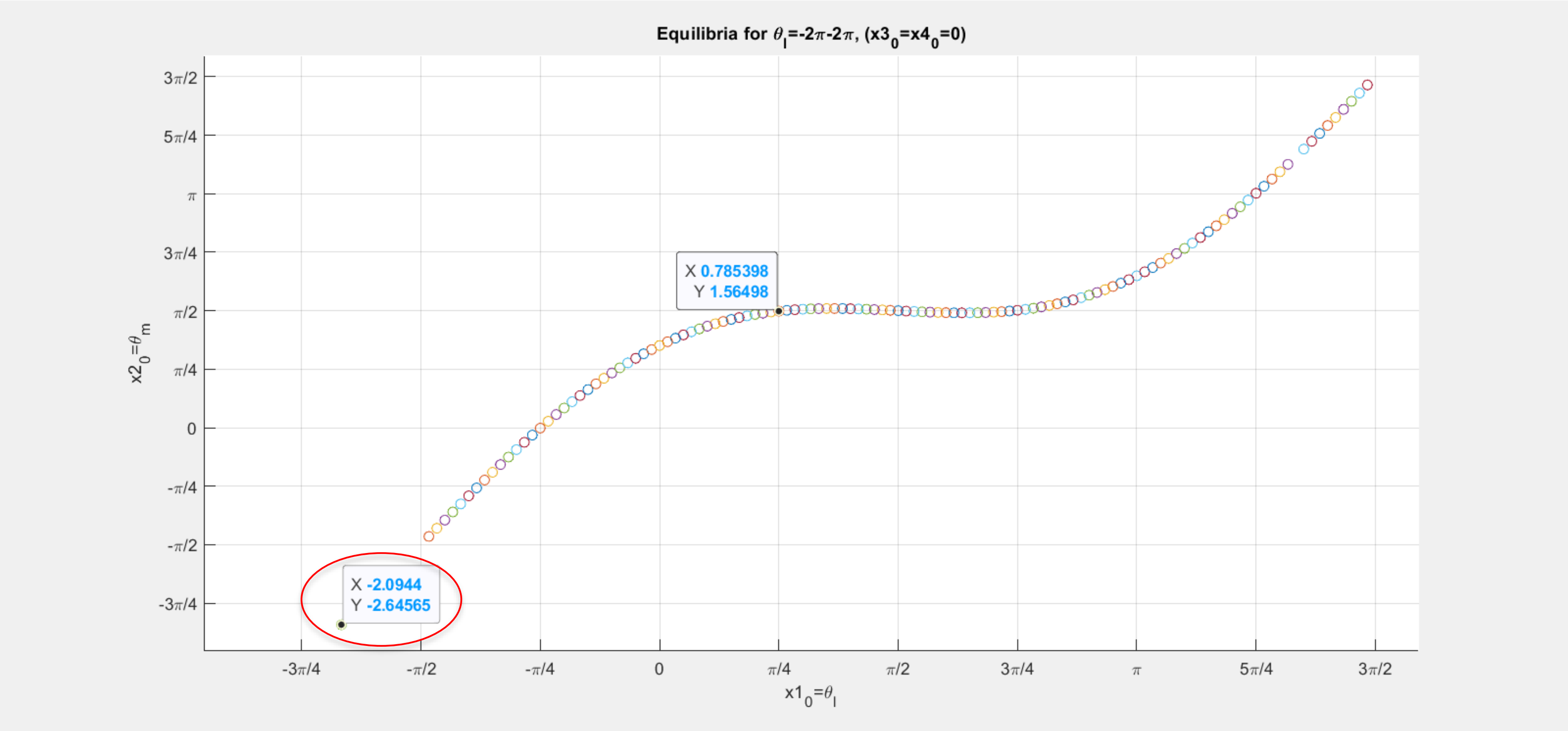


Sin: amplitude= $\pi/12$, bias=$\vartheta_{l0} = \pi/4$

Tangent linearization

Simulation results for tangent-linearized system: NON LINEAR EFFECT

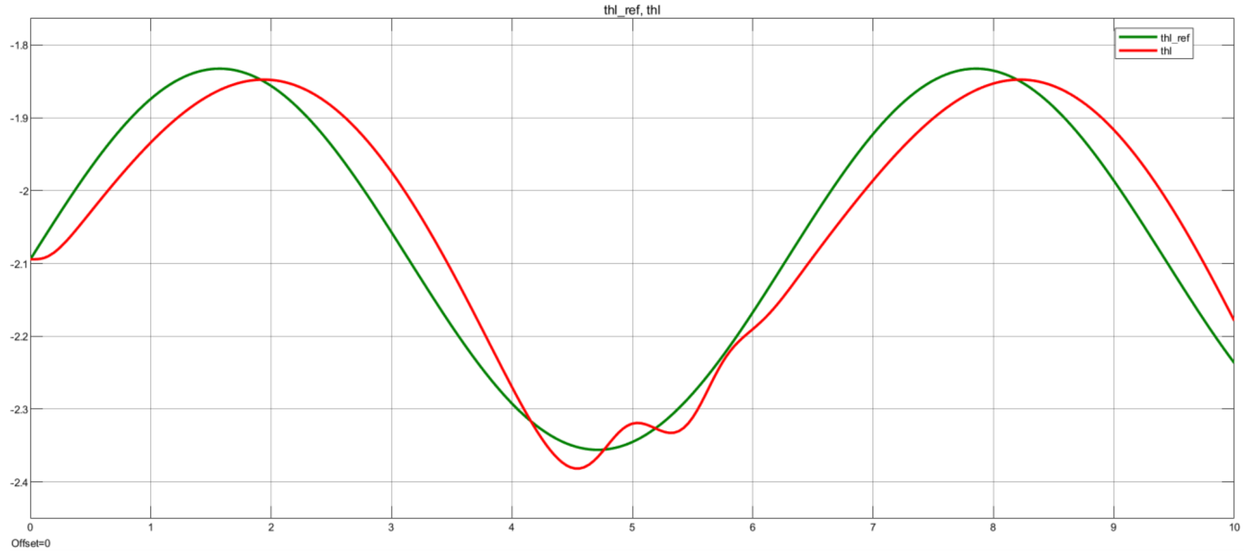
We then computed the equilibria to check their distribution density and verify the previous statement about the non-linear effect. We then analyzed the highlighted isolated equilibrium.



Tangent linearization

Simulation results for tangent-linearized system: NON LINEAR EFFECT

We see indeed that for a more isolated equilibrium we better preserve stability beacuse there are less nearby equilibria the state can approach.



Sin: amplitude= $\pi/12$, bias=ϑ_{l0} =-2.0944 [rad]
(same condition that for $\vartheta_{l0} = \pi/4$ led to instability)

The system can be rewritten as:

$$\dot{x} = \underbrace{\begin{bmatrix} x_3 \\ x_4 \\ -\frac{B_l x_3 + k(x_1 - x_2) + mgl\cos x_1}{J_l} \\ -\frac{B_m x_4 - k(x_1 - x_2)}{J_m} \end{bmatrix}}_{a(x)} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_m} \end{bmatrix}}_{b(x)} u$$

$$y = \underbrace{\frac{\pi}{4} - x_1}_{c(x)}$$

I/O linearizability conditions

By recursively applying the Lie derivative, it is possible to find out that the system’s relative degree is r = 4:

$$L_b L_a^i c = 0 \; \forall x, i = 0,1,2$$
$$L_b L_a^3 c = L_b L_a^{r-1} c = -\frac{k}{J_l J_m} \neq 0 \; \forall x \Rightarrow r = n = 4$$

The system has relative degree equal to the order of the system, so the system is fully I/O linearizable.

Control law implementation

The control law is then:

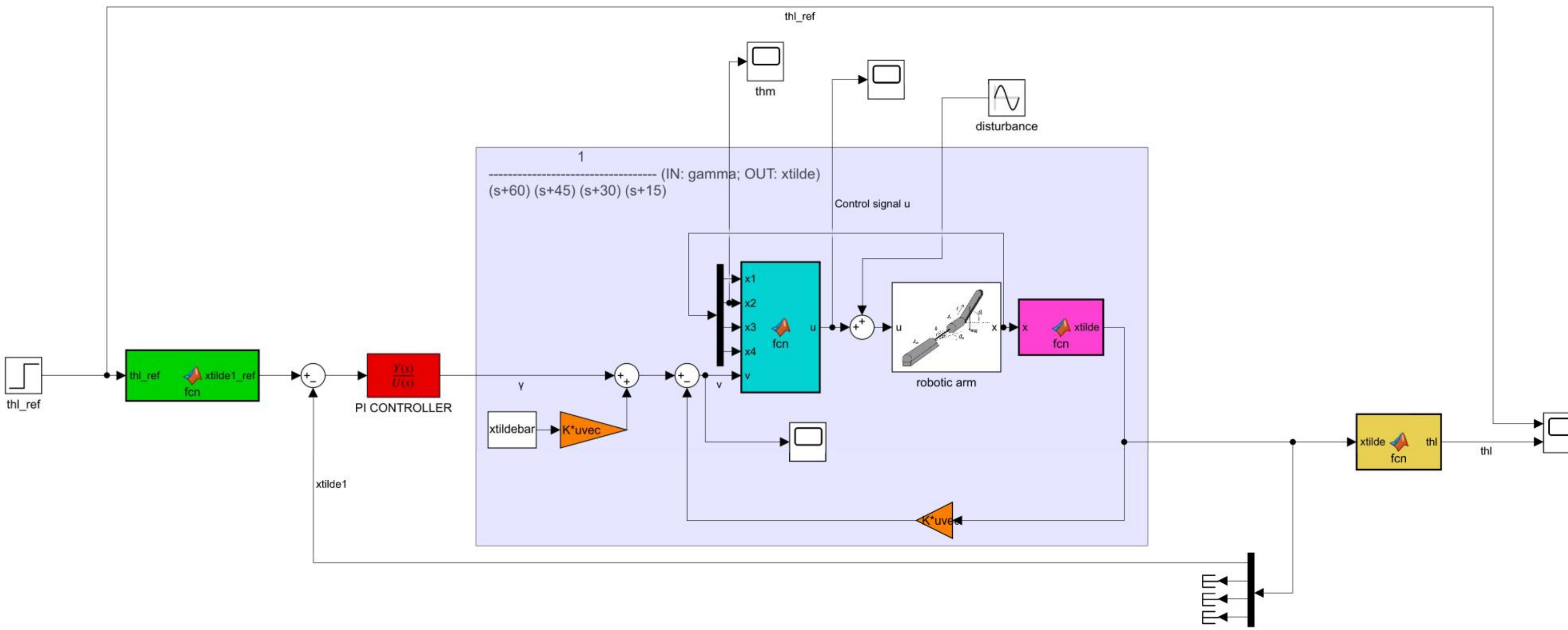
$$u = \frac{1}{L_b L_a^3 c}\left(v - L_a^4 c\right)$$

The closed loop system is now just a 4-th order integrator, with output y and input v.

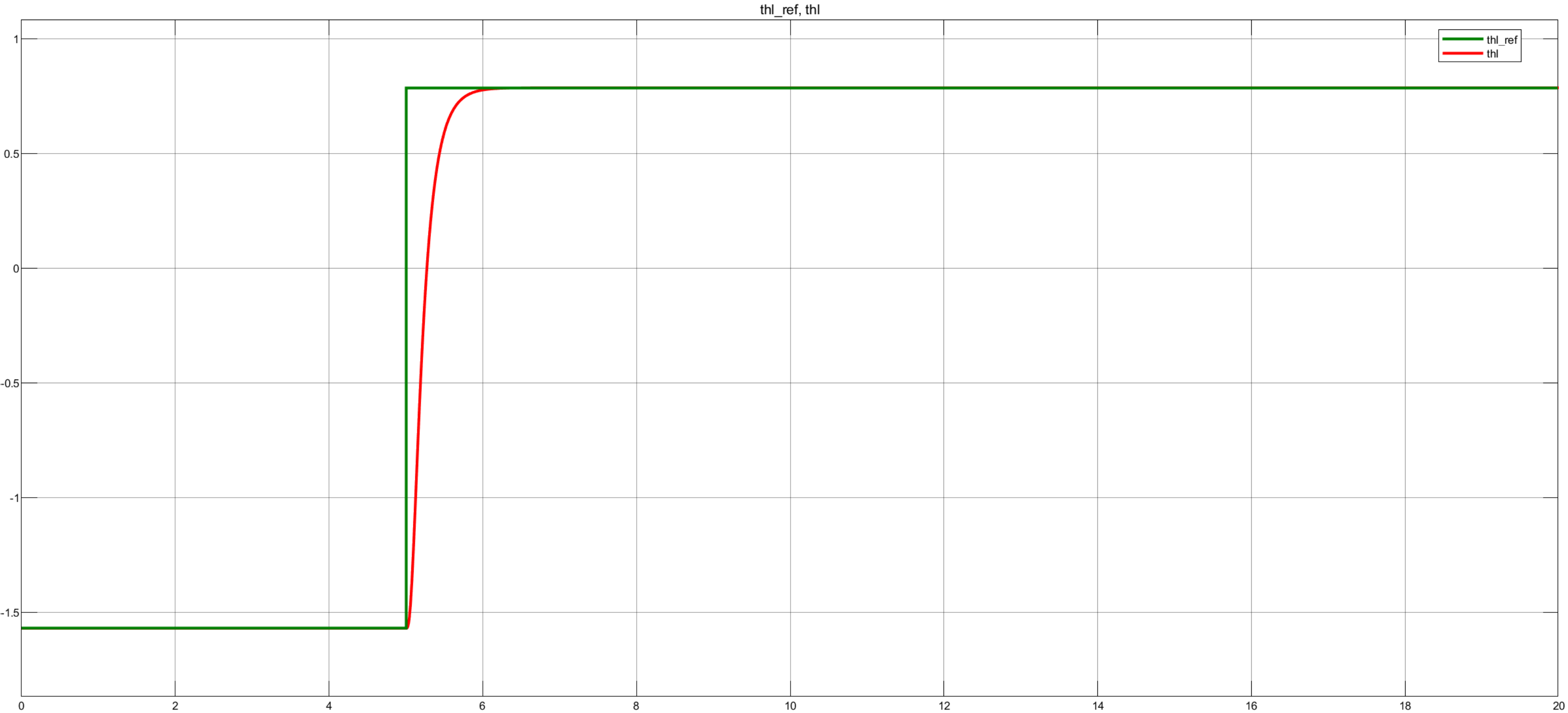
Can close another loop to have an asymptotically stable system (with pole placement).

Can finally close another loop to improve the performance.

State feedback linearization
Simulation scheme for feedback-linearized system

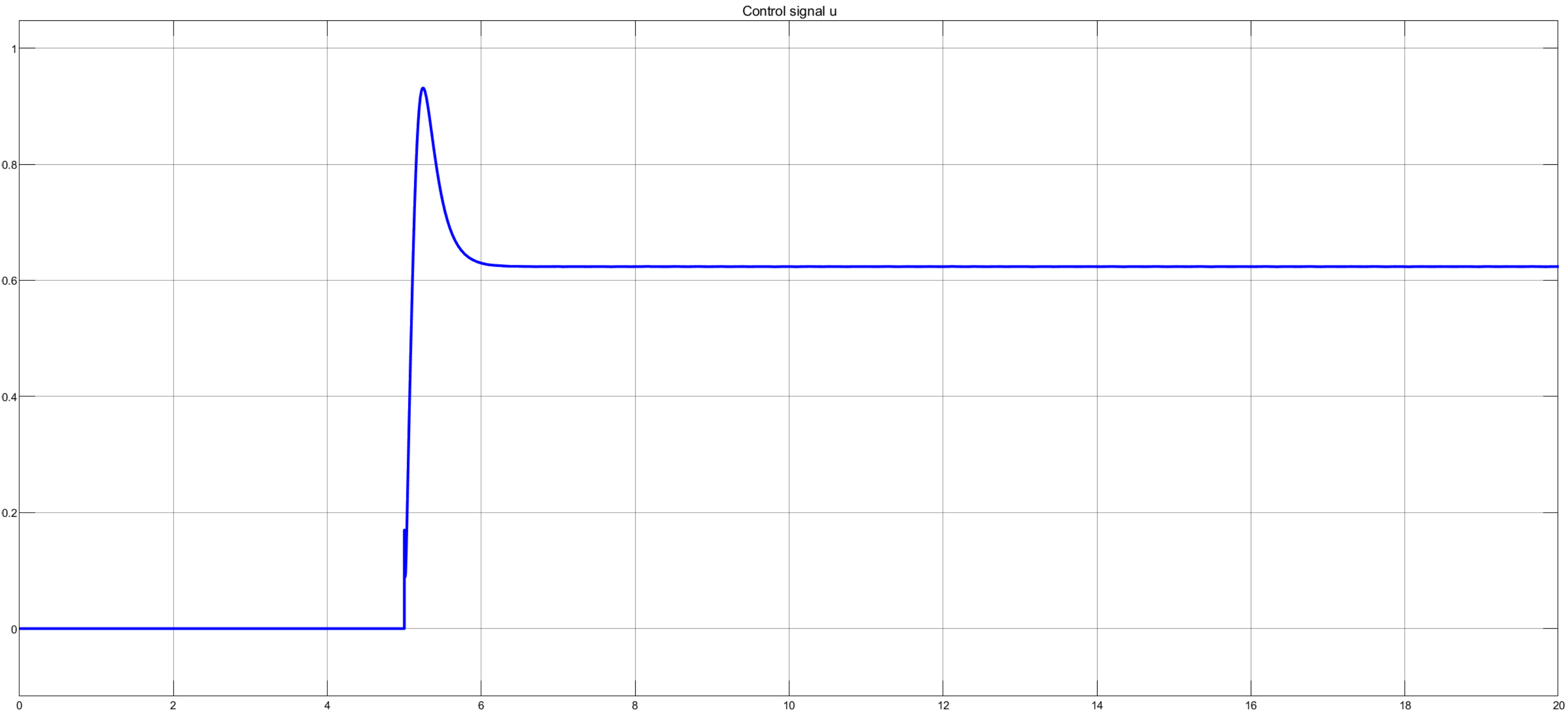


State feedback linearization
Simulation results for feedback-linearized system



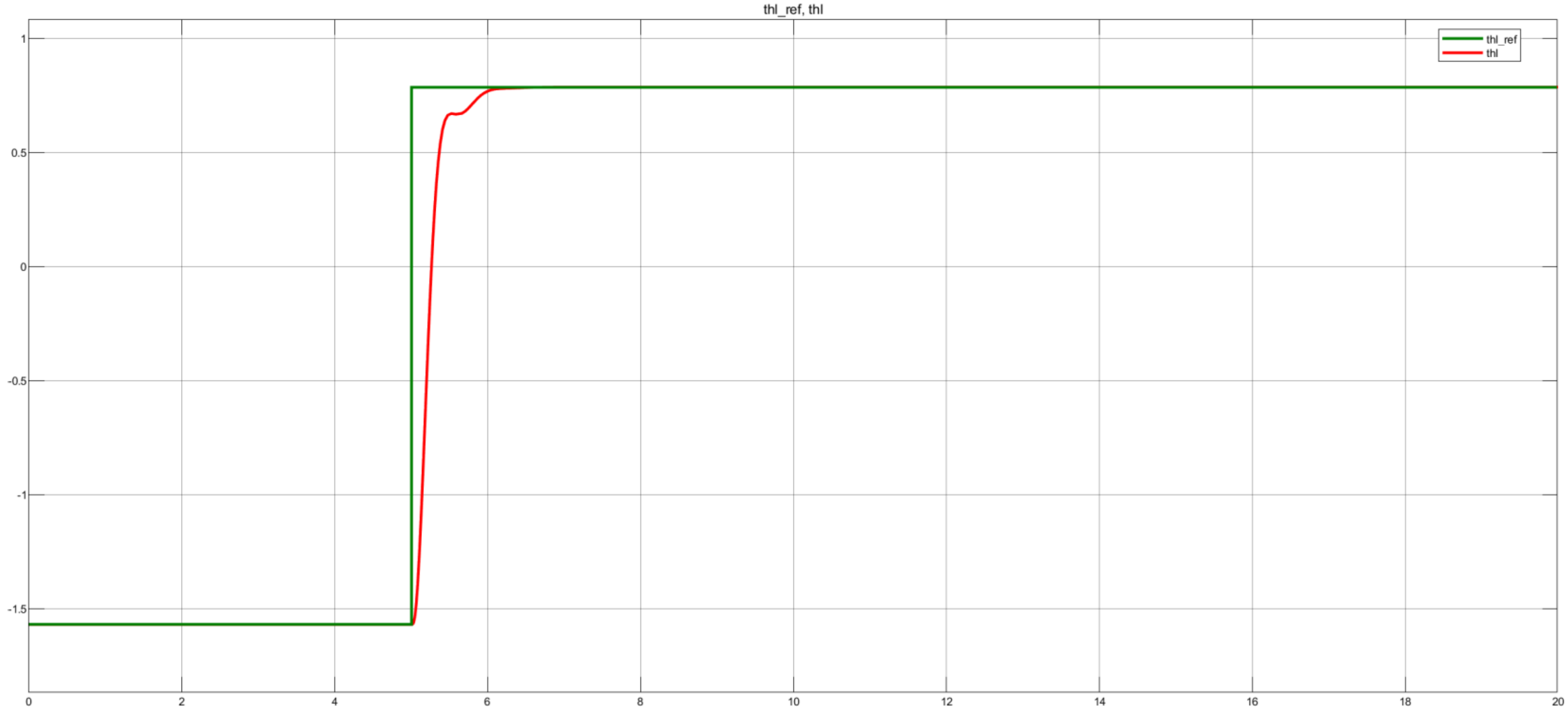
Settling Time: 1.36 s

State feedback linearization
Simulation results for feedback-linearized system



State feedback linearization
Simulation results for feedback-linearized system: ROBUSTNESS

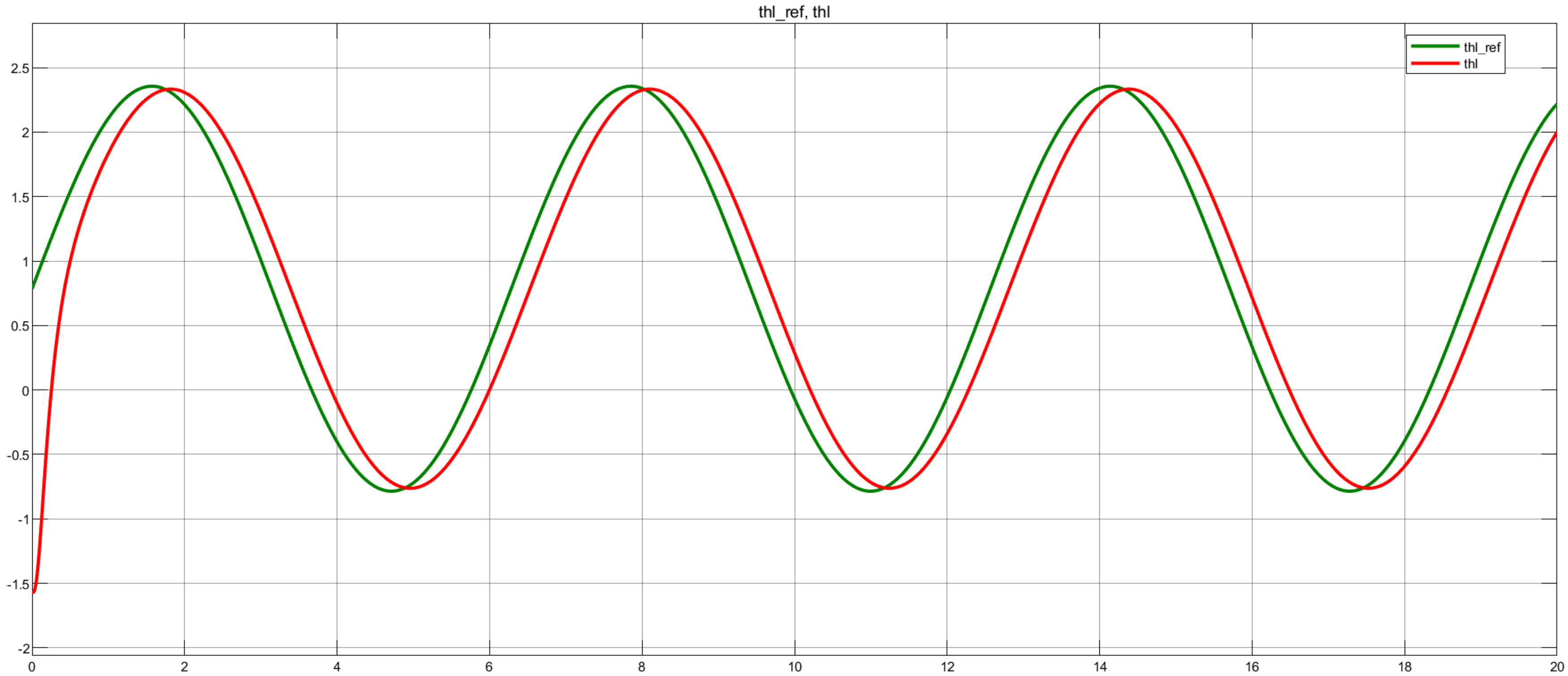
Results with an uncertainty of 50% on the values of the physical parameters J_l, J_m



The controller for the feedback-linearized system slightly deteriorates the performance in precence of big uncertainties, this is due to the fact that the control law is dependent from the model parameters.

State feedback linearization
Simulation results for feedback-linearized system: ROBUSTNESS

Results with a sinusoidal reference around $\pi/4$ (but works for any bias) with amplitude $\pi/2$



The controller is able to follow sinusoidal references more robustly than the tangent method (the delay between the reference and the real output is approximately 280 ms)

Conclusions
Performance conclusions

Applying the same control strategies and using the same tuning parameters:

- The feedback linearization scheme provides a faster response than the scheme with the tangent linearization.
- Indeed, the tangent linearization scheme could achieve a performance similar to the feedback linearization using faster poles than the ones originally applied.
- Moreover the tangent linearization approach leads to good results just in the neighbourhood of the chosen equilibrium
- In conclusion the feedback linearization approach provides a wider applicability range and better robustness' properties wrt the tangent method.

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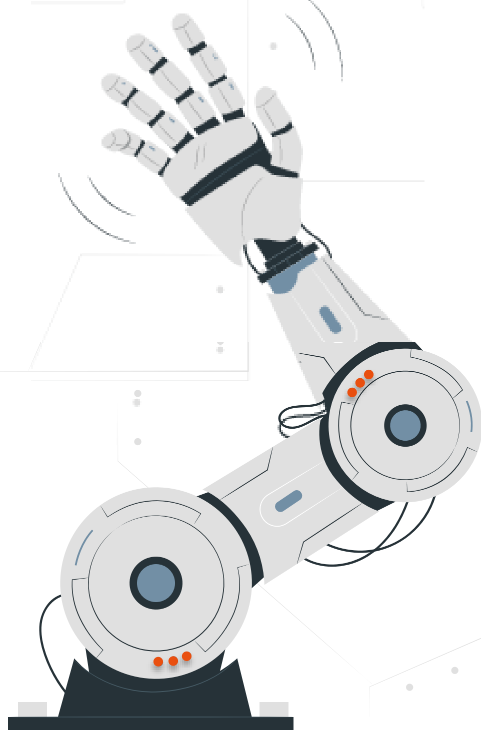

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THANK YOU FOR YOUR ATTENTION
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MERCI

"The Popov's Cops": R.Cazzato, C. Inviti, M. Molinaro, D. Ruiz