VYSOKÉ UČENÍ TECHNICKÉ V BRNĚ FAKULTA INFORMAČNÍCH TECHNOLOGIÍ

# IEL – protokol k projektu

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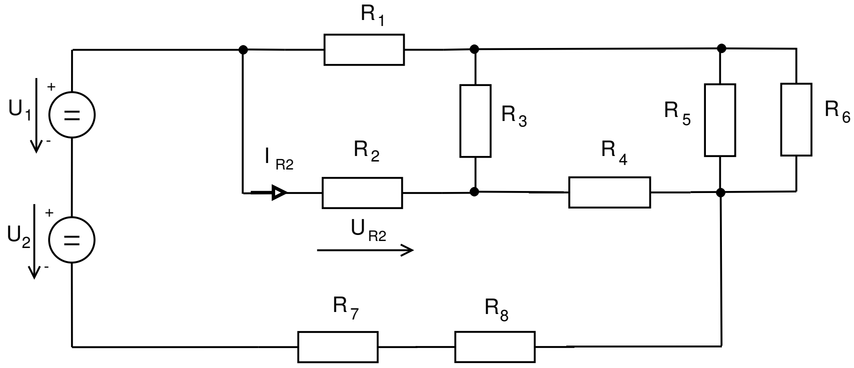
## Obsah

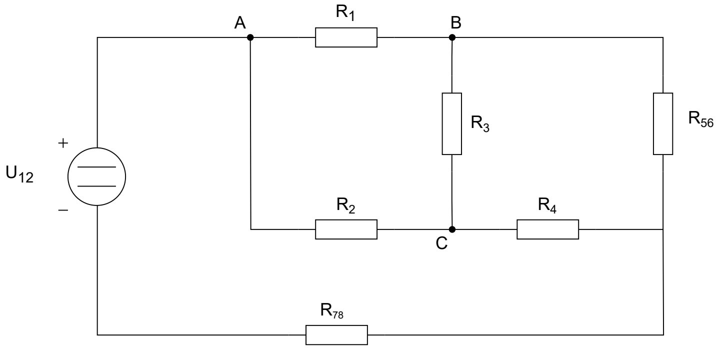
# Příklad 1

Stanovte napětí $U_{R2}$ a proud $I_{R2}$. Použijte metodu postupného zjednodušování obvodu.

| sk. | $U_1$ [V] | $U_2$ [V] | $R_1$ [Ω] | $R_2$ [Ω] | $R_3$ [Ω] | $R_4$ [Ω] | $R_5$ [Ω] | $R_6$ [Ω] | $R_7$ [Ω] | $R_8$ [Ω] |
|---|---|---|---|---|---|---|---|---|---|---|
| C | 100 | 80 | 450 | 810 | 190 | 220 | 220 | 720 | 260 | 180 |

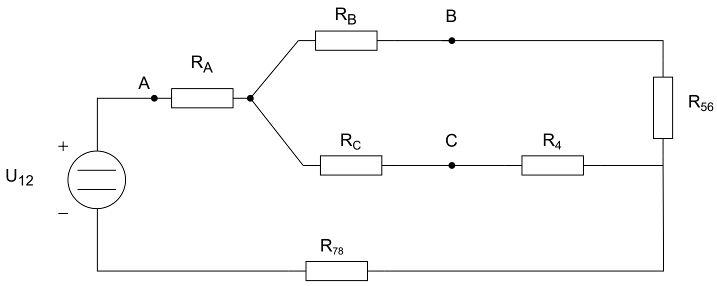



$$U_{12} = U_1 + U_2 = 100 + 80 = 180V$$
$$R_{56} = \frac{R_5 R_6}{R_5 + R_6} = \frac{220 \cdot 720}{220 + 720} = 168.51063829787233\Omega$$
$$R_{78} = R_7 + R_8 = 260 + 180 = 440\Omega$$




Nyní provedeme transfiguraci trojuhelník hvězda

$$R_A = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{450 \cdot 810}{450 + 810 + 190} = \frac{364500}{1450} = 251.37931034482768276\Omega$$
$$R_B = \frac{R_1 R_3}{R_1 + R_2 + R_3} = \frac{450 \cdot 190}{450 + 810 + 190} = \frac{85500}{1450} = 58.965517241379310\Omega$$
$$R_C = \frac{R_2 R_3}{R_1 + R_2 + R_3} = \frac{810 \cdot 190}{450 + 810 + 190} = \frac{153900}{1450} = 106.13793103448276\Omega$$

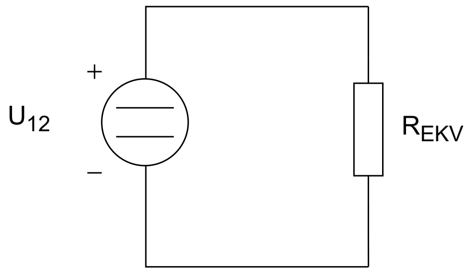



$$R_{B56} = R_B + R_{56} = 58.96551724137931 + 168.51063829787233 = 227.47615553925164\Omega$$

$$R_{C4} = R_C + R_4 = 106.13793103448276 + 220 = 326.13793103448276\Omega$$

$$R_{B56C4} = \frac{R_{B56}R_{C4}}{R_{B56} + R_{C4}} = \frac{227.47615553925164 \cdot 326.13793103448276}{227.47615553925164 + 326.13793103448276} = 134.00779446635116\Omega$$

$$R_{EKV} = R_A + R_{B56C4} + R_{78} = 251.3793103448276 + 134.00779446635116 + 440 = 825.3871048111787\Omega$$




Celkový proud I

$$I = \frac{U}{R_{EKV}} = \frac{180}{825.3871048111787} = 0.21807949136929883A$$

$$U_{RA} = I \cdot R_A = 0.21807949136929883 \cdot 251.37931034482768276 = 54.82067214076514V$$

$$U_{B56C4} = I \cdot R_{B56C4} = 0.21807949136929883 \cdot 134.00779446635116 = 29.224351656743398V$$

$$I_{RC4} = \frac{U_{B56C4}}{R_{C4}} = \frac{29.224351656743398}{326.13793103448276} = 0.08960733749688714A$$

$$U_{RC} = I_{RC4} \cdot R_C = 0.08960733749688714 \cdot 106.13793103448276 = 9.510737407428229V$$

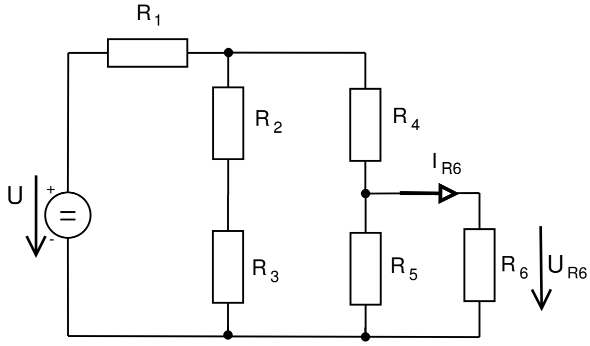
$$U_{R2} = U_{RA} + U_{RC} = 9.510737407428229 + 54.82067214076514 = \mathbf{64.33140954819336V}$$

$$I_{R2} = \frac{U_{R2}}{R_2} = \mathbf{0.07942149326937452A}$$

## Příklad 2

Stanovte napětí $U_{R6}$ a proud $I_{R6}$. Použijte metodu Théveninovy věty.

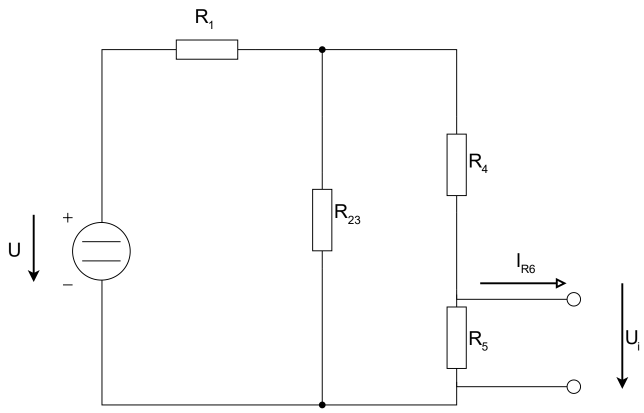
| sk. | $U$ [V] | $R_1$ [Ω] | $R_2$ [Ω] | $R_3$ [Ω] | $R_4$ [Ω] | $R_5$ [Ω] | $R_6$ [Ω] |
|---|---|---|---|---|---|---|---|
| G | 180 | 250 | 315 | 615 | 180 | 460 | 120 |




Vyřešíme za využití Theveninovy věty.

$$R_{23} = R_2 + R_3 = 315 + 615 = 930\Omega$$

$$R_{45} = R_4 + R_5 = 180 + 460 = 640\Omega$$




$$R_{EKV} = R_1 + \frac{R_{23}R_{45}}{R_{23} + R_{45}} = 250 + \frac{930 \cdot 640}{930 + 640} = 629.1082802547771\Omega$$

$$I = \frac{U}{R_{EKV}} = \frac{180}{629.1082802547771} = 0.28611926698390194A$$

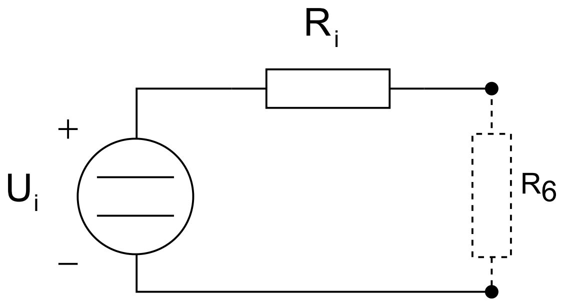
$$U_{R45} = U - (IR_1) = 180 - (0.28611926698390194 \cdot 250) = 108.47018325402452V$$

$$I_{R45} = \frac{U_{R45}}{R_{45}} = \frac{108.47018325402452}{640} = 0.1694846613344133A$$

$$U_i = I_{R45}R_5 = 0.1694846613344133 \cdot 460 = 77.96294421383013V$$

$$R_i = \frac{(\frac{R_{23}R_1}{R_{23}R_1} + R_4)R_5}{\frac{R_1R_{23}}{R_1 + R_{23}} + R_{45}} = \frac{(\frac{930 \cdot 250}{930 + 250} + 180) \cdot 460}{\frac{250 \cdot 930}{250 + 930} + 640} = 207.20259188012557\Omega$$

Ekvivalentní obvod dle Théveninovy věty



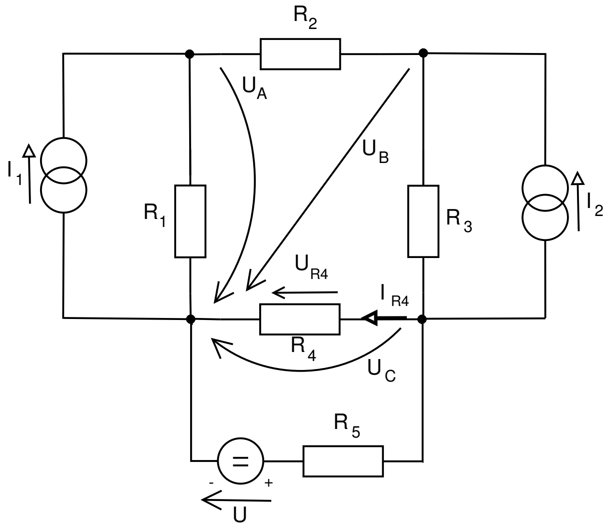

$$I_{R6} = \frac{U_i}{R_i + R_6} = \frac{77.96294421383013}{207.20259188012557 + 120} = \mathbf{0.23827116944841545} A$$

$$U_{R6} = R_6 I_{R6} = 120 \cdot 0.23827116944841545 = \mathbf{28.592540333809854} V$$

## Příklad 3

Stanovte napětí $U_{R4}$ a proud $I_{R4}$. Použijte metodu uzlových napětí ($U_A$, $U_B$, $U_C$).

| sk. | $U$ [V] | $I_1$ [A] | $I_2$ [A] | $R_1$ [Ω] | $R_2$ [Ω] | $R_3$ [Ω] | $R_4$ [Ω] | $R_5$ [Ω] |
|---|---|---|---|---|---|---|---|---|
| H | 130 | 0.95 | 0.50 | 47 | 39 | 58 | 28 | 25 |




$$G_1 = \frac{1}{R_1} = \frac{1}{47}S$$

$$G_2 = \frac{1}{R_2} = \frac{1}{39}S$$

$$G_3 = \frac{1}{R_3} = \frac{1}{58}S$$

$$G_4 = \frac{1}{R_4} = \frac{1}{28}S$$

$$G_5 = \frac{1}{R_5} = \frac{1}{25}S$$

$$I_3 = \frac{U}{R_4 + R_5} = \frac{130}{28 + 25} = \frac{130}{53}A = 2.452830188679245A$$

Nyní sestavíme rovnici pro jednotlivé uzly

$$A)G_1U_A + G_2(U_A - U_B) = -I_1$$

Nyní si vytvoříme matici za jejiž pomocí vypočteme U

$$\begin{pmatrix} -G_1 - G_2 & G_2 & 0 \\ G_2 & -G_2 - G_3 & G_3 \\ 0 & G_3 & -G_3 - G_4 - G_5 \end{pmatrix}^{-1} \times \begin{pmatrix} -I_1 \\ -I_2 \\ I_2 - \frac{U}{R_5} \end{pmatrix}$$

Dosadíme hodnoty a vypočteme U

$$\begin{pmatrix} -\frac{1}{47} - \frac{1}{39} & \frac{1}{39} & 0 \\ \frac{1}{39} & -\frac{1}{39} - \frac{1}{58} & \frac{1}{58} \\ 0 & \frac{1}{58} & -\frac{1}{58} - \frac{1}{28} - \frac{1}{25} \end{pmatrix}^{-1} \times \begin{pmatrix} -0.95 \\ -0.50 \\ 0.50 - \frac{130}{25} \end{pmatrix} =$$

$$= \begin{pmatrix} -32.94851176 & -21.2887662 & -3.94863178 \\ -21.2887662 & -38.95391263 & -7.22515602 \\ -3.94863178 & -7.22515602 & -12.09793567 \end{pmatrix} \times \begin{pmatrix} -0.95 \\ -0.5 \\ -4.7 \end{pmatrix} = \begin{pmatrix} 60.50403865 \\ 73.65951752 \\ 64.22407585 \end{pmatrix}$$

$$U_A = 60.50403865V$$

$$U_B = 73.65951752V$$

$$U_C = U_{R4} = \mathbf{64.22407585V}$$

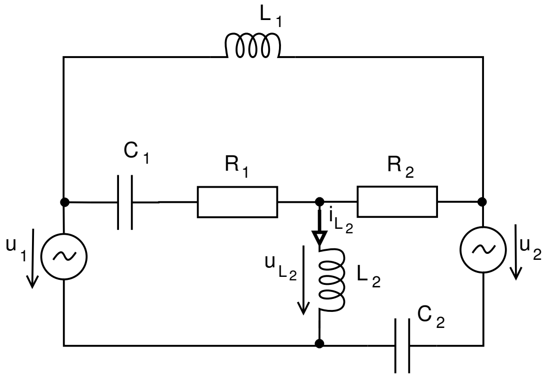
$$I_{R4} = \frac{U_{R4}}{R_4} = \frac{64.22407585}{28} = \mathbf{2.2937169946428573A}$$

## Příklad 4

Pro napájecí napětí platí: $u_1 = U_1 \cdot \sin(2\pi f t)$, $u_2 = U_2 \cdot \sin(2\pi f t)$.
Ve vztahu pro napětí $u_{L_2} = U_{L_2} \cdot \sin(2\pi f t + \varphi_{L_2})$ určete $|U_{L_2}|$ a $\varphi_{L_2}$. Použijte metodu smyčkových proudů.

Pozn: Pomocné směry šipek napájecích zdrojů platí pro speciální časový okamžik ($t = \frac{\pi}{2\omega}$).

| sk. | $U_1$ [V] | $U_2$ [V] | $R_1$ [Ω] | $R_2$ [Ω] | $L_1$ [mH] | $L_2$ [mH] | $C_1$ [µF] | $C_2$ [µF] | $f$ [Hz] |
|---|---|---|---|---|---|---|---|---|---|
| C | 3 | 4 | 10 | 13 | 220 | 70 | 230 | 85 | 75 |




Nejprve si převedeme hodnoty na zakladní jednotky

$$L_1 = 220mH = 0.22H$$

$$L_2 = 70mH = 0.07H$$

$$C_1 = 230mH = 0.00023F$$

$$C_2 = 85mH = 0.000085F$$

$$\omega = 2\pi f = 2 \cdot \pi 75 = 471.23889803846896 // Addunits$$

$$Z_{L1} = 1j\omega L_1 = 1j \cdot 471.23889803846896 \cdot 0.22 = 103.67255756846318j \ \Omega$$

$$Z_{L2} = 1j\omega L_2 = 1j \cdot 471.23889803846896 \cdot 0.07 = 32.98672286269283j \ \Omega$$

$$Z_{C1} = -\frac{1j}{\omega C_1} = -\frac{1j}{471.23889803846896 \cdot 0.00023} = 9.226373512573643j \ \Omega$$

$$Z_{C2} = -\frac{1j}{\omega C_2} = -\frac{1j}{471.23889803846896 \cdot 0.000085} = 24.965481269316918j \ \Omega$$

$$M = \begin{pmatrix} R_2 + R_1 + Z_{C1} + Z_{L1} & -Z_{C1} - R_1 & -R_2 \\ -R_1 - Z_{C1} & Z_{C1} + R_1 + Z_{L2} & -Z_{L2} \\ -R_2 & -Z_{L2} & Z_{L2} + R_2 + Z_{C2} \end{pmatrix} =$$

$$= \begin{pmatrix} 23 + 112.89893108j & -10 - 9.22637351j & -13 \\ -10 - 9.22637351j & 10 + 42.21309638j & -32.98672286j \\ -13 & -32.98672286j & 13 + 57.95220413j \end{pmatrix}$$

$$M^{-1} = \begin{pmatrix} 0.00060133 - 0.00869261j & -0.00252111 - 0.00573255j & -0.0024971 - 0.00395805j \\ -0.00252111 - 0.00573255j & 0.01424775 - 0.03366823j & 0.01046923 - 0.01625012j \\ -0.0024971 - 0.00395805j & 0.01046923 - 0.01625012j & 0.01036954 - 0.02361898j \end{pmatrix}$$

$$P = M^{-1} \cdot \begin{pmatrix} 0 \\ U_1 \\ -U_2 \end{pmatrix} = M^{-1} \cdot \begin{pmatrix} 0 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 0.00242508 - 0.00136546j \\ 0.00086633 - 0.03600422j \\ -0.01007047 + 0.04572556j \end{pmatrix}$$

$$I_A = 0.00242508 - 0.00136546jA$$

$$I_B = 0.00086633 - 0.03600422jA$$

$$I_C = -0.01007047 + 0.04572556jA$$

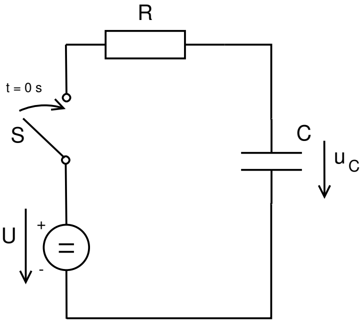
$U_{L2} = |Z_{L2} * (I_B - I_C)| =$
$= |32.98672286269283j * ((0.00086633 - 0.03600422j) - (-0.01007047 + 0.04572556j))|=$
$= |2.695997602488855 + 0.3607691906046989j| =$**2.720028948653897** V

$$\varphi = arctan\frac{Im(U_{L2})}{Re(U_{L2})} = arctan\frac{0.3607691906046989}{2.695997602488855} = 0.13302631216320196rad = \mathbf{7.621846}^{\circ}$$

## Příklad 5

V obvodu na obrázku níže v čase $t = 0$ [s] sepne spínač $S$. Sestavte diferenciální rovnici popisující chování obvodu na obrázku, dále ji upravte dosazením hodnot parametrů. Vypočítejte analytické řešení $u_C = f(t)$. Proveďte kontrolu výpočtu dosazením do sestavené diferenciální rovnice.

| sk. | $U$ [V] | $C$ [F] | $R$ [$\Omega$] | $u_C(0)$ [V] |
|---|---|---|---|---|
| G | 20 | 8 | 100 | 5 |




Známe U, R, C, $U_{C(0)}$
1)Ohmův zákon:
I = $\frac{U_R}{R}$
2)Kirchhochův zákon:
U = $U_R + U_C$
$U - U_C - U_R = 0$
3)Axiom:
$u'_C = \frac{1}{C} \times I$
$U_C(0) = U_{CP}$
1.krok

$$u'_C = \frac{1}{C} \times \frac{1}{R} \times U_R$$
$$U_R = U - U_C$$
$$u'_C = \frac{1}{RC} \times (U - u_C)$$

Jedná se o diferenciální rovnici 1. řádu
*počáteční podmínka:* $u_C(0) = U_{CP}$

$$u'_C + \frac{u_C}{RC} = \frac{U}{RC}$$

Charakteristická rovnice:

$$\lambda + \frac{1}{RC} = 0$$

$$\lambda = -\frac{1}{RC}$$

Očekávané řešení $u_C(t) = K(t)e^{\lambda t} = K(t)e^{-\frac{t}{RC}}$
Zderivujeme $u_c(t)$:

$$u_c(t) = K(t)e^{-\frac{t}{RC}}$$

$$u'_c(t) = K'(t)e^{-\frac{t}{RC}} - \frac{1}{RC} \times K(t)e^{-\frac{t}{RC}}$$

Nyní $u_C(t)$ a $u'_c(t)$ dosadíme do $u'_C + \frac{u_C}{RC} = \frac{U}{RC}$

$$K'(t)e^{-\frac{t}{RC}} - \frac{K(t)}{RC}e^{-\frac{t}{RC}} + \frac{K(t)e^{-\frac{t}{RC}}}{RC} = \frac{U}{RC}$$

$$K'(t)e^{-\frac{t}{RC}} = \frac{U}{RC}$$

Nyní zjistíme K(t)

$$K'(t)e^{-\frac{t}{RC}} = \frac{U}{RC} \ /e^{\frac{t}{RC}}$$

$$K'(t) = \frac{U}{RC}e^{\frac{t}{RC}} \ / \int$$

$$K(t) = \frac{U}{RC}(RCe^{\frac{t}{RC}})$$

$$K(t) = Ue^{\frac{t}{RC}} + k$$

*k je integrační konstanta*

Nyní dosadíme do původní rovnice (TODO:change this text to reflect reality)

$$u_C(t) = (Ue^{\frac{t}{RC}} + k)e^{-\frac{t}{RC}}$$

$$u_C(t) = U + ke^{-\frac{t}{RC}}$$

Dosadíme $u_C(0) = u_{CP} \implies k = 0$

$$u_{CP} = U + ke^0$$

$$u_{CP} - U = k$$

Analytické řešení

$$u_C(t) = U + (u_{CP} - U)e^{-\frac{t}{RC}}$$

Kontrola dosazením hodnot v $u_C(0)$

$$u_c(0) = 20 + (5 - 20)e^{-\frac{0}{100 \times 8}}$$

$$u_c(0) = 20 + (-15)e^{-\frac{0}{800}}$$

$$u_c(0) = 20 - 15 \times e^0$$

$$u_c(0) = 5$$

$$5 = 5$$

## Shrnutí výsledků

| **Příklad** | **Skupina** | **Výsledky** | |
|---|---|---|---|
| 1 | C | $U_{R2} = 64.33140954819336V$ | $I_{R2} = 0.07942149326937452A$ |
| 2 | G | $U_{R6} = 28.592540333809854V$ | $I_{R6} = 0.23827116944841545A$ |
| 3 | H | $U_{R4} = 64.22407585V$ | $I_{R4} = 2.2937169946428573A$ |
| 4 | C | $\|U_{L_2}\| =$ | $\varphi_{L_2} =$ |
| 5 | G | $u_C = 5V$ | |