Septiembre 21, 2018

Tarea Semana 7

2nd Parcial

Martin Noboa - A01704052
Aldrin Hernandez - A01704040

1.

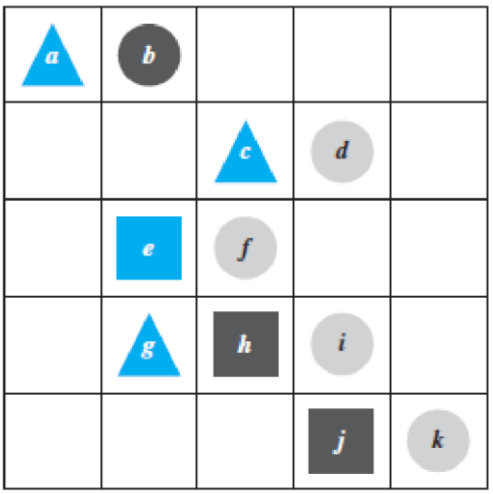

Figure 3.1.1

Determine the truth or falsity of each of the following statements. The domain for all variables is the set of objects in the Tarski world shown above.

a. $\forall t$, Triangle(t) $\rightarrow$ Blue(t).

b. $\forall x$, Blue(x) $\rightarrow$ Triangle(x).

c. $\exists y$ such that Square(y) $\wedge$ RightOf(d, y).

d. $\exists z$ such that Square(z) $\wedge$ Gray(z).

a. T
b. F
c. F
d. T

Septiembre 21, 2018

2.

1. If $P(x)$ is a predicate with domain D, the truth set of $P(x)$ is denoted ____. We read these symbols out loud as ____.
2. Some ways to express the symbol $\forall$ in words are ____.
3. Some ways to express the symbol $\exists$ in words are ____.
4. A statement of the form $\forall x \in D$, $Q(x)$ is true if, and only if, $Q(x)$ is ____ for ____.
5. A statement of the form $\exists x \in D$ such that $Q(x)$ is true if, and only if, $Q(x)$ is ____ for ____.

1. If P(x) is a predicate with domain D, the truth set of P(x) is denoted: {x ∈ D, P(x)}. We read these symbols out loud as *x belongs to domain D, for which P of x is true.*
2. Some ways to express the symbol ∀ in words are *for all values of x in the domain D.*
3. Some ways to express the symbol ∃ in words are *there is at least 1 value of x in the domain D.*
4. A statement in the form **∀x ∈ D, Q(x)** is true if and only if Q(x) is *true* for *all values of x.*
5. A statement in the form **∃x ∈ D, Q(x)** is true if, and only if, Q(x) is *true* for *at least one value of x.*

3.

1. A menagerie consists of seven brown dogs, two black dogs, six gray cats, ten black cats, five blue birds, six yellow birds, and one black bird. Determine which of the following statements are true and which are false.
   a. There is an animal in the menagerie that is red.
   b. Every animal in the menagerie is a bird or a mammal.
   c. Every animal in the menagerie is brown or gray or black.
   d. There is an animal in the menagerie that is neither a cat nor a dog.
   e. No animal in the menagerie is blue.
   f. There are in the menagerie a dog, a cat, and a bird that all have the same color.

2. Indicate which of the following statements are true and which are false. Justify your answers as best as you can.
   a. Every integer is a real number.
   b. 0 is a positive real number.
   c. For all real numbers r, $-r$ is a negative real number.
   d. Every real number is an integer.

3. Let $P(x)$ be the predicate "$x > 1/x$."
   a. Write $P(2)$, $P(\frac{1}{2})$, $P(-1)$, $P(-\frac{1}{2})$, and $P(-8)$, and indicate which of these statements are true and which are false.
   b. Find the truth set of $P(x)$ if the domain of x is $\mathbf{R}$, the set of all real numbers.
   c. If the domain is the set $\mathbf{R}^+$ of all positive real numbers, what is the truth set of $P(x)$?

4. Let $Q(n)$ be the predicate "$n^2 \leq 30$."
   a. Write $Q(2)$, $Q(-2)$, $Q(7)$, and $Q(-7)$, and indicate which of these statements are true and which are false.
   b. Find the truth set of $Q(n)$ if the domain of n is $\mathbf{Z}$, the set of all integers.
   c. If the domain is the set $\mathbf{Z}^+$ of all positive integers, what is the truth set of $Q(n)$?

5. Let $Q(x, y)$ be the predicate "If $x < y$ then $x^2 < y^2$" with domain for both x and y being the set $\mathbf{R}$ of real numbers.
   a. Explain why $Q(x, y)$ is false if $x = -2$ and $y = 1$.
   b. Give values different from those in part (a) for which $Q(x, y)$ is false.
   c. Explain why $Q(x, y)$ is true if $x = 3$ and $y = 8$.
   d. Give values different from those in part (c) for which $Q(x, y)$ is true.

1.

- **a. F**
- **b. T**
- **c. F**
- **d. T**
- **e. F**
- **f. T**

2.

- **a.** T. Since integers belong to the domain of real numbers, all integers are real numbers.
- **b.** F. Since positive integers are defined as numbers greater than 0, 0 can't be greater than itself so it can't be neither positive nor negative.
- c. T. Since the real numbers are both positive and negative, it is a correct assumption to state for every x in the real number domain, there is a -x.
- **d.** F. Since integers are defined as whole numbers, other subsets in the real number domain do not meet this condition, such as decimals and fractions.

3.

a.

- **i.** P(2) = T
- **ii.** P(1/2) = F
- iii. P(-1) = F
- iv. P(-1/2) = F
- v. P(-8) = F

b. $\text{Domain}_x = \{x \in R, x > 1\}$

c. All positive real numbers greater than one.

4.

a.

- i. Q(2) = T
- ii. Q(-2) = T
- iii. Q(7) = F
- iv. Q(-7) = F

b. $\text{Domain}_x = \{x \in Z, (-5 <= x <= 5\}$

c. All positive integers less than or equal to 5.

5.

a. Since this is an implication, the only scenario where it is false is if the predecessor is true and the outcome false, which happens here.
IF -2 < 1 (true), THEN 4 < 1(false).

b. Q(-3,2)

c. Since both conditions are true, the implication returns true.

d. Q(4,6)