Markov Chains exercises

Which of the following matrices are **stochastic matrices**? (Argument your decision)

$$\begin{pmatrix} 0.25 & 0.75 \\ 0.75 & 0.15 \end{pmatrix}$$

$$\begin{pmatrix} 2/7 & 2/5 \\ 4/7 & 3/5 \end{pmatrix}$$

Identify if the following transition matrices are **regular** or not (and state why)

$$\begin{pmatrix} 0.10 & 0.50 & 0.40 \\ 0.30 & 0.60 & 0.10 \\ 0.05 & 0.05 & 0.90 \end{pmatrix}$$

$$\begin{pmatrix} 0.50 & 0.40 & 0.10 \\ 0 & 0.30 & 0.70 \\ 0.40 & 0.20 & 0.40 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0.30 & 0.20 & 0.50 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Find all absorbing states for the transition matrices in Exercises 31–34. Which are transition matrices for absorbing Markov chains?

31. $\begin{bmatrix} 0.15 & 0.05 & 0.8 \\ 0 & 1 & 0 \\ 0.4 & 0.6 & 0 \end{bmatrix}$

32. $\begin{bmatrix} 0.4 & 0 & 0.6 \\ 0 & 1 & 0 \\ 0.9 & 0 & 0.1 \end{bmatrix}$

33. $\begin{bmatrix} 0.32 & 0.41 & 0.16 & 0.11 \\ 0.42 & 0.30 & 0 & 0.28 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

34. $\begin{bmatrix} 0.2 & 0.5 & 0.1 & 0.2 \\ 0 & 1 & 0 & 0 \\ 0.9 & 0.02 & 0.04 & 0.04 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Global equations

Obtain and simplify the global equations of the following transition matrix:

$$P = \begin{pmatrix} 0.05 & 0.10 & 0.85 \\ 0.40 & 0.20 & 0.40 \\ 0.01 & 0.09 & 0.90 \end{pmatrix}$$

Remember the global equations are obtained by setting

$$vP = v$$

where v is a vector such that $v = < v1, v2, v3 >$

Solve the following problems:

Medical Research *A medical researcher is studying the risk of heart attack in men. She first divides men into three weight categories: thin, normal, and overweight. By studying the male ancestors, sons, and grandsons of these men, the researcher comes up with the following transition matrix.*

	Thin	Normal	Overweight
Thin	0.3	0.5	0.2
Normal	0.2	0.6	0.2
Overweight	0.1	0.5	0.4

Find the probabilities of the following for a man of normal weight.

52. Thin son

53. Thin grandson

54. Thin great-grandson

Find the probabilities of the following for an overweight man.

55. Overweight son

56. Overweight grandson

57. Overweight great-grandson

Suppose that the distribution of men by weight is initially given by $[0.2 \quad 0.55 \quad 0.25]$. *Find each of the following distributions.*

58. After 1 generation

59. After 2 generations

60. After 3 generations

61. Find the long-range prediction for the distribution of weights.

69. ***Voting Trends*** At the end of June in a presidential election year, 40% of the voters were registered as liberal, 45% as conservative, and 15% as independent. Over a one-month period, the liberals retained 80% of their constituency, while 15% switched to conservative and 5% to independent. The conservatives retained 70% and lost 20% to the liberals. The independents retained 60% and lost 20% each to the conservatives and liberals. Assume that these trends continue.

a. Write a transition matrix using this information.