

Given the following functions, calculate 'k' such that f(x) is a probability density function (P.D.F) and calculate the given probabilities.

Expressions in red are the answers

$$\text{a) } f(x) = kx^2 + \frac{1}{30}; x \in [0, 3]$$

$$k = 1/10$$

$$P(X < 1) = 1/15$$

$$P(0.2 \leq X \leq 2.6) = 0.6656$$

$$\text{b) } f(x) = \begin{cases} k + x, & -1 < x < 0 \\ k - x, & 0 \leq x < 1 \end{cases}$$

$$k = 1$$

$$P(0.2 < X < 0.6) = 0.24$$

$$P(-0.5 < X < 0.2) = 0.875$$

$$\text{c) } f(x) = \frac{3}{1000} x^2, x \in [0, k]$$

$$k = 10$$

$$P(X > 8.5) = 0.385875$$

$$P(X = 2) = 0$$

$$\text{d) } f(x) = 2ke^{-kx}, 0 \leq x \leq 4$$

$$k = -\frac{1}{4} \ln \left( \frac{1}{2} \right) \approx 0.1733$$

$$P(1 \leq X \leq 2) = 0.2675$$