QUANTITATIVE METHODS & SIMULATION — AUGUST 16, 2021

- CALCULATE k SUCH THAT $f(x)$ IS A PROBABILITY DENSITY FUNCTION

(a) $f(x) = kx^2 + \frac{1}{30}$; $x \in [0,3]$

$$P(X) = \int_0^3 kx^2 + \frac{1}{30}\,dx = \frac{kx^3}{3} + \frac{x}{30}\Big|_0^3 = 1$$

$$\therefore \left(\frac{k(3)^3}{3} + \frac{3}{30}\right) - \left(\frac{k(0)^3}{3} + \frac{0}{30}\right) = 1$$

$$9k + \frac{1}{10} - 0 = 1 \quad \therefore k = \frac{1}{10} \quad \therefore P(X) = 0.1x^2 + \frac{1}{30};\ x \in [0,3]$$

⊙ $P(X<1) = \int_0^1 0.1x^2 + \frac{1}{30}\,dx = \frac{1}{15} \simeq 0.0\overline{6}$

⊙ $P(0.2 \le X \le 2.6) = \int_{0.2}^{2.6} P(x)\,dx = \frac{0.1x^3}{3} + \frac{x}{30}\Big|_{0.2}^{2.6} = \frac{416}{425} \simeq 0.6656$

(b) $f(x) \begin{cases} k + x, & -1 < x < 0 \\ k - x, & 0 \le x \le 1 \end{cases} \quad \therefore P(X) = \int_{-1}^{0} P(x)\,dx + \int_0^1 P(x)\,dx$

$$P(X) = kx + \frac{x^2}{2}\Big|_{-1}^{0} + kx - \frac{x^2}{2}\Big|_0^1 = 1$$

$$P(X) = \left(0 - \left(-k + \frac{1}{2}\right)\right) + \left(k - \frac{1}{2}\right) = 1 \rightarrow 2k - 1 = 1 \quad \therefore k = 1$$

$$P(X) = \begin{cases} 1x + \frac{x^2}{2}, & -1 < x < 0 \\ 1x - \frac{x^2}{2}, & 0 \le x \le 1 \end{cases}$$

⊙ $P(0.2 < x < 0.6) = \int_{0.2}^{0.6} P(x)\,dx = \cancel{\frac{x^2}{2} - \frac{x^3}{6}}\ x - \frac{x^2}{2}\Big|_{0.2}^{0.6} = \frac{6}{25} \simeq 0.24$

⊙ $P(-0.5 < X < 0.2) = \int_{-.5}^{0} P(x)\,dx + \int_0^{0.2} P(x)\,dx = \frac{111}{200} \simeq 0.555$

(c) $f(x) = \frac{3x^2}{1000}$, $x \in [0, k]$

$$P(X) = \int_0^k \frac{3x^2}{1000}\,dx = \frac{3}{1000}\left(\frac{x^3}{3}\right)\Big|_0^k = \frac{k^3}{1000} = 1 \therefore k = 10$$

$P(X) \mid x > 8.5 \rightarrow P(X > 8.5) = \int_{8.5}^{\infty} P(x)\,dx = \int_{8.5}^{\infty} \frac{3x^2}{1000}\,dx$

$= \lim_{t \to \infty} \int_{8.5}^{t} \frac{3x^2}{1000}\,dx = \lim_{t\to\infty} \frac{x^3}{1000}\Big|_{8.5}^{t} = \lim_{t\to\infty}\left(\frac{t^3}{1000} - \frac{8.5}{1000}\right)$

• PERDON PROFE ME EMOCIONÉ, NO LE HAGA CASO A ESTA PARTE

$P(X) \mid x > 8.5 \rightarrow P(X > 8.5) = \int_{8.5}^{10} P(x)\,dx = \frac{x^3}{1000}\Big|_{8.5}^{10} = \frac{3087}{8000} \approx 0.3859$

• $P(X=2) = 0$, A PROBABILITY DENSITY FUNCTION (PDF) must be integrated to yield a probability.

(d) $f(x) = 2ke^{-kx}$, $0 \leq x \leq 4$

$$P(X) = \int_0^4 f(x)\,dx = \int_0^4 2ke^{-kx} = 2k\int_0^4 e^{-kx} = \frac{-2ke^{-kx}}{k}\Big|_0^4$$

$$P(X) = \frac{-2ke^{-4k}}{4} - \left(\frac{-2k}{k}\right) = 1 \therefore 2 - \frac{ke^{-4k}}{2} = 1$$

$= ke^{-4k} = 2 = \ln k + (-4k)\ln e = \ln 2$

$= \ln k - 4k = \ln 2$

$$P(X) = \frac{-2\cancel{k}e^{-kx}}{\cancel{k}}\Big|_0^4 = -2e^{-kx}\Big|_0^4 = -2e^{-4k} - (-2e^{-k(0)})$$

$= -2(e^{-4k} - 1) = 1 \therefore e^{-4k} = \frac{1}{2}$

$-4k \ln e = \ln\left(\frac{1}{2}\right) \therefore k = -\frac{1}{4}\ln\left(\frac{1}{2}\right) \approx 0.1733$

$\odot$ $P(1 \leq x \leq 2) = \int_1^2 f(x)\,dx = -2e^\wedge\left(\frac{-1}{4}\ln\left(\frac{1}{2}\right)x\right)\Big|_1^2$

$= -2e^\wedge\left(\frac{-2}{4}\ln\left(\frac{1}{2}\right)\right) - \left(-2e^\wedge\left(\frac{-1}{4}\ln\left(\frac{1}{2}\right)\right)\right)$

$= 0.26758$

~~• Recordar que $\ln\left(\frac{1}{2}\right)$~~