Quantitative Methods & Simulation

Activity 08

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1. For each of the following matrices determine:
   a) If it represents a regular or non-regular Markov chain.
   b) If it's an absorbing Markov chain.
   c) The long trend or steady state of the matrix (if that's the case)

(a)

$$\mathbf{P} = \begin{pmatrix} .5 & .5 \\ .5 & .5 \end{pmatrix}$$

a) Regular
b) Not an absorbing Markov chain
c) .

(b)

$$\text{(b) } \mathbf{P} = \begin{pmatrix} .5 & .5 \\ 1 & 0 \end{pmatrix}$$

a) Irregular
b) Absorbing Markov chain
c) .

(c)

$$\mathbf{P} = \begin{pmatrix} 1/3 & 0 & 2/3 \\ 0 & 1 & 0 \\ 0 & 1/5 & 4/5 \end{pmatrix}$$

a) Irregular
b) Absorbing Markov chain
c) .

(d)

$$\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

a) Irregular
b) Non absorbing Markov chain
c) .

(e)

$$\mathbf{P} = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

a) Regular
b) Non absorbing Markov chain
c) .

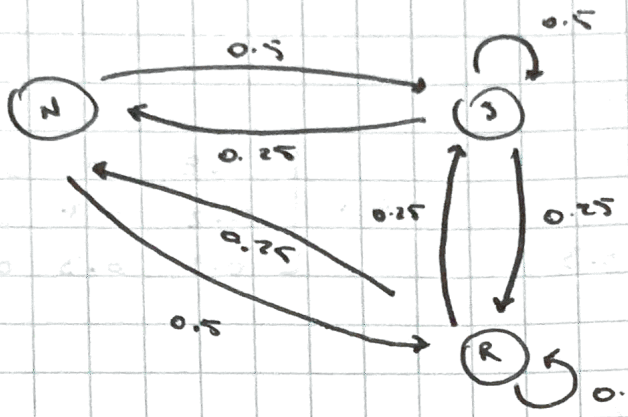
(f)

$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 0 & 1 \end{pmatrix}$$

a) Irregular
b) Absorbing Markov chain
c) .

EXAMPLE 1.1

(a)



$$P = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.25 & 0.5 & 0.25 \\ 0.25 & 0.5 & 0.25 \end{bmatrix}$$

(b) Calculate P^4 for rainy day from a nice day

$$P^4 = \begin{array}{c} \\ N \\ S \\ R \end{array} \begin{array}{c} \begin{array}{ccc} N & S & R \end{array} \\ \begin{bmatrix} 0.2031 & 0.5 & 0.2969 \\ 0.1992 & 0.5 & 0.3008 \\ 0.1992 & 0.5 & 0.3008 \end{bmatrix} \end{array}$$

Probability = 0.2969 //

(c) Long term probability

$$P^{10} = \begin{bmatrix} 0.2 & 0.5 & 0.2\overline{99} \\ 0.1\overline{99} & 0.5 & 0.3 \\ 0.1\overline{99} & 0.5 & 0.3 \end{bmatrix}$$

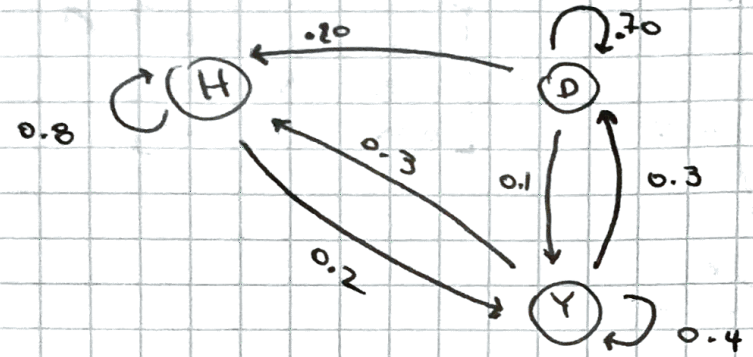
P(Nice day) = 0.2

P (Snowy day) = 0.5

P (Rainy day) = 0.3

EXAMPLE 11.6

(a)



$$P = \begin{bmatrix} 0.8 & 0 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}$$

(b) Calculate P^3 from $S_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ (H, D, Y)

$$P^3 = \begin{matrix} H \\ D \\ Y \end{matrix} \begin{bmatrix} 0.644 & 0.114 & 0.242 \\ 0.413 & 0.409 & 0.178 \\ 0.477 & 0.306 & \boxed{0.217} \end{bmatrix} \quad ; \text{ PROBABILITY} = 0.217 \text{ //}$$

(c) ~~Same~~ P^{3+4}, $S_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

PROBABILITY of great-granddaughter going to Dartmouth given the great grandfather went to Harvard

$$P^4 = \begin{bmatrix} 0.6106 & \boxed{0.1524} & 0.237 \\ 0.4656 & 0.3397 & 0.1942 \\ 0.5079 & 0.2793 & 0.2128 \end{bmatrix}$$

PROBABILITY = 0.1524 OR 15.24%.