Simulation of airflow through a window with an anti-bug grid

Final Project
Course: Modelling and Simulation

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1 Introduction

Figure 1.1: *Anti-bug grid (Flickr.com)*

Bug grids can be placed over open windows to allow air to flow through without letting bugs into someone's house. Presumably, when people open their windows, they want air to flow through their house. However, the installation of the grid decreases the functional cross-sectional area of the window, which might decrease the airflow. If the properties of the grid cause the flow to become turbulent, the reduction in airflow would be more drastic, than due to the reduction in the cross-sectional area alone. Using a fluid simulation, this report aims to capture this phenomenon, and investigate how the grid's thickness and density affect the airflow through a window. In doing so, an investigation into the optimal parameters of the grid that maximise airflow while "keeping the bugs out" will be conducted.

1.1 Research question

Main:

- How much does the air flux through a window with a bug grid change with the grid's thickness and density?

Subquestions:

- What are the optimal parameters of the grid that maximise airflow while keeping the bugs out?
- In which section of the window has the airflow decreased the most due to the presence of the bug grid?

2 Model and Methodology

We aim to implement a numerical 3D simulation of airflow based on Navier-Stokes equations in Python. The numerical simulation will include discretising space and time. The cells that make up the bug grid will be modelled using a no-slip boundary condition.

2.1 Assumptions:

1. Mass conservation
2. Isotropy (i.e., no gravity)
3. Compressible fluid
4. Dynamic (μ) and bulk (ζ) viscosities are uniform in space
5. Stoke's hypothesis ($\zeta = 0$)
6. Newtonian fluid

2.2 Math

The assumptions above yield the Navier-Stokes equation(s) [1] in the following form:

$$\left(\partial_t + \mathbf{u}\cdot\nabla - \nu\,\nabla^2 - \left(\tfrac{1}{3}\nu + \xi\right)\nabla(\nabla\cdot)\right)\mathbf{u} = -\frac{1}{\rho}\nabla p + \mathbf{f}, \tag{2.1}$$

where ρ is the mass density, $\mathbf{u}$ is velocity, p is the pressure, ν is a shear kinematic viscosity parameter, ξ is the bulk kinematic viscosity, and $\mathbf{f}$ is the body force (which is 0 due to isotropy). Further "massaging" the equation:

$$\partial_t\mathbf{u} + \underbrace{(\mathbf{u}\cdot\nabla)\mathbf{u} - \nu\,\nabla^2\mathbf{u} - \left(\tfrac{1}{3}\nu + \xi\right)\nabla(\nabla\cdot\mathbf{u})}_{f(\mathbf{u})} = -\frac{1}{\rho}\nabla p, \tag{2.2}$$

where $f(\mathbf{u})$ is a function of $\mathbf{u}$ and its spatial (partial) derivatives.

$$\partial_t\mathbf{u} = -f(\mathbf{u}) - \frac{1}{\rho}\nabla p \tag{2.3}$$

Applying the finite difference method to the time derivative in eq. 2.3, we get:

$$\frac{\mathbf{u}_{ijk}^{n+1} - \mathbf{u}_{ijk}^{n}}{\Delta t} = -f(\mathbf{u}^n) - \frac{1}{\rho}\nabla p \tag{2.4}$$

$$\mathbf{u}_{ijk}^{n+1} = \mathbf{u}_{ijk}^{n} - \Delta t\left(f(\mathbf{u}^n) + \frac{1}{\rho}\nabla p\right), \tag{2.5}$$

where Δt is the discrete time-step size, n is the time-step index, and $i, j,$ and k are the indices in the $x, y,$ and z spatial coordinates respectively. We can apply the finite difference method to all the (spatial) derivatives in f in the same way as we did above.
We also impose the no-slip boundary condition at the location of the rectangular bug grid, as well as on the walls around the window:

$$\forall i, j, k \in [\text{grid, window}]\ \forall n:\ \mathbf{u}_{ijk}^{n} = \mathbf{0} \tag{2.6}$$

3 Simulation setup

There are two ways of approaching the numerical simulation of the equations above:

1. Giving eq. (2.3) into a PDE solver
2. Implement and perform a numerical integration using eq. (2.5)

Option 1. is the preferred option, assuming a PDE-solve Python package exists that can accommodate eqs. (2.5) and (2.6). Option 2. is a backup option, and will only be used if no existing package fits our criteria well enough.

4 Results

5 Discussion

6 Conclusion

7 References

[1] G. K. Batchelor, *An introduction to fluid dynamics*. Cambridge university press, 2000, pp. 131–170.