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1 Setup**1.1 header.h**

```

1 #pragma once // Delete this when copying this file
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 #define ll long long
6 #define ull unsigned ll
7 #define ld long double
8 #define pl pair<ll, ll>
9 #define pi pair<int, int> // use pl where possible/necessary
10 #define vl vector<ll>
11 #define vi vector<int> // change to vl where possible/necessary
12 #define vb vector<bool>
13 #define vvi vector<vi>
14 #define vvl vector<vl>
15 #define vpl vector<pl>
16 #define vpi vector<pi>
17 #define vld vector<ld>
18 #define vvpv vector<vpi>
19 #define in_fast(el, cont) (cont.find(el) != cont.end())
20 #define in(el, cont) (find(cont.begin(), cont.end(), el) != cont.end())
21
22 constexpr int INF = 2000000010;
23 constexpr ll LLINF = 90000000000000000010LL;
24
25 template <typename T, template <typename ELEM, typename ALLOC = std::
    allocator<ELEM> > class Container>
26 std::ostream& operator<<(std::ostream& o, const Container<T>& container) {
27     typename Container<T>::const_iterator beg = container.begin();
28     if (beg != container.end()) {
29         o << *beg++;
30         while (beg != container.end()) {
31             o << " " << *beg++;
32         }
33     }
34     return o;
35 }
36
37 // int main() {
38 //     ios::sync_with_stdio(false); // do not use cout + printf
39 //     cin.tie(NULL);
40 //     cout << fixed << setprecision(12);
41 //     return 0;
42 // }

```

1.2 Bash for c++ compile with header.h

```

1 #!/bin/bash
2 if [ $# -ne 1 ];then echo "Usage: $0 <input_file>"; exit 1;fi
3 f="$1";d=code;/o=a.out
4 [ -f $d/$f ] || { echo "Input file not found: $f"; exit 1; }
5 g++ -I$d $d/$f -o $o && echo "Compilation successful. Executable '$o'
   created." || echo "Compilation failed."

```

1.3 Bash for run tests c++

```

1 g++ $1/$1.cpp -o $1/$1.out
2 for file in $1/*.in; do diff <($1/$1.out < "$file") "${file%.in}.ans"; done

```

1.4 Bash for run tests python

```

1 for file in $1/*.in; do diff <(python3 $1/$1.py < "$file") "${file%.in}.ans
   "; done

```

1.4.1 Aux. helper C++

```

1 #include "header.h"
2
3 int main() {
4     // Read in a line including white space
5     string line;
6     getline(cin, line);
7     // When doing the above read numbers as follows:
8     int n;
9     getline(cin, line);
10    stringstream ss(line);
11    ss >> n;
12
13    // Count the number of 1s in binary represnatation of a number
14    ull number;
15    __builtin_popcountll(number);
16 }

```

1.4.2 Aux. helper python

```

1 # Read until EOF
2 while True:
3     try:
4         pattern = input()
5     except EOFError:
6         break

```

2 Python

2.1 Graphs

2.1.1 BFS

```

1 from collections import deque
2 def bfs(g, roots, n):
3     q = deque(roots)
4     explored = set(roots)
5     distances = [float("inf")]*n
6     distances[0][0] = 0
7
8     while len(q) != 0:
9         node = q.popleft()
10        if node in explored: continue
11        explored.add(node)
12        for neigh in g[node]:
13            if neigh not in explored:
14                q.append(neigh)
15                distances[neigh] = distances[node] + 1
16    return distances

```

2.1.2 Dijkstra

```

1 from heapq import *
2 def dijkstra(n, root, g): # g = {node: (cost, neigh)}
3     dist = [float("inf")]*n
4     dist[root] = 0
5     prev = [-1]*n
6
7     pq = [(0, root)]
8     heapify(pq)
9     visited = set([])
10
11    while len(pq) != 0:
12        _, node = heappop(pq)
13
14        if node in visited: continue
15        visited.add(node)
16
17        # In case of disconnected graphs
18        if node not in g:
19            continue
20
21        for cost, neigh in g[node]:
22            alt = dist[node] + cost
23            if alt < dist[neigh]:
24                dist[neigh] = alt
25                prev[neigh] = node
26                heappush(pq, (alt, neigh))
27    return dist

```

2.2 Num. Th. / Comb.

2.2.1 nCk % prime

```

1 # Note: p must be prime and k < p
2 def fermat_binom(n, k, p):
3     if k > n:
4         return 0
5     # calculate numerator
6     num = 1
7     for i in range(n-k+1, n+1):
8         num *= i % p
9     num %= p
10    # calculate denominator
11    denom = 1
12    for i in range(1, k+1):
13        denom *= i % p
14    denom %= p
15    # numerator * denominator^(p-2) (mod p)
16    return (num * pow(denom, p-2, p)) % p

```

2.2.2 Sieve of E. $O(n)$ so actually faster than C++ version, but more memory

```

1 MAX_SIZE = 10**8+1
2 isprime = [True] * MAX_SIZE
3 prime = []
4 SPF = [None] * (MAX_SIZE)
5
6 def manipulated_seive(N): # Up to N (not included)
7     isprime[0] = isprime[1] = False
8     for i in range(2, N):
9         if isprime[i] == True:
10            prime.append(i)
11            SPF[i] = i
12            j = 0
13            while (j < len(prime) and
14                   i * prime[j] < N and
15                   prime[j] <= SPF[i]):
16                isprime[i * prime[j]] = False
17                SPF[i * prime[j]] = prime[j]
18                j += 1

```

2.3 Strings

2.3.1 LCS

```

1 def longestCommonSubsequence(text1, text2): # O(m*n) time, O(m) space
2     n = len(text1)
3     m = len(text2)
4
5     # Initializing two lists of size m
6     prev = [0] * (m + 1)
7     cur = [0] * (m + 1)

```

```

8
9     for idx1 in range(1, n + 1):
10        for idx2 in range(1, m + 1):
11            # If characters are matching
12            if text1[idx1 - 1] == text2[idx2 - 1]:
13                cur[idx2] = 1 + prev[idx2 - 1]
14            else:
15                # If characters are not matching
16                cur[idx2] = max(cur[idx2 - 1], prev[idx2])
17
18        prev = cur.copy()
19
20    return cur[m]

```

2.3.2 KMP

```

1 class KMP:
2     def partial(self, pattern):
3         """ Calculate partial match table: String -> [Int] """
4         ret = [0]
5         for i in range(1, len(pattern)):
6             j = ret[i - 1]
7             while j > 0 and pattern[j] != pattern[i]: j = ret[j - 1]
8             ret.append(j + 1 if pattern[j] == pattern[i] else j)
9         return ret
10
11    def search(self, T, P):
12        """KMP search main algorithm: String -> String -> [Int]
13        Return all the matching position of pattern string P in T"""
14        partial, ret, j = self.partial(P), [], 0
15        for i in range(len(T)):
16            while j > 0 and T[i] != P[j]: j = partial[j - 1]
17            if T[i] == P[j]: j += 1
18            if j == len(P):
19                ret.append(i - (j - 1))
20                j = partial[j - 1]
21        return ret

```

2.4 Other Algorithms

2.4.1 Rotate matrix

```

1 def rotate_matrix(m):
2     return [[m[j][i] for j in range(len(m))] for i in range(len(m[0])
3             -1, -1, -1)]

```

2.5 Other Data Structures

2.5.1 Segment Tree

```

1 N = 100000 # limit for array size
2 tree = [0] * (2 * N) # Max size of tree
3
4 def build(arr, n): # function to build the tree
5     # insert leaf nodes in tree
6     for i in range(n):
7         tree[n + i] = arr[i]
8
9     # build the tree by calculating parents
10    for i in range(n - 1, 0, -1):
11        tree[i] = tree[i << 1] + tree[i << 1 | 1]
12
13 def updateTreeNode(p, value, n): # function to update a tree node
14     # set value at position p
15     tree[p + n] = value
16     p = p + n
17
18     i = p # move upward and update parents
19     while i > 1:
20         tree[i >> 1] = tree[i] + tree[i ^ 1]
21         i >>= 1
22
23 def query(l, r, n): # function to get sum on interval [l, r]
24     res = 0
25     # loop to find the sum in the range
26     l += n
27     r += n
28     while l < r:
29         if l & 1:
30             res += tree[l]
31             l += 1
32         if r & 1:
33             r -= 1
34             res += tree[r]
35         l >>= 1
36         r >>= 1
37     return res

```

2.5.2 Trie

```

1 class TrieNode:
2     def __init__(self):
3         self.children = [None]*26
4         self.isEndOfWord = False
5
6 class Trie:
7     def __init__(self):
8         self.root = self.getNode()
9
10    def getNode(self):
11        return TrieNode()
12
13    def _charToIndex(self, ch):
14        return ord(ch)-ord('a')

```

```

15
16
17    def insert(self, key):
18        pCrawl = self.root
19        length = len(key)
20        for level in range(length):
21            index = self._charToIndex(key[level])
22            if not pCrawl.children[index]:
23                pCrawl.children[index] = self.getNode()
24            pCrawl = pCrawl.children[index]
25            pCrawl.isEndOfWord = True
26
27    def search(self, key):
28        pCrawl = self.root
29        length = len(key)
30        for level in range(length):
31            index = self._charToIndex(key[level])
32            if not pCrawl.children[index]:
33                return False
34            pCrawl = pCrawl.children[index]
35
36        return pCrawl.isEndOfWord

```

3 C++

3.1 Graphs

3.1.1 BFS

```

1 #include "header.h"
2 #define graph unordered_map<ll, unordered_set<ll>>
3 vi bfs(int n, graph& g, vi& roots) {
4     vi parents(n+1, -1); // nodes are 1..n
5     unordered_set<int> visited;
6     queue<int> q;
7     for (auto x: roots) {
8         q.emplace(x);
9         visited.insert(x);
10    }
11    while (not q.empty()) {
12        int node = q.front();
13        q.pop();
14
15        for (auto neigh: g[node]) {
16            if (not in(neigh, visited)) {
17                parents[neigh] = node;
18                q.emplace(neigh);
19                visited.insert(neigh);
20            }
21        }
22    }
23    return parents;
24 }

```

```

25 vi reconstruct_path(vi parents, int start, int goal) {
26     vi path;
27     int curr = goal;
28     while (curr != start) {
29         path.push_back(curr);
30         if (parents[curr] == -1) return vi(); // No path, empty vi
31         curr = parents[curr];
32     }
33     path.push_back(start);
34     reverse(path.begin(), path.end());
35     return path;
36 }

```

3.1.2 DFS Cycle detection / removal

```

1 #include "header.h"
2 void removeCyc(ll node, unordered_map<ll, vector<pair<ll, ll>>>& neighs,
   vector<bool>& visited,
3 vector<bool>& recStack, vector<ll>& ans) {
4     if (!visited[node]) {
5         visited[node] = true;
6         recStack[node] = true;
7         auto it = neighs.find(node);
8         if (it != neighs.end()) {
9             for (auto util: it->second) {
10                 ll nnode = util.first;
11                 if (recStack[nnode]) {
12                     ans.push_back(util.second);
13                 } else if (!visited[nnode]) {
14                     removeCyc(nnode, neighs, visited, recStack, ans);
15                 }
16             }
17         }
18     }
19     recStack[node] = false;
20 }

```

3.1.3 Dijkstra

```

1 #include "header.h"
2 vector<int> dijkstra(int n, int root, map<int, vector<pair<int, int>>>& g) {
3     unordered_set<int> visited;
4     vector<int> dist(n, INF);
5     priority_queue<pair<int, int>> pq;
6     dist[root] = 0;
7     pq.push({0, root});
8     while (!pq.empty()) {
9         int node = pq.top().second;
10        int d = -pq.top().first;
11        pq.pop();
12
13        if (in(node, visited)) continue;
14        visited.insert(node);

```

```

15        for (auto e : g[node]) {
16            int neigh = e.first;
17            int cost = e.second;
18            if (dist[neigh] > dist[node] + cost) {
19                dist[neigh] = dist[node] + cost;
20                pq.push({-dist[neigh], neigh});
21            }
22        }
23    }
24    return dist;
25 }
26 }

```

3.1.4 Floyd-Warshall

```

1 #include "header.h"
2 // g[i][j] = inf if not path from i to j
3 // if g[i][i] < 0, i is contained in a negative cycle
4 void warshall(vvl g) {
5     for (int i=0; i<g.size(); ++i) {
6         for (int j=0; j<g.size(); ++j) {
7             for (int k=0; k<g.size(); ++k) {
8                 if (g[i][k] < LLINF and g[k][j] < LLINF and g[i][j] > g[i][k]
9                     + g[k][j]) {
10                    g[i][j] = g[i][k] + g[k][j];
11                }
12            }
13        }
14    }
15 }

```

3.1.5 Kruskal Minimum spanning tree of undirected weighted graph

```

1 #include "header.h"
2 #include "disjoint_set.h"
3 // O(E log E)
4 pair<set<pair<ll, ll>>, ll> kruskal(vector<tuple<ll, ll, ll>>& edges, ll n)
5 {
6     set<pair<ll, ll>> ans;
7     ll cost = 0;
8
9     sort(edges.begin(), edges.end());
10    DisjointSet<ll> fs(n);
11
12    ll dist, i, j;
13    for (auto edge: edges) {
14        dist = get<0>(edge);
15        i = get<1>(edge);
16        j = get<2>(edge);
17
18        if (fs.find_set(i) != fs.find_set(j)) {
19            fs.union_sets(i, j);
20            ans.insert({i, j});
21            cost += dist;
22        }
23    }
24    return pair<set<pair<ll, ll>>, ll> {ans, cost};
25 }

```

24 }

3.1.6 Hungarian algorithm

```

1 #include "header.h"
2
3 template <class T> bool ckmin(T &a, const T &b) { return b < a ? a = b, 1 :
    0; }
4 /**
5  * Given J jobs and W workers (J <= W), computes the minimum cost to assign
    each
6  * prefix of jobs to distinct workers.
7  * @tparam T a type large enough to represent integers on the order of J *
8  * max(|C|)
9  * @param C a matrix of dimensions JxW such that C[j][w] = cost to assign j-
    th
10 * job to w-th worker (possibly negative)
11 *
12 * @return a vector of length J, with the j-th entry equaling the minimum
    cost
13 * to assign the first (j+1) jobs to distinct workers
14 */
15 template <class T> vector<T> hungarian(const vector<vector<T>> &C) {
16     const int J = (int)size(C), W = (int)size(C[0]);
17     assert(J <= W);
18     // job[w] = job assigned to w-th worker, or -1 if no job assigned
19     // note: a W-th worker was added for convenience
20     vector<int> job(W + 1, -1);
21     vector<T> ys(J), yt(W + 1); // potentials
22     // -yt[W] will equal the sum of all deltas
23     vector<T> answers;
24     const T inf = numeric_limits<T>::max();
25     for (int j_cur = 0; j_cur < J; ++j_cur) { // assign j_cur-th job
26         int w_cur = W;
27         job[w_cur] = j_cur;
28         // min reduced cost over edges from Z to worker w
29         vector<T> min_to(W + 1, inf);
30         vector<int> prv(W + 1, -1); // previous worker on alternating path
31         vector<bool> in_Z(W + 1); // whether worker is in Z
32         while (job[w_cur] != -1) { // runs at most j_cur + 1 times
33             in_Z[w_cur] = true;
34             const int j = job[w_cur];
35             T delta = inf;
36             int w_next;
37             for (int w = 0; w < W; ++w) {
38                 if (!in_Z[w]) {
39                     if (ckmin(min_to[w], C[j][w] - ys[j] - yt[w]))
40                         prv[w] = w_cur;
41                     if (ckmin(delta, min_to[w])) w_next = w;
42                 }
43             }
44             // delta will always be non-negative,
45             // except possibly during the first time this loop runs
46             // if any entries of C[j_cur] are negative

```

```

47         for (int w = 0; w <= W; ++w) {
48             if (in_Z[w]) ys[job[w]] += delta, yt[w] -= delta;
49             else min_to[w] -= delta;
50         }
51         w_cur = w_next;
52     }
53     // update assignments along alternating path
54     for (int w; w_cur != W; w_cur = w) job[w_cur] = job[w = prv[w_cur]];
55     answers.push_back(-yt[W]);
56 }
57 return answers;
58 }

```

3.1.7 Suc. shortest path Calculates max flow, min cost

```

1 #include "header.h"
2 // map<node, map<node, pair<cost, capacity>>>
3 #define graph unordered_map<int, unordered_map<int, pair<ld, int>>>
4 graph g;
5 const ld inf = 1e60; // Change if necessary
6 ld fill(int n, ld& potential) { // Finds max flow, min cost
7     priority_queue<pair<ld, int>> pq;
8     vector<bool> visited(n+2, false);
9     vi parent(n+2, 0);
10    vld dist(n+2, inf);
11    dist[0] = 0.1;
12    pq.emplace(make_pair(0.1, 0));
13    while (not pq.empty()) {
14        int node = pq.top().second;
15        pq.pop();
16        if (visited[node]) continue;
17        visited[node] = true;
18        for (auto& x : g[node]) {
19            int neigh = x.first;
20            int capacity = x.second.second;
21            ld cost = x.second.first;
22            if (capacity and not visited[neigh]) {
23                ld d = dist[node] + cost + potential[node] - potential[neigh];
24                if (d + 1e-101 < dist[neigh]) {
25                    dist[neigh] = d;
26                    pq.emplace(make_pair(-d, neigh));
27                    parent[neigh] = node;
28                }
29            }
30        }
31        for (int i = 0; i < n+2; i++) {
32            potential[i] = min(inf, potential[i] + dist[i]);
33        }
34        if (not parent[n+1]) return inf;
35        ld ans = 0.1;
36        for (int x = n+1; x; x = parent[x]) {
37            ans += g[parent[x]][x].first;
38            g[parent[x]][x].second--;
39            g[x][parent[x]].second++;
40        }

```

```

40 return ans;
41 }

```

3.1.8 Bipartite check

```

1 #include "header.h"
2 int main() {
3     int n;
4     vvi adj(n);
5
6     vi side(n, -1);    // will have 0's for one side 1's for other side
7     bool is_bipartite = true;    // becomes false if not bipartite
8     queue<int> q;
9     for (int st = 0; st < n; ++st) {
10         if (side[st] == -1) {
11             q.push(st);
12             side[st] = 0;
13             while (!q.empty()) {
14                 int v = q.front();
15                 q.pop();
16                 for (int u : adj[v]) {
17                     if (side[u] == -1) {
18                         side[u] = side[v] ^ 1;
19                         q.push(u);
20                     } else {
21                         is_bipartite &= side[u] != side[v];
22                     }
23                 }
24             }
25         }
26     }
27 }

```

3.1.9 Find cycle directed

```

1 #include "header.h"
2 int n;
3 const int mxN = 2e5+5;
4 vvi adj(mxN);
5 vector<char> color;
6 vi parent;
7 int cycle_start, cycle_end;
8 bool dfs(int v) {
9     color[v] = 1;
10    for (int u : adj[v]) {
11        if (color[u] == 0) {
12            parent[u] = v;
13            if (dfs(u)) return true;
14        } else if (color[u] == 1) {
15            cycle_end = v;
16            cycle_start = u;
17            return true;
18        }
19    }
20    color[v] = 2;
21    return false;
22 }

```

```

23 void find_cycle() {
24     color.assign(n, 0);
25     parent.assign(n, -1);
26     cycle_start = -1;
27     for (int v = 0; v < n; v++) {
28         if (color[v] == 0 && dfs(v)) break;
29     }
30     if (cycle_start == -1) {
31         cout << "Acyclic" << endl;
32     } else {
33         vector<int> cycle;
34         cycle.push_back(cycle_start);
35         for (int v = cycle_end; v != cycle_start; v = parent[v])
36             cycle.push_back(v);
37         cycle.push_back(cycle_start);
38         reverse(cycle.begin(), cycle.end());
39
40         cout << "Cycle Found: ";
41         for (int v : cycle) cout << v << " ";
42         cout << endl;
43     }
44 }

```

3.1.10 Find cycle directed

```

1 #include "header.h"
2 int n;
3 const int mxN = 2e5 + 5;
4 vvi adj(mxN);
5 vector<bool> visited;
6 vi parent;
7 int cycle_start, cycle_end;
8 bool dfs(int v, int par) { // passing vertex and its parent vertex
9     visited[v] = true;
10    for (int u : adj[v]) {
11        if (u == par) continue; // skipping edge to parent vertex
12        if (visited[u]) {
13            cycle_end = v;
14            cycle_start = u;
15            return true;
16        }
17        parent[u] = v;
18        if (dfs(u, parent[u]))
19            return true;
20    }
21    return false;
22 }
23 void find_cycle() {
24     visited.assign(n, false);
25     parent.assign(n, -1);
26     cycle_start = -1;
27     for (int v = 0; v < n; v++) {
28         if (!visited[v] && dfs(v, parent[v])) break;
29     }

```

```

30     if (cycle_start == -1) {
31         cout << "Acyclic" << endl;
32     } else {
33         vector<int> cycle;
34         cycle.push_back(cycle_start);
35         for (int v = cycle_end; v != cycle_start; v = parent[v])
36             cycle.push_back(v);
37         cycle.push_back(cycle_start);
38         cout << "Cycle Found: ";
39         for (int v : cycle) cout << v << " ";
40         cout << endl;
41     }
42 }

```

3.1.11 Tarjan's SCC

```

1 #include "header.h"
2
3 struct Tarjan {
4     vvi &edges;
5     int V, counter = 0, C = 0;
6     vi n, l;
7     vector<bool> vs;
8     stack<int> st;
9     Tarjan(vvi &e) : edges(e), V(e.size()), n(V, -1), l(V, -1), vs(V, false)
10    {}
11 void visit(int u, vi &com) {
12     l[u] = n[u] = counter++;
13     st.push(u);
14     vs[u] = true;
15     for (auto &v : edges[u]) {
16         if (n[v] == -1) visit(v, com);
17         if (vs[v]) l[u] = min(l[u], l[v]);
18     }
19     if (l[u] == n[u]) {
20         while (true) {
21             int v = st.top();
22             st.pop();
23             vs[v] = false;
24             com[v] = C; //<== ACT HERE
25             if (u == v) break;
26         }
27         C++;
28     }
29 }
30 int find_sccs(vi &com) { // component indices will be stored in 'com'
31     com.assign(V, -1);
32     C = 0;
33     for (int u = 0; u < V; ++u)
34         if (n[u] == -1) visit(u, com);
35     return C;
36 }
37 // scc is a map of the original vertices of the graph to the vertices
38 // of the SCC graph, scc_graph is its adjacency list.

```

```

38 // SCC indices and edges are stored in 'scc' and 'scc_graph'.
39 void scc_collapse(vi &scc, vvi &scc_graph) {
40     find_sccs(scc);
41     scc_graph.assign(C, vi());
42     set<pi> rec; // recorded edges
43     for (int u = 0; u < V; ++u) {
44         assert(scc[u] != -1);
45         for (int v : edges[u]) {
46             if (scc[v] == scc[u] ||
47                 rec.find({scc[u], scc[v]}) != rec.end()) continue;
48             scc_graph[scc[u]].push_back(scc[v]);
49             rec.insert({scc[u], scc[v]});
50         }
51     }
52 }
53 // Function to find sources and sinks in the SCC graph
54 // The number of edges needed to be added is max(sources.size(), sinks.size())
55 void findSourcesAndSinks(const vvi &scc_graph, vi &sources, vi &sinks) {
56     vi in_degree(C, 0), out_degree(C, 0);
57     for (int u = 0; u < C; ++u) {
58         for (auto v : scc_graph[u]) {
59             in_degree[v]++;
60             out_degree[u]++;
61         }
62     }
63     for (int i = 0; i < C; ++i) {
64         if (in_degree[i] == 0) sources.push_back(i);
65         if (out_degree[i] == 0) sinks.push_back(i);
66     }
67 }
68 };

```

3.1.12 SCC edges Prints out the missing edges to make the input digraph strongly connected

```

1 #include "header.h"
2 const int N=1e5+10;
3 int n,a[N],cnt[N],vis[N];
4 vector<int> hd,tl;
5 int dfs(int x){
6     vis[x]=1;
7     if(!vis[a[x]])return vis[x]=dfs(a[x]);
8     return vis[x]=x;
9 }
10 int main(){
11     scanf("%d",&n);
12     for(int i=1;i<=n;i++){
13         scanf("%d",&a[i]);
14         cnt[a[i]]++;
15     }
16     int k=0;
17     for(int i=1;i<=n;i++){
18         if(!cnt[i]){
19             k++;

```



```

20         hd.push_back(i);
21         tl.push_back(dfs(i));
22     }
23 }
24 int tk=k;
25 for(int i=1;i<=n;i++){
26     if(!vis[i]){
27         k++;
28         hd.push_back(i);
29         tl.push_back(dfs(i));
30     }
31 }
32 if(k==1&&!tk)k=0;
33 printf("%d\n",k);
34 for(int i=0;i<k;i++)printf("%d_ %d\n",tl[i],hd[(i+1)%k]);
35 return 0;
36 }

```

3.1.13 Find Bridges

```

1 #include "header.h"
2 int n; // number of nodes
3 vvi adj; // adjacency list of graph
4 vector<bool> visited;
5 vi tin, low;
6 int timer;
7 void dfs(int v, int p = -1) {
8     visited[v] = true;
9     tin[v] = low[v] = timer++;
10    for (int to : adj[v]) {
11        if (to == p) continue;
12        if (visited[to]) {
13            low[v] = min(low[v], tin[to]);
14        } else {
15            dfs(to, v);
16            low[v] = min(low[v], low[to]);
17            if (low[to] > tin[v])
18                IS_BRIDGE(v, to);
19        }
20    }
21 }
22 void find_bridges() {
23     timer = 0;
24     visited.assign(n, false);
25     tin.assign(n, -1);
26     low.assign(n, -1);
27     for (int i = 0; i < n; ++i) {
28         if (!visited[i]) dfs(i);
29     }
30 }

```

3.1.14 Artic. points (i.e. cut off points)

```

1 #include "header.h"
2 int n; // number of nodes
3 vvi adj; // adjacency list of graph
4 vector<bool> visited;
5 vi tin, low;
6 int timer;
7 void dfs(int v, int p = -1) {
8     visited[v] = true;
9     tin[v] = low[v] = timer++;
10    int children=0;
11    for (int to : adj[v]) {
12        if (to == p) continue;
13        if (visited[to]) {
14            low[v] = min(low[v], tin[to]);
15        } else {
16            dfs(to, v);
17            low[v] = min(low[v], low[to]);
18            if (low[to] >= tin[v] && p!=-1) IS_CUTPOINT(v);
19            ++children;
20        }
21    }
22    if(p == -1 && children > 1)
23        IS_CUTPOINT(v);
24 }
25 void find_cutpoints() {
26     timer = 0;
27     visited.assign(n, false);
28     tin.assign(n, -1);
29     low.assign(n, -1);
30     for (int i = 0; i < n; ++i) {
31         if (!visited[i]) dfs(i);
32     }
33 }

```

3.1.15 Topological sort

```

1 #include "header.h"
2 int n; // number of vertices
3 vvi adj; // adjacency list of graph
4 vector<bool> visited;
5 vi ans;
6 void dfs(int v) {
7     visited[v] = true;
8     for (int u : adj[v]) {
9         if (!visited[u]) dfs(u);
10    }
11    ans.push_back(v);
12 }
13 void topological_sort() {
14     visited.assign(n, false);
15     ans.clear();
16     for (int i = 0; i < n; ++i) {
17         if (!visited[i]) dfs(i);
18     }

```

```

19     reverse(ans.begin(), ans.end());
20 }

```

3.1.16 Bellmann-Ford Same as Dijkstra but allows neg. edges

```

1 #include "header.h"
2 // Switch vi and vvpj to vl and vvpl if necessary
3 void bellmann_ford_extended(vvpj &e, int source, vi &dist, vb &cyc) {
4     dist.assign(e.size(), INF);
5     cyc.assign(e.size(), false); // true when u is in a <0 cycle
6     dist[source] = 0;
7     for (int iter = 0; iter < e.size() - 1; ++iter){
8         bool relax = false;
9         for (int u = 0; u < e.size(); ++u)
10             if (dist[u] == INF) continue;
11             else for (auto &e : e[u])
12                 if (dist[u]+e.second < dist[e.first])
13                     dist[e.first] = dist[u]+e.second, relax = true;
14         if(!relax) break;
15     }
16     bool ch = true;
17     while (ch) { // keep going untill no more changes
18         ch = false; // set dist to -INF when in cycle
19         for (int u = 0; u < e.size(); ++u)
20             if (dist[u] == INF) continue;
21             else for (auto &e : e[u])
22                 if (dist[e.first] > dist[u] + e.second
23                     && !cyc[e.first]) {
24                     dist[e.first] = -INF;
25                     ch = true; //return true for cycle detection only
26                     cyc[e.first] = true;
27                 }
28     }
29 }

```

3.2 Dynamic Programming

3.2.1 Longest Increasing Subsequence

```

1 #include "header.h"
2 template<class T>
3 vector<T> index_path_lis(vector<T>& nums) {
4     int n = nums.size();
5     vector<T> sub;
6     vector<int> subIndex;
7     vector<T> path(n, -1);
8     for (int i = 0; i < n; ++i) {
9         if (sub.empty() || sub[sub.size() - 1] < nums[i]) {
10             path[i] = sub.empty() ? -1 : subIndex[sub.size() - 1];
11             sub.push_back(nums[i]);
12             subIndex.push_back(i);
13         } else {
14             int idx = lower_bound(sub.begin(), sub.end(), nums[i]) - sub.begin();

```

```

15         path[i] = idx == 0 ? -1 : subIndex[idx - 1];
16         sub[idx] = nums[i];
17         subIndex[idx] = i;
18     }
19 }
20 vector<T> ans;
21 int t = subIndex[subIndex.size() - 1];
22 while (t != -1) {
23     ans.push_back(t);
24     t = path[t];
25 }
26 reverse(ans.begin(), ans.end());
27 return ans;
28 }
29 // Length only
30 template<class T>
31 int length_lis(vector<T> &a) {
32     set<T> st;
33     typename set<T>::iterator it;
34     for (int i = 0; i < a.size(); ++i) {
35         it = st.lower_bound(a[i]);
36         if (it != st.end()) st.erase(it);
37         st.insert(a[i]);
38     }
39     return st.size();
40 }

```

3.2.2 0-1 Knapsack

```

1 #include "header.h"
2 // given a number of coins, calculate all possible distinct sums
3 int main() {
4     int n;
5     vi coins(n); // all possible coins to use
6     int sum = 0; // sum of the coins
7     vi dp(sum + 1, 0); // dp[x] = 1 if sum x can be made
8     dp[0] = 1; // sum 0 can be made
9     for (int c = 0; c < n; ++c) // first iteration: sums with first
10         for (int x = sum; x >= 0; --x) // coin, next first 2 coins etc
11             if (dp[x]) dp[x + coins[c]] = 1; // if sum x valid, x+c valid
12 }

```

3.3 Trees

3.3.1 Tree diameter

```

1 #include "header.h"
2 const int mxN = 2e5 + 5;
3 int n, d[mxN]; // distance array
4 vi adj[mxN]; // tree adjacency list
5 void dfs(int s, int e) {
6     d[s] = 1 + d[e]; // recursively calculate the distance from the
7                     // starting node to each node

```

```

7   for (auto u : adj[s]) { // for each adjacent node
8       if (u != e) dfs(u, s); // don't move backwards in the tree
9   }
10 }
11 int main() {
12     // read input, create adj list
13     dfs(0, -1); // first dfs call to find farthest node from
                  // arbitrary node
14     dfs(distance(d, max_element(d, d + n)), -1); // second dfs call to find
                  // farthest node from that one
15     cout << *max_element(d, d + n) - 1 << '\n'; // distance from second node
                  // to farthest is the diameter
16 }

```

3.3.2 Tree Node Count

```

1 #include "header.h"
2 // calculate amount of nodes in each node's subtree
3 const int mxN = 2e5 + 5;
4 int n, cnt[mxN];
5 vi adj[mxN];
6 void dfs(int s = 0, int e = -1) {
7     cnt[s] = 1; // count leaves as one
8     for (int u : adj[s]) {
9         dfs(u, s);
10        cnt[s] += cnt[u]; // add up nodes of the subtrees
11    }
12 }

```

3.4 Num. Th. / Comb.

3.4.1 Basic stuff

```

1 #include "header.h"
2 ll gcd(ll a, ll b) { while (b) { a %= b; swap(a, b); } return a; }
3 ll lcm(ll a, ll b) { return (a / gcd(a, b)) * b; }
4 ll mod(ll a, ll b) { return ((a % b) + b) % b; }
5 // Finds x, y s.t. ax + by = d = gcd(a, b).
6 void extended_euclid(ll a, ll b, ll &x, ll &y, ll &d) {
7     ll xx = y = 0;
8     ll yy = x = 1;
9     while (b) {
10        ll q = a / b;
11        ll t = b; b = a % b; a = t;
12        t = xx; xx = x - q * xx; x = t;
13        t = yy; yy = y - q * yy; y = t;
14    }
15    d = a;
16 }
17 // solves ab = 1 (mod n), -1 on failure
18 ll mod_inverse(ll a, ll n) {
19     ll x, y, d;
20     extended_euclid(a, n, x, y, d);

```

```

21     return (d > 1 ? -1 : mod(x, n));
22 }
23 // All modular inverses of [1..n] mod P in O(n) time.
24 vi inverses(ll n, ll P) {
25     vi I(n+1, 1LL);
26     for (ll i = 2; i <= n; ++i)
27         I[i] = mod(-(P/i) * I[P%i], P);
28     return I;
29 }
30 // (a*b)%m
31 ll mulmod(ll a, ll b, ll m){
32     ll x = 0, y=a%m;
33     while(b>0){
34         if(b&1) x = (x+y)%m;
35         y = (2*y)%m, b /= 2;
36     }
37     return x % m;
38 }
39 // Finds b^e % m in O(lg n) time, ensure that b < m to avoid overflow!
40 ll powmod(ll b, ll e, ll m) {
41     ll p = e<2 ? 1 : powmod((b*b)%m,e/2,m);
42     return e&1 ? p*b%m : p;
43 }
44 // Solve ax + by = c, returns false on failure.
45 bool linear_diophantine(ll a, ll b, ll c, ll &x, ll &y) {
46     ll d = gcd(a, b);
47     if (c % d) {
48         return false;
49     } else {
50         x = c / d * mod_inverse(a / d, b / d);
51         y = (c - a * x) / b;
52         return true;
53     }
54 }

```

3.4.2 Mod. exponentiation Or use pow() in python

```

1 #include "header.h"
2 ll mod_pow(ll base, ll exp, ll mod) {
3     if (mod == 1) return 0;
4     if (exp == 0) return 1;
5     if (exp == 1) return base;
6
7     ll res = 1;
8     base %= mod;
9     while (exp) {
10        if (exp % 2 == 1) res = (res * base) % mod;
11        exp >>= 1;
12        base = (base * base) % mod;
13    }
14
15    return res % mod;
16 }

```

3.4.3 GCD Or math.gcd in python, std::gcd in C++

```
1 #include "header.h"
2 ll gcd(ll a, ll b) {
3     if (a == 0) return b;
4     return gcd(b % a, a);
5 }
```

3.4.4 Sieve of Eratosthenes

```
1 #include "header.h"
2 vl primes;
3 void getprimes(ll n) { // Up to n (not included)
4     vector<bool> p(n, true);
5     p[0] = false;
6     p[1] = false;
7     for(ll i = 0; i < n; i++) {
8         if(p[i]) {
9             primes.push_back(i);
10            for(ll j = i*2; j < n; j+=i) p[j] = false;
11 }}}}
```

3.4.5 Fibonacci % prime

```
1 #include "header.h"
2 const ll MOD = 1000000007;
3 unordered_map<ll, ll> Fib;
4 ll fib(ll n) {
5     if (n < 2) return 1;
6     if (Fib.find(n) != Fib.end()) return Fib[n];
7     Fib[n] = (fib((n + 1) / 2) * fib(n / 2) + fib((n - 1) / 2) * fib((n - 2) / 2)) % MOD;
8     return Fib[n];
9 }
```

3.4.6 nCk % prime

```
1 #include "header.h"
2 ll binom(ll n, ll k) {
3     ll ans = 1;
4     for(ll i = 1; i <= min(k, n-k); ++i) ans = ans*(n+1-i)/i;
5     return ans;
6 }
7 ll mod_nCk(ll n, ll k, ll p){
8     ll ans = 1;
9     while(n){
10        ll np = n%p, kp = k%p;
11        if(kp > np) return 0;
12        ans *= binom(np, kp);
13        n /= p; k /= p;
14    }
15    return ans;
```

```
16 }
```

3.4.7 Chin. rem. th.

```
1 #include "header.h"
2 #include "elementary.cpp"
3 // Solves x = a1 mod m1, x = a2 mod m2, x is unique modulo lcm(m1, m2).
4 // Returns {0, -1} on failure, {x, lcm(m1, m2)} otherwise.
5 pair<ll, ll> crt(ll a1, ll m1, ll a2, ll m2) {
6     ll s, t, d;
7     extended_euclid(m1, m2, s, t, d);
8     if (a1 % d != a2 % d) return {0, -1};
9     return {mod(s*a2 %m2 * m1 + t*a1 %m1 * m2, m1 * m2) / d, m1 / d * m2};
10 }
11
12 // Solves x = ai mod mi. x is unique modulo lcm mi.
13 // Returns {0, -1} on failure, {x, lcm mi} otherwise.
14 pair<ll, ll> crt(vector<ll> &a, vector<ll> &m) {
15     pair<ll, ll> res = {a[0], m[0]};
16     for (ull i = 1; i < a.size(); ++i) {
17         res = crt(res.first, res.second, mod(a[i], m[i]), m[i]);
18         if (res.second == -1) break;
19     }
20     return res;
21 }
```

3.5 Strings

3.5.1 Z alg. KMP alternative

```
1 #include "../header.h"
2 void Z_algorithm(const string &s, vi &Z) {
3     Z.assign(s.length(), -1);
4     int L = 0, R = 0, n = s.length();
5     for (int i = 1; i < n; ++i) {
6         if (i > R) {
7             L = R = i;
8             while (R < n && s[R - L] == s[R]) R++;
9             Z[i] = R - L; R--;
10        } else if (Z[i - L] >= R - i + 1) {
11            L = i;
12            while (R < n && s[R - L] == s[R]) R++;
13            Z[i] = R - L; R--;
14        } else Z[i] = Z[i - L];
15    }
16 }
```

3.5.2 KMP

```
1 #include "header.h"
2 void compute_prefix_function(string &w, vi &prefix) {
```

```

3  prefix.assign(w.length(), 0);
4  int k = prefix[0] = -1;
5
6  for(int i = 1; i < w.length(); ++i) {
7      while(k >= 0 && w[k + 1] != w[i]) k = prefix[k];
8      if(w[k + 1] == w[i]) k++;
9      prefix[i] = k;
10 }
11 }
12 void knuth_morris_pratt(string &s, string &w) {
13     int q = -1;
14     vi prefix;
15     compute_prefix_function(w, prefix);
16     for(int i = 0; i < s.length(); ++i) {
17         while(q >= 0 && w[q + 1] != s[i]) q = prefix[q];
18         if(w[q + 1] == s[i]) q++;
19         if(q + 1 == w.length()) {
20             // Match at position (i - w.length() + 1)
21             q = prefix[q];
22         }
23     }
24 }

```

3.5.3 Aho-Corasick Also can be used as Knuth-Morris-Pratt algorithm

```

1  #include "header.h"
2
3  map<char, int> cti;
4  int cti_size;
5  template <int ALPHABET_SIZE, int (*mp)(char)>
6  struct AC_FSM {
7      struct Node {
8          int child[ALPHABET_SIZE], failure = 0, match_par = -1;
9          vi match;
10         Node() { for (int i = 0; i < ALPHABET_SIZE; ++i) child[i] = -1; }
11     };
12     vector<Node> a;
13     vector<string> &words;
14     AC_FSM(vector<string> &words) : words(words) {
15         a.push_back(Node());
16         construct_automaton();
17     }
18     void construct_automaton() {
19         for (int w = 0, n = 0; w < words.size(); ++w, n = 0) {
20             for (int i = 0; i < words[w].size(); ++i) {
21                 if (a[n].child[mp(words[w][i])] == -1) {
22                     a[n].child[mp(words[w][i])] = a.size();
23                     a.push_back(Node());
24                 }
25                 n = a[n].child[mp(words[w][i])];
26             }
27             a[n].match.push_back(w);
28         }
29         queue<int> q;

```

```

30         for (int k = 0; k < ALPHABET_SIZE; ++k) {
31             if (a[0].child[k] == -1) a[0].child[k] = 0;
32             else if (a[0].child[k] > 0) {
33                 a[a[0].child[k]].failure = 0;
34                 q.push(a[0].child[k]);
35             }
36         }
37         while (!q.empty()) {
38             int r = q.front(); q.pop();
39             for (int k = 0, arck; k < ALPHABET_SIZE; ++k) {
40                 if ((arck = a[r].child[k]) != -1) {
41                     q.push(arck);
42                     int v = a[r].failure;
43                     while (a[v].child[k] == -1) v = a[v].failure;
44                     a[arck].failure = a[v].child[k];
45                     a[arck].match_par = a[v].child[k];
46                     while (a[arck].match_par != -1
47                         && a[a[arck].match_par].match.empty())
48                         a[arck].match_par = a[a[arck].match_par].match_par;
49                 }
50             }
51         }
52     }
53     void aho_corasick(string &sentence, vvi &matches){
54         matches.assign(words.size(), vi());
55         int state = 0, ss = 0;
56         for (int i = 0; i < sentence.length(); ++i, ss = state) {
57             while (a[ss].child[mp(sentence[i])] == -1)
58                 ss = a[ss].failure;
59             state = a[state].child[mp(sentence[i])]
60                 = a[ss].child[mp(sentence[i])];
61             for (ss = state; ss != -1; ss = a[ss].match_par)
62                 for (int w : a[ss].match)
63                     matches[w].push_back(i + 1 - words[w].length());
64         }
65     }
66 };
67 int char_to_int(char c) {
68     return cti[c];
69 }

```

3.5.4 Long. palin. subs Manacher - $O(n)$

```

1  #include "header.h"
2  void manacher(string &s, vi &pal) {
3      int n = s.length(), i = 1, l, r;
4      pal.assign(2 * n + 1, 0);
5      while (i < 2 * n + 1) {
6          if ((i & 1) && pal[i] == 0) pal[i] = 1;
7          l = i / 2 - pal[i] / 2; r = (i - 1) / 2 + pal[i] / 2;
8
9          while (l - 1 >= 0 && r + 1 < n && s[l - 1] == s[r + 1])
10              --l, ++r, pal[i] += 2;
11

```

```

12 for (l = i - 1, r = i + 1; l >= 0 && r < 2 * n + 1; --l, ++r) {
13     if (l <= i - pal[l]) break;
14     if (l / 2 - pal[l] / 2 > i / 2 - pal[i] / 2)
15         pal[r] = pal[l];
16     else { if (l >= 0)
17         pal[r] = min(pal[l], i + pal[i] - r);
18         break;
19     }
20 }
21 i = r;
22 } }

```

3.6 Geometry

3.6.1 essentials.cpp

```

1 #include "../header.h"
2 using C = ld; // could be long long or long double
3 constexpr C EPS = 1e-10; // change to 0 for C=ll
4 struct P { // may also be used as a 2D vector
5     C x, y;
6     P(C x = 0, C y = 0) : x(x), y(y) {}
7     P operator+ (const P &p) const { return {x + p.x, y + p.y}; }
8     P operator- (const P &p) const { return {x - p.x, y - p.y}; }
9     P operator* (C c) const { return {x * c, y * c}; }
10    P operator/ (C c) const { return {x / c, y / c}; }
11    C operator* (const P &p) const { return x*p.x + y*p.y; }
12    C operator^ (const P &p) const { return x*p.y - p.x*y; }
13    P perp() const { return P{y, -x}; }
14    C lensq() const { return x*x + y*y; }
15    ld len() const { return sqrt((ld)lensq()); }
16    static ld dist(const P &p1, const P &p2) {
17        return (p1-p2).len(); }
18    bool operator==(const P &r) const {
19        return ((*this)-r).lensq() <= EPS*EPS; }
20 };
21 C det(P p1, P p2) { return p1^p2; }
22 C det(P p1, P p2, P o) { return det(p1-o, p2-o); }
23 C det(const vector<P> &ps) {
24     C sum = 0; P prev = ps.back();
25     for(auto &p : ps) sum += det(p, prev), prev = p;
26     return sum;
27 }
28 // Careful with division by two and C=ll
29 C area(P p1, P p2, P p3) { return abs(det(p1, p2, p3))/C(2); }
30 C area(const vector<P> &poly) { return abs(det(poly))/C(2); }
31 int sign(C c){ return (c > C(0)) - (c < C(0)); }
32 int ccw(P p1, P p2, P o) { return sign(det(p1, p2, o)); }
33
34 // Only well defined for C = ld.
35 P unit(const P &p) { return p / p.len(); }
36 P rotate(P p, ld a) { return P{p.x*cos(a)-p.y*sin(a), p.x*sin(a)+p.y*cos(a)}; }

```

3.6.2 Two segs. itersec.

```

1 #include "header.h"
2 #include "essentials.cpp"
3 bool intersect(P a1, P a2, P b1, P b2) {
4     if (max(a1.x, a2.x) < min(b1.x, b2.x)) return false;
5     if (max(b1.x, b2.x) < min(a1.x, a2.x)) return false;
6     if (max(a1.y, a2.y) < min(b1.y, b2.y)) return false;
7     if (max(b1.y, b2.y) < min(a1.y, a2.y)) return false;
8     bool l1 = ccw(a2, b1, a1) * ccw(a2, b2, a1) <= 0;
9     bool l2 = ccw(b2, a1, b1) * ccw(b2, a2, b1) <= 0;
10    return l1 && l2;
11 }

```

3.6.3 Convex Hull

```

1 #include "header.h"
2 #include "essentials.cpp"
3 struct ConvexHull { // O(n lg n) monotone chain.
4     size_t n;
5     vector<size_t> h, c; // Indices of the hull are in 'h', ccw.
6     const vector<P> &p;
7     ConvexHull(const vector<P> &p) : n(p.size()), c(n), p(p) {
8         std::iota(c.begin(), c.end(), 0);
9         std::sort(c.begin(), c.end(), [this](size_t l, size_t r) -> bool {
10             return p[l].x != p[r].x ? p[l].x < p[r].x : p[l].y < p[r].y; });
11         c.erase(std::unique(c.begin(), c.end(), [this](size_t l, size_t r) {
12             return p[l] == p[r]; }), c.end());
13         for (size_t s = 1, r = 0; r < 2; ++r, s = h.size()) {
14             for (size_t i : c) {
15                 while (h.size() > s && ccw(p[h.end()[-2]], p[h.end()[-1]], p[i]) <= 0)
16                     h.pop_back();
17                 h.push_back(i);
18             }
19             reverse(c.begin(), c.end());
20         }
21         if (h.size() > 1) h.pop_back();
22     }
23     size_t size() const { return h.size(); }
24     template <class T, void U(const P &, const P &, const P &, T &)>
25     void rotating_calipers(T &ans) {
26         if (size() <= 2)
27             U(p[h[0]], p[h.back()], p[h.back()], ans);
28         else
29             for (size_t i = 0, j = 1, s = size(); i < 2 * s; ++i) {
30                 while (det(p[h[(i + 1) % s]], p[h[i % s]], p[h[(j + 1) % s]] - p[h[j % s]]) >= 0)
31                     j = (j + 1) % s;
32                 U(p[h[i % s]], p[h[(i + 1) % s]], p[h[j % s]], ans);
33             }
34     };
35 // Example: furthest pair of points. Now set ans = 0LL and call
36 // ConvexHull(pts).rotating_calipers<ll, update>(ans);

```

```

36 void update(const P &p1, const P &p2, const P &o, ll &ans) {
37     ans = max(ans, (ll)max((p1 - o).lensq(), (p2 - o).lensq()));
38 }

```

3.7 Other Algorithms

3.7.1 2-sat

```

1 #include "../header.h"
2 #include "../Graphs/tarjan.cpp"
3 struct TwoSAT {
4     int n;
5     vvi imp; // implication graph
6     Tarjan tj;
7
8     TwoSAT(int _n) : n(_n), imp(2 * _n, vi()), tj(imp) {}
9
10    // Only copy the needed functions:
11    void add_implies(int c1, bool v1, int c2, bool v2) {
12        int u = 2 * c1 + (v1 ? 1 : 0),
13            v = 2 * c2 + (v2 ? 1 : 0);
14        imp[u].push_back(v); // u => v
15        imp[v^1].push_back(u^1); // -v => -u
16    }
17    void add_equivalence(int c1, bool v1, int c2, bool v2) {
18        add_implies(c1, v1, c2, v2);
19        add_implies(c2, v2, c1, v1);
20    }
21    void add_or(int c1, bool v1, int c2, bool v2) {
22        add_implies(c1, !v1, c2, v2);
23    }
24    void add_and(int c1, bool v1, int c2, bool v2) {
25        add_true(c1, v1); add_true(c2, v2);
26    }
27    void add_xor(int c1, bool v1, int c2, bool v2) {
28        add_or(c1, v1, c2, v2);
29        add_or(c1, !v1, c2, !v2);
30    }
31    void add_true(int c1, bool v1) {
32        add_implies(c1, !v1, c1, v1);
33    }
34
35    // on true: a contains an assignment.
36    // on false: no assignment exists.
37    bool solve(vb &a) {
38        vi com;
39        tj.find_sccs(com);
40        for (int i = 0; i < n; ++i)
41            if (com[2 * i] == com[2 * i + 1])
42                return false;
43
44        vvi bycom(com.size());
45        for (int i = 0; i < 2 * n; ++i)
46            bycom[com[i]].push_back(i);

```

```

47
48        a.assign(n, false);
49        vb vis(n, false);
50        for(auto &&component : bycom){
51            for (int u : component) {
52                if (vis[u / 2]) continue;
53                vis[u / 2] = true;
54                a[u / 2] = (u % 2 == 1);
55            }
56        }
57        return true;
58    }
59 };

```

3.7.2 Matrix Solve

```

1 #include "header.h"
2 #define REP(i, n) for(auto i = decltype(n)(0); i < (n); i++)
3 using T = double;
4 constexpr T EPS = 1e-8;
5 template<int R, int C>
6 using M = array<array<T,C>,R>; // matrix
7 template<int R, int C>
8 T ReducedRowEchelonForm(M<R,C> &m, int rows) { // return the determinant
9     int r = 0; T det = 1; // MODIFIES the input
10    for(int c = 0; c < rows && r < rows; c++) {
11        int p = r;
12        for(int i=r+1; i<rows; i++) if(abs(m[i][c]) > abs(m[p][c])) p=i;
13        if(abs(m[p][c]) < EPS){ det = 0; continue; }
14        swap(m[p], m[r]); det = -det;
15        T s = 1.0 / m[r][c], t; det *= m[r][c];
16        REP(j,C) m[r][j] *= s; // make leading term in row 1
17        REP(i,rows) if (i!=r){ t = m[i][c]; REP(j,C) m[i][j] -= t*m[r][j]; }
18        ++r;
19    }
20    return det;
21 }
22 bool error, inconst; // error => multiple or inconsistent
23 template<int R,int C> // Mx = a; M:R*R, v:R*C => x:R*C
24 M<R,C> solve(const M<R,R> &m, const M<R,C> &a, int rows){
25     M<R,R+C> q;
26     REP(r,rows){
27         REP(c,rows) q[r][c] = m[r][c];
28         REP(c,C) q[r][R+c] = a[r][c];
29     }
30     ReducedRowEchelonForm<R,R+C>(q,rows);
31     M<R,C> sol; error = false, inconst = false;
32     REP(c,C) for(auto j = rows-1; j >= 0; --j){
33         T t=0; bool allzero=true;
34         for(auto k = j+1; k < rows; ++k)
35             t += q[j][k]*sol[k][c], allzero &= abs(q[j][k]) < EPS;
36         if(abs(q[j][j]) < EPS)
37             error = true, inconst |= allzero && abs(q[j][R+c]) > EPS;
38         else sol[j][c] = (q[j][R+c] - t) / q[j][j]; // usually q[j][j]=1

```

3.7.3 Matrix Exp.

```

39 }
40 return sol;
41 }

```

```

1 #include "header.h"
2 #define ITERATE_MATRIX(w) for (int r = 0; r < (w); ++r) \
3     for (int c = 0; c < (w); ++c)
4 template <class T, int N>
5 struct M {
6     array<array<T,N>,N> m;
7     M() { ITERATE_MATRIX(N) m[r][c] = 0; }
8     static M id() {
9         M I; for (int i = 0; i < N; ++i) I.m[i][i] = 1; return I;
10    }
11    M operator*(const M &rhs) const {
12        M out;
13        ITERATE_MATRIX(N) for (int i = 0; i < N; ++i)
14            out.m[r][c] += m[r][i] * rhs.m[i][c];
15        return out;
16    }
17    M raise(ll n) const {
18        if(n == 0) return id();
19        if(n == 1) return *this;
20        auto r = (*this**this).raise(n / 2);
21        return (n%2 ? *this*r : r);
22    }
23 };

```

3.7.4 Finite field For FFT

```

1 #include "header.h"
2 #include "../Number_Theory/elementary.cpp"
3 template<ll p,ll w> // prime, primitive root
4 struct Field { using T = Field; ll x; Field(ll x=0) : x{x} {}
5     T operator+(T r) const { return {(x+r.x)%p}; }
6     T operator-(T r) const { return {(x-r.x+p)%p}; }
7     T operator*(T r) const { return {(x*r.x)%p}; }
8     T operator/(T r) const { return (*this)*r.inv(); }
9     T inv() const { return {mod_inverse(x,p)}; }
10    static T root(ll k) { assert( (p-1)%k==0 ); // (p-1)%k == 0?
11        auto r = powmod(w,(p-1)/abs(k),p); // k-th root of unity
12        return k>0 ? T{r} : T{r}.inv();
13    }
14    bool zero() const { return x == 0LL; }
15 };
16 using F1 = Field<1004535809,3 >;
17 using F2 = Field<1107296257,10>; // 1<<30 + 1<<25 + 1
18 using F3 = Field<2281701377,3 >; // 1<<31 + 1<<27 + 1

```

3.7.5 Complex field For FFR

```

1 #include "header.h"
2 const double m_pi = M_PI/64;
3 struct Complex { using T = Complex; double u,v;
4     Complex(double u=0, double v=0) : u{u}, v{v} {}
5     T operator+(T r) const { return {u+r.u, v+r.v}; }
6     T operator-(T r) const { return {u-r.u, v-r.v}; }
7     T operator*(T r) const { return {u*r.u - v*r.v, u*r.v + v*r.u}; }
8     T operator/(T r) const {
9         auto norm = r.u*r.u+r.v*r.v;
10        return {(u*r.u + v*r.v)/norm, (v*r.u - u*r.v)/norm};
11    }
12    T operator*(double r) const { return T{u*r, v*r}; }
13    T operator/(double r) const { return T{u/r, v/r}; }
14    T inv() const { return T{1,0}/ *this; }
15    T conj() const { return T{u, -v}; }
16    static T root(ll k){ return {cos(2*m_pi/k), sin(2*m_pi/k)}; }
17    bool zero() const { return max(abs(u), abs(v)) < 1e-6; }
18 };

```

3.7.6 FFT

```

1 #include "header.h"
2 #include "complex_field.cpp"
3 #include "fin_field.cpp"
4 void brinc(int &x, int k) {
5     int i = k - 1, s = 1 << i;
6     x ^= s;
7     if ((x & s) != s) {
8         --i; s >>= 1;
9         while (i >= 0 && ((x & s) == s))
10             x = x &~ s, --i, s >>= 1;
11         if (i >= 0) x |= s;
12     }
13 }
14 using T = Complex; // using T=F1,F2,F3
15 vector<T> roots;
16 void root_cache(int N) {
17     if (N == (int)roots.size()) return;
18     roots.assign(N, T{0});
19     for (int i = 0; i < N; ++i)
20         roots[i] = ((i&1) == 1)
21             ? T{cos(2.0*m_pi*i/N), sin(2.0*m_pi*i/N)}
22             : roots[i&1] * roots[i-(i&1)];
23 }
24 void fft(vector<T> &A, int p, bool inv = false) {
25     int N = 1<<p;
26     for(int i = 0, r = 0; i < N; ++i, brinc(r, p))
27         if (i < r) swap(A[i], A[r]);
28     // Uncomment to precompute roots (for T=Complex). Slower but more precise.
29     // root_cache(N);
30     // , sh=p-1 , --sh
31     for (int m = 2; m <= N; m <= 1) {
32         T w, w_m = T::root(inv ? -m : m);

```



```

33 for (int k = 0; k < N; k += m) {
34     w = T{1};
35     for (int j = 0; j < m/2; ++j) {
36 //         T w = (!inv ? roots[j<<sh] : roots[j<<sh].conj());
37         T t = w * A[k + j + m/2];
38         A[k + j + m/2] = A[k + j] - t;
39         A[k + j] = A[k + j] + t;
40         w = w * w_m;
41     }
42 }
43 }
44 if(inv){ T inverse = T(N).inv(); for(auto &x : A) x = x*inverse; }
45 }
46 // convolution leaves A and B in frequency domain state
47 // C may be equal to A or B for in-place convolution
48 void convolution(vector<T> &A, vector<T> &B, vector<T> &C){
49     int s = A.size() + B.size() - 1;
50     int q = 32 - __builtin_clz(s-1), N=1<<q; // fails if s=1
51     A.resize(N,{}); B.resize(N,{}); C.resize(N,{});
52     fft(A, q, false); fft(B, q, false);
53     for (int i = 0; i < N; ++i) C[i] = A[i] * B[i];
54     fft(C, q, true); C.resize(s);
55 }
56 void square_inplace(vector<T> &A) {
57     int s = 2*A.size()-1, q = 32 - __builtin_clz(s-1), N=1<<q;
58     A.resize(N,{}); fft(A, q, false);
59     for(auto &x : A) x = x*x;
60     fft(A, q, true); A.resize(s);
61 }

```

3.7.7 Polyn. inv. div.

```

1 #include "header.h"
2 #include "fft.cpp"
3 vector<T> &rev(vector<T> &A) { reverse(A.begin(), A.end()); return A; }
4 void copy_into(const vector<T> &A, vector<T> &B, size_t n) {
5     std::copy(A.begin(), A.begin()+min({n, A.size(), B.size()}), B.begin());
6 }
7
8 // Multiplicative inverse of A modulo x^n. Requires A[0] != 0!!
9 vector<T> inverse(const vector<T> &A, int n) {
10     vector<T> Ai{A[0].inv()};
11     for (int k = 0; (1<<k) < n; ++k) {
12         vector<T> As(4<<k, T(0)), Ais(4<<k, T(0));
13         copy_into(A, As, 2<<k); copy_into(Ai, Ais, Ai.size());
14         fft(As, k+2, false); fft(Ais, k+2, false);
15         for (int i = 0; i < (4<<k); ++i) As[i] = As[i]*Ais[i]*Ais[i];
16         fft(As, k+2, true); Ai.resize(2<<k, {});
17         for (int i = 0; i < (2<<k); ++i) Ai[i] = T(2) * Ai[i] - As[i];
18     }
19     Ai.resize(n);
20     return Ai;
21 }
22 // Polynomial division. Returns {Q, R} such that A = QB+R, deg R < deg B.

```

```

23 // Requires that the leading term of B is nonzero.
24 pair<vector<T>, vector<T>> divmod(const vector<T> &A, const vector<T> &B) {
25     size_t n = A.size()-1, m = B.size()-1;
26     if (n < m) return {vector<T>(1, T(0)), A};
27
28     vector<T> X(A), Y(B), Q, R;
29     convolution(rev(X), Y = inverse(rev(Y), n-m+1), Q);
30     Q.resize(n-m+1); rev(Q);
31
32     X.resize(Q.size()), copy_into(Q, X, Q.size());
33     Y.resize(B.size()), copy_into(B, Y, B.size());
34     convolution(X, Y, X);
35
36     R.resize(m), copy_into(A, R, m);
37     for (size_t i = 0; i < m; ++i) R[i] = R[i] - X[i];
38     while (R.size() > 1 && R.back().zero()) R.pop_back();
39     return {Q, R};
40 }
41 vector<T> mod(const vector<T> &A, const vector<T> &B) {
42     return divmod(A, B).second;
43 }

```

3.7.8 Linear recurs. Given a linear recurrence of the form

$$a_n = \sum_{i=0}^{k-1} c_i a_{n-i-1}$$

this code computes a_n in $O(k \log k \log n)$ time.

```

1 #include "header.h"
2 #include "poly.cpp"
3 // x^k mod f
4 vector<T> xmod(const vector<T> f, ll k) {
5     vector<T> r{f[1]};
6     for (int b = 62; b >= 0; --b) {
7         if (r.size() > 1)
8             square_inplace(r), r = mod(r, f);
9         if ((k>>b)&1) {
10             r.insert(r.begin(), T(0));
11             if (r.size() == f.size()) {
12                 T c = r.back() / f.back();
13                 for (size_t i = 0; i < f.size(); ++i)
14                     r[i] = r[i] - c * f[i];
15                 r.pop_back();
16             }
17         }
18     }
19     return r;
20 }
21 // Given A[0,k) and C[0, k), computes the n-th term of:
22 // A[n] = \sum_i C[i] * A[n-i-1]
23 T nth_term(const vector<T> &A, const vector<T> &C, ll n) {
24     int k = (int)A.size();

```

```

25 if (n < k) return A[n];
26
27 vector<T> f(k+1, T{1});
28 for (int i = 0; i < k; ++i)
29     f[i] = T{-1} * C[k-i-1];
30 f = xmod(f, n);
31
32 T r = T{0};
33 for (int i = 0; i < k; ++i)
34     r = r + f[i] * A[i];
35 return r;
36 }

```

3.7.9 Convolution Precise up to 9e15

```

1 #include "header.h"
2 #include "fft.cpp"
3 void convolution_mod(const vi &A, const vi &B, ll MOD, vi &C) {
4     int s = A.size() + B.size() - 1; ll m15 = (1LL<<15)-1LL;
5     int q = 32 - __builtin_clz(s-1), N=1<<q; // fails if s=1
6     vector<T> Ac(N), Bc(N), R1(N), R2(N);
7     for (size_t i = 0; i < A.size(); ++i) Ac[i] = T{A[i]&m15, A[i]>>15};
8     for (size_t i = 0; i < B.size(); ++i) Bc[i] = T{B[i]&m15, B[i]>>15};
9     fft(Ac, q, false); fft(Bc, q, false);
10    for (int i = 0, j = 0; i < N; ++i, j = (N-1)&(N-i)) {
11        T as = (Ac[i] + Ac[j].conj()) / 2;
12        T al = (Ac[i] - Ac[j].conj()) / T{0, 2};
13        T bs = (Bc[i] + Bc[j].conj()) / 2;
14        T bl = (Bc[i] - Bc[j].conj()) / T{0, 2};
15        R1[i] = as*bs + al*bl*T{0,1}, R2[i] = as*bl + al*bs;
16    }
17    fft(R1, q, true); fft(R2, q, true);
18    ll p15 = (1LL<<15)%MOD, p30 = (1LL<<30)%MOD; C.resize(s);
19    for (int i = 0; i < s; ++i) {
20        ll l = llround(R1[i].u), m = llround(R2[i].u), h = llround(R1[i].v);
21        C[i] = (l + m*p15 + h*p30) % MOD;
22    }
23 }

```

3.8 Other Data Structures

3.8.1 Disjoint set (i.e. union-find)

```

1 template <typename T>
2 class DisjointSet {
3     typedef T * iterator;
4     T *parent, n, *rank;
5     public:
6         // O(n), assumes nodes are [0, n)
7         DisjointSet(T n) {
8             this->parent = new T[n];
9             this->n = n;
10            this->rank = new T[n];

```

```

11
12        for (T i = 0; i < n; i++) {
13            parent[i] = i;
14            rank[i] = 0;
15        }
16    }
17
18    // O(log n)
19    T find_set(T x) {
20        if (x == parent[x]) return x;
21        return parent[x] = find_set(parent[x]);
22    }
23
24    // O(log n)
25    void union_sets(T x, T y) {
26        x = this->find_set(x);
27        y = this->find_set(y);
28
29        if (x == y) return;
30
31        if (rank[x] < rank[y]) {
32            T z = x;
33            x = y;
34            y = z;
35        }
36
37        parent[y] = x;
38        if (rank[x] == rank[y]) rank[x]++;
39    }
40 }

```

3.8.2 Fenwick tree (i.e. BIT) eff. update + prefix sum calc.

```

1 #include "header.h"
2 #define maxn 200010
3 int t,n,m,tree[maxn],p[maxn];
4
5 void update(int k, int z) {
6     while (k <= maxn) {
7         tree[k] += z;
8         k += k & (-k);
9     }
10 }
11
12 int sum(int k) {
13     int ans = 0;
14     while(k) {
15         ans += tree[k];
16         k -= k & (-k);
17     }
18     return ans;
19 }

```

3.8.3 Fenwick2d tree

```

1 #include "header.h"
2 template <class T>
3 struct FenwickTree2D {
4     vector< vector<T> > tree;
5     int n;
6     FenwickTree2D(int n) : n(n) { tree.assign(n + 1, vector<T>(n + 1, 0)); }
7     T query(int x1, int y1, int x2, int y2) {
8         return query(x2,y2)+query(x1-1,y1-1)-query(x2,y1-1)-query(x1-1,y2);
9     }
10    T query(int x, int y) {
11        T s = 0;
12        for (int i = x; i > 0; i -= (i & (-i)))
13            for (int j = y; j > 0; j -= (j & (-j)))
14                s += tree[i][j];
15        return s;
16    }
17    void update(int x, int y, T v) {
18        for (int i = x; i <= n; i += (i & (-i)))
19            for (int j = y; j <= n; j += (j & (-j)))
20                tree[i][j] += v;
21    }
22 };

```

3.8.4 Trie

```

1 #include "header.h"
2 const int ALPHABET_SIZE = 26;
3 inline int mp(char c) { return c - 'a'; }
4
5 struct Node {
6     Node* ch[ALPHABET_SIZE];
7     bool isleaf = false;
8     Node() {
9         for(int i = 0; i < ALPHABET_SIZE; ++i) ch[i] = nullptr;
10    }
11
12    void insert(string &s, int i = 0) {
13        if (i == s.length()) isleaf = true;
14        else {
15            int v = mp(s[i]);
16            if (ch[v] == nullptr)
17                ch[v] = new Node();
18            ch[v]->insert(s, i + 1);
19        }
20    }
21
22    bool contains(string &s, int i = 0) {
23        if (i == s.length()) return isleaf;
24        else {
25            int v = mp(s[i]);
26            if (ch[v] == nullptr) return false;
27            else return ch[v]->contains(s, i + 1);
28        }

```

```

29    }
30
31    void cleanup() {
32        for (int i = 0; i < ALPHABET_SIZE; ++i)
33            if (ch[i] != nullptr) {
34                ch[i]->cleanup();
35                delete ch[i];
36            }
37    }
38 };

```

3.8.5 Treap A binary tree whose nodes contain two values, a key and a priority, such that the key keeps the BST property

```

1 #include "header.h"
2 struct Node {
3     ll v;
4     int sz, pr;
5     Node *l = nullptr, *r = nullptr;
6     Node(ll val) : v(val), sz(1) { pr = rand(); }
7 };
8 int size(Node *p) { return p ? p->sz : 0; }
9 void update(Node* p) {
10     if (!p) return;
11     p->sz = 1 + size(p->l) + size(p->r);
12     // Pull data from children here
13 }
14 void propagate(Node *p) {
15     if (!p) return;
16     // Push data to children here
17 }
18 void merge(Node *&t, Node *l, Node *r) {
19     propagate(l), propagate(r);
20     if (!l) t = r;
21     else if (!r) t = l;
22     else if (l->pr > r->pr)
23         merge(l->r, l->r, r), t = l;
24     else merge(r->l, l, r->l), t = r;
25     update(t);
26 }
27 void spliti(Node *t, Node *&l, Node *&r, int index) {
28     propagate(t);
29     if (!t) { l = r = nullptr; return; }
30     int id = size(t->l);
31     if (index <= id) // id \in [index, \infty), so move it right
32         spliti(t->l, l, t->l, index), r = t;
33     else
34         spliti(t->r, t->r, r, index - id), l = t;
35     update(t);
36 }
37 void splitv(Node *t, Node *&l, Node *&r, ll val) {
38     propagate(t);
39     if (!t) { l = r = nullptr; return; }
40     if (val <= t->v) // t->v \in [val, \infty), so move it right

```

```

41     splitv(t->l, l, t->l, val), r = t;
42     else
43         splitv(t->r, t->r, r, val), l = t;
44     update(t);
45 }
46 void clean(Node *p) {
47     if (p) { clean(p->l), clean(p->r); delete p; }
48 }

```

4 Other Mathematics

4.1 Helpful functions

4.1.1 Euler's Totient Fuction $n = p_1^{k_1-1} \cdot (p_1 - 1) \cdot \dots \cdot p_r^{k_r-1} \cdot (p_r - 1)$, where $p_1^{k_1} \cdot \dots \cdot p_r^{k_r}$ is the prime factorization of n .

```

1 # include "header.h"
2 ll phi(ll n) { // \Phi(n)
3     ll ans = 1;
4     for (ll i = 2; i*i <= n; i++) {
5         if (n % i == 0) {
6             ans *= i-1;
7             n /= i;
8             while (n % i == 0) {
9                 ans *= i;
10                n /= i;
11            }
12        }
13    }
14    if (n > 1) ans *= n-1;
15    return ans;
16 }
17 vi phis(int n) { // All \Phi(i) up to n
18     vi phi(n+1, 0LL);
19     iota(phi.begin(), phi.end(), 0LL);
20     for (ll i = 2LL; i <= n; ++i)
21         if (phi[i] == i)
22             for (ll j = i; j <= n; j += i)
23                 phi[j] -= phi[j] / i;
24     return phi;
25 }

```

Formulas $\Phi(n)$ counts all numbers in $1, \dots, n-1$ coprime to n .

$a^{\varphi(n)} \equiv 1 \pmod n$, a and n are coprimes.

$\forall e > \log_2 m : n^e \pmod m = n^{\Phi(m)+e \pmod \Phi(m)} \pmod m$.

$\gcd(m, n) = 1 \Rightarrow \Phi(m \cdot n) = \Phi(m) \cdot \Phi(n)$.

4.2 Theorems and definitions

Fermat's little theorem $a^p \equiv a \pmod p$

Subfactorial $!n = n! \sum_{i=0}^n \frac{(-1)^i}{i!}$, $!(0) = 1$, $!n = n \cdot !(n-1) + (-1)^n$

Least common multiple $\text{lcm}(a, b) = a \cdot b / \gcd(a, b)$

Binomials and other partitionings We have $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1} = \prod_{i=1}^k \frac{n-i+1}{i}$. This last product may be computed incrementally since any product of k' consecutive values is divisible by $k'!$. Basic identities: The hockeystick identity: $\sum_{k=r}^n \binom{k}{r} = \binom{n+1}{r+1}$ or $\sum_{k \leq n} \binom{r+k}{k} = \binom{r+n+1}{n}$. Also $\sum_{k=0}^n \binom{k}{m} = \binom{n+1}{m+1}$.

For $n, m \geq 0$ and p prime. Write n, m in base p , i.e. $n = n_k p^k + \dots + n_1 p + n_0$ and $m = m_k p^k + \dots + m_1 p + m_0$. Then by Lucas theorem we have $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod p$, with the convention that $n_i < m_i \Rightarrow \binom{n_i}{m_i} = 0$.

Fibonacci (See also number theory section)

$\sum_{0 \leq k \leq n} \binom{n-k}{k} = F_{n+1}$, $F_n = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^n$,
 $\sum_{i=1}^n F_i = F_{n+2} - 1$, $\sum_{i=1}^n F_i^2 = F_n F_{n+1}$,
 $\gcd(F_m, F_n) = F_{\gcd(m, n)}$, $\gcd(F_n, F_{n+1}) = \gcd(F_n, F_{n+2}) = 1$

Bit stuff $a + b = a \oplus b + 2(a \& b) = a|b + a \& b$.

k th bit is set in x iff $x \pmod{2^{k-1}} \geq 2^{k-1}$, or iff $x \pmod{2^{k-1}} - x \pmod{2^k} \neq 0$ (i.e. $= 2^k$). It comes handy when you need to look at the bits of the numbers which are pair sums or subset sums etc.

$n \pmod{2^i} = n \& (2^i - 1)$.

$\forall k : 1 \oplus 2 \oplus \dots \oplus (4k-1) = 0$

Stirling's numbers First kind: $S_1(n, k)$ count permutations on n items with k cycles. $S_1(n, k) = S_1(n-1, k-1) + (n-1)S_1(n-1, k)$ with $S_1(0, 0) = 1$. Note $\sum_{k=0}^n S_1(n, k)x^k = x(x+1)\dots(x+n-1)$.

Second kind: $S_2(n, k)$ count partitions of n distinct elements into exactly k non-empty groups. $S_2(n, k) = S_2(n-1, k-1) + kS_2(n-1, k)$ with $S_2(n, 1) = S_2(n, n) = 1$ and $S_2(n, k) = \frac{1}{k!} \sum_{i=0}^k (-1)^{k-i} \binom{k}{i} i^n$

4.3 Geometry Formulas

$$[ABC] = rs = \frac{1}{2}ab \sin \gamma = \frac{abc}{4R} = \sqrt{s(s-a)(s-b)(s-c)} = \frac{1}{2} |(B-A, C-A)^T|$$

$$\begin{array}{ll}
s = \frac{a+b+c}{2} & 2R = \frac{a}{\sin \alpha} \\
\text{cosine rule:} & c^2 = a^2 + b^2 - 2ab \cos \gamma \\
\text{Euler:} & 1 + CC = V - E + F \\
\text{Pick:} & \text{Area} = \text{interior points} + \frac{\text{boundary points}}{2} - 1 \\
p \cdot q = |p||q| \cos(\theta) & |p \times q| = |p||q| \sin(\theta)
\end{array}$$

Given a non-self-intersecting closed polygon on n vertices, given as (x_i, y_i) , its centroid (C_x, C_y) is given as:

$$C_x = \frac{1}{6A} \sum_{i=0}^{n-1} (x_i + x_{i+1})(x_i y_{i+1} - x_{i+1} y_i), \quad C_y = \frac{1}{6A} \sum_{i=0}^{n-1} (y_i + y_{i+1})(x_i y_{i+1} - x_{i+1} y_i)$$

$$A = \frac{1}{2} \sum_{i=0}^{n-1} (x_i y_{i+1} - x_{i+1} y_i) = \text{polygon area}$$