TCR - Balloon Addicts (NWERC 2023) - University of Groningen

1 Setup

1.1 header.h

```
1 #pragma once  // Delete this when copying this file
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 #define ll long long
6 #define ull unsigned ll
7 #define ld long double
8 #define pl pair<ll, ll>
9 #define pi pair<int, int>
10 #define vl vector<ll>
11 #define vi vector<int>
12 #define vvi vector<vi>
13 #define vvl vector<vl>
14 #define vpl vector<pl>
15 #define vpi vector<pi>
16 #define vld vector<ld>
17 #define in_fast(el, cont) (cont.find(el) != cont.end())
18 #define in(el, cont) (find(cont.begin(), cont.end(), el) != cont.end())
19
20 constexpr int INF  = 2000000010;
21 constexpr ll LLINF = 9000000000000000010LL;
22
23 template <typename T, template <typename ELEM, typename ALLOC = std::
      allocator<ELEM> > class Container>
24 std::ostream& operator<<(std::ostream& o, const Container<T>& container) {
25   typename Container<T>::const_iterator beg = container.begin();
26   if (beg != container.end()) {
27     o << *beg++;
28     while (beg != container.end()) {
29       o << " " << *beg++;
30     }
31   }
32   return o;
33 }
34
35 // int main() {
36 //   ios::sync_with_stdio(false);  // do not use cout + printf
37 //   cin.tie(NULL);
38 //   cout << fixed << setprecision(12);
39 //   return 0;
40 // }
```

1.2 Bash for c++ compile with header.h

```
1 #!/bin/bash
2 if [ $# -ne 1 ];then echo "Usage: $0 <input_file>"; exit 1;fi
3 f="$1";d=code/;o=a.out
4 [ -f $d/$f ] || { echo "Input file not found: $f"; exit 1; }
5 g++ -I$d $d/$f -o $o && echo "Compilation successful. Executable '$o'
      created." || echo "Compilation failed."
```

1.3 Bash for run tests c++

```
1 g++ $1/$1.cpp -o $1/$1.out
2 for file in $1/*.in; do diff <($1/$1.out < "$file") "${file%.in}.ans"; done
```

1.4 Bash for run tests python

```bash
for file in $1/*.in; do diff <(python3 $1/$1.py < "$file") "${file%.in}.ans"; done
```

1.4.1 Auxiliary helper C++

```cpp
#include "header.h"

int main() {
    // Read in a line including white space
    string line;
    getline(cin, line);
    // When doing the above read numbers as follows:
    int n;
    getline(cin, line);
    stringstream ss(line);
    ss >> n;

    // Count the number of 1s in binary represnatation of a number
    ull number;
    __builtin_popcountll(number);
}
```

1.4.2 Auxiliary helper python

```python
# Read until EOF
while True:
    try:
        pattern = input()
    except EOFError:
        break
```

2 Python

2.1 Graphs

2.1.1 BFS

```python
from collections import deque
def bfs(g, roots, n):
    q = deque(roots)
    explored = set(roots)
    distances = [float("inf")]*n
    distances[0][0] = 0

    while len(q) != 0:
        node = q.popleft()
        if node in explored: continue
        explored.add(node)
        for neigh in g[node]:
            if neigh not in explored:
                q.append(neigh)
                distances[neigh] = distances[node] + 1
    return distances
```

2.1.2 Dijkstra

```python
from heapq import *
def dijkstra(n, root, g):   # g = {node: (cost, neigh)}
    dist = [float("inf")]*n
    dist[root] = 0
    prev = [-1]*n

    pq = [(0, root)]
    heapify(pq)
    visited = set([])

    while len(pq) != 0:
        _, node = heappop(pq)

        if node in visited: continue
        visited.add(node)

        # In case of disconnected graphs
        if node not in g:
            continue

        for cost, neigh in g[node]:
            alt = dist[node] + cost
            if alt < dist[neigh]:
                dist[neigh] = alt
                prev[neigh] = node
                heappush(pq, (alt, neigh))
    return dist
```

2.2 Dynamic Programming

2.3 Trees

2.4 Number Theory / Combinatorics

2.4.1 nCk % prime

```python
# Note: p must be prime and k < p
def fermat_binom(n, k, p):
    if k > n:
        return 0
    # calculate numerator
    num = 1
    for i in range(n-k+1, n+1):
        num *= i % p
    num %= p
    # calculate denominator
    denom = 1
    for i in range(1,k+1):
        denom *= i % p
    denom %= p
```

```
15      # numerator * denominator^(p-2) (mod p)
16      return (num * pow(denom, p-2, p)) % p
```

2.4.2 Sieve of Eratosthenes

$O(n)$ so actually faster than C++ version, but more memory

```
1  MAX_SIZE = 10**8+1
2  isprime = [True] * MAX_SIZE
3  prime = []
4  SPF = [None] * (MAX_SIZE)
5
6  def manipulated_seive(N):   # Up to N (not included)
7    isprime[0] = isprime[1] = False
8    for i in range(2, N):
9      if isprime[i] == True:
10       prime.append(i)
11       SPF[i] = i
12     j = 0
13     while (j < len(prime) and
14       i * prime[j] < N and
15         prime[j] <= SPF[i]):
16       isprime[i * prime[j]] = False
17       SPF[i * prime[j]] = prime[j]
18       j += 1
```

2.5 Strings

2.5.1 LCS

```
1  def longestCommonSubsequence(text1, text2):   # O(m*n) time, O(m) space
2      n = len(text1)
3      m = len(text2)
4
5      # Initializing two lists of size m
6      prev = [0] * (m + 1)
7      cur = [0] * (m + 1)
8
9      for idx1 in range(1, n + 1):
10         for idx2 in range(1, m + 1):
11             # If characters are matching
12             if text1[idx1 - 1] == text2[idx2 - 1]:
13                 cur[idx2] = 1 + prev[idx2 - 1]
14             else:
15                 # If characters are not matching
16                 cur[idx2] = max(cur[idx2 - 1], prev[idx2])
17
18         prev = cur.copy()
19
20     return cur[m]
```

2.5.2 KMP

```
1  class KMP:
2      def partial(self, pattern):
3          """ Calculate partial match table: String -> [Int]"""
4          ret = [0]
5          for i in range(1, len(pattern)):
6              j = ret[i - 1]
7              while j > 0 and pattern[j] != pattern[i]: j = ret[j - 1]
8              ret.append(j + 1 if pattern[j] == pattern[i] else j)
9          return ret
10
11     def search(self, T, P):
12         """KMP search main algorithm: String -> String -> [Int]
13         Return all the matching position of pattern string P in T"""
14         partial, ret, j = self.partial(P), [], 0
15         for i in range(len(T)):
16             while j > 0 and T[i] != P[j]: j = partial[j - 1]
17             if T[i] == P[j]: j += 1
18             if j == len(P):
19                 ret.append(i - (j - 1))
20                 j = partial[j - 1]
21         return ret
```

2.6 Geometry

2.7 Other Algorithms

2.7.1 Rotate matrix

```
1  def rotate_matrix(m):
2      return [[m[j][i] for j in range(len(m))] for i in range(len(m[0])
           -1,-1,-1)]
```

2.8 Other Data Structures

2.8.1 Segment Tree

```
1  N = 100000  # limit for array size
2  tree = [0] * (2 * N)  # Max size of tree
3
4  def build(arr, n):  # function to build the tree
5      # insert leaf nodes in tree
6      for i in range(n):
7          tree[n + i] = arr[i]
8
9      # build the tree by calculating parents
10     for i in range(n - 1, 0, -1):
11         tree[i] = tree[i << 1] + tree[i << 1 | 1]
12
13 def updateTreeNode(p, value, n):  # function to update a tree node
14     # set value at position p
15     tree[p + n] = value
16     p = p + n
17
18     i = p  # move upward and update parents
19     while i > 1:
20         tree[i >> 1] = tree[i] + tree[i ^ 1]
21         i >>= 1
22
```

```python
23  def query(l, r, n):  # function to get sum on interval [l, r)
24      res = 0
25      # loop to find the sum in the range
26      l += n
27      r += n
28      while l < r:
29          if l & 1:
30              res += tree[l]
31              l += 1
32          if r & 1:
33              r -= 1
34              res += tree[r]
35          l >>= 1
36          r >>= 1
37      return res
```

3   C++

3.1   Graphs

3.1.1   BFS

```cpp
#include "header.h"
#define graph unordered_map<ll, unordered_set<ll>>
vi bfs(int n, graph& g, vi& roots) {
    vi parents(n+1, -1); // nodes are 1..n
    unordered_set<int> visited;
    queue<int> q;
    for (auto x: roots) {
        q.emplace(x);
        visited.insert(x);
    }
    while (not q.empty()) {
        int node = q.front();
        q.pop();

        for (auto neigh: g[node]) {
            if (not in(neigh, visited)) {
                parents[neigh] = node;
                q.emplace(neigh);
                visited.insert(neigh);
            }
        }
    }
    return parents;
}
vi reconstruct_path(vi parents, int start, int goal) {
    vi path;
    int curr = goal;
    while (curr != start) {
        path.push_back(curr);
        if (parents[curr] == -1) return vi(); // No path, empty vi
        curr = parents[curr];
    }
    path.push_back(start);
    reverse(path.begin(), path.end());
```

```cpp
    return path;
}
```

3.1.2   DFS Cycle detection / removal

```cpp
#include "header.h"
void removeCyc(ll node, unordered_map<ll, vector<pair<ll, ll>>>& neighs,
    vector<bool>& visited,
vector<bool>& recStack, vector<ll>& ans) {
    if (!visited[node]) {
        visited[node] = true;
        recStack[node] = true;
        auto it = neighs.find(node);
        if (it != neighs.end()) {
            for (auto util: it->second) {
                ll nnode = util.first;
                if (recStack[nnode]) {
                    ans.push_back(util.second);
                } else if (!visited[nnode]) {
                    removeCyc(nnode, neighs, visited, recStack, ans);
                }
            }
        }
    }
    recStack[node] = false;
}
```

3.1.3   Dijkstra

```cpp
#include "header.h"
vector<int> dijkstra(int n, int root, map<int, vector<pair<int, int>>>& g) {
  unordered_set<int> visited;
  vector<int> dist(n, INF);
    priority_queue<pair<int, int>> pq;
    dist[root] = 0;
    pq.push({0, root});
    while (!pq.empty()) {
        int node = pq.top().second;
        int d = -pq.top().first;
        pq.pop();

        if (in(node, visited)) continue;
        visited.insert(node);

        for (auto e : g[node]) {
            int neigh = e.first;
            int cost = e.second;
            if (dist[neigh] > dist[node] + cost) {
                dist[neigh] = dist[node] + cost;
                pq.push({-dist[neigh], neigh});
            }
        }
    }
    return dist;
}
```

3.1.4 Floyd-Warshall

```cpp
#include "header.h"
// g[i][j] = infty if not path from i to j
// if g[i][i] < 0, i is contained in a negative cycle
void warshall(vvl g) {
    for (int i=0; i<g.size(); ++i) {
        for (int j=0; j<g.size(); ++j) {
            for (int k=0; k<g.size(); ++k) {
                if (g[i][k] < LLINF and g[k][j] < LLINF and g[i][j] > g[i][k
                    ] + g[k][j]) {
                    g[i][j] = g[i][k] + g[k][j];
}}}}}
```

3.1.5 Kruskal Minimum spanning tree of undirected weighted graph

```cpp
#include "header.h"
#include "disjoint_set.h"
// O(E log E)
pair<set<pair<ll, ll>>, ll>  kruskal(vector<tuple<ll, ll, ll>>& edges, ll n)
    {
    set<pair<ll, ll>> ans;
    ll cost = 0;

    sort(edges.begin(), edges.end());
    DisjointSet<ll> fs(n);

    ll dist, i, j;
    for (auto edge: edges) {
        dist = get<0>(edge);
        i = get<1>(edge);
        j = get<2>(edge);

        if (fs.find_set(i) != fs.find_set(j)) {
            fs.union_sets(i, j);
            ans.insert({i, j});
            cost += dist;
        }
    }
    return pair<set<pair<ll, ll>>, ll> {ans, cost};
}
```

3.1.6 Hungarian algorithm

```cpp
#include "header.h"

template <class T> bool ckmin(T &a, const T &b) { return b < a ? a = b, 1 :
    0; }
/**
 * Given J jobs and W workers (J <= W), computes the minimum cost to assign
     each
 * prefix of jobs to distinct workers.
 * @tparam T a type large enough to represent integers on the order of J *
 * max(|C|)
 * @param C a matrix of dimensions JxW such that C[j][w] = cost to assign j-
     th
 * job to w-th worker (possibly negative)
 *
 * @return a vector of length J, with the j-th entry equaling the minimum
     cost
 * to assign the first (j+1) jobs to distinct workers
 */
template <class T> vector<T> hungarian(const vector<vector<T>> &C) {
    const int J = (int)size(C), W = (int)size(C[0]);
    assert(J <= W);
    // job[w] = job assigned to w-th worker, or -1 if no job assigned
    // note: a W-th worker was added for convenience
    vector<int> job(W + 1, -1);
    vector<T> ys(J), yt(W + 1);  // potentials
    // -yt[W] will equal the sum of all deltas
    vector<T> answers;
    const T inf = numeric_limits<T>::max();
    for (int j_cur = 0; j_cur < J; ++j_cur) {  // assign j_cur-th job
        int w_cur = W;
        job[w_cur] = j_cur;
        // min reduced cost over edges from Z to worker w
        vector<T> min_to(W + 1, inf);
        vector<int> prv(W + 1, -1);  // previous worker on alternating path
        vector<bool> in_Z(W + 1);    // whether worker is in Z
        while (job[w_cur] != -1) {    // runs at most j_cur + 1 times
            in_Z[w_cur] = true;
            const int j = job[w_cur];
            T delta = inf;
            int w_next;
            for (int w = 0; w < W; ++w) {
                if (!in_Z[w]) {
                    if (ckmin(min_to[w], C[j][w] - ys[j] - yt[w]))
                        prv[w] = w_cur;
                    if (ckmin(delta, min_to[w])) w_next = w;
                }
            }
            // delta will always be non-negative,
            // except possibly during the first time this loop runs
            // if any entries of C[j_cur] are negative
            for (int w = 0; w <= W; ++w) {
                if (in_Z[w]) ys[job[w]] += delta, yt[w] -= delta;
                else min_to[w] -= delta;
            }
            w_cur = w_next;
        }
        // update assignments along alternating path
        for (int w; w_cur != W; w_cur = w) job[w_cur] = job[w = prv[w_cur]];
        answers.push_back(-yt[W]);
    }
    return answers;
}
```

3.1.7 Successive shortest path Calculates max flow, min cost

```cpp
#include "header.h"
// map<node, map<node, pair<cost, capacity>>>
#define graph unordered_map<int, unordered_map<int, pair<ld, int>>>
graph g;
const ld infty = 1e60l;  // Change if necessary
ld fill(int n, vld& potential) {  // Finds max flow, min cost
```

```
7     priority_queue<pair<ld, int>> pq;
8     vector<bool> visited(n+2, false);
9     vi parent(n+2, 0);
10    vld dist(n+2, infty);
11    dist[0] = 0.l;
12    pq.emplace(make_pair(0.l, 0));
13    while (not pq.empty()) {
14      int node = pq.top().second;
15      pq.pop();
16      if (visited[node]) continue;
17      visited[node] = true;
18      for (auto& x : g[node]) {
19        int neigh = x.first;
20        int capacity = x.second.second;
21        ld cost = x.second.first;
22        if (capacity and not visited[neigh]) {
23          ld d = dist[node] + cost + potential[node] - potential[neigh];
24          if (d + 1e-10l < dist[neigh]) {
25            dist[neigh] = d;
26            pq.emplace(make_pair(-d, neigh));
27            parent[neigh] = node;
28    }}}}

29
30    for (int i = 0; i < n+2; i++) {
31      potential[i] = min(infty, potential[i] + dist[i]);
32    }
33    if (not parent[n+1]) return infty;
34    ld ans = 0.l;
35    for (int x = n+1; x; x=parent[x]) {
36      ans += g[parent[x]][x].first;
37      g[parent[x]][x].second--;
38      g[x][parent[x]].second++;
39    }
40    return ans;
41 }
```

3.1.8 Bipartite check

```
1 #include "header.h"
2 int main() {
3     int n;
4     vvi adj(n);
5
6     vi side(n, -1);    // will have 0's for one side 1's for other side
7     bool is_bipartite = true;   // becomes false if not bipartite
8     queue<int> q;
9     for (int st = 0; st < n; ++st) {
10        if (side[st] == -1) {
11            q.push(st);
12            side[st] = 0;
13            while (!q.empty()) {
14                int v = q.front();
15                q.pop();
16                for (int u : adj[v]) {
17                    if (side[u] == -1) {
18                        side[u] = side[v] ^ 1;
19                        q.push(u);
20                    } else {
```

```
21                        is_bipartite &= side[u] != side[v];
22                }
23 }}}}}
```

3.1.9 Find cycle directed

```
1 #include "header.h"
2 int n;
3 const int mxN = 2e5+5;
4 vvi adj(mxN);
5 vector<char> color;
6 vi parent;
7 int cycle_start, cycle_end;
8 bool dfs(int v) {
9     color[v] = 1;
10    for (int u : adj[v]) {
11        if (color[u] == 0) {
12            parent[u] = v;
13            if (dfs(u)) return true;
14        } else if (color[u] == 1) {
15            cycle_end = v;
16            cycle_start = u;
17            return true;
18        }
19    }
20    color[v] = 2;
21    return false;
22 }
23 void find_cycle() {
24    color.assign(n, 0);
25    parent.assign(n, -1);
26    cycle_start = -1;
27    for (int v = 0; v < n; v++) {
28        if (color[v] == 0 && dfs(v))break;
29    }
30    if (cycle_start == -1) {
31        cout << "Acyclic" << endl;
32    } else {
33        vector<int> cycle;
34        cycle.push_back(cycle_start);
35        for (int v = cycle_end; v != cycle_start; v = parent[v])
36            cycle.push_back(v);
37        cycle.push_back(cycle_start);
38        reverse(cycle.begin(), cycle.end());
39
40        cout << "Cycle Found: ";
41        for (int v : cycle) cout << v << " ";
42        cout << endl;
43    }
44 }
```

3.1.10 Find cycle directed

```
1 #include "header.h"
2 int n;
3 const int mxN = 2e5 + 5;
```

```
 4  vvi adj(mxN);
 5  vector<bool> visited;
 6  vi parent;
 7  int cycle_start, cycle_end;
 8  bool dfs(int v, int par) { // passing vertex and its parent vertex
 9      visited[v] = true;
10      for (int u : adj[v]) {
11          if(u == par) continue; // skipping edge to parent vertex
12          if (visited[u]) {
13              cycle_end = v;
14              cycle_start = u;
15              return true;
16          }
17          parent[u] = v;
18          if (dfs(u, parent[u]))
19              return true;
20      }
21      return false;
22  }
23  void find_cycle() {
24      visited.assign(n, false);
25      parent.assign(n, -1);
26      cycle_start = -1;
27      for (int v = 0; v < n; v++) {
28          if (!visited[v] && dfs(v, parent[v])) break;
29      }
30      if (cycle_start == -1) {
31          cout << "Acyclic" << endl;
32      } else {
33          vector<int> cycle;
34          cycle.push_back(cycle_start);
35          for (int v = cycle_end; v != cycle_start; v = parent[v])
36              cycle.push_back(v);
37          cycle.push_back(cycle_start);
38          cout << "Cycle␣Found:␣";
39          for (int v : cycle) cout << v << "␣";
40          cout << endl;
41      }
42  }
```

3.1.11   Tarjan's SCC

```
 1  #include "header.h"
 2
 3  struct Tarjan {
 4    vvi &edges;
 5    int V, counter = 0, C = 0;
 6    vi n, l;
 7    vector<bool> vs;
 8    stack<int> st;
 9    Tarjan(vvi &e) : edges(e), V(e.size()), n(V, -1), l(V, -1), vs(V, false)
        {}
10    void visit(int u, vi &com) {
11      l[u] = n[u] = counter++;
12      st.push(u);
13      vs[u] = true;
14      for (auto &&v : edges[u]) {
15        if (n[v] == -1) visit(v, com);
```

```
16        if (vs[v]) l[u] = min(l[u], l[v]);
17      }
18      if (l[u] == n[u]) {
19        while (true) {
20          int v = st.top();
21          st.pop();
22          vs[v] = false;
23          com[v] = C;   //<== ACT HERE
24          if (u == v) break;
25        }
26        C++;
27      }
28    }
29    int find_sccs(vi &com) {  // component indices will be stored in 'com'
30      com.assign(V, -1);
31      C = 0;
32      for (int u = 0; u < V; ++u)
33        if (n[u] == -1) visit(u, com);
34      return C;
35    }
36    // scc is a map of the original vertices of the graph to the vertices
37    // of the SCC graph, scc_graph is its adjacency list.
38    // SCC indices and edges are stored in 'scc' and 'scc_graph'.
39    void scc_collapse(vi &scc, vvi &scc_graph) {
40      find_sccs(scc);
41      scc_graph.assign(C, vi());
42      set<pi> rec;  // recorded edges
43      for (int u = 0; u < V; ++u) {
44        assert(scc[u] != -1);
45        for (int v : edges[u]) {
46          if (scc[v] == scc[u] ||
47            rec.find({scc[u], scc[v]}) != rec.end()) continue;
48          scc_graph[scc[u]].push_back(scc[v]);
49          rec.insert({scc[u], scc[v]});
50        }
51      }
52    }
53    // Function to find sources and sinks in the SCC graph
54    // The number of edges needed to be added is max(sources.size(), sinks.())
55    void findSourcesAndSinks(const vvi &scc_graph, vi &sources, vi &sinks) {
56      vi in_degree(C, 0), out_degree(C, 0);
57      for (int u = 0; u < C; u++) {
58        for (auto v : scc_graph[u]) {
59          in_degree[v]++;
60          out_degree[u]++;
61        }
62      }
63      for (int i = 0; i < C; ++i) {
64        if (in_degree[i] == 0) sources.push_back(i);
65        if (out_degree[i] == 0) sinks.push_back(i);
66      }
67    }
68  };
```

3.1.12   SCC edges   Prints out the missing edges to make the input digraph strongly connected

```
 1  #include "header.h"
```

Left column:

```
const int N=1e5+10;
int n,a[N],cnt[N],vis[N];
vector<int> hd,tl;
int dfs(int x){
    vis[x]=1;
    if(!vis[a[x]])return vis[x]=dfs(a[x]);
    return vis[x]=x;
}
int main(){
    scanf("%d",&n);
    for(int i=1;i<=n;i++){
        scanf("%d",&a[i]);
        cnt[a[i]]++;
    }
    int k=0;
    for(int i=1;i<=n;i++){
        if(!cnt[i]){
            k++;
            hd.push_back(i);
            tl.push_back(dfs(i));
        }
    }
    int tk=k;
    for(int i=1;i<=n;i++){
        if(!vis[i]){
            k++;
            hd.push_back(i);
            tl.push_back(dfs(i));
        }
    }
    if(k==1&&!tk)k=0;
    printf("%d\n",k);
    for(int i=0;i<k;i++)printf("%d %d\n",tl[i],hd[(i+1)%k]);
    return 0;
}
```

3.2 Dynamic Programming

3.2.1 Longest Increasing Subsequence

```
#include "header.h"
template<class T>
vector<T> index_path_lis(vector<T>& nums) {
  int n = nums.size();
  vector<T> sub;
    vector<int> subIndex;
  vector<T> path(n, -1);
  for (int i = 0; i < n; ++i) {
      if (sub.empty() || sub[sub.size() - 1] < nums[i]) {
    path[i] = sub.empty() ? -1 : subIndex[sub.size() - 1];
    sub.push_back(nums[i]);
    subIndex.push_back(i);
      } else {
    int idx = lower_bound(sub.begin(), sub.end(), nums[i]) - sub.begin();
    path[i] = idx == 0 ? -1 : subIndex[idx - 1];
    sub[idx] = nums[i];
    subIndex[idx] = i;
      }
```

Right column:

```
  }
  vector<T> ans;
    int t = subIndex[subIndex.size() - 1];
    while (t != -1) {
        ans.push_back(t);
        t = path[t];
    }
    reverse(ans.begin(), ans.end());
    return ans;
}
// Length only
template<class T>
int length_lis(vector<T> &a) {
  set<T> st;
  typename set<T>::iterator it;
  for (int i = 0; i < a.size(); ++i) {
    it = st.lower_bound(a[i]);
    if (it != st.end()) st.erase(it);
    st.insert(a[i]);
  }
  return st.size();
}
```

3.2.2 0-1 Knapsack

```
#include "header.h"
// given a number of coins, calculate all possible distinct sums
int main() {
  int n;
  vi coins(n);  // all possible coins to use
  int sum = 0;        // sum of the coins
  vi dp(sum + 1, 0);        // dp[x] = 1 if sum x can be made
  dp[0] = 1;              // sum 0 can be made
  for (int c = 0; c < n; ++c)        // first iteration: sums with first
    for (int x = sum; x >= 0; --x)      // coin, next first 2 coins etc
      if (dp[x]) dp[x + coins[c]] = 1;  // if sum x valid, x+c valid
}
```

3.3 Trees

3.3.1 Tree diameter

```
#include "header.h"
const int mxN = 2e5 + 5;
int n, d[mxN];  // distance array
vi adj[mxN];  // tree adjacency list
void dfs(int s, int e) {
  d[s] = 1 + d[e];      // recursively calculate the distance from the
        starting node to each node
  for (auto u : adj[s]) { // for each adjacent node
    if (u != e) dfs(u, s);  // don't move backwards in the tree
  }
}
int main() {
  // read input, create adj list
```

```
13    dfs(0, -1);                        // first dfs call to find farthest node from
              arbitrary node
14    dfs(distance(d, max_element(d, d + n)), -1);  // second dfs call to find
          farthest node from that one
15    cout << *max_element(d, d + n) - 1 << '\n';  // distance from second node
          to farthest is the diameter
16 }
```

3.3.2 Tree Node Count

```
1 #include "header.h"
2 // calculate amount of nodes in each node's subtree
3 const int mxN = 2e5 + 5;
4 int n, cnt[mxN];
5 vi adj[mxN];
6 void dfs(int s = 0, int e = -1) {
7   cnt[s] = 1;  // count leaves as one
8   for (int u : adj[s]) {
9     dfs(u, s);
10    cnt[s] += cnt[u];  // add up nodes of the subtrees
11  }
12 }
```

3.4 Number Theory / Combinatorics

3.4.1 Modular exponentiation Or use pow() in python

```
1 #include "header.h"
2 ll mod_pow(ll base, ll exp, ll mod) {
3   if (mod == 1) return 0;
4     if (exp == 0) return 1;
5     if (exp == 1) return base;
6
7   ll res = 1;
8   base %= mod;
9   while (exp) {
10    if (exp % 2 == 1) res = (res * base) % mod;
11    exp >>= 1;
12    base = (base * base) % mod;
13  }
14
15  return res % mod;
16 }
```

3.4.2 GCD Or math.gcd in python, std::gcd in C++

```
1 #include "header.h"
2 ll gcd(ll a, ll b) {
3   if (a == 0) return b;
4   return gcd(b % a, a);
5 }
```

3.4.3 Sieve of Eratosthenes

```
1 #include "header.h"
2 vl primes;
3 void getprimes(ll n) {  // Up to n (not included)
4     vector<bool> p(n, true);
5     p[0] = false;
6     p[1] = false;
7     for(ll i = 0; i < n; i++) {
8         if(p[i]) {
9             primes.push_back(i);
10            for(ll j = i*2; j < n; j+=i) p[j] = false;
11 }}}
```

3.4.4 Fibonacci % prime

```
1 #include "header.h"
2 const ll MOD = 1000000007;
3 unordered_map<ll, ll> Fib;
4 ll fib(ll n) {
5     if (n < 2) return 1;
6     if (Fib.find(n) != Fib.end()) return Fib[n];
7     Fib[n] = (fib((n + 1) / 2) * fib(n / 2) + fib((n - 1) / 2) * fib((n - 2)
          / 2)) % MOD;
8     return Fib[n];
9 }
```

3.4.5 nCk % prime

```
1 #include "header.h"
2 ll binom(ll n, ll k) {
3     ll ans = 1;
4     for(ll i = 1; i <= min(k,n-k); ++i) ans = ans*(n+1-i)/i;
5     return ans;
6 }
7 ll mod_nCk(ll n, ll k, ll p ){
8     ll ans = 1;
9     while(n){
10        ll np = n%p, kp = k%p;
11        if(kp > np) return 0;
12        ans *= binom(np,kp);
13        n /= p; k /= p;
14    }
15    return ans;
16 }
```

3.5 Strings

3.5.1 Aho-Corasick algorithm Also can be used as Knuth-Morris-Pratt algorithm

```
1 #include "header.h"
2
3 map<char, int> cti;
4 int cti_size;
5 template <int ALPHABET_SIZE, int (*mp)(char)>
```

```cpp
struct AC_FSM {
  struct Node {
    int child[ALPHABET_SIZE], failure = 0, match_par = -1;
    vi match;
    Node() { for (int i = 0; i < ALPHABET_SIZE; ++i) child[i] = -1; }
  };
  vector<Node> a;
  vector<string> &words;
  AC_FSM(vector<string> &words) : words(words) {
    a.push_back(Node());
    construct_automaton();
  }
  void construct_automaton() {
    for (int w = 0, n = 0; w < words.size(); ++w, n = 0) {
      for (int i = 0; i < words[w].size(); ++i) {
        if (a[n].child[mp(words[w][i])] == -1) {
          a[n].child[mp(words[w][i])] = a.size();
          a.push_back(Node());
        }
        n = a[n].child[mp(words[w][i])];
      }
      a[n].match.push_back(w);
    }
    queue<int> q;
    for (int k = 0; k < ALPHABET_SIZE; ++k) {
      if (a[0].child[k] == -1) a[0].child[k] = 0;
      else if (a[0].child[k] > 0) {
        a[a[0].child[k]].failure = 0;
        q.push(a[0].child[k]);
      }
    }
    while (!q.empty()) {
      int r = q.front(); q.pop();
      for (int k = 0, arck; k < ALPHABET_SIZE; ++k) {
        if ((arck = a[r].child[k]) != -1) {
          q.push(arck);
          int v = a[r].failure;
          while (a[v].child[k] == -1) v = a[v].failure;
          a[arck].failure = a[v].child[k];
          a[arck].match_par = a[v].child[k];
          while (a[arck].match_par != -1
                 && a[a[arck].match_par].match.empty())
            a[arck].match_par = a[a[arck].match_par].match_par;
        }
      }
    }
  }
  void aho_corasick(string &sentence, vvi &matches){
    matches.assign(words.size(), vi());
    int state = 0, ss = 0;
    for (int i = 0; i < sentence.length(); ++i, ss = state) {
      while (a[ss].child[mp(sentence[i])] == -1)
        ss = a[ss].failure;
      state = a[state].child[mp(sentence[i])]
            = a[ss].child[mp(sentence[i])];
      for (ss = state; ss != -1; ss = a[ss].match_par)
        for (int w : a[ss].match)
          matches[w].push_back(i + 1 - words[w].length());
    }
  }
};
int char_to_int(char c) {
    return cti[c];
}
```

3.5.2 KMP

```cpp
#include "header.h"
void compute_prefix_function(string &w, vi &prefix) {
  prefix.assign(w.length(), 0);
  int k = prefix[0] = -1;

  for(int i = 1; i < w.length(); ++i) {
    while(k >= 0 && w[k + 1] != w[i]) k = prefix[k];
    if(w[k + 1] == w[i]) k++;
    prefix[i] = k;
  }
}
void knuth_morris_pratt(string &s, string &w) {
  int q = -1;
  vi prefix;
  compute_prefix_function(w, prefix);
  for(int i = 0; i < s.length(); ++i) {
    while(q >= 0 && w[q + 1] != s[i]) q = prefix[q];
    if(w[q + 1] == s[i]) q++;
    if(q + 1 == w.length()) {
      // Match at position (i - w.length() + 1)
      q = prefix[q];
    }
  }
}
```

3.6 Geometry

3.6.1 essentials.cpp

```cpp
#include "../header.h"
using C = ld; // could be long long or long double
constexpr C EPS = 1e-10;  // change to 0 for C=ll
struct P {     // may also be used as a 2D vector
  C x, y;
  P(C x = 0, C y = 0) : x(x), y(y) {}
  P operator+ (const P &p) const { return {x + p.x, y + p.y}; }
  P operator- (const P &p) const { return {x - p.x, y - p.y}; }
  P operator* (C c) const { return {x * c, y * c}; }
  P operator/ (C c) const { return {x / c, y / c}; }
  C operator* (const P &p) const { return x*p.x + y*p.y; }
  C operator^ (const P &p) const { return x*p.y - p.x*y; }
  P perp() const { return P{y, -x}; }
  C lensq() const { return x*x + y*y; }
  ld len() const { return sqrt((ld)lensq()); }
  static ld dist(const P &p1, const P &p2) {
    return (p1-p2).len(); }
  bool operator==(const P &r) const {
    return ((*this)-r).lensq() <= EPS*EPS; }
```

```cpp
20 };
21 C det(P p1, P p2) { return p1^p2; }
22 C det(P p1, P p2, P o) { return det(p1-o, p2-o); }
23 C det(const vector<P> &ps) {
24   C sum = 0; P prev = ps.back();
25   for(auto &p : ps) sum += det(p, prev), prev = p;
26   return sum;
27 }
28 // Careful with division by two and C=ll
29 C area(P p1, P p2, P p3) { return abs(det(p1, p2, p3))/C(2); }
30 C area(const vector<P> &poly) { return abs(det(poly))/C(2); }
31 int sign(C c){ return (c > C(0)) - (c < C(0)); }
32 int ccw(P p1, P p2, P o) { return sign(det(p1, p2, o)); }
33
34 // Only well defined for C = ld.
35 P unit(const P &p) { return p / p.len(); }
36 P rotate(P p, ld a) { return P{p.x*cos(a)-p.y*sin(a), p.x*sin(a)+p.y*cos(a)
       }; }
```

3.6.2 Convex Hull

```cpp
1 #include "header.h"
2 #include "essentials.cpp"
3 struct ConvexHull {  // O(n lg n) monotone chain.
4   size_t n;
5   vector<size_t> h, c;  // Indices of the hull are in 'h', ccw.
6   const vector<P> &p;
7   ConvexHull(const vector<P> &_p) : n(_p.size()), c(n), p(_p) {
8     std::iota(c.begin(), c.end(), 0);
9     std::sort(c.begin(), c.end(), [this](size_t l, size_t r) -> bool {
           return p[l].x != p[r].x ? p[l].x < p[r].x : p[l].y < p[r].y; });
10    c.erase(std::unique(c.begin(), c.end(), [this](size_t l, size_t r) {
           return p[l] == p[r]; }), c.end());
11    for (size_t s = 1, r = 0; r < 2; ++r, s = h.size()) {
12      for (size_t i : c) {
13        while (h.size() > s && ccw(p[h.end()[-2]], p[h.end()[-1]], p[i]) <=
               0)
14          h.pop_back();
15        h.push_back(i);
16      }
17      reverse(c.begin(), c.end());
18    }
19    if (h.size() > 1) h.pop_back();
20  }
21  size_t size() const { return h.size(); }
22  template <class T, void U(const P &, const P &, const P &, T &)>
23  void rotating_calipers(T &ans) {
24    if (size() <= 2)
25      U(p[h[0]], p[h.back()], p[h.back()], ans);
26    else
27      for (size_t i = 0, j = 1, s = size(); i < 2 * s; ++i) {
28        while (det(p[h[(i + 1) % s]] - p[h[i % s]], p[h[(j + 1) % s]] - p[h[
               j]]) >= 0)
29          j = (j + 1) % s;
30        U(p[h[i % s]], p[h[(i + 1) % s]], p[h[j]], ans);
31      }
32  }
33 };
```

```cpp
34 // Example: furthest pair of points. Now set ans = 0LL and call
35 // ConvexHull(pts).rotating_calipers<ll, update>(ans);
36 void update(const P &p1, const P &p2, const P &o, ll &ans) {
37   ans = max(ans, (ll)max((p1 - o).lensq(), (p2 - o).lensq()));
38 }
```

3.7 Other Algorithms

3.8 Other Data Structures

3.8.1 Disjoint set (i.e. union-find)

```cpp
1 template <typename T>
2 class DisjointSet {
3     typedef T * iterator;
4     T *parent, n, *rank;
5     public:
6     // O(n), assumes nodes are [0, n)
7     DisjointSet(T n) {
8         this->parent = new T[n];
9         this->n = n;
10        this->rank = new T[n];
11
12        for (T i = 0; i < n; i++) {
13            parent[i] = i;
14            rank[i] = 0;
15        }
16    }
17
18    // O(log n)
19    T find_set(T x) {
20        if (x == parent[x]) return x;
21        return parent[x] = find_set(parent[x]);
22    }
23
24    // O(log n)
25    void union_sets(T x, T y) {
26        x = this->find_set(x);
27        y = this->find_set(y);
28
29        if (x == y) return;
30
31        if (rank[x] < rank[y]) {
32            T z = x;
33            x = y;
34            y = z;
35        }
36
37        parent[y] = x;
38        if (rank[x] == rank[y]) rank[x]++;
39    }
40 };
```

3.8.2 Fenwick tree (i.e. BIT) eff. update + prefix sum calc.

```cpp
1 #include "header.h"
```

```
2  #define maxn 200010
3  int t,n,m,tree[maxn],p[maxn];
4
5  void update(int k, int z) {
6      while (k <= maxn) {
7          tree[k] += z;
8          k += k & (-k);
9          // cout << "k: " << k << endl;
10     }
11 }
12
13 int sum(int k) {
14     int ans = 0;
15     while(k) {
16         ans += tree[k];
17         k -= k & (-k);
18     }
19     return ans;
20 }
```

4 Other Mathematics

4.1 Helpful functions

4.1.1 Euler's Totient Fucntion $n = p_1^{k_1-1} \cdot (p_1 - 1) \cdot \ldots \cdot p_r^{k_r-1} \cdot (p_r - 1)$, where $p_1^{k_1} \cdot \ldots \cdot p_r^{k_r}$ is the prime factorization of n.

```
1  # include "header.h"
2  ll phi(ll n) {   // \Phi(n)
3      ll ans = 1;
4      for (ll i = 2; i*i <= n; i++) {
5          if (n % i == 0) {
6              ans *= i-1;
7              n /= i;
8              while (n % i == 0) {
9                  ans *= i;
10                 n /= i;
11             }
12         }
13     }
14     if (n > 1) ans *= n-1;
15     return ans;
16 }
17 vi phis(int n) {  // All \Phi(i) up to n
18   vi phi(n + 1, 0LL);
19   iota(phi.begin(), phi.end(), 0LL);
20   for (ll i = 2LL; i <= n; ++i)
21     if (phi[i] == i)
22       for (ll j = i; j <= n; j += i)
23         phi[j] -= phi[j] / i;
24   return phi;
25 }
```

Formulas $\Phi(n)$ counts all numbers in $1, \ldots, n-1$ coprime to n.
$a^{\varphi(n)} \equiv 1 \mod n$, a and n are coprimes.
$\forall e > \log_2 m : n^e \mod m = n^{\Phi(m)+e \mod \Phi(m)} \mod m$.
$\gcd(m, n) = 1 \Rightarrow \Phi(m \cdot n) = \Phi(m) \cdot \Phi(n)$.

4.2 Theorems and definitions

Fermat's little theorem $a^p \equiv a \mod p$

Subfactorial $!n = n! \sum_{i=0}^{n} \frac{(-1)^i}{i!}$, $!(0) = 1$, $!n = n \cdot !(n-1) + (-1)^n$

Least common multiple $\text{lcm}(a, b) = a \cdot b / \gcd(a, b)$

Binomials and other partitionings We have $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1} = \prod_{i=1}^{k} \frac{n-i+1}{i}$. This last product may be computed incrementally since any product of k' consecutive values is divisibleby $k'!$. Basic identities: The hockeystick identity: $\sum_{k=r}^{n} \binom{k}{r} = \binom{n+1}{r+1}$ or $\sum_{k \leq n} \binom{r+k}{k} = \binom{r+n+1}{n}$. Also $\sum_{k=0}^{n} \binom{k}{m} = \binom{n+1}{m+1}$.

For $n, m \geq 0$ and p prime. Write n, m in base p, i.e. $n = n_k p^k + \cdots + n_1 p + n_0$ and $m = m_k p^k + \ldots m_1 p + m_0$. Then by Lucas theorem we have $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \mod p$, with the convention that $n_i < m_i \implies \binom{n_i}{m_i} = 0$.

Fibonacci (See also number theory section)
$\sum_{0 \leq k \leq n} \binom{n-k}{k} = F_{n+1}$, $F_n = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^n$,
$\sum_{i=1}^{n} F_i = F_{n+2} - 1$, $\sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$,
$\gcd(F_m, F_n) = F_{\gcd(m,n)}$, $\gcd(F_n, F_{n+1}) = \gcd(F_n, F_{n+2}) = 1$

Bit stuff $a + b = a \oplus b + 2(a\&b) = a|b + a\&b$.
kth bit is set in x iff $x \mod 2^{k-1} \geq 2^k$, or iff $x \mod 2^{k-1} - x \mod 2^k \neq 0$ (i.e. $= 2^k$)
It comes handy when you need to look at the bits of the numbers which are pair sums or subset sums etc.
$n \mod 2^i = n\&(2^i - 1)$.
$\forall k : 1 \oplus 2 \oplus \ldots \oplus (4k - 1) = 0$