3.10.13

1 a) $D_{AB} = \sqrt{(-8m-(-11m))^2 + (19m - 13{,}5m)^2} = 6{,}265\,m$

$D_{BC} = \sqrt{(-11m-(-5m))^2 + (13{,}5m - 13{,}5m)^2} = 6m$

$D_{AC} = \sqrt{(-8m-(-5m))^2 + (19m - 13{,}5m)^2} = 6{,}265\,m$

$D = \sqrt[3]{D_{AB} \cdot D_{BC} \cdot D_{AC}} = \sqrt[3]{6{,}265m \cdot 6m \cdot 6{,}265m} = 6{,}175\,m$

$r_B = \sqrt[n]{n \cdot \sqrt{\frac{A}{\pi}} \cdot \left(\frac{d}{2}\right)^{n-1}} = \sqrt{2 \sqrt{\frac{314{,}159mm^2}{\pi}} \left(\frac{300mm}{2}\right)} = 54{,}77\,mm$

$L_B' = \frac{\mu_0}{2\pi}\left(\ln\left(\frac{D}{r_B}\right) + \frac{1}{4n}\right) = \frac{4\pi 10^{-7} \frac{H}{m}}{2\pi}\left(\ln\left(\frac{6{,}175m}{54{,}77mm}\right) + \frac{1}{4 \cdot 2}\right)$

$= 970{,}0\,\frac{nH}{m} = 970{,}0\,\frac{\mu H}{km}$

$C_B' = \frac{2\pi \varepsilon_0 \varepsilon_r}{\ln\left(\frac{D}{r_B}\right)} = \frac{2\pi\, 8{,}854 \cdot 10^{-12} \cdot 1}{\ln\left(\frac{6{,}175m}{54{,}77mm}\right)} = 11{,}77\,\frac{pF}{m} = 11{,}77\,\frac{nF}{km}$

b) $\alpha = 0$

$\beta = \omega \sqrt{L_B' C_B'} = 2\pi\, 50Hz \sqrt{970{,}0\frac{\mu H}{km} \cdot 11{,}77\,\frac{nF}{km}} = 1{,}062 \cdot 10^{-3}\,\frac{1}{km}$

$\gamma = \alpha + j\beta = j1{,}062 \cdot 10^{-3}\,\frac{1}{km}$

c) $\underline{U}_2 = 0$

$\underline{U}_1 = \sinh(\gamma l)\, \underline{Z}_W\, \underline{I}_2$ $\qquad \underline{Z}_W = \sqrt{\frac{L_B'}{C_B'}} = \sqrt{\frac{970{,}0\frac{\mu H}{km}}{11{,}77\,\frac{nF}{km}}} = 287{,}1\,\Omega$

$\sinh(j\beta l) = j\sin(\beta l)$

$\underline{I}_2 = \underline{I}_K = \frac{\underline{U}_1}{j\sin(\beta l)\, \underline{Z}_W} = \frac{220kV}{j\sin(1{,}062 \cdot 10^{-3}\frac{1}{km} \cdot 250km)\; 287{,}1\,\Omega}$

$= -j2{,}920\,kA$

d) $\underline{U}_1 = j \sin(\beta l) \underline{Z}_W \underline{I}_2$

$\cosh(j\beta l) = \cos(\beta l)$

$\underline{I}_1 = \cos(\beta l) \underline{I}_2$

$$\underline{Z}_1 = \frac{\underline{U}_1}{\underline{I}_1} = \frac{j \sin(\beta l)}{\cos(\beta l)} \underline{Z}_W = j \tan\left(1{,}062 \cdot 10^{-3} \frac{1}{km} \cdot 250km\right) 287{,}1\Omega$$

$= j\, 78{,}07\Omega$

e) $\underline{U}_1 = \cos(\beta l) \underline{U}_2 + j \sin(\beta l) \cdot \frac{\underline{Z}_W}{\underline{Z}_2} \underline{U}_2$

$$\underline{I}_1 = j \sin(\beta l) \frac{\underline{U}_2}{\underline{Z}_W} + \cos(\beta l) \cdot \frac{\underline{U}_2}{\underline{Z}_2}$$

$$\underline{Z}_1 = \frac{\underline{U}_1}{\underline{I}_1} = \frac{\cos(\beta l) + j \sin(\beta l) \frac{\underline{Z}_W}{\underline{Z}_2}}{j \sin(\beta l) \frac{1}{\underline{Z}_W} + \cos(\beta l) \frac{1}{\underline{Z}_2}}$$

$$\underline{Z}_1 j \underline{Z}_2 \sin(\beta l) \frac{1}{\underline{Z}_W} + \underline{Z}_1 \cos(\beta l) = \underline{Z}_2 \cos(\beta l) + j \sin(\beta l) \underline{Z}_W$$

$$\underline{Z}_2 = \frac{j \sin(\beta l) \underline{Z}_W - \underline{Z}_1 \cos(\beta l)}{j \underline{Z}_1 \sin(\beta l) \frac{1}{\underline{Z}_W} - \cos(\beta l)}$$

$$= \frac{j \sin\left(1{,}062 \cdot 10^{-3} \frac{1}{km} 250km\right) 287{,}1\Omega - j134{,}39 \cos\left(1{,}062 \cdot 10^{-3} \frac{1}{km} \cdot 250km\right)}{j \cdot j134{,}39\Omega \sin\left(1{,}062 \cdot 10^{-3} \frac{1}{km} \cdot 250km\right) \frac{1}{287{,}1\Omega} - \cos\left(1{,}062 \cdot 10^{-3} \frac{1}{km} \cdot 250km\right)}$$

$= j\, 49{,}96\Omega$

f) Die Leitung muss mit $\underline{Z}_W$ abgeschlossen werden.

2a) $$\underline{Z}_{(0)} = \frac{\underline{U}_{(0)}}{\underline{I}_{(0)}} = \frac{\frac{1}{3}(\underline{U}_{aN} + \underline{U}_{bN} + \underline{U}_{cN})}{\frac{1}{3}(\underline{I}_a + \underline{I}_b + \underline{I}_c)} = \frac{3\,\underline{U}_{aN}}{\underline{I}_a(1+2+2)}$$

Vereinfachungen: S.49 u. S.50 im Skriptum

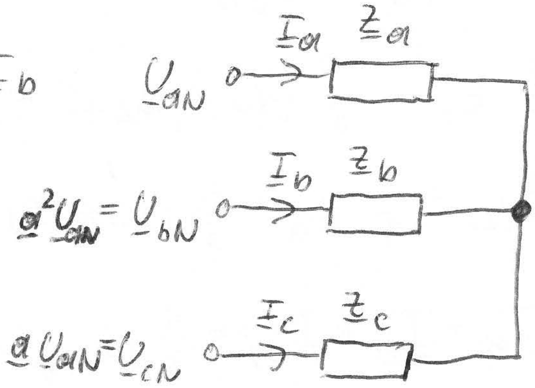
$$\frac{\underline{I}_a}{\underline{I}_b} = \frac{\underline{Z}_b}{\underline{Z}_a} = \frac{1}{2}$$

$$= \frac{3\left(\underline{I}_a \cdot \underline{Z}_a + (\underline{I}_a + \underline{I}_b + \underline{I}_c)\,\underline{Z}_N\right)}{5\,\underline{I}_a} = \frac{3\,\underline{I}_a\left(2\,\underline{Z}_b + 5 \cdot \frac{1}{5}\underline{Z}_b\right)}{5\,\underline{I}_a}$$

$$\underline{Z}_{(0)} = \frac{9}{5}\underline{Z}_b$$

$$\underline{Z}_{(1)} = \frac{\underline{U}_{(1)}}{\underline{I}_{(1)}} = \frac{\frac{1}{3}(\underline{U}_{aN} + \underline{a}\,\underline{U}_{bN} + \underline{a}^2\underline{U}_{cN})}{\frac{1}{3}(\underline{I}_a + \underline{a}\,\underline{I}_b + \underline{a}^2\underline{I}_c)}$$

$$\underline{U}_{aN} - \underline{Z}_a\underline{I}_a = \underline{U}_{bN} - \underline{Z}_b\underline{I}_b$$



$$\Rightarrow \underline{I}_b = \frac{\underline{U}_{aN}(\underline{a}^2 - 1) + \underline{Z}_a\underline{I}_a}{\underline{Z}_b}$$

$$\underline{U}_{aN} - \underline{Z}_a\underline{I}_a = \underline{U}_{cN} - \underline{Z}_c\underline{I}_c$$

$$\Rightarrow \underline{I}_c = \frac{\underline{U}_{aN}(\underline{a} - 1) + \underline{Z}_a\underline{I}_a}{\underline{Z}_c}$$

$$\underline{I}_a = -\underline{I}_b - \underline{I}_c = \frac{\underline{U}_{aN}}{\underline{Z}_b}(-\underline{a}^2 + 1 - \underline{a} + 1) - 4\,\underline{I}_a$$

$$5\,\underline{I}_a = 3\,\frac{\underline{U}_{aN}}{\underline{Z}_b}$$

$$\underline{I}_a = \frac{3}{5}\frac{\underline{U}_{aN}}{\underline{Z}_b}$$

$$\underline{Z}_{(1)} = \frac{3\,\underline{U}_{aN}}{\underline{I}_a + \frac{\underline{U}_{aN}}{\underline{Z}_b}\underline{a}(\underline{a}^2 - 1) + 2\underline{a}\,\underline{I}_a + \frac{\underline{U}_{aN}}{\underline{Z}_b}\underline{a}^2(\underline{a} - 1) + 2\underline{a}^2\underline{I}_a}$$

2a) ff $\underline{Z}_{(1)} = \dfrac{3\,\underline{U}_{aN}}{-\underline{I}_a + \dfrac{\underline{U}_{aN}}{\underline{Z}_b}\cdot 3} = \dfrac{3\,\underline{U}_{aN}}{-\frac{3}{5}\dfrac{\underline{U}_{aN}}{\underline{Z}_b} + 3\dfrac{\underline{U}_{aN}}{\underline{Z}_b}}$

$= \frac{5}{4}\underline{Z}_b$

$\underline{Z}_{(2)} = \dfrac{\underline{U}_{(2)}}{\underline{I}_{(2)}} = \dfrac{\frac{1}{3}(\underline{U}_{aN} + \underline{a}^2\underline{U}_{bN} + \underline{a}\,\underline{U}_{cN})}{\frac{1}{3}(\underline{I}_a + \underline{a}^2\underline{I}_b + \underline{a}\,\underline{I}_c)} = \dfrac{3\,\underline{U}_{aN}}{\underline{I}_a + \underline{a}^2\underline{I}_b + \underline{a}\,\underline{I}_c}$

wenn man $\underline{I}_b$ und $\underline{I}_c$ miteinander vertauscht, was man machen darf weil $\underline{Z}_b = \underline{Z}_c$, dann hat man die selbe Gleichung wie für $\underline{Z}_{(1)}$

$\Rightarrow \underline{Z}_{(2)} = \underline{Z}_{(1)}$

b) $\underline{Z}_{(0)} = \frac{9}{5}\underline{Z}_b = \frac{9}{5}(15+j5)\,\Omega = (27+j9)\,\Omega$

$\underline{Z}_{(1)} = \underline{Z}_{(2)} = \frac{5}{4}\underline{Z}_b = \frac{5}{4}(15+j5)\,\Omega = (18{,}75+j6{,}25)\,\Omega$

c) $\underline{I}_{(0)} = \frac{1}{3}(\underline{I}_a + \underline{I}_b + \underline{I}_c) = \frac{1}{3}(k\,\underline{I}_{ph} + \underline{a}^2\underline{I}_{ph} + \underline{a}\,\underline{I}_{ph}) = \frac{(k-1)}{3}\underline{I}_{ph}$

$\underline{I}_{(1)} = \frac{1}{3}(\underline{I}_a + \underline{a}\,\underline{I}_b + \underline{a}^2\underline{I}_c) = \frac{1}{3}(k\,\underline{I}_{ph} + \underline{I}_{ph} + \underline{I}_{ph}) = \frac{(k+2)}{3}\underline{I}_{ph}$

$\underline{I}_{(2)} = \frac{1}{3}(\underline{I}_a + \underline{a}^2\underline{I}_b + \underline{a}\,\underline{I}_c) = \frac{1}{3}(k\,\underline{I}_{ph} + \underline{a}\,\underline{I}_{ph} + \underline{a}^2\underline{I}_{ph}) = \frac{(k-1)}{3}\underline{I}_{ph}$

d) $\underline{U}_{(0)} = \underline{I}_{(0)}\cdot\underline{Z}_{(0)} = \frac{(k-1)}{3}\cdot\underline{I}_{ph}\frac{9}{5}\underline{Z}_b = \frac{3(k-1)}{5}\underline{I}_{ph}\underline{Z}_b$

$\underline{U}_{(1)} = \underline{I}_{(1)}\cdot\underline{Z}_{(1)} = \frac{(k+2)}{3}\underline{I}_{ph}\cdot\frac{5}{4}\underline{Z}_b = \frac{5(k+2)}{12}\underline{I}_{ph}\underline{Z}_b$

$\underline{U}_{(2)} = \underline{I}_{(2)}\cdot\underline{Z}_{(2)} = \frac{(k-1)}{3}\underline{I}_{ph}\cdot\frac{5}{4}\underline{Z}_b = \frac{5(k-1)}{12}\underline{I}_{ph}\underline{Z}_b$

e) $\underline{U}_a = \underline{U}_{(0)} + \underline{U}_{(1)} + \underline{U}_{(2)} = \left(\frac{3}{5}(k-1) + \frac{5}{12}(k+2) + \frac{5}{12}(k-1)\right)\underline{I}_{ph}\underline{Z}_b = \left(\frac{43}{30}k - \frac{11}{60}\right)\underline{I}_{ph}\underline{Z}_b$

f) $\underline{U}_a = \left(\frac{43}{30}k - \frac{11}{60}\right)\underline{I}_{ph}\underline{Z}_b = \left(\frac{43}{30}0{,}619 - \frac{11}{60}\right)\cdot 13{,}05\,A\,(15+j5)\,\Omega = (137{,}8+j45{,}93)\,V$

3 a) $E = \rho \cdot V \cdot g \cdot \Delta h = 1000 \frac{kg}{m^3} \cdot 0{,}4 \cdot 50 \cdot 10^6 m^3 \cdot 9{,}81 \frac{m}{s^2} \cdot 250 m$

$= 49{,}05 \, TJ$

b) $E = c \cdot m_{us} \, \Delta T = \rho \cdot V_{us}' \cdot g \cdot \Delta h$

$$\Delta T = \frac{\rho \cdot V_{us}' \cdot g \cdot \Delta h}{c \cdot \rho \cdot V_{us}} = \frac{(1-0{,}85) \cdot 25 \cdot 10^6 m^3 \cdot 9{,}81 \frac{m}{s^2} \cdot 250 m}{4{,}18 \frac{kWs}{kgK} \cdot 0{,}85 \cdot 25 \cdot 10^6 m^3}$$

$\Delta T = 0{,}1035 K$

c) $$P_{el} = \frac{\rho \cdot Q \cdot g \cdot \Delta h}{\eta_H \cdot \eta_P \cdot \eta_{el} \cdot (1-\varepsilon)} = \frac{1000 \frac{kg}{m^3} \cdot 90 \frac{m^3}{s} \cdot 9{,}81 \frac{m}{s^2} \cdot 250 m}{0{,}94 \cdot 0{,}88 \cdot 0{,}96 \cdot (1-0{,}02)}$$

$= 283{,}6 \, MW$

d) $(1-0{,}4) \cdot V_{os} = (1-0{,}4) \cdot 50 \cdot 10^6 m^3 = 30 \cdot 10^6 m^3$

$0{,}85 \cdot V_{us} = 0{,}85 \cdot 25 \cdot 10^6 m^3 = 21{,}25 \cdot 10^6 m^3$ $\leftarrow$ ist weniger, daher relevant

$$t = \frac{0{,}85 \cdot V_{us}}{Q} = \frac{0{,}85 \cdot 25 \cdot 10^6 m^3}{90 \frac{m^3}{s}} = 65{,}59 h$$

e) $E_{el} = P_{el} \cdot t = 283{,}6 \, MW \cdot 65{,}59 h = 18{,}60 \, GWh$

f) $E = \rho \cdot 0{,}85 \cdot V_{us} \cdot g \cdot \Delta h = 1000 \frac{kg}{m^3} \cdot 0{,}85 \cdot 25 \cdot 10^6 m^3 \cdot 9{,}81 \frac{m}{s^2} \cdot 250 m$

$= 52{,}12 \, TJ = 14{,}48 \, GWh$

5 a) $Z_Q = c \cdot \frac{U_2^2}{S_{kQ}''} = 1{,}1 \frac{(30kV)^2}{5GVA} = 0{,}198\,\Omega$

$Z_Q = \sqrt{R_Q^2 + X_Q^2} = X_Q \sqrt{0{,}35^2 + 1^2}$

$X_Q = \frac{Z_Q}{\sqrt{0{,}35^2+1^2}} = \frac{0{,}198\,\Omega}{\sqrt{0{,}35^2+1^2}} = 0{,}1869\,\Omega$

$R_Q = \sqrt{Z_Q^2 - X_Q^2} = \sqrt{(0{,}198\,\Omega)^2 - (0{,}1869\,\Omega)^2} = 0{,}06536\,\Omega$

$Z_T = u_k \frac{U_2^2}{S_N} = 0{,}14 \cdot \frac{(30kV)^2}{40MVA} = 3{,}15\,\Omega$

$R_T = P_k \frac{U_2^2}{S_N^2} = 300kW \frac{(30kV)^2}{(40MVA)^2} = 0{,}1688\,\Omega$

$X_T = \sqrt{Z_T^2 - R_T^2} = \sqrt{(3{,}15\,\Omega)^2 - (0{,}1688\,\Omega)^2} = 3{,}145\,\Omega$

$R_L = R' \cdot l = 0{,}1 \frac{\Omega}{km} \cdot 60km = 6\,\Omega$

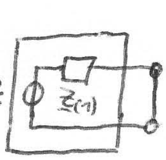
$X_L = \omega L' l = 2\pi \cdot 50Hz \cdot 0{,}75 \frac{mH}{km} \cdot 60km = 14{,}14\,\Omega$

$\underline{Z}_G = R_Q + R_T + R_L + j(X_Q + X_T + X_L) = 0{,}06536\,\Omega + 0{,}1688\,\Omega + 6\,\Omega + j(0{,}1869\,\Omega + 3{,}145\,\Omega + 14{,}14\,\Omega) = (6{,}234 + j\,17{,}47)\,\Omega$

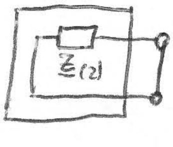
b) $\underline{U}_{aN,F} = \underline{U}_{bN,F} = \underline{U}_{cN,F}$

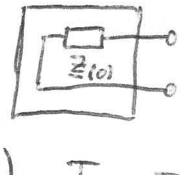
$\underline{I}_{a,F} + \underline{I}_{b,F} + \underline{I}_{c,F} = 0 \Rightarrow$ $\underline{I}_{(0)} = 0$, $\underline{U}_{(1)} = 0$, $\underline{U}_{(2)} = 0$

$E_1 = c\frac{U_n}{\sqrt{3}}$



$\underline{Z}_{(1)} = \underline{Z}_G$





c) $\underline{I}_{(1)} = \frac{E_1}{\underline{Z}_{(1)}} = \frac{1{,}1 \cdot 30kV}{\sqrt{3}(6{,}234 + j17{,}47)\,\Omega} = (345{,}2 - j\,967{,}4)\,A$

$I_{k3p}'' = |\underline{I}_{(a)}| = |\underline{I}_{(1)}| = 1{,}027\,kA$

d) $i_p = \sqrt{2}\left(1 + e^{-t \cdot \frac{R}{\frac{X}{\omega}}}\right) I''_{k3p} = \sqrt{2}\left(1 + e^{-10ms \cdot \frac{6,234\Omega}{\frac{17,47\Omega}{2\pi 50Hz}}}\right) \cdot 1,027kA$

$= 1,926\ kA$

e) $Z_Q' = \ddot{u}^2 Z_Q = \left(\frac{110kV}{30kV}\right)^2 \cdot 0,198\Omega = 2,662\Omega$

$I''_{k3p} = c \cdot \frac{U_1}{\sqrt{3}\, Z_Q'} = 1,1 \frac{110kV}{\sqrt{3}\, 2,662\Omega} = 26,24kA$