Circular Motion and Gravity
Orbital Simulation
PHYS 442

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Partners:	Whole class
Instructor:	Dr. Schultz

1 Objective

Explored the motion of a particle under the influence of a gravitational force. Specifically we look at attractive inverse square distance forces, Hookean forces, escape velocity, circular orbits, kinetic energy, potential energy and elliptical orbits. These are defined in 1.1:

1.1 Definitions

Law of Universal Gravitation The law of universal gravitation states the force of gravity between two point masses is directly proportional to each mass and inversely proportional to the distance between them. This is also true for masses outside of spherically symmetric mass distributions.Homer (2014)

$$F_g = \frac{mMG}{r^2}$$

Hookean Forces Inside a uniformly dense sphere of mass the force is Hookean, with an attractive force proportional to the displacement from equilibrium. The effective spring constant is $K = \frac{mMG}{R^3}$.

$$F_g = \frac{mMG}{R^3} r$$

Gravitational Constant The universal gravitation constant G determines the strength of the gravity force from a given mass. It may also be considered as the force that 1 kg exerts on another 1 kg mass separated by 1 meter.

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

Escape Velocity Escape velocity is the initial velocity required to escape gravitational attraction. An object launched at the escape velocity will never come back (escape).

$$v_{escape} = \sqrt{\frac{2MG}{r}}$$

Kinetic Energy Kinetic energy is the energy associated with motion.

$$KE = \frac{mv^2}{2}$$

Potential Energy The potential associated with the universal gravitation force is written as follows.

$$PE = -\frac{mMG}{r}$$

Circular Orbit A circular orbit is an orbit with a constant radius r.

Elliptic Orbit An elliptic orbit is a closed orbit with changing radius r.

2 Simulation

The simulation applies a central acceleration to the orbiting particle. Outside the boundary of the central mass we have the following acceleration.

$$a = \frac{K}{r^2}$$

Inside the boundary of the central mass ($r < R$) we have the following acceleration.

$$a = \frac{K}{R^3} r$$

Here R is the radius of the central mass and K is a constant determined by the user of the simulation. K is MG. For this simulation the radius was set to $R = 6$ and the constant was set to $K = -0.1$ making the acceleration attractive.

The initial position $\overrightarrow{r}_0$ and initial velocity $\overrightarrow{v}_0$ are set by the user.

3 Sample Calculation

3.1 Circular Orbit

Given $\overrightarrow{r}_0 = (10, 0)$ and $K = -0.1$ we find the $\overrightarrow{v}_0$ for circular orbit.

$$F_{net} = ma$$

$$\frac{mMG}{r^2} = m\frac{v^2}{r}$$

$$\frac{K}{r^2} = \frac{v^2}{r}$$

$$v = \sqrt{\frac{K}{r}}$$

$$v = \sqrt{\frac{0.1}{10}} = 0.1$$

The velocity must be tangential and therefore $\overrightarrow{v}_0$ must be perpendicular to $\overrightarrow{r}_0$.

$$\overrightarrow{v}_0 = (0, 0.1)$$

3.2 Escape Velocity

Given $\overrightarrow{r}_0 = (10, 0)$ and $K = -0.1$ we find the $\overrightarrow{v}_0$ for escape from the central mass' gravitational attraction. Escape is associated with a total mechanical energy of zero.

$$PE + KE = 0$$

$$-\frac{mMG}{r} + \frac{mv^2}{2} = 0$$

$$-\frac{K}{r} + \frac{v^2}{2} = 0$$

$$v_{escape} = \sqrt{\frac{2K}{r}}$$

$$v_{escape} = \sqrt{\frac{2(0.1)}{10}} = 0.14$$

4 Results and Conclusions

4.1 Circular Orbit

For the conditions $\overrightarrow{r}_0 = (10, 0)$ and $K = -0.1$ we calculate an initial velocity $\overrightarrow{v}_0 = (0, 0.1)$ will give a circular orbit. Running the simulation yields the following orbit. We can see it is circular.

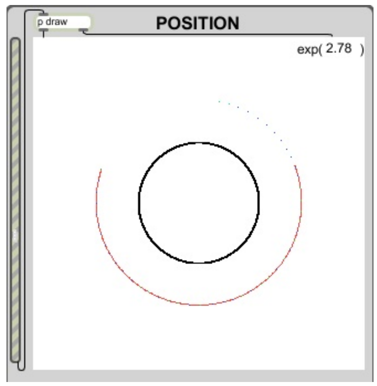

Figure 1: Circular Orbit

4.2 Escape Velocity

For the conditions $\overrightarrow{r}_0 = (10, 0)$ and $K = -0.1$ we calculate an initial velocity $\overrightarrow{v}_0 = (0, 0.14)$ will give an escape from the gravitational attraction. We can observe in the simulation the object indeed escapes and the total energy is zero. See the following figure.

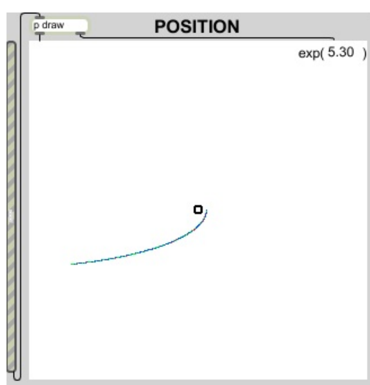

Figure 2: Escape Velocity

4.3 Elliptical Orbit

For the conditions $\overrightarrow{r}_0 = (10, 0)$ and $K = -0.1$ we use an initial velocity $\overrightarrow{v}_0 = (0, 0.12)$ that is between the one for circular orbit and escape. As the figure shows the resulting orbit is elliptical.

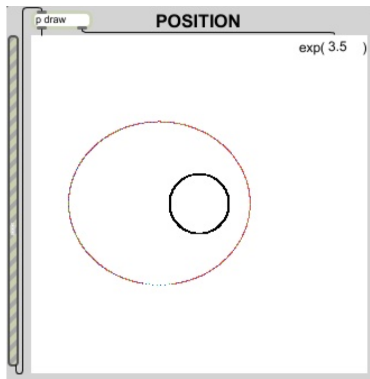

Figure 3: Elliptic Orbit

5 Discussion

I believe, Mechanics describes an orbit as one object in free fall around another where projectile paths become elliptical. It describes an orbit as an object taking the shortest distance through curved space or for instance if the cannonball is fired with sufficient velocity, the ground curves away from the at least as much as the ball falls, so the ball never strikes the ground. It is now in what could be called a non-interrupted orbit. For any specific combination of height above the center of gravity and mass of the planet, there is one specific firing velocity.

References

Homer, J. (2014). *Physics*. Oxford, 3rd edition.