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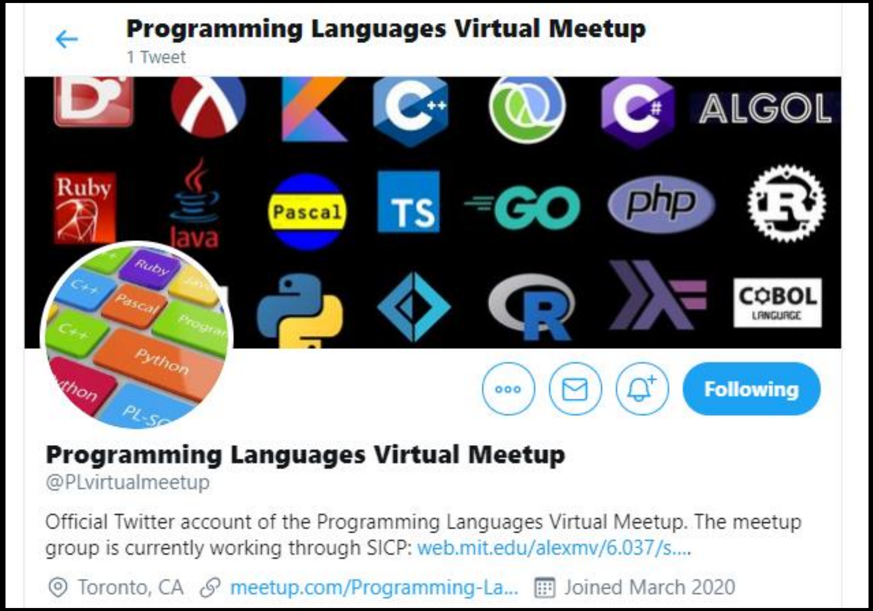
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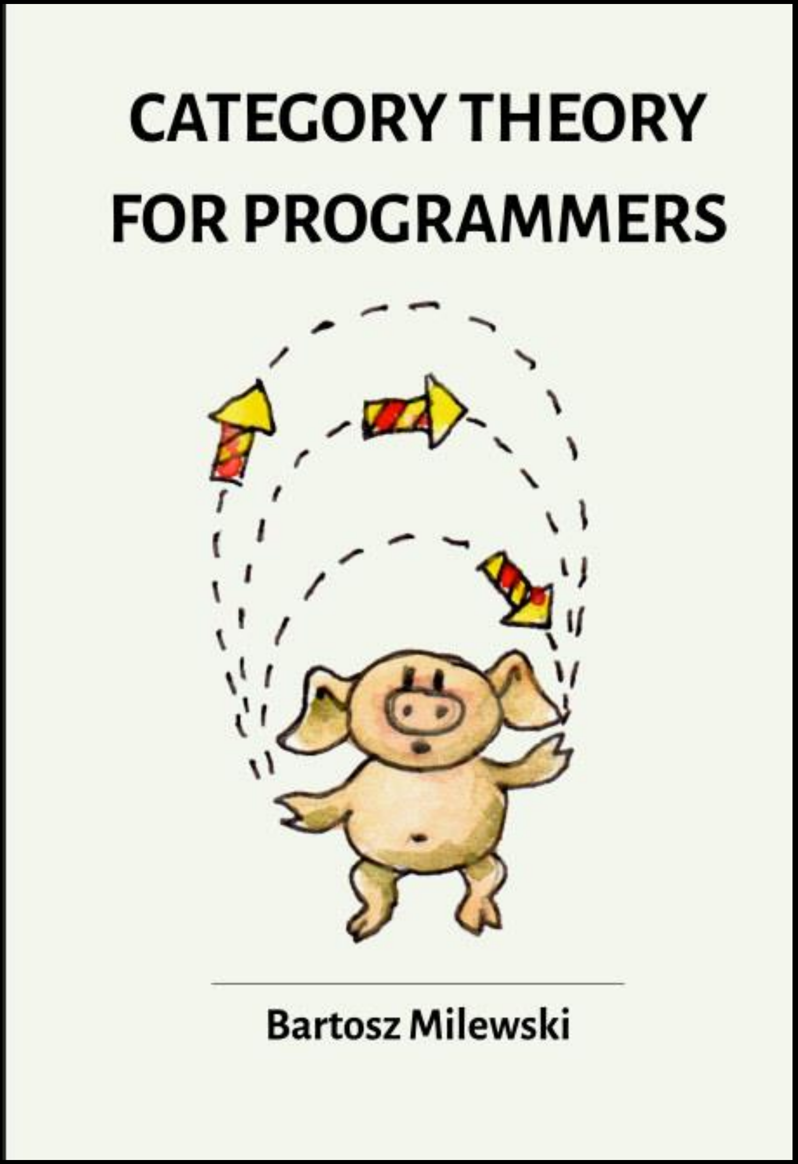
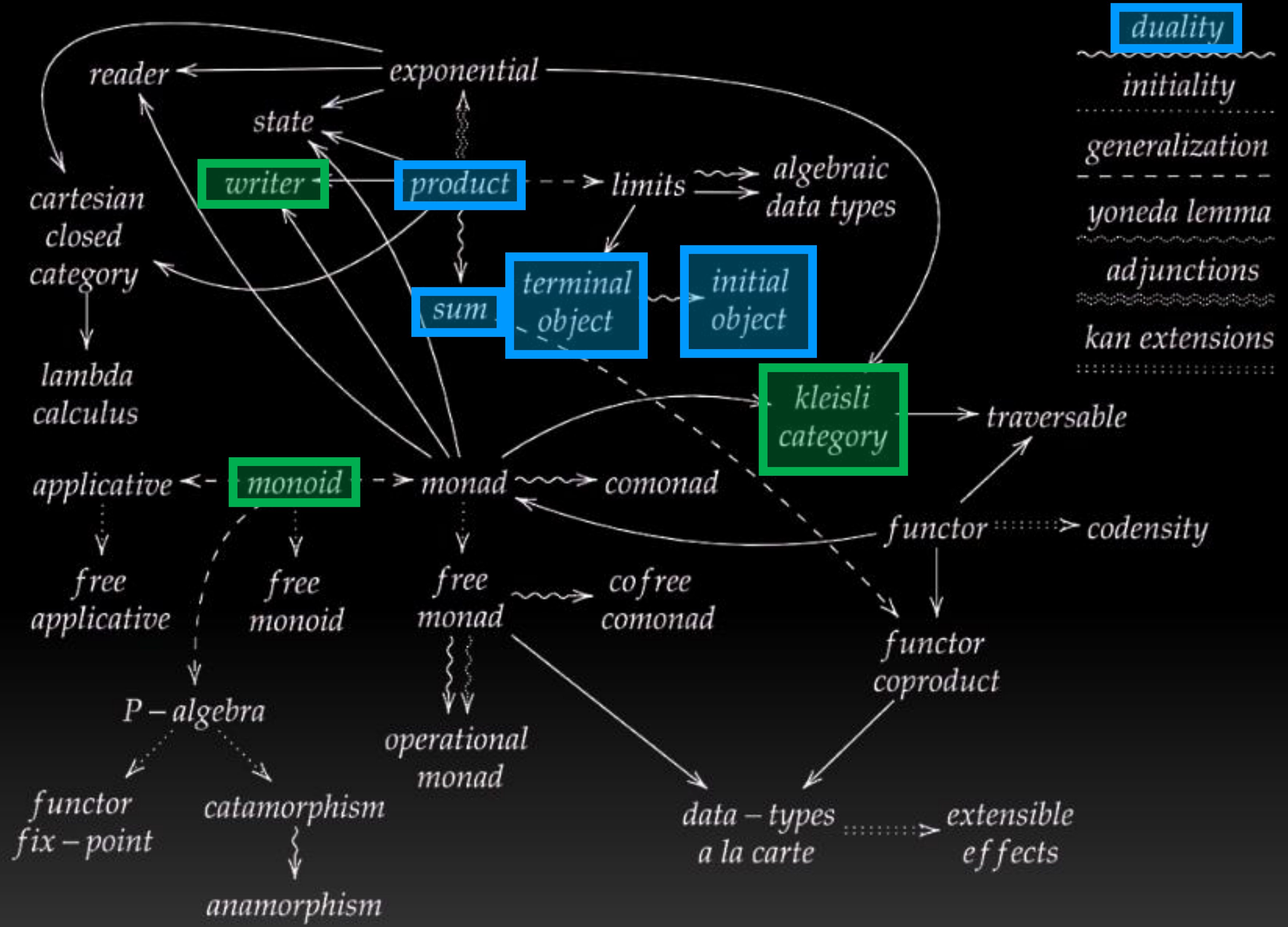


Category Theory for Programmers

Chapter 5:

Products & Coproducts

The Tools for Thought



The **initial object** is the object that has one and only one morphism going to any object in the category.

The **terminal object** is the object with one and only one morphism coming to it from any object in the category.

5.3 Duality

You can't help but to notice the symmetry between the way we defined the initial object and the terminal object. The only difference between the two was the direction of morphisms. It turns out that for any category $\mathbf{C}$ we can define the *opposite category* $\mathbf{C}^{op}$ just by reversing all the arrows.

Mathematically it means that there is a mapping from object a to object b, and there is a mapping from object b back to object a, and they are the inverse of each other. In category theory we replace mappings with morphisms. An isomorphism is an invertible morphism; or a pair of morphisms, one being the inverse of the other.

Now let’s forget about sets and define a product of two objects in any category using the same universal construction. Such a product doesn’t always exist, but when it does, it is unique up to a unique isomorphism.

When I said that the initial (terminal) object was unique up to isomorphism, I meant that any two initial (terminal) objects are isomorphic.

A **product** of two objects a and b is the object c equipped with two projections such that for any other object c' equipped with two projections there is a unique morphism m from c' to c that factorizes those projections.

A **coproduct** of two objects a and b is the object c equipped with two injections such that for any other object c' equipped with two injections there is a unique morphism m from c to c' that factorizes those injections.

1. Show that the terminal object is unique up to unique isomorphism.

What does 'unique up to isomorphism' really mean?

Many thanks to Jens Seeber for comments and corrections!

Phrases like "*Terminal objects are unique up to isomorphism*" are everywhere in category theory. In this post, I'll explain the concept of "uniqueness up to isomorphism", and its best buddy, "uniqueness up to *unique* isomorphism".

I'll also talk about the philosophy behind these concepts, and how they allow category theorists to define objects in terms of their **relationship to other objects** rather than by some internal properties. In other words, how can we avoid asking "what *is* this object?", but instead ask "how does this object *behave*?".

I will assume only a minimal background in category theory, but you should at least know what a category is, and what is meant by "objects" and "morphisms".

We'll begin with some intuition for how isomorphism mathematically means "same shape". Next, we'll make that intuition precise, before examining the example of terminal objects, which are both unique up to isomorphism, and up to *unique* isomorphism.

$$\begin{tikzcd} A \arrow[r, bend left, "\alpha"] \arrow["id_A", loop left] & B \arrow[l, bend left, "\alpha^{-1}"] \arrow["id_B", loop right] \end{tikzcd}$$

4. Implement the equivalent of Haskell `Either` as a generic type in your favorite language (other than Haskell).
5. Show that `Either` is a "better" coproduct than `int` equipped with two injections:

```
left      ← 1,⊢
right     ← 2,⊢
isLeft    ← 1=⊃
isRight   ← 2=⊃
fromLeft  ← { isLeft  ⍵: ⊃⌽⍵ ⋄ 'Fail' ⎕SIGNAL 999 }
fromRight ← { isRight ⍵: ⊃⌽⍵ ⋄ 'Fail' ⎕SIGNAL 999 }
```

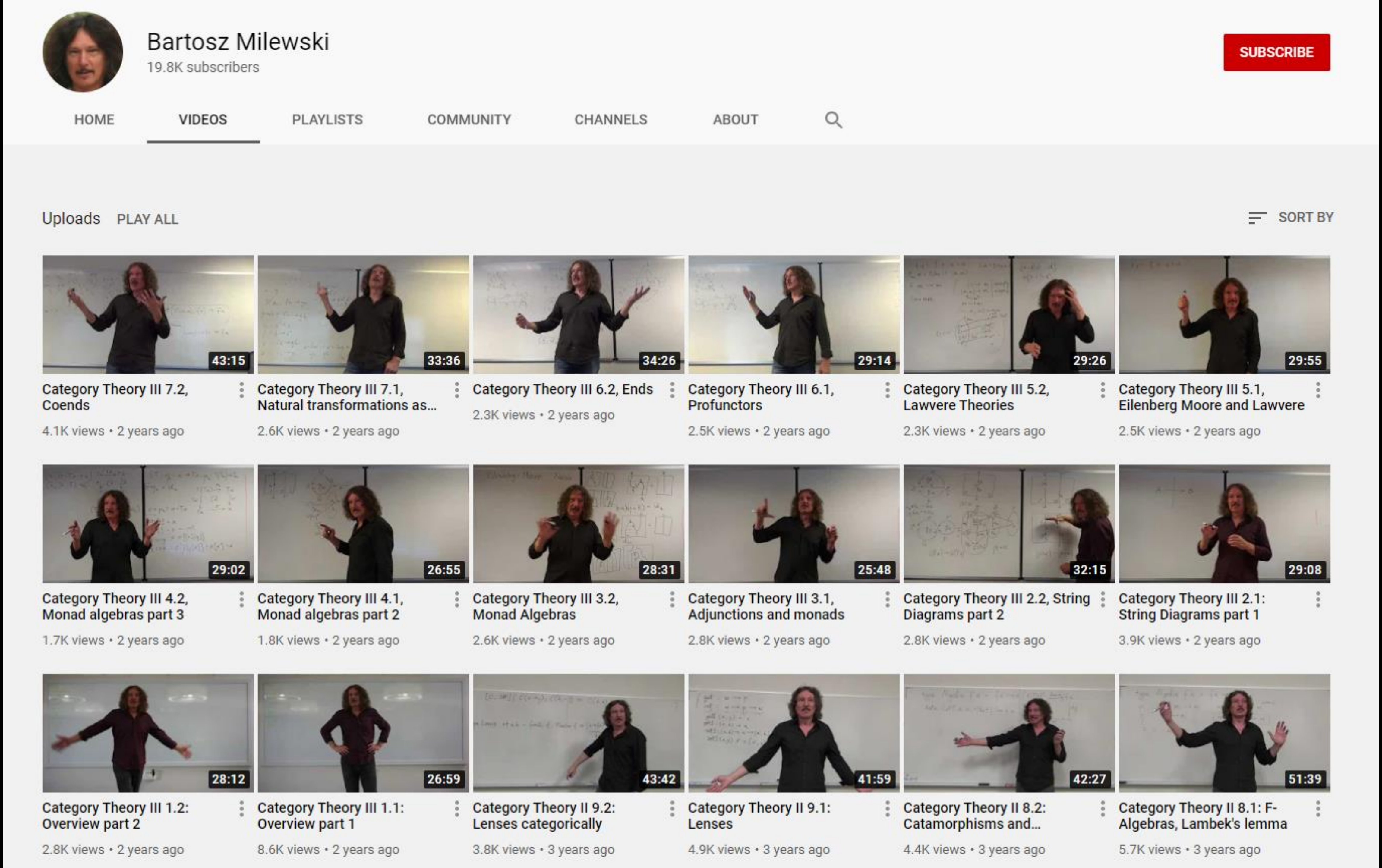
```
left      ← 1,⊢
right     ← 2,⊢
isLeft    ← 1=⊃
isRight   ← 2=⊃
fromLeft  ← { isLeft  ⍵: ⊃⌽⍵ ⋄ 'Fail' ⎕SIGNAL 999 }
fromRight ← { isRight ⍵: ⊃⌽⍵ ⋄ 'Fail' ⎕SIGNAL 999 }

l ← left  42
r ← right 1729

isLeft    l ⍝ 1
isRight   r ⍝ 1
fromLeft  l ⍝ 42
fromRight r ⍝ 1729
fromLeft  r ⍝ Fail
fromRight l ⍝ Fail
```

```
i ← ⊢
j ← 0∘≠

m ← { isLeft ⍵: fromLeft     ⍵
              ⋄ 0≠fromRight ⍵ }
```



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$$\begin{tikzcd}
 & c' \arrow[ddl, bend right, "p'"'] \arrow[ddr, bend left, "q'"] \arrow[d, dashed, "m"] & \\
 & c \arrow[dl, "p"] \arrow[dr, "q"'] & \\
a & & b
\end{tikzcd}$$

$p' = p \circ m$

$q' = q \circ m$

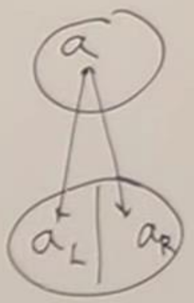
(a, b)

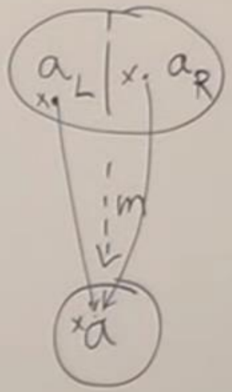
pair $\langle a, b \rangle$

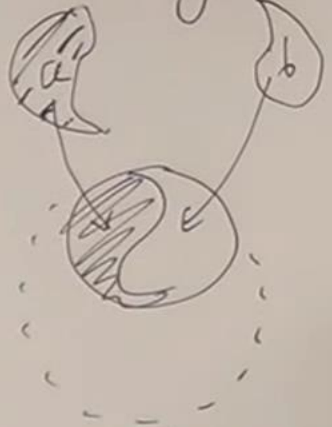
$$\begin{tikzcd}
a \arrow[dr, "i"] \arrow[ddr, bend right, "i'"'] & & b \arrow[dl, "j"'] \arrow[ddl, bend left, "j'"] \\
 & c \arrow[d, dashed, "m"] & \\
 & c' &
\end{tikzcd}$$

$i' = m \circ i$

$j' = m \circ j$







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