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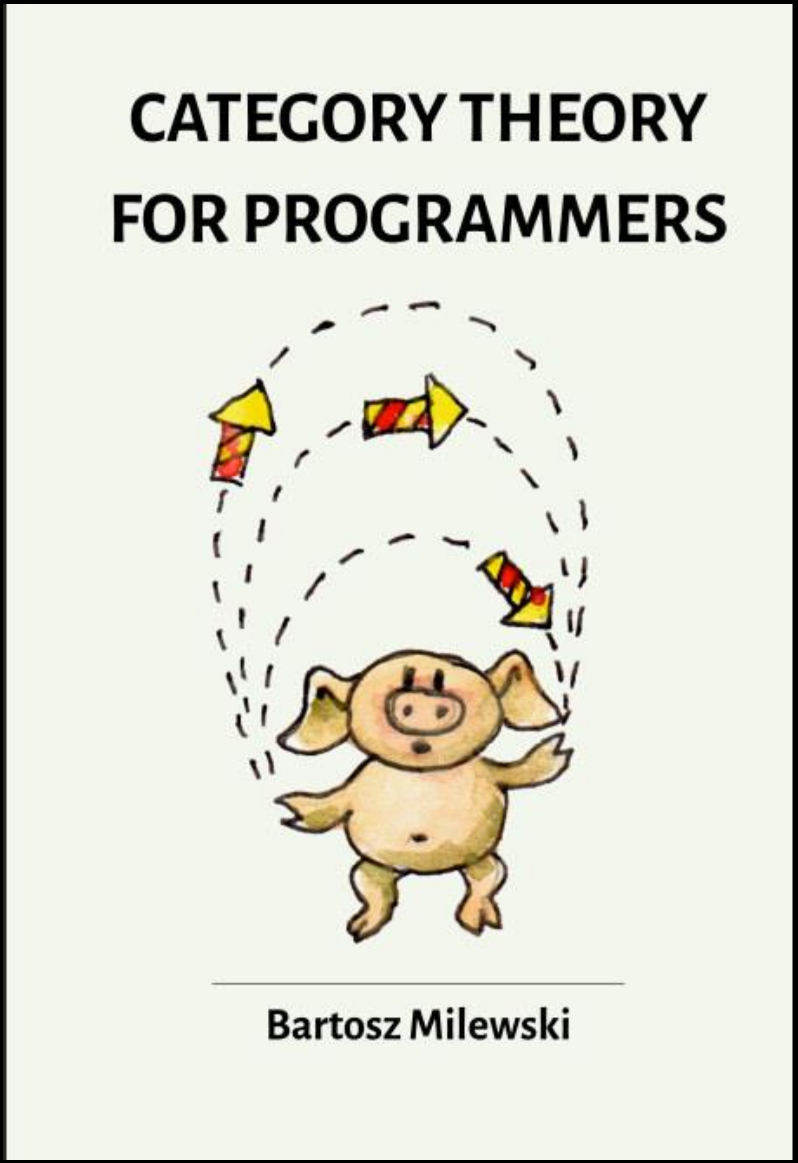
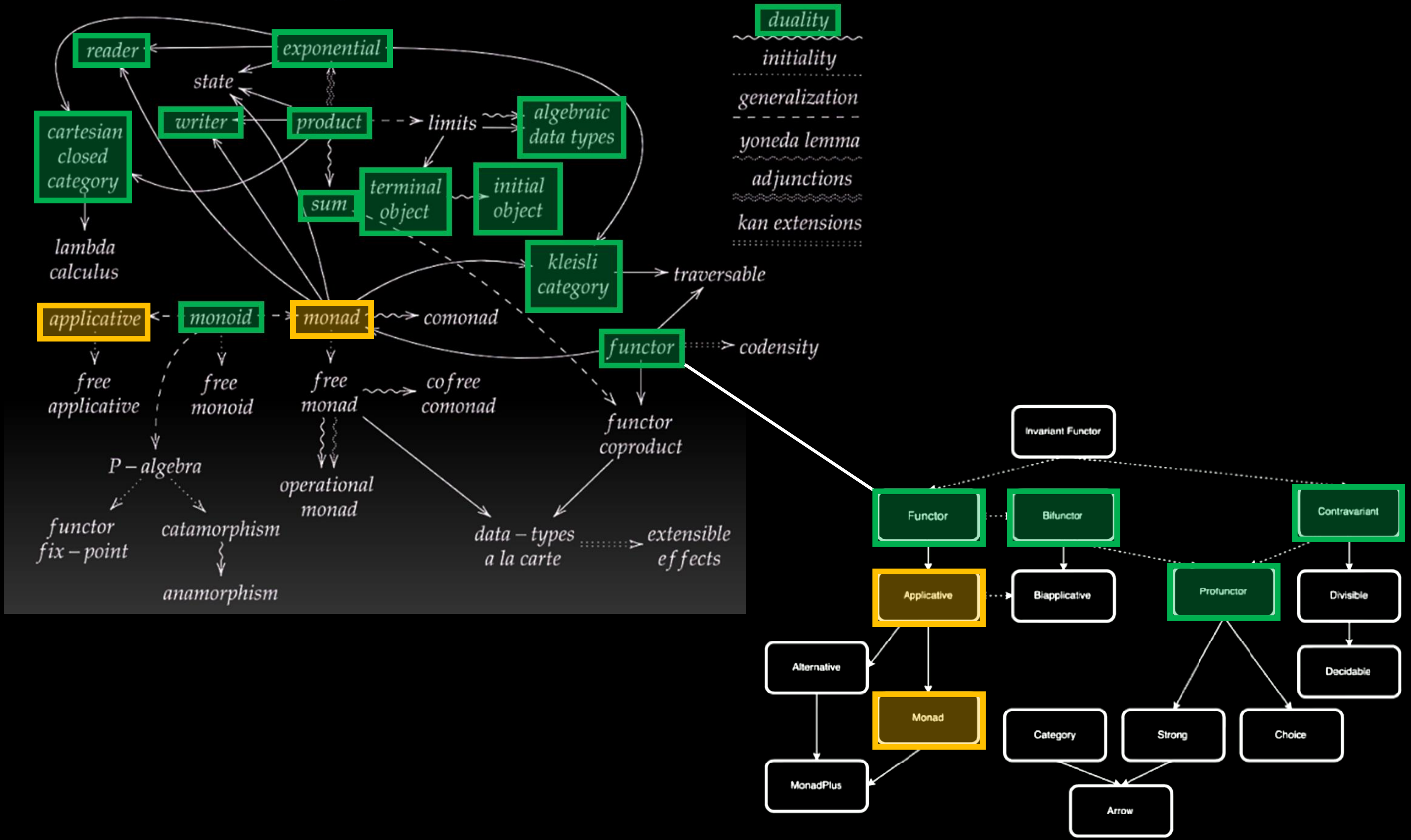
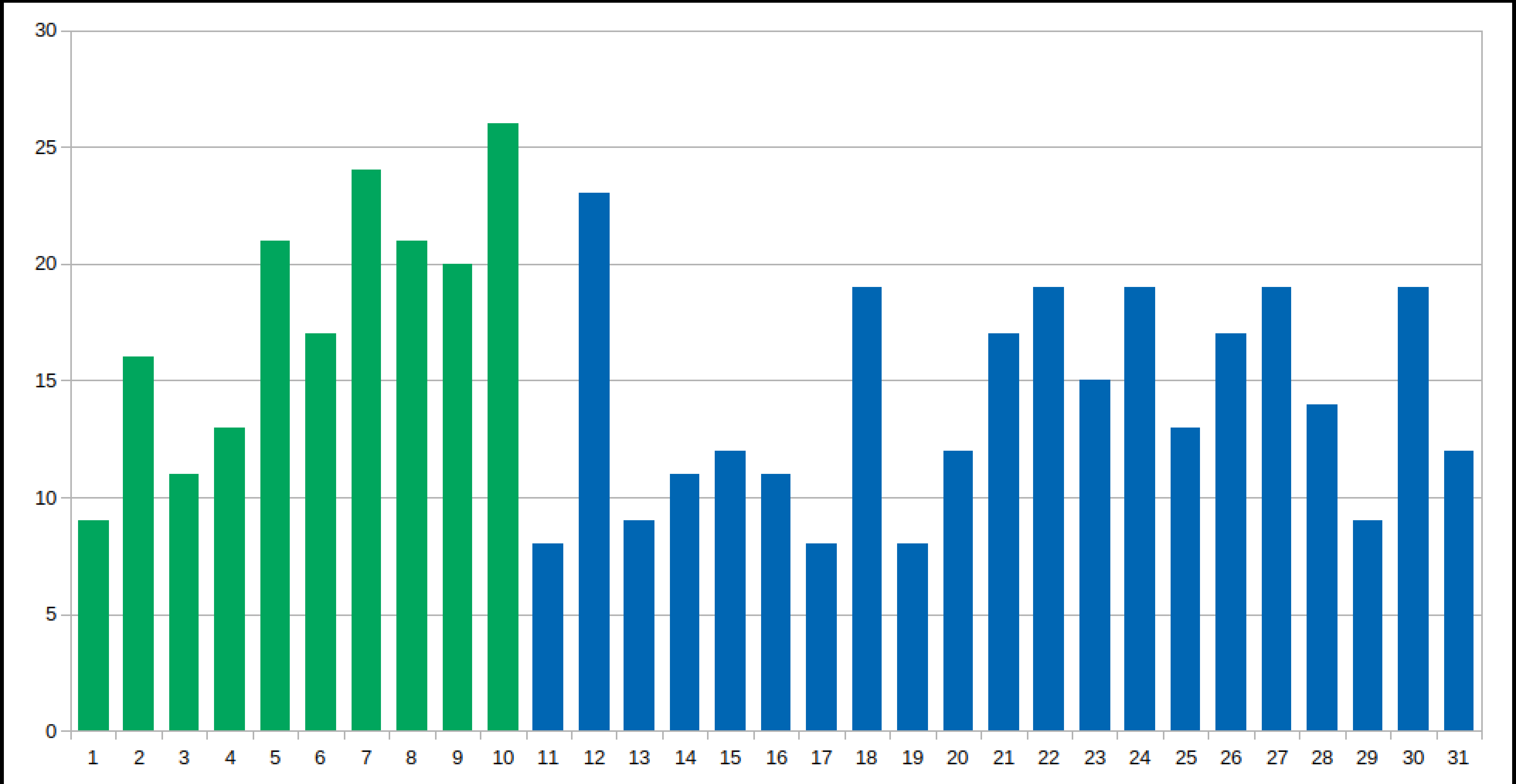


Category Theory for Programmers

Chapter 10: Natural Transformations

The Tools for Thought





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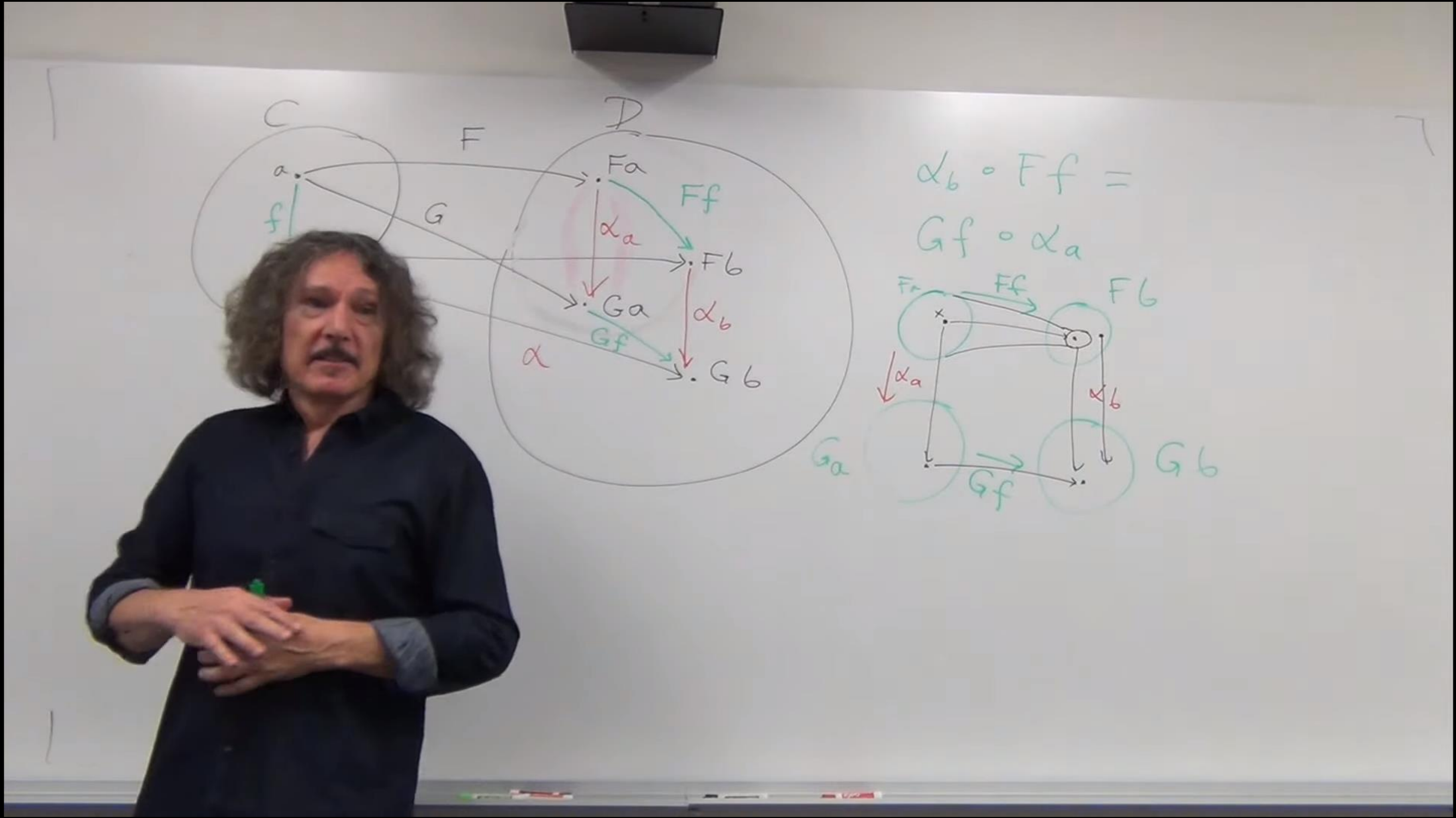
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A natural transformation is a polymorphic function.

Bartosz Milewski – Lecture 9.1

10.1 Polymorphic Functions

We talked about the role of functors (or, more specifically, endofunctors) in programming. They correspond to type constructors that map types to types. They also map functions to functions, and this mapping is implemented by a higher order function `fmap` (or `transform`, `then`, and the like in C++).

To construct a natural transformation we start with an object, here a type, `a`. One functor, `F`, maps it to the type Fa. Another functor, `G`, maps it to Ga. The component of a natural transformation `alpha` at `a` is a function from Fa to Ga. In pseudo-Haskell:

```
alpha_a :: F a -> G a
```

There is a more profound difference between Haskell's polymorphic functions and C++ generic functions, and it's reflected in the way these functions are implemented and type-checked. In Haskell, a polymorphic function must be defined uniformly for all types. One formula must work across all types. This is called *parametric polymorphism*.

C++, on the other hand, supports by default *ad hoc polymorphism*, which means that a template doesn't have to be well-defined for all types. Whether a template will work for a given type is decided at instantiation time, where a concrete type is substituted for the type parameter. Type checking is deferred, which unfortunately often leads to incomprehensible error messages.

In C++, there is also a mechanism for function overloading and template specialization, which allows different definitions of the same function for different types. In Haskell this functionality is provided by type classes and type families.

Concepts vs Typeclasses vs Traits vs Protocols vs Type Constraints

Conor Hoekstra

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How to make *ad-hoc* polymorphism less *ad hoc*

Philip Wadler and Stephen Blott
University of Glasgow*

October 1988

Abstract

This paper presents *type classes*, a new approach to *ad-hoc* polymorphism. Type classes permit overloading of arithmetic operators such as multiplication, and generalise the "eqtype variables" of Standard ML. Type classes extend the Hindley/Milner polymorphic type system, and provide a new approach to issues that arise in object-oriented programming, bounded type quantification, and abstract data types. This paper provides an informal introduction to type classes, and defines them formally by means of type inference rules.

1 Introduction

Strachey chose the adjectives *ad-hoc* and *parametric* to distinguish two varieties of *polymorphism* [Str67].

Ad-hoc polymorphism occurs when a function is defined over several different types, acting in a different way for each type. A typical example is overloaded multiplication: the same symbol may be used to denote multiplication of integers (as in `3*3`) and multiplication of floating point values (as in `3.14*3.14`).

Parametric polymorphism occurs when a function is defined over a range of types, acting in the same way for each type. A typical example is the `length` function, which acts in the same way on a list of integers and a list of floating point numbers.

One widely accepted approach to parametric polymorphism is the Hindley/Milner type system [Hin69, Mil78, DM82], which is used in Standard ML [HMM86, Mil87], Miranda[1][Tur85], and other languages. On the other hand, there is no widely accepted approach to *ad-hoc* polymorphism, and so its name is doubly appropriate.

This paper presents *type classes*, which extend the Hindley/Milner type system to include certain kinds of overloading, and thus bring together the two sorts of polymorphism that Strachey separated.

The type system presented here is a generalisation of the Hindley/Milner type system. As in that system, type declarations can be inferred, so explicit type declarations for functions are not required. During the inference process, it is possible to translate a program using type classes to an equivalent program that does not use overloading. The translated programs are typable in the (ungeneralised) Hindley/ Milner type system.

The body of this paper gives an informal introduction to type classes and the translation rules, while an appendix gives formal rules for typing and translation, in the form of inference rules (as in [DM82]). The translation rules provide a semantics for type classes. They also provide one possible implementation technique: if desired, the new system could be added to an existing language with Hindley/Milner types simply by writing a pre-processor.

introduction to type classes, and defines them formally by means of type inference rules.

1 Introduction

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Parametric polymorphism occurs when a function is defined over a range of types, acting in the same

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Ad Hoc vs Parametric Polymorphism

	Function Name	Types	Behavior
Parametric	Same	Different	Same
Ad Hoc	Same	Different	Different

Let’s see a few examples of natural transformations in Haskell. The first is between the list functor, and the `Maybe` functor. It returns the head of the list, but only if the list is non-empty:

```
safeHead :: [a] -> Maybe a
safeHead [] = Nothing
safeHead (x:xs) = Just x
```

An interesting case is when one of the functors is the trivial `Const` functor. A natural transformation from or to a `Const` functor looks just like a function that's either polymorphic in its return type or in its argument type.

For instance, `length` can be thought of as a natural transformation from the list functor to the `Const Int` functor:

```
length :: [a] -> Const Int a
length [] = Const 0
length (x:xs) = Const (1 + unConst (length xs))
```

Another common functor that we've seen already, and which will play an important role in the Yoneda lemma, is the `Reader` functor. I will rewrite its definition as a `newtype`:

```
newtype Reader e a = Reader (e -> a)
```

It is parameterized by two types, but is (covariantly) functorial only in the second one:

```
instance Functor (Reader e) where
    fmap f (Reader g) = Reader (\x -> f (g x))
```

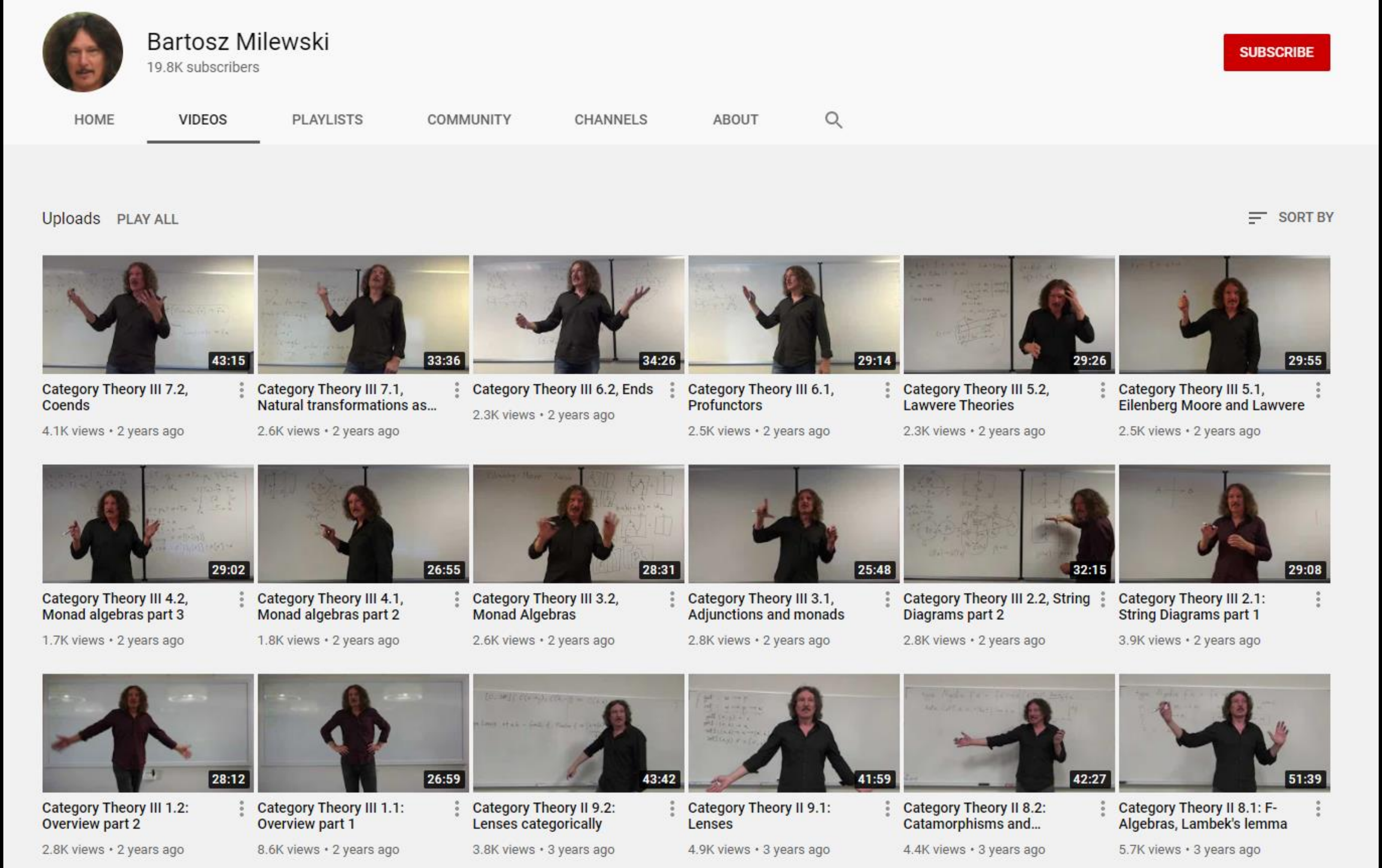
For every type `e`, you can define a family of natural transformations from `Reader e` to any other functor `f`. We'll see later that the members of this family are always in one to one correspondence with the elements of `f e` (the Yoneda lemma).

In the case of **Cat** seen as a **2**-category we have:

- Objects: (Small) categories
- 1-morphisms: Functors between categories
- 2-morphisms: Natural transformations between functors.

1. Define a natural transformation from the `Maybe` functor to the list functor. Prove the naturality condition for it.

```haskell
maybeToList :: Maybe a -> [a]
maybeToList (Just x) = [x]
maybeToList Nothing  = []
```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