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Threshold selection for regional peaks-over-threshold data

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General info

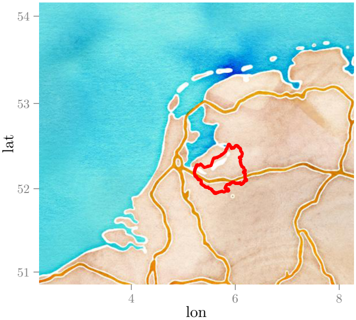
EVT related problems

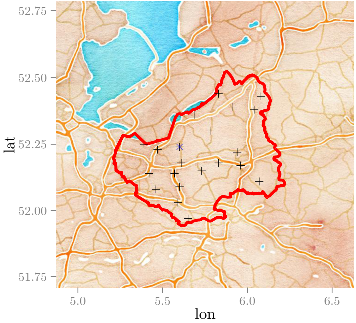
- point-wise extreme rainfall, temperature, wind gusts
- combined extremes (Friesland example)
- EVT for climate models (spatial extent)
- EVT conditional on predictions (Jasper & Juan)
- EVT for earthquakes

Myself

- This is work on point-wise extreme precipitation
- In my master thesis I focused on more high dimensional extremes, i.e. ...
- before the analysis started I had to rescale (conceptually) the margins to unit Frechet
- now I am stuck with the rescaling (and a long time has passed since my master thesis - so I am excited to refresh and expand my knowledge at this workshop)
- but it is not only point-wise because we want to borrow strength from neighbors ...

Area of Interest





Waterboard Vallei & Veluwe
21 daily precipitation series for 1951–2009

Buishand et al. (2013): Homogeneity of precipitation series in the Netherlands and their trends in the past century. *Int. J. Climatol.,*

Introduction to RFA

RFA and threshold selection

short recap of threshold selection procedures and Figure 4.1

Extend visual diagnostics

Extend GOF diagnostics

Marginal model

Short recap on marginal models - often not even continuous

Our marginal model

Threshold plot

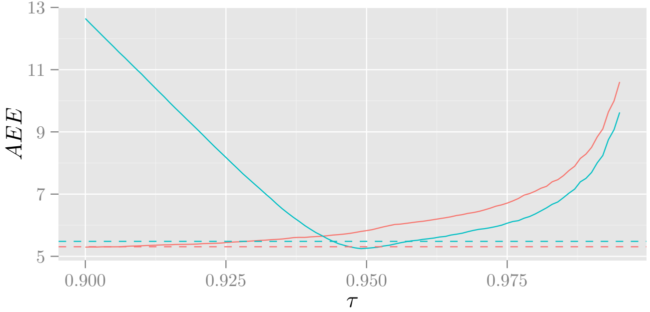
Mean excess plot

note something on the shape estimate based on the slope

Violin plots

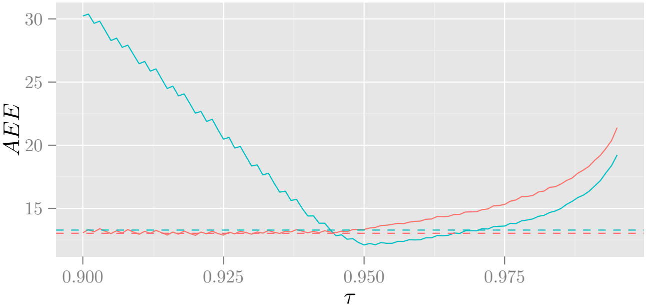
criterion!

Effect of Bulk–Tail Transition



AEE in the 5-year return level as a function of the probability τ for the simulated data (blue - bulk and tail are different, red - similar). The dashed horizontal lines indicate the AEE of the selected threshold.

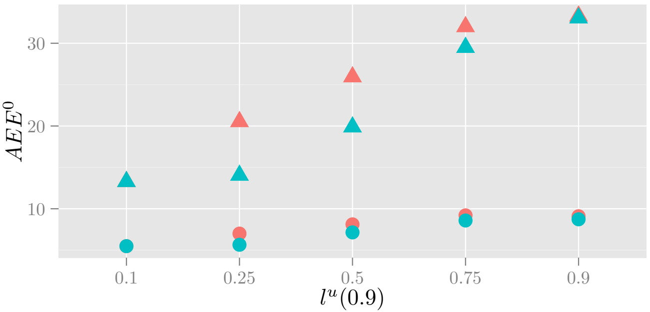
Effect of Bulk–Tail Transition



AEE in the 5-year return level as a function of the probability τ for the simulated data (blue - bulk and tail are different, red - similar). The dashed horizontal lines indicate the AEE of the selected threshold.

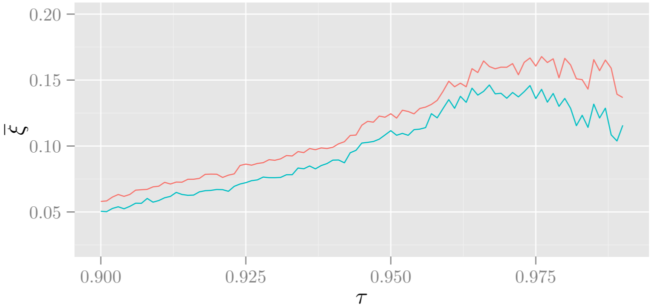
Spatial dependence model

Effect of Spatial Dependence



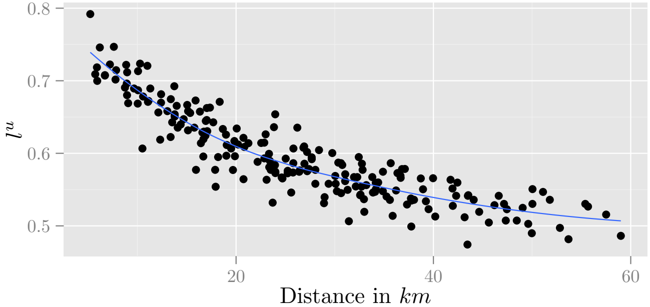
Averaged Euclidean error of the 5- (dots) and 50-year (triangles) return level for simulated data (red - Gumbel copula, blue - normal copula)

Spatially averaged TS plot



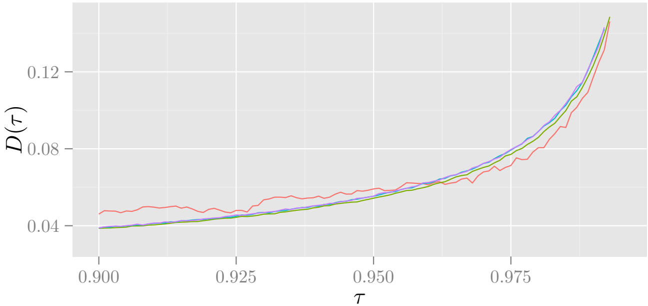
Threshold stability plot Say something on the linear increase and the stable region afterwards In particular we can reject too low thresholds

Quantile dependence in the data



Dependence in the 0.9 quantile of the rainfall data versus distance. Say something that the decay with distance indicates tail independent data.

Averaged KS statistic



Spatially averaged KS statistic for the three different dependence models

Summary

References

Your ideas? Comments?

Why Threshold Selection?

Two main approaches for extreme value analysis

- Block maxima – one value per year
  $\Rightarrow$ Generalized extreme value distribution
- Peaks-over-threshold – all peak values above a threshold
  $\Rightarrow$ Generalized Pareto distribution (GPD)

Trade-off situation

- Low thresholds lead to bias in the analysis of the excesses
- High thresholds result in high parameter estimation uncertainty

Preliminaries

At-site methods for threshold selection

- Visual inspection: mean excess plot, threshold stability plot
- Goodness of Fit tests: Anderson–Darling or Kolmogorov–Smirnov statistic
- Many more ... [1]

Regional frequency analysis

- Reduces parameter estimation uncertainty
- Often regional constant exceedance probability (e.g. 5%)
- But inference on threshold level only on at-site basis

[1] Scarrott and MacDonald (2012): A review of extreme value threshold estimation and uncertainty quantification. *REVSTAT*

Simulation: Smooth Marginal Distribution

Any continuous distribution F on $[0, \infty)$ can be written as

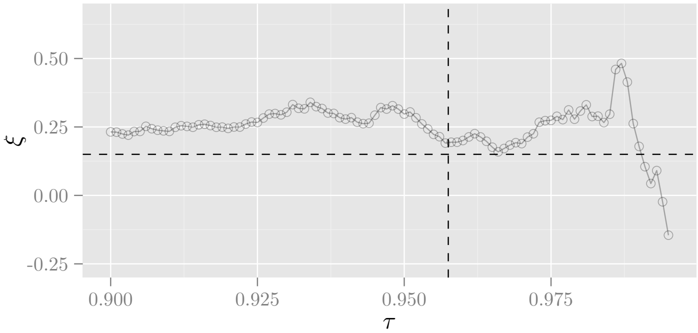
$$F(x) = 1 - \exp\Big(- \int_0^x h(u)\, du\Big),$$

where h is the hazard rate of the distribution. Define

$$h(x) := \eta\left(\frac{x-u}{\varepsilon}\right) h_1(x) + \left(1 - \eta\left(\frac{x-u}{\varepsilon}\right)\right) h_2(x),$$

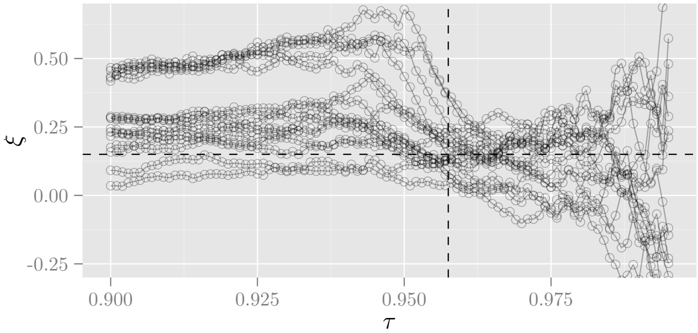
with h_1 the hazard rate of some bulk distribution, h_2 the hazard rate of the GPD, and η some smooth transition function.

Threshold Stability Plot



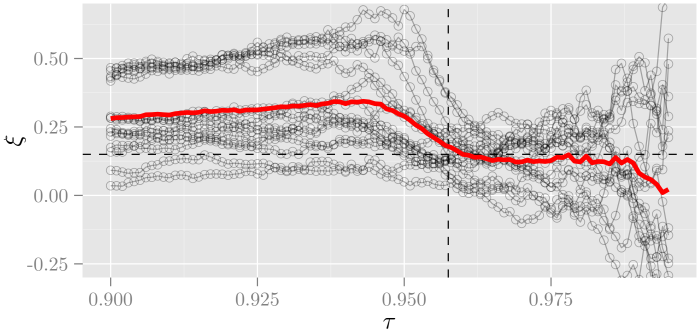
τ is the non-exceedance probability. The vertical line indicates the start of the GPD tail and the horizontal line shows the true shape parameter of the simulated data.

Threshold Stability Plot



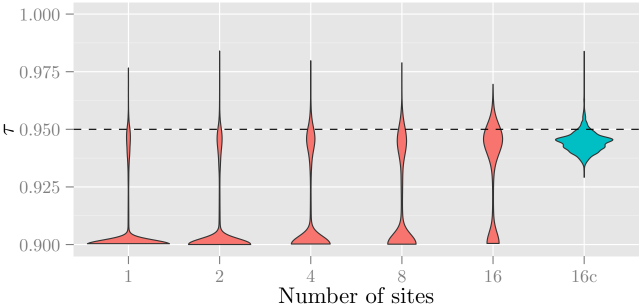
τ is the non-exceedance probability. The vertical line indicates the start of the GPD tail and the horizontal line shows the true shape parameter of the simulated data.

Threshold Stability Plot – Averaged



τ is the non-exceedance probability. The vertical line indicates the start of the GPD tail and the horizontal line shows the true shape parameter of the simulated data.

GOF Test and Automatic Selection



Violin plots of the selected threshold based on the lowest value of τ for which the average KS statistic is not significant at the 5% level.

Simulation: Spatial Dependence

Copula approach

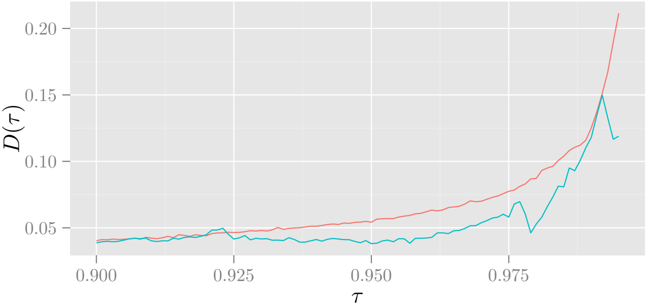
- Normal copula $\Rightarrow$ weak tail dependence
- Gumbel copula $\Rightarrow$ strong tail dependence

Quantile based measure of tail dependence

$$l^{\mathrm{u}}(\tau) := P\left(X_1 > F_1^{-1}(\tau) | X_2 > F_2^{-1}(\tau)\right)$$

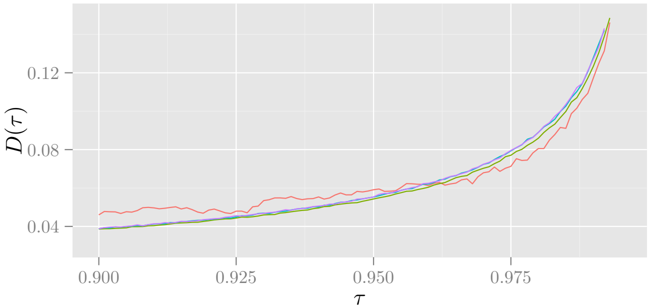
Copula parameter can be chosen such that $l^{\mathrm{u}}(0.9)$ is the same.

At-site KS Statistic



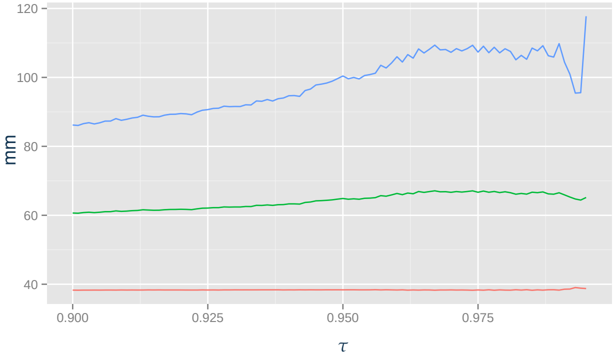
The blue line gives the KS statistic and the red the 95% critical values.

Regional KS Statistic



The red line gives the regionally averaged KS statistic, the three other lines give the 95% critical values based on the Normal (blue), distance-dependent Normal (purple), and Gumbel copula (green).

Effect on Return Levels



Average return level as a function of the non-exceedance probability of the selected threshold for return periods of 5 (red), 50 (green) and 500 (blue) years.

Conclusion

Use RFA principles also for threshold selection!

GoF based approach not restricted to the GPD.

Outlook

- Inspect the role of selection criterion
- Extend the approach to incorporate trends

Roth, M., G. Jongbloed, and T. A. Buishand (2016),
Threshold selection for regional peaks-over-threshold data.
Journal of Applied Statistics, 43:1291–1309.

Kolmogrov-Smirnov statistic

(1) Fix τ_0 and the corresponding threshold u.

(2) Estimate the parameters based on the n excesses above u.

(3a) Simulate n values from the corresponding GPD.

(3b) Sample $\tilde{n}$ from $B(T, 1-\tau_0)$. Simulate $\tilde{n}$ values from the corresponding GPD. Scales to regional setting.

(4) Calculate KS statistic for the simulated data.

(5) Repeat steps 3 and 4 a thousand times and take the 0.95-quantile of the bootstrapped statistic as critical value for D_n at τ_0.

(6) Repeat procedure for every τ in a reasonable range.

Averaged Euclidean Error (AEE)

$$AEE(\tau) := \frac{1}{B} \sum_{i=1}^{B} ||\hat{\boldsymbol{r}}_i(\tau) - \boldsymbol{r}||_2,$$

where $||.||_2$ denotes the Euclidean (or l_2) norm, B gives the number of bootstrap samples and $\boldsymbol{r}$ specifies some return level.