1. Get started with Python (alternatively R or Matlab). Read the OSEBX data into the work space and plot the time series. You find the data and some Python code in Canvas.

2. a) Let $\{w_t\}$ be Gaussian (normal distributed) white noise WN(0,1). Simulate 500 observations from this process. In Python:

```
import random
import matplotlib.pyplot as plt
w = [random.gauss(0, 1) for i in range(500)]
plt.plot(w)
```

In R:

```
w<-rnorm(500)
plot(w,type="l")
```

b) Define $\{X_t\}$ by

$$X_t = \sum_{k=1}^{t} w_k, \quad t = 1, 2, \dots \tag{1}$$

What is this model called?
Simulate 500 observations from $\{X_t\}$:

```
import numpy as np
x = np.cumsum(w)
plt.plot(x)
```

Or in R:

```
x<-cumsum(w)
plot(x,type="l")
```

What is the expectation of X_t?

c) What class of stochastic processes are the increments $Y_t = X_t - X_{t-1}$?

d) Repeat (a) with WN(0.2,1), note that $\mathbb{E}[w_t] = 0.2$. How does this affect the mean, i.e., what is the expectation of X_t?

3. Let X and Y be rv's with means u_X and u_Y respectively. Show that

$$\mathrm{Cov}(X, Y) = \mathbb{E}[XY] - \mu_X \mu_Y$$

4. In 1905 Louis Bachelier, in his phd thesis "Theory of speculation", proposed Brownian motion as a model for financial prices. Brownian motion is the model[1] X_t in problem 2b. Draw sample paths of the Brownian motion (same as you did in problem 2b), using same sample length as for the OSEBX time series. Plot the logarithm of the OSEBX

[1] To be precise, Brownian motion is a continuous time stochastic process. Discretising results in the model in 2b)

time series and compare with the sample path from the Brownian motion. Do you think Brownian motion is a good model for logarithmic prices (log OSEBX)?

Plot the increments from the simulated data and compare with the log-return of OSEBX (increments of log OSEBX). Does this change your opinion of the model?

5. Problem 1.3 in the book `http://www.stat.pitt.edu/stoffer/tsa4/tsa4.pdf`

6. Let $X_1, \ldots, X_n$ be independent and identically distributed random variables, with mean $\mathbb{E}[X_1] = \theta$ and $\mathrm{Var}(X_1) = \sigma^2$. The sample mean is defined as

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i.$$

What is the expectation and variance of the sample mean?