1

a) Show that $\text{DTFT}\{x[n-k]\} = e^{-i\omega k}\,\text{DTFT}\{x[n]\}$, where

$$\text{DTFT}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]e^{i\omega n}$$

b) Prove the convolution property for the Discrete Time Fourier Transform (DTFT), i.e. show that if

$$y[n] = a[n] * x[n] = \sum_{n=-\infty}^{\infty} a_k x_{n-k}$$

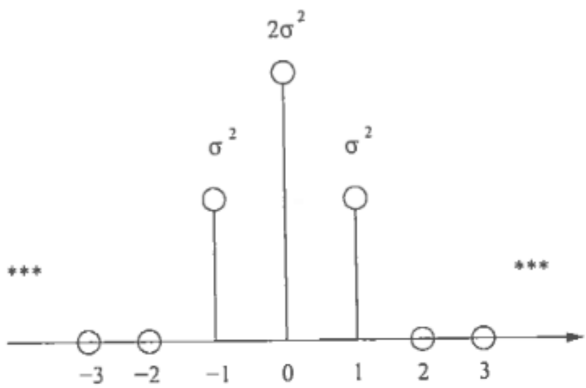
then

$$\text{DTFT}\{y_n\} = \text{DTFT}\{a_n\}\text{DTFT}\{x_n\}$$

Hint: Use the result from **a)**.

2

We assume a stationary time series $X[n]$, with $E[X[n]] = \mu_X = 0$.



Figur 1: Autocovariance function.

a) Give an interpretation of the autocovariance function $\gamma_x(h)$ shown in Figure 1. Is the associated stochastic process uncorrelated?

b) Why can't we define the Fourier transform of a stochastic process? Why can't an energy spectral density be defined either?

c) Show that the power spectral density $S(\omega)$ for the ACVF shown in Figure 1 is given by

$$S(\omega) = 2\sigma^2(1 + \cos(\omega)) \tag{1}$$

Sketch $S(\omega)$ and discuss the result.

d) Explain why the average power of $X[n]$ is given by $\gamma_x(0)$. Hint: Think of the inverse Fourier transform.

e) Let $w_t \sim wn(0, \sigma^2)$ and let $x_t = w_t + w_{t-1}$. Show that $\gamma_x(h)$ corresponds to Figure 1.

3

We assume a stationary time series $X[n]$, with only real values.

a) Prove that the power spectral density (PSD) is a symmetric function: $S(\omega) = S(-\omega)$.

b) Prove that the PSD is always non-negative: $S(\omega) \geq 0$. You may prove this by showing that $|X(\omega)|^2 \geq 0$, where $X(\omega) = \sum_{n=-\infty}^{\infty} X[n]e^{-i\omega n}$ is the DTFT.
Hint: remember that the autocovariance function is positive semi-definite.

4

(Exercise 4.8 in the book) Suppose $x[n]$ and $y[n]$ are stationary zero-mean time series with $x[n]$ independent of $y[k]$ for all n, k. Consider the product series $z[n] = x[n] \cdot y[n]$. Prove the PSD for $z[n]$ can be written as

$$S_Z(f) = \int_{-1/2}^{1/2} S_X(f - f')S_Y(f')df'$$

where $S_Z(f)$ is the PSD of $Z[n]$ and $S_X(f), S_Y(f)$ the PSD of $X[n], Y[n]$, respectively.