Tarea 10

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Ley De Benford

Sea $X \sim Benford$ donde $D_X = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

d) Defino aquí varias funciones que utilizaré luego

```r
# Función de probabilidad
dben = function(x){
  if(x >= 1 && x <= 9){
    sapply(x,function(x){log10(x + 1) - log10(x);});
  }else{
    0;
  }
}

# Función de distribución
pben = function(x){
  sapply(x,FUN=function(y){if(y < 1){return(0);}
    else if(y >= 9){
      return(1);
    }else{
      return(sum(dben(c(1:y))));}
    });
}

# Media
mben = function(){
  sum(sapply(c(1:9),function(x){x*(log10(x + 1) - log10(x));}));
}

# Varianza
vben = function(){
  s <- sum(sapply(c(1:9),function(x){x^2*(log10(x + 1) - log10(x));}));
  s - mben()^2
}
```

a)

$$E(X) = \sum_{x \in X(\Omega)} x \cdot P_X(x)$$

```r
mben()
```

```
## [1] 3.440237
```

$$Var(X) = E(X^2) - (E(X))^2$$

```r
vben()
```

```
## [1] 6.056513
```

b)

$$F_X(x_0) = \sum_{x \leq x_0} P_X(x)$$

Por ejemplo, calculamos $P(X \leq 7)$

```r
pben(7)
```

```
## [1] 0.90309
```

c)

```r
dben(c(1:9))
```

```
## [1] 0.30103000 0.17609126 0.12493874 0.09691001 0.07918125 0.06694679 0.05799195
## [8] 0.05115252 0.04575749
```

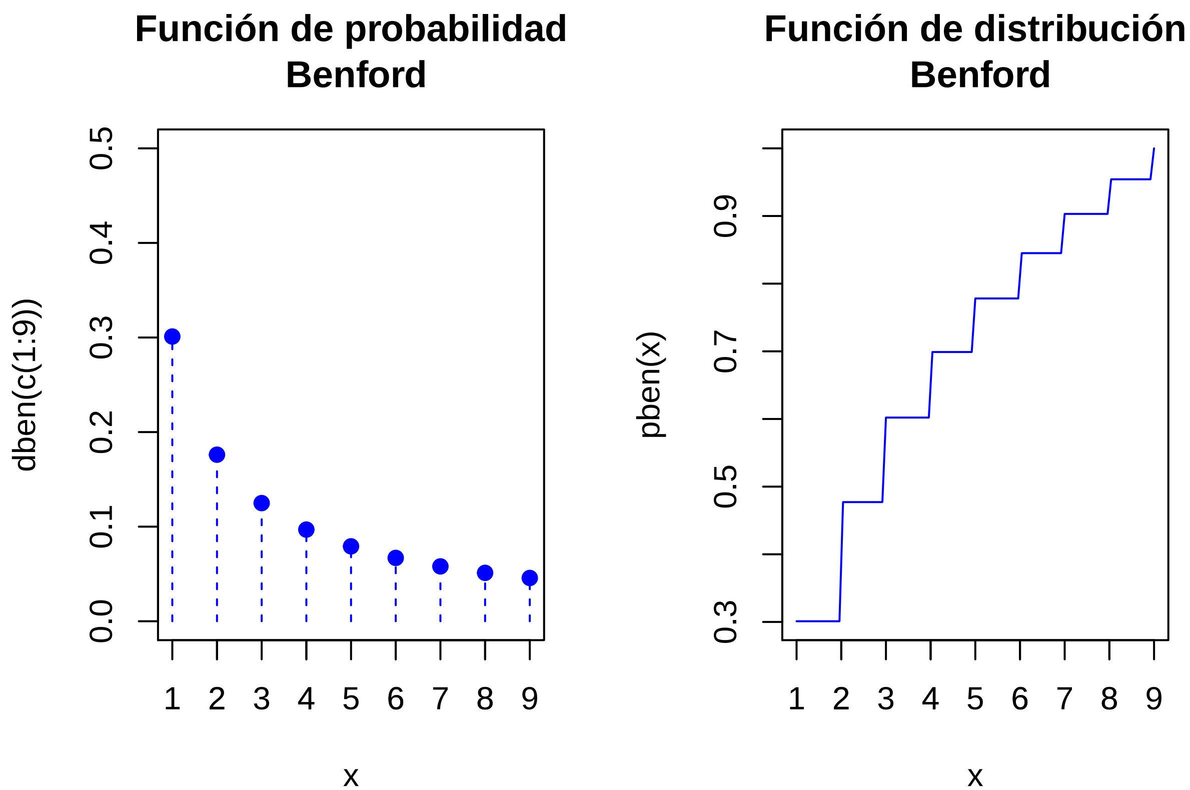
Claramente vemos que el dígito con mayor probabilidad (y muy posiblemente la moda en una muestra) es el 1.

e)

```r
par(mfrow=c(1,2))

plot(x=c(1:9),
     y=dben(c(1:9)),
     ylim=c(0,0.5),
     xlim=c(1,9),
     xlab="x",
     pch = 19,
     col = 'blue',
     main="Función de probabilidad\n Benford")
lines(x=c(1,2,3,4,5,6,7,8,9),
      y=c(dben(1),dben(2),dben(3),dben(4),dben(5),dben(6),dben(7),dben(8),dben(9)),
      type = "h",
      lty = 2,
      col="blue")
axis(1,seq(1,9,1))

curve(pben(x),
      xlim=c(1,9),
      col="blue",
      main="Función de distribución\n Benford")
axis(1,seq(1,9,1))
```




```
par(mfrow=c(1,1))
```