

Data Structures

Linear Probing

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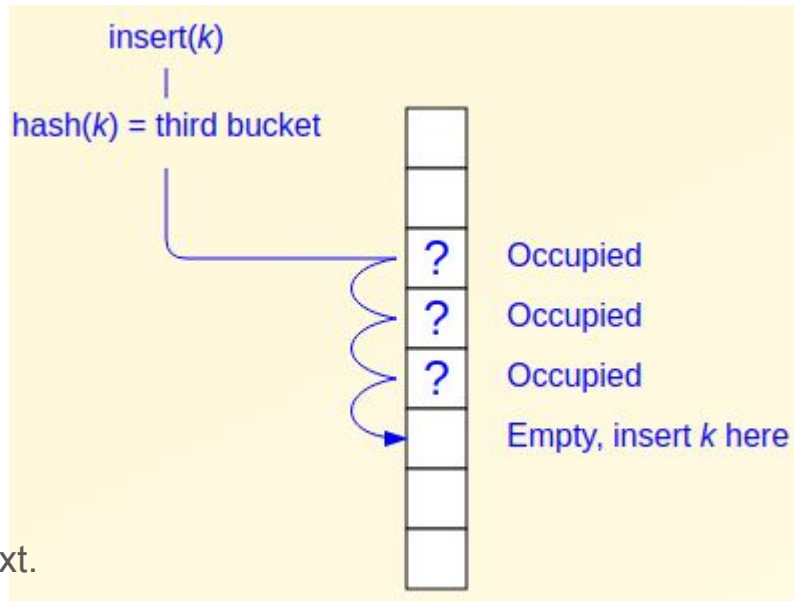
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Probing

- Another way to handle collision
- Create only a single 1D array
- Given a hash index:
 - If it is empty, add the item
 - If not, move to the next element (**linear** probing)
 - Again, if empty use it. If not, move to the next.
 - We may go circular in the array
 - In the worst case, all array content will be used
- Observe
 - Chaining technique: memory is expanding all the time
 - Probing: Fixed memory as limited to initial array
 - In both, we can rehash if we have to



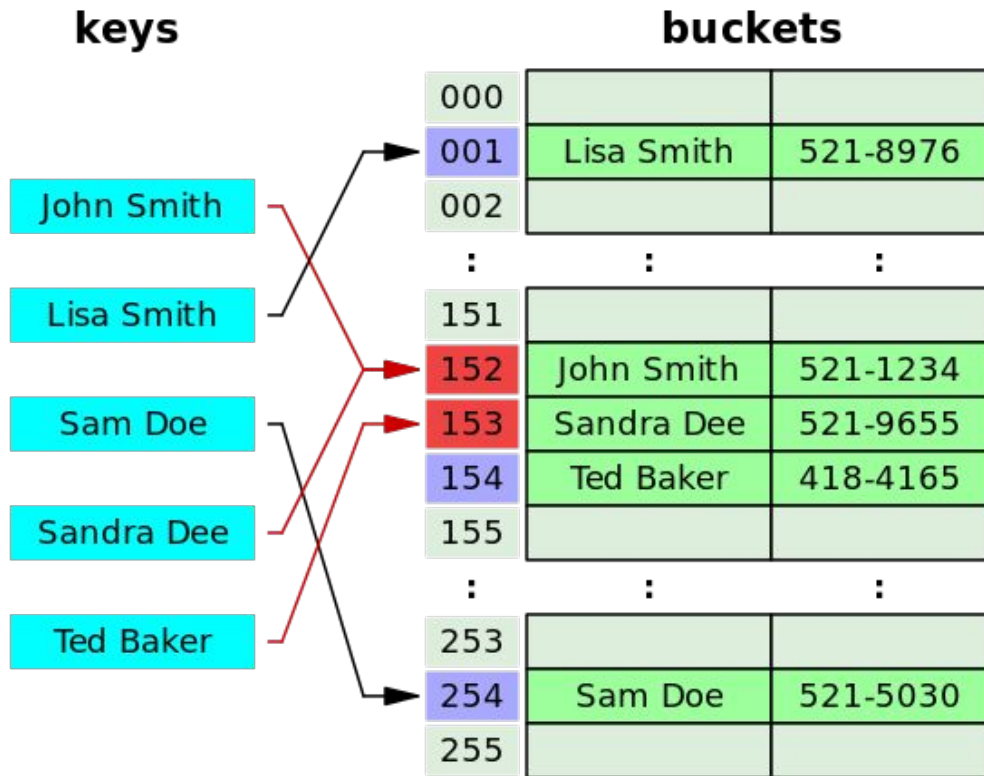
Recall Example

- Assume we use these entries and computed their final hash function

Name (as search key)	Attached data (phone #)	Hash Function code
John Smith	5211234	152
Lisa Smith	5218976	1
Sam Doe	5215030	254
Sandata Dee	5219655	152 (collision)
Ted Baker	4184165	153

Put in order (linear probing)

- John Smith (152)
 - Empty: add
- Lisa Smith (001)
 - Empty: add
- Sam Doe (254)
 - Empty: add
- Sandra Dee (152)
 - 152 **not empty**. Move
 - 153 empty: add
- Ted Baker (153)
 - 153 **not empty**. Move
 - 154 empty: add
 - Observe originally was not a collision



Search and deletion

- Search (same as put)
 - Compute hash idx. As long as not empty and not target key, move 1 step
- Deletion is tricky!
 - Imagine we inserted items that ended up 5 consecutive elements in table
 - E.g. A, B, C, D, E
 - Search(D) \Rightarrow Exist
 - Delete C \Rightarrow A, B, Empty, D, E
 - Search(D) \Rightarrow not exist!
 - The problem search stops when finds an empty item! But this empty because deletion
 - Trivial solution: Mark a cell as deleted so that we keep going in search

Example on integer

- Assume we have following integers
 - [18, 41, 22, 44, 59, 32, 31, 73] and their hash index: [5, 2, 9, 5, 7, 6, 5, 8] and our array is 13 cells

	0	1	2	3	4	5	6	7	8	9	10	11	12
Insert 18 (5)						18							
Insert 41 (2)			41			18							
Insert 22 (9)			41			18				22			

Example on integer

- Assume we have following integers
 - [18, 41, 22, **44**, 59, 32, 31, 73] and their hash index: [5, 2, 9, **5**, 7, 6, 5, 8] and our array is 13 cells

	0	1	2	3	4	5	6	7	8	9	10	11	12
Insert 44 (5)			41			18	44			22			

5? not empty

6? Empty. Use it

Example on integer

- Assume we have following integers
 - [18, 41, 22, 44, **59**, 32, 31, 73] and their hash index: [5, 2, 9, 5, **7**, 6, 5, 8] and our array is 13 cells

	0	1	2	3	4	5	6	7	8	9	10	11	12
Insert 59 (7)			41			18	44	59		22			

Example on integer

- Assume we have following integers
 - [18, 41, 22, 44, 59, **32**, 31, 73] and their hash index: [5, 2, 9, 5, 7, **6**, 5, 8] and our array is 13 cells

0	1	2	3	4	5	6	7	8	9	10	11	12
		41			18	44	59	32	22			

Insert 32 (6)

6? not empty

7? not empty

8? Empty. Use it

Example on integer

- Assume we have following integers
 - [18, 41, 22, 44, 59, 32, **31**, 73] and their hash index: [5, 2, 9, 5, 7, 6, **5**, 8] and our array is 13 cells

0	1	2	3	4	5	6	7	8	9	10	11	12
		41			18	44	59	32	22	31		

Insert 31 (5)

5? not empty

6? not empty

7? not empty

8? not empty

9? not empty

10? Empty. Use it

Example on integer

- Assume we have following integers
 - [18, 41, 22, 44, 59, 32, 31, **73**] and their hash index: [5, 2, 9, 5, 7, 6, 5, **8**] and our array is 13 cells

0	1	2	3	4	5	6	7	8	9	10	11	12
		41			18	44	59	32	22	31	73	

Insert 73 (8)

8? not empty

9? not empty

10? not empty

11? Empty. Use it

Example on integer

- Search 115 (hash = 8)
 - Idx = 8: 32 == 115? No. Move
 - Idx = 9: 22 == 115? No. Move
 - Idx = 10: 31 == 115? No. Move
 - Idx = 11: 73 == 115? No. Move
 - Idx = 12: None. **Not found**

0	1	2	3	4	5	6	7	8	9	10	11	12
		41			18	44	59	32	22	31	73	

Example on integer

- Search 32 (hash = 6)
 - Idx = 6: 44 == 32? No. Move
 - Idx = 7: 59 == 32? No. Move
 - Idx = 8: 32 == 32? Found

0	1	2	3	4	5	6	7	8	9	10	11	12
		41			18	44	59	32	22	31	73	

Example on integer

- Delete 59 (hash = 7)
 - Idx = 7: 59 == 59? Yes, delete (mark as deleted, but leave it showing something was here)

0	1	2	3	4	5	6	7	8	9	10	11	12
		41			18	44	X	32	22	31	73	

Example on integer

- Search 32 (hash = 6)
 - Idx = 6: 44 == 32? No. Move
 - Idx = 7: X == 32? No. Move [without mark, we fail here!]
 - Idx = 8: 32 == 32? Found

0	1	2	3	4	5	6	7	8	9	10	11	12
		41			18	44	X	32	22	31	73	

Example on integer

- Delete 32 (hash = 6)
 - Idx = 6: 44 == 32? No. Move
 - Idx = 7: X == 32? No. Move
 - Idx = 8: 32 == 32? Mark as deleted

0	1	2	3	4	5	6	7	8	9	10	11	12
		41			18	44	X	X	22	31	73	

Example on integer

- Insert 200 (hash = 6)
 - Idx 6: empty? No. move
 - Idx 7: is marked as deleted. Just use it

0	1	2	3	4	5	6	7	8	9	10	11	12
		41			18	44	200	X	22	31	73	

Example on integer

- Insert 211 (hash = 12)
 - Idx 12: empty? Use it

0	1	2	3	4	5	6	7	8	9	10	11	12
		41			18	44	200	X	22	31	73	211

Example on integer

- Insert 315 (hash = 12)
 - Idx 12: empty? No move. Go circular to 0
 - Idx 0: empty? Use it
- Note: there are only 5 elements remaining in this table (4 + X)

0	1	2	3	4	5	6	7	8	9	10	11	12
315		41			18	44	200	X	22	31		211

Rehashing

- Similar to chaining, after some `load_factor` limit we can rehash
- But there is another reason with probing technique
- Imagine we have several deletion. No array has many marks X
- But this means our search will take much time
 - Better rehash at this stage
- With a good hash function, it can be shown that the expected number of insertion approximately $1 / (1 - \text{load_factor})$ steps
 - E.g. for a load factor of 0.75, we take 4 steps $\Rightarrow O(1)$

Probing Techniques

- Linear probing (today)
 - $\text{Index} = (\text{initial_hash} + i) \% \text{table_size}$ for $i = \{0, 1, 2, 3, 4\} \Rightarrow$ move to next position
 - If $\text{initial_hash} = 10 \Rightarrow$ Try as long as not empty $\{10, 11, 12, 13, 14, 15, \dots\}$
 - Issue: Create blocks of consecutive values \Rightarrow More time for search
- Quadratic Probing
 - $\text{Index} = (\text{initial_hash} + i * i) \% \text{table_size}$
 - We jump to square positions. E.g. $10 + 0$, then $10 + 1$, then $10 + 4$, then $10 + 9$, then $10 + 16$
- Double Hashing
 - Use 2 independent hash functions
 - $\text{Index} = (\text{initial_hash1} + \text{initial_hash2} * i) \% \text{table_size}$
 - The 2nd hash function should have several characteristics

“Acquire knowledge and impart it to the people.”

“Seek knowledge from the Cradle to the Grave.”