MA0001 - Martin Skatvedt - Øving 3

2.2.1

$f(x) = 3 + \frac{1}{x}$ $\quad D_f = (0, \infty)$

a) hvis $g(x) = \frac{1}{x}$ og $\lim_{x \to 0} \frac{1}{x}$ vil $g(x)$ gå mot uendelig.
Derfor vil $f(x) \to \infty$ når $x \to 0$

b) $\lim_{x \to 0} f(x) = \infty$ $\quad \lim_{x \to \infty} f(x) = 3$ $\quad V_f = (3, \infty)$

$f'(x) = -\frac{1}{x^2}$ $\quad \lim_{x \to 0} f'(x) = -\infty$ $\quad \lim_{x \to \infty} f'(x) = 0$

siden $f'(x) < 0$ for hele D_f, vil $f(x)$ være strekt avtagende og injektiv.

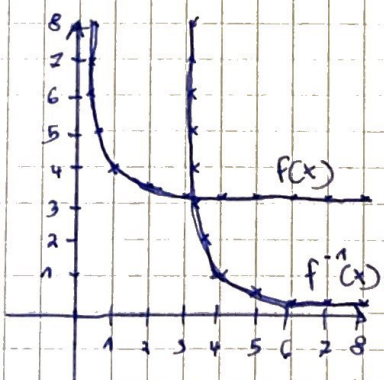
c) $y = 3 + \frac{1}{x} \to x = 3 + \frac{1}{y} \to y = \frac{1}{x - 3}$

$f^{-1}(x) = \frac{1}{x-3}$ $\quad D_{f^{-1}} = (3, \infty)$ $\quad V_{f^{-1}} = (0, \infty)$

d) $\lim_{x \to \infty} f^{-1}(x) = \frac{1}{\infty} = 0$

$f^{-1}(x)$ vil gå mot 0

e)



f) $f^{-1}(f(x)) = \frac{1}{f(x) - 3} = \frac{1}{(3 + \frac{1}{x}) - 3} = \frac{1}{\frac{3x+1}{x} - 3} = \frac{1}{\frac{3x+1-3x}{x}} = \frac{1}{\frac{1}{x}}$

$= x$

$f(f^{-1}(x)) = 3 + \frac{1}{\frac{1}{x-3}} = 3 + x - 3 = x$

$f^{-1}(f(x)) = x$ og $f(f^{-1}(x)) = x$

2

$$g(x) = \frac{6-10x}{8x+7}$$

$$y = \frac{6-10x}{8x+7} \rightarrow x = \frac{6-10y}{8y+7}$$

$$8yx + 7x = 6 - 10y \rightarrow 8yx + 10y = 6 - 7x$$

$$y(8x+10) = 6 - 7x \rightarrow y = \frac{6-7x}{8x+10}$$

$$\underline{\underline{g^{-1}(x) = \frac{6-7x}{2(4x+5)}}}$$

$$g(g^{-1}(x)) = \frac{6-10\left(\frac{6-7x}{2(4x+5)}\right)}{7+8\left(\frac{6-7x}{2(4x+5)}\right)}$$

$$= \frac{6 - \frac{60-70x}{8x+10}}{7 + \frac{48-56x}{8x+10}} = \frac{\frac{48x+60-60+70x}{8x+10}}{\frac{56x+70+48-56x}{8x+10}}$$

$$= \frac{\frac{118x}{8x+10}}{\frac{118}{8x+10}} = \frac{118x}{118} = \underline{x}$$

$$g^{-1}(g(x)) = \frac{6-7\left(\frac{6-10x}{8x+7}\right)}{10+8\left(\frac{6-10x}{8x+7}\right)} = \frac{6 - \frac{42-70x}{8x+7}}{10 + \frac{48-80x}{8x+7}}$$

$$= \frac{\frac{48x+42-42+70x}{8x+7}}{\frac{80x+70+48-80x}{8x+7}} = \frac{\frac{118x}{8x+7}}{\frac{118}{8x+7}} = \frac{118x}{118} = \underline{x}$$

$$\underline{\underline{g(g^{-1}(x)) = x \quad \text{og} \quad g^{-1}(g(x)) = x}}$$

3 $g^{-1}(2) = 6$ $g(3) = 4$ $f^{-1}(-2) = 4$ $f(3) = 6$

$f^{-1}(9) = 2$

$f^{-1}(6) = 3$ siden $f(3) = 6$

$g(3) = 4$

$f(4) = -2$ siden $f^{-1}(-2) = 4$

$f(g(f^{-1}(6))) = -2$

3.3.6 0,5 m stokk kaster 0,7 m → $\frac{50}{70}$

x m mast kaster 36 m → $\frac{x}{3600}$

$\frac{50}{70} = \frac{x}{3600}$ → $70x = 50 \cdot 360$ → $x = \frac{50 \cdot 3600}{70}$

$x = 2571{,}42 \text{ cm} \approx 25{,}71 \text{ m}$

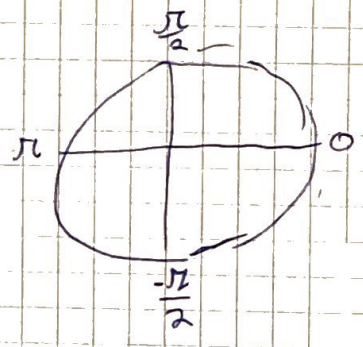
Masten er ca 25,71 m høy

3.B.6 a) $f(v) = \sin v$ $f'(v) = \cos v$

$f'(v)$ er strengt voksende for $v \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$f(v) = 0$ for $v = 0$ og $v = \pi$

Der for $[\frac{-\pi}{2}, \frac{\pi}{2}]$



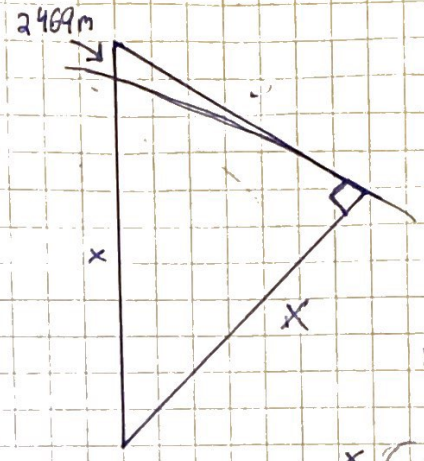
b) $g(v) = \cos v$ $g'(v) = -\sin v$

$g'(v)$ er strengt voksende for $v \in [-\pi, 0]$

$g(v) = 0$ for $v = \frac{\pi}{2}$ og $v = -\frac{\pi}{2}$

Derfor $[-\pi, 0]$

6



$r = 2469\,m + x$

$\sin a = \frac{\text{mot katet}}{\text{radius}}$

$\sin 88,41° = \frac{x}{2469 + x}$

$x \sin(88,41°) + 2469 \cdot \sin(88,41°) = x$

$x\,(1 - \sin(88,41°)) = 2469 \cdot \sin(88,41°)$

$x = \frac{2469 \cdot \sin(88,41°)}{1 - \sin(88,41°)}$

$x = 6410069,31\,m$

$x \approx 6410\,km$

Jordens radius er ca 6410 km