Martin Skatvedt, Øving 3

1) $\neg((\neg p \land q) \lor (\neg p \land \neg q)) \lor (p \land q)$

$\equiv \neg(\neg p \land (q \lor \neg q)) \lor (p \land q)$ Distributive law

$\equiv \neg(\neg p \land T_0) \lor (p \land q)$ Inverse law

$\equiv \neg(\neg p) \lor (p \land q)$ Identity law

$\equiv p \lor (p \land q)$ Double negation

$\equiv \underline{\underline{p}}$ Absorption law

2) $[(p \land q) \lor (p \land \neg r) \lor \neg(\neg p \lor q)] \lor [(r \lor s \lor \neg r) \land \neg q]$

$\equiv [(p \land q) \lor (p \land \neg r) \lor (p \land \neg q)] \lor [(r \lor \neg r \lor s) \land \neg q]$ D.M.G commutative law

$\equiv [(p \land q) \lor (p \land \neg q) \lor (p \land \neg r)] \lor [(T_0 \lor s) \land \neg q]$ commutative law inverse law

$\equiv [(p \land (q \lor \neg q)) \lor (p \land \neg r)] \lor [T_0 \land \neg q]$ Distributive law Domination law

$\equiv [(p \land T_0) \lor (p \land \neg r)] \lor [\neg q]$ Inverse law identity law

$\equiv [p \lor (p \land \neg r)] \lor [\neg q]$ Identity law

$\equiv [p] \lor [\neg q]$ Absorption law

$\equiv \underline{\underline{p \lor \neg q}}$

3) i)

a) $\{\{2,4,6\} \cup \{6,4\}\} \cap \{4,6,8\} = \underline{\underline{\{4,6\}}}$

b) $P(\{7,8,9\}) - P(\{7,9\}) = \underline{\underline{P(\{8\}) = \{\{8\}, \varnothing\}}}$

c) $P(\varnothing) = \underline{\underline{\{\varnothing\}}}$

d) $\{1,3,5\} \times \{0\} = \underline{\underline{\{\{1,0\},\{3,0\},\{5,0\}\}}}$

e) $\{2,4,6\} \times \varnothing = \underline{\underline{\{\{2\},\{4\},\{6\}\}}}$

f) $P(\{0\}) \times P\{1\} = \{\{0\},\varnothing\} \times \{\{1\},\varnothing\}$

$= \underline{\underline{\{\{\{0\},\{1\}\},\{\{0\}\},\{\{1\}\}\}}}$

g) $P(P(\{2\})) = \underline{\underline{\{\{\{2\},\varnothing\},\{\{2\}\},\{\varnothing\},\varnothing\}}}$

3) jj) Hvis $|A| = n$ da blir

$$|P(A) - \{\{x\} : x \in A\}| = ?$$

Eksempel $A ::= \{1,2,3\}$ $B ::= \{\{x\} : x \in A\} = \{\{1\},\{2\},\{3\}\}$

$P(A) ::= \{\{1,2,3\},\{1,2\},\{1,3\},\{1\},\{2,3\},\{2\},\{3\},\emptyset\}$

D.v.s $P(A) - B = \{\{1,2,3\},\{1,2\},\{1,3\},\{2,3\},\emptyset\}$

$|P(A) - B| = 5$ og $|P(A) - \{\{x\} : x \in A\}| = \underline{\underline{2^n - n}}$

4) a) False $\emptyset = \{\}$

b) False $\emptyset = \{\}$

c) True $|\{\}| = 0$

d) False $|\{\emptyset\}| = 1$

e) False $\emptyset = \{\}$, $\emptyset \notin \{\}$

f) True $\{x \in \mathbb{N} : x \leq 0 \land x > 0\} = \{\} = \emptyset$

5) a) $A \cap (A \cup B) = A$

$A \cap (A \cup B) ::= \{x, x \in A \land (x \in A \lor x \in B)\}$ $A ::= \{x, x \in A\}$

$x \in A$	$x \in B$	$x \in A \lor x \in B$	$x \in A \land (x \in A \lor x \in B)$
F	F	F	F
F	T	T	F
T	F	T	T
T	T	T	T

b) $A - (B \cap C) = (A - B) \cup (A - C)$

$B \cap C = \{x : x \in B \land x \in C\}$

$[A - (B \cap C) ::= \{x, x \in A \land \neg(x \in B \land x \in C)\}]$

$[(A - B) \cup (A - C) ::= \{x, (x \in A \land x \notin B) \lor (x \in A \land x \notin C)\}]$

$x \in A$	$x \in B$	$x \in C$	$\overbrace{x \in A \land x \notin B}^{p}$	$\overbrace{x \in A \land x \notin C}^{q}$	$p \lor q$	$\overbrace{x \in B \land x \in C}^{s}$	$x \in A \land \neg s$
F	F	F	F	F	F	F	F
F	F	T	F	F	F	F	F
F	T	F	F	F	F	F	F
F	T	T	F	F	F	T	F
T	F	F	T	T	T	F	T

F	T	F	F	F	F	F	F
F	T	T	F	F	F	T	F
T	F	F	T	T	T	F	T
T	F	T	T	F	T	F	T
T	T	F	F	T	T	F	T
T	T	T	F	F	F	T	F

6) $A \Delta B = (A - B) \cup (B - A)$

$x \in A \Delta B \Leftrightarrow x \in (A \cup B) - (A \cap B)$

b) $U = \mathbb{N}$

$A ::= \{1, 2, 3, 4, 5, 6, 7, 9, 10\}$

$B ::= \{1, 3, 5, 8, 9, 10\}$

$A - B = \{2, 4, 6, 7\}$

$B - A = \{8\}$

$A \Delta B = \underline{\underline{\{2, 4, 6, 7, 8\}}}$

7) $x ::= \{\{1, 2, 3\}, \{2, 3\}, \{e, f\}, \{e\}\}$

$y ::= \{\{1, 2, 3, e, f\}\}$

$P(x) = \{\{\{1,2,3\}, \{2,3\}, \{e,f\}, \{e\}\},$
$\{\{1,2,3\}, \{2,3\}, \{e,f\}\}, \{\{1,2,3\}, \{2,3\}, \{e\}\},$
$\{\{1,2,3\}, \{2,3\}\}, \{\{1,2,3\}, \{e,f\}, \{e\}\}, \{\{1,2,3\}, \{e,f\}\},$
$\{\{1,2,3\}, \{e\}\}, \{\{1,2,3\}\}, \{\{2,3\}, \{e,f\}, \{e\}\},$
$\{\{2,3\}, \{e,f\}\}, \{\{2,3\}, \{e\}\}, \{\{2,3\}\},$
$\{\{e,f\}, \{e\}\}, \{\{e,f\}\}, \{\{e\}\}, \varnothing\}$

$\underline{\underline{P(x \cap y) = \{\varnothing\}}}$

8) 1)

1) $A_1^{\epsilon_1} \cap A_2^{\epsilon_2} \cap A_3^{\epsilon_3}$

2) $\bar{A}_1^{\epsilon_1} \cap \bar{A}_2^{\epsilon_2} \cap \bar{A}_3^{\epsilon_3}$

3) $\bar{A}_1^{\epsilon_1} \cap A_2^{\epsilon_2} \cap A_3^{\epsilon_3}$

4) $A_1^{\epsilon_1} \cap \bar{A}_2^{\epsilon_2} \cap A_3^{\epsilon_3}$

5) $A_1^{\epsilon_1} \cap A_2^{\epsilon_2} \cap \bar{A}_3^{\epsilon_3}$

6) $\bar{A}_1^{\epsilon_1} \cap \bar{A}_2^{\epsilon_2} \cap A_3^{\epsilon_3}$

7) $\bar{A}_1^{\epsilon_1} \cap A_2^{\epsilon_2} \cap \bar{A}_3^{\epsilon_3}$

8) $A_1^{\epsilon_1} \cap \bar{A}_2^{\epsilon_2} \cap \bar{A}_3^{\epsilon_3}$

2) since it can be A_i or $\bar{A}_i$

$\underline{\underline{\text{\# fundamental products} = 2^n}}$

9) $(A - (B - C)) - ((A - B) - C) = A \cap C$

$= (A \cap \overline{(B \cap \bar{C})}) \cap \overline{((A \cap \bar{B}) \cap \bar{C})}$

$= (A \cap (\bar{B} \cup C)) \cap (\overline{(A \cap \bar{B})} \cup C)$

$= (A \cap (\bar{B} \cup C)) \cap ((\bar{A} \cup B) \cup C)$

$= A \cap (\bar{B} \cup C) \cap (\bar{A} \cup B \cup C)$

$= A \cap (\bar{A} \cup B \cup C) \cap (\bar{B} \cup C)$

$= A \cap (B \cup C) \cap (\bar{B} \cup C)$

$= A \cap (C \cup B) \cap (C \cup \bar{B})$

$= A \cap C \cup (B \cap \bar{B})$

$= A \cap [C \cup \emptyset]$

$= \underline{\underline{A \cap C}}$

10) $(A \cup B) \Delta (A \cup C) = (B \Delta C) - A$

$A \Delta B = (A - B) \cup (B - A) = (A \cap \bar{B}) \cup (B \cap \bar{A})$

$= [(A \cup B) \cap \overline{(A \cup C)}] \cup [(A \cup C) \cap \overline{(A \cup B)}]$

$= (A \cup B) \cap (\bar{A} \cap \bar{C}) \cup (A \cup C) \cap (\bar{A} \cap \bar{B})$

$= (A \cup B) \cap (\bar{A} \cap \bar{C}) \cup (\bar{A} \cap \bar{B}) \cap (A \cup C)$

$= (A \cup B) \cap \bar{A} \cap (\bar{C} \cup \bar{B}) \cap (A \cup C)$

$= (A \cup B) \cap (A \cup C) \cap \bar{A} \cap (\bar{C} \cup \bar{B})$

$= A \cup (B \cap C) \cap \bar{A} \cap (\bar{C} \cup \bar{B})$

$= A \cup \bar{A} \cap (B \cap C) \cap (\bar{C} \cup \bar{B})$

$= U \cap (B \cap C) \cap (\bar{C} \cup B)$

$= (B \cap C) \cap (\bar{C} \cup B)$

$(B \cap \bar{C}) \cup (C \cap \bar{B}) \cap \bar{B}$

$\bar{B} \cup (B \cap \bar{C}) \cap (C \cap \bar{B})$

$= (B \cap \bar{C}) \cap C \cap \bar{B})$

$= (B \cap C) \cap C \bar{C} \cup B)$