Set-11 Martin skatvedt
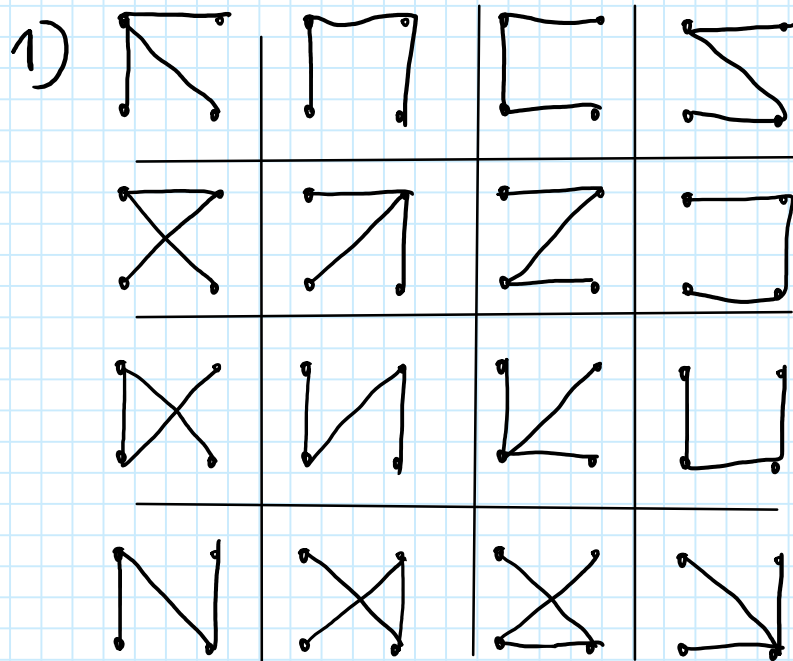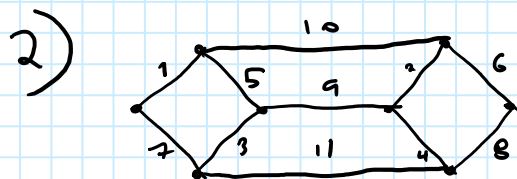
1)

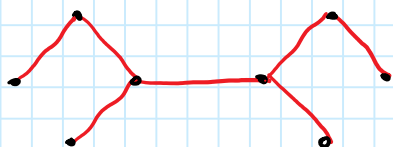

n spannin trees $= k^{k-2}$

$4^2 = 16$ spanning trees

2)

minimum spanning tree

3) Eulers theorem $r = e - v + 2$

a) Verticies $= 16$
   Edges $= 30$
   Regions $= 16$

   $r = 30 - 16 + 2 = 16$

b) Verticies $= 10$
   Edges $= 24$
   Regions $= 16$

   $r = 24 - 10 + 2 = 16$

4) Since each edge is at the boundry of exactly two regions (only two regions share an edge)

And each region has at least 5 edges
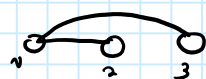
$$e \geq \frac{53 \cdot 5}{2} = 132,5 \approx 133$$
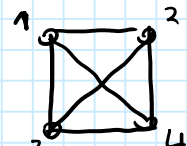
Euters formula gives us

$$v = 2 - r + e$$
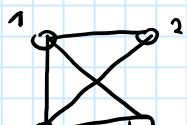
$$\hookrightarrow v = 2 - 53 + 133 = \underline{82}$$

which means that $|V| \geq 82$

5) a)

$$\begin{array}{c} \\ {}^1 \\ {}^2 \\ {}^3 \end{array}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = $$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} = $$

They are isomorphic

b)

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} = $$

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = $$

They are not isomorphic

6) $U = abab$
$V = bab$

a) $UV = \underline{ababbab}$ $|UV| = \underline{\underline{7}}$

b) $VU = \underline{bababab}$ $|VU| = \underline{\underline{7}}$

c) $V^2 = \underline{babbab}$ $|V^2| = \underline{\underline{6}}$

7) $K = \{a, ab, a^2\}$ $A = \{a, b\}$
$L = \{b^2, aba\}$

a) $KL = \{ab^2, a^2ba, ab^3, ababa, a^2b^2, a^2aba\}$

b) $LL = \{b^4, b^2aba, abab^2, abaaba\}$

8) $A = \{a, b, c\}$

a) $L = \{b^2\}$

$L^* = \{(b^2)^n : n \geq 0\}$

b) $L = \{a, b\}$

$L^* = \{x^n, n \geq 0 \text{ and } x = a \cup x = b\}$

c) $L = \{a, b, c^2\}$

$L^* = ?$

9)

a) no

b) yes