TMA4110 - Martin Skatvedt - Innlevering 3

tirsdag 11. oktober 2022 12:29

1) $$0 \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

0 : skalaren 0

$\mathbf{0}$ = nullvektor

2) 1. Nullvektor i M_n ligger i alle nedre triangulære nxn matriser

2. summen av to nedre triangulære nxn matriser er fortsatt en nedre triangulær nxn matrise

3. Produktet av to nedre triangulære nxn matriser er forsatt en nedre triangulær nxn matrise

3)

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4)

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$$\mathrm{Row}_A = \mathrm{Sp}\left\{\begin{bmatrix}1\\-1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\2\\-4\\1\end{bmatrix}, \begin{bmatrix}3\\0\\-3\\2\end{bmatrix}\right\}$$

$$A = \begin{bmatrix}1 & -1 & 1 & 0\\0 & 2 & -4 & 1\\3 & 0 & -3 & 2\end{bmatrix}$$

$$\begin{bmatrix}1 & -1 & 1 & 0\\0 & 2 & -4 & 1\\0 & 3 & -6 & 2\end{bmatrix} \quad \mathrm{I} \cdot -3$$

$$\begin{bmatrix}1 & -1 & 1 & 0\\0 & 1 & -2 & \frac{1}{2}\\0 & 3 & -6 & 2\end{bmatrix} \quad \mathrm{II} \cdot \frac{1}{2}$$

$$\begin{bmatrix}1 & 0 & -1 & \frac{1}{2}\\0 & 1 & -2 & \frac{1}{2}\\0 & 0 & 0 & \frac{1}{2}\end{bmatrix} \quad \begin{matrix}\mathrm{II}\\ \\ \mathrm{II} \cdot -3\end{matrix}$$

$$\begin{bmatrix}1 & 0 & -1 & \frac{1}{2}\\0 & 1 & -2 & \frac{1}{2}\\0 & 0 & 0 & 1\end{bmatrix} \quad \mathrm{III} \cdot 2$$

$$\begin{bmatrix}1 & 0 & -1 & 0\\0 & 1 & -2 & 0\\0 & 0 & 0 & 1\end{bmatrix} \quad \begin{matrix}\mathrm{III} \cdot -\frac{1}{2}\\ \mathrm{III} \cdot -\frac{1}{2}\end{matrix}$$

$$x = r\begin{bmatrix}1\\2\\1\\0\end{bmatrix}$$

$$\mathrm{Null}_A = \left\{\begin{bmatrix}1\\2\\1\\0\end{bmatrix}\right\}$$

$\begin{bmatrix}1 & 2 & 1 & 0\end{bmatrix}^T$ ligger i Null_A

$$\dim \mathrm{Col}_A = 3$$

5)

$$A = \begin{array}{c} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \end{array} \begin{bmatrix} 3 & -1 & 1 & 14 & -8 \\ 0 & 2 & -1 & 1 & 5 \\ 2 & 0 & 1 & 13 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{14}{3} & -\frac{8}{3} \\ 0 & 2 & -1 & 1 & 5 \\ 2 & 0 & 1 & 13 & -3 \end{bmatrix} \quad \mathrm{I} \cdot \frac{1}{3}$$

$$\begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{14}{3} & -\frac{8}{3} \\ 0 & 2 & -1 & 1 & 5 \\ 0 & \frac{2}{3} & \frac{1}{3} & \frac{11}{3} & \frac{7}{3} \end{bmatrix} \quad \mathrm{I} \cdot -2$$

$$\begin{bmatrix} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{14}{3} & -\frac{8}{3} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & \frac{5}{2} \\ 0 & \frac{2}{3} & \frac{1}{3} & \frac{11}{3} & \frac{7}{3} \end{bmatrix} \quad \mathrm{II} \cdot \frac{1}{2}$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{6} & \frac{29}{6} & -\frac{11}{6} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & \frac{5}{2} \\ 0 & \frac{2}{3} & \frac{1}{3} & \frac{11}{3} & \frac{7}{3} \end{bmatrix} \quad \mathrm{II} \cdot \frac{1}{3}$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{6} & \frac{29}{6} & -\frac{11}{6} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & \frac{5}{2} \\ 0 & 0 & \frac{2}{3} & \frac{10}{3} & \frac{2}{3} \end{bmatrix} \quad \mathrm{II} \cdot -\frac{2}{3}$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{6} & \frac{29}{6} & -\frac{11}{6} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 1 & 5 & 1 \end{bmatrix} \quad \mathrm{III} \cdot \frac{3}{2}$$

$$\begin{bmatrix} 1 & 0 & 0 & 4 & -2 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 1 & 5 & 1 \end{bmatrix} \quad \mathrm{III} \cdot -\frac{1}{6}$$

$$\begin{bmatrix} 1 & 0 & 0 & 4 & -2 \\ 0 & 1 & 0 & 3 & 3 \\ 0 & 0 & 1 & 5 & 1 \end{bmatrix} \quad \mathrm{III} \cdot \frac{1}{2}$$

$$\mathrm{Row}\,A = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 5 \\ 1 \end{bmatrix} \right\}$$

$$\mathrm{Col}\,A = \left\{ \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$$

$$x = r \begin{bmatrix} -4 \\ -3 \\ -5 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 2 \\ -3 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathrm{Null}_A = \left\{ \begin{bmatrix} -4 \\ -3 \\ -5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & 4 \\ 3 & -4 & 5 \\ 4 & -5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 7 & 10 \\ 0 & -10 & -4 \\ 0 & -13 & -6 \end{bmatrix} \begin{array}{l} \\ I \cdot 2 \\ I \cdot -3 \\ I \cdot -4 \end{array}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & \frac{10}{7} \\ 0 & -10 & -4 \\ 0 & -13 & -6 \end{bmatrix} \begin{array}{l} \\ II \cdot \frac{1}{7} \\ \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{7} \\ 0 & 1 & \frac{10}{7} \\ 0 & 0 & \frac{72}{7} \\ 0 & 0 & \frac{88}{7} \end{bmatrix} \begin{array}{l} II \cdot -2 \\ \\ II \cdot 10 \\ II \cdot 13 \end{array}$$

$$\begin{bmatrix} 1 & 0 & \frac{1}{7} \\ 0 & 1 & \frac{10}{7} \\ 0 & 0 & 1 \\ 0 & 0 & \frac{88}{7} \end{bmatrix} \begin{array}{l} \\ \\ III \cdot \frac{7}{72} \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{array}{l} III \cdot -\frac{1}{7} \\ III \cdot -\frac{10}{7} \\ \\ III \cdot -\frac{88}{7} \end{array}$$

$$Row_B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$Col_B = \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -4 \\ -5 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} \right\}$$

$$x = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$Null_B = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

6)
$$A = \begin{bmatrix} a & b & c & d & e \\ f & g & h & i & j \\ k & l & m & n & o \\ p & q & r & s & t \end{bmatrix}$$

a) Maks $\dim \mathrm{Col} A = 4$, siden det kan være maks 4 pivot elementer

b) $\dim \mathrm{Col}_A = n - \dim \mathrm{Null}_A = 3$

vi har 3 kolonner med pivot elementer, så 3 lineært uavhegige kolonner

c)

d) $\mathrm{Rank}_{A^T} = 4 - 3 = \underline{1}$

dvs $\dim \mathrm{Row}_{A^T} = 1$

e) $\dim \mathrm{Null}_A > 0$ siden maks $\mathrm{Rank}_A = 4$.

Null_A kan <u>ikke</u> være $\{0\}$

7)

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$B = n \times n$

Siden B er et kvadrat har det full rang, dvs $\text{Rank}\,B = n$, da må

$\dim \text{Null}\,B = 0$ så

$\text{Null}\,B =$ nullvektor med n elementer

9)

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$$T\left(\begin{bmatrix}1\\2\end{bmatrix}\right)=\begin{bmatrix}5\\2\\-1\end{bmatrix} \quad \text{og} \quad T\left(\begin{bmatrix}-1\\3\end{bmatrix}\right)=\begin{bmatrix}0\\1\\1\end{bmatrix}$$

a)

$$\begin{bmatrix}3\\1\end{bmatrix}=2\begin{bmatrix}1\\2\end{bmatrix}-1\begin{bmatrix}-1\\3\end{bmatrix}$$

$$T\left(\begin{bmatrix}3\\1\end{bmatrix}\right)=2\,T\left(\begin{bmatrix}1\\2\end{bmatrix}\right)-1\,T\left(\begin{bmatrix}-1\\3\end{bmatrix}\right)$$

$$=2\begin{bmatrix}5\\2\\-1\end{bmatrix}-1\begin{bmatrix}0\\1\\1\end{bmatrix}=\underline{\underline{\begin{bmatrix}10\\3\\-3\end{bmatrix}}}$$

b)

$$\begin{bmatrix}1\\0\end{bmatrix}=\frac{3}{5}\begin{bmatrix}1\\2\end{bmatrix}-\frac{2}{5}\begin{bmatrix}-1\\3\end{bmatrix}$$

$$\begin{bmatrix}0\\1\end{bmatrix}=\frac{1}{5}\begin{bmatrix}1\\2\end{bmatrix}+\frac{1}{5}\begin{bmatrix}-1\\3\end{bmatrix}$$

$$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)=\frac{3}{5}\begin{bmatrix}5\\2\\-1\end{bmatrix}-\frac{2}{5}\begin{bmatrix}0\\1\\1\end{bmatrix}=\begin{bmatrix}\frac{15}{5}\\\frac{6}{5}\\-\frac{3}{5}\end{bmatrix}+\begin{bmatrix}0\\-\frac{2}{5}\\-\frac{2}{5}\end{bmatrix}=\begin{bmatrix}3\\\frac{4}{5}\\-1\end{bmatrix}$$

$$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)=\frac{1}{5}\begin{bmatrix}5\\2\\-1\end{bmatrix}+\frac{1}{5}\begin{bmatrix}0\\1\\1\end{bmatrix}=\begin{bmatrix}1\\\frac{2}{5}\\-\frac{1}{5}\end{bmatrix}+\begin{bmatrix}0\\\frac{1}{5}\\\frac{1}{5}\end{bmatrix}=\begin{bmatrix}1\\\frac{3}{5}\\0\end{bmatrix}$$

$$A=\begin{bmatrix}3 & 1\\\frac{4}{5} & \frac{3}{5}\\-1 & 0\end{bmatrix}$$

$$A = \begin{bmatrix} \frac{\;}{5} & \frac{\;}{5} \\ -1 & 0 \end{bmatrix}$$

10)

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$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = 3x_1 + 2x_2 \qquad R(x) = \begin{bmatrix} x \\ -x \\ 4x \end{bmatrix}$$

$$R(3x_1 + 2x_2) = \begin{bmatrix} 3x_1 + 2x_2 \\ -3x_1 - 2x_2 \\ 12x_1 + 8x_2 \end{bmatrix}$$

$$R\left(T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)\right) = \begin{bmatrix} 3\cdot 1 + 2\cdot 0 \\ -3\cdot 1 - 2\cdot 0 \\ 12\cdot 1 + 8\cdot 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 12 \end{bmatrix}$$

$$R\left(T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)\right) = \begin{bmatrix} 3\cdot 0 + 2\cdot 1 \\ -3\cdot 0 - 2\cdot 1 \\ 12\cdot 0 + 8\cdot 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 8 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 2 \\ -3 & -2 \\ 12 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 1 & 0 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & \frac{2}{3} & 1 & 0 \end{bmatrix}$$

$$\ker T = \left\{ \begin{bmatrix} 1 \\ -\frac{2}{3} \end{bmatrix} \right\}$$

11)

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$$B = \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix} \right) \quad C = \left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$$

$$[b_1 \; b_2 \mid c_1 \; c_2] \sim \left[\begin{array}{cc|cc} 1 & 0 & 2 & 1 \\ 1 & 2 & 3 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & 2 & 1 \\ 0 & 2 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & 0 & 2 & 1 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \end{array}\right]$$

$$\underline{\underline{A = \begin{bmatrix} 2 & 1 \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}}}$$

12)

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13)

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$B = (1, \cos x, \cos 2x)$

$T : V \to \mathbb{R}^3$

$T : a + b\cos x + c\cos 2x \to \begin{bmatrix} c \\ 2a+b \\ b-3c \end{bmatrix}$

a) $\cos 2x = 2\cos^2 x - 1$

$$\cos^2 x = \frac{1+\cos 2x}{2} = \underline{\overset{a_1}{\frac{1}{2}} + \overset{a_2}{0} \cdot \cos x + \overset{a_3}{\frac{1}{2}} \cos 2x}$$

$$\underline{\underline{[\cos^2 x]_B = \begin{bmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \end{bmatrix}}}$$

$\cos 2x = 1 - 2\sin^2 x$

$$\sin^2 x = \frac{1-\cos 2x}{2} = \underline{\overset{a_1}{\frac{1}{2}} + \overset{a_2}{0} \cdot \cos x \overset{a_3}{- \frac{1}{2}} \cos 2x}$$

$$\underline{\underline{[\sin^2 x]_B = \begin{bmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{bmatrix}}}$$

b) $[u]_B = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ $[v]_B = \begin{bmatrix} d \\ e \\ f \end{bmatrix}$ $[v]_B + [u]_B = \begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix}$

$T([u]_B) = \begin{bmatrix} c \\ 2a+b \\ b-3c \end{bmatrix}$ $T([v]_B) = \begin{bmatrix} f \\ 2d+e \\ e-3f \end{bmatrix}$

$T([u]_B + [v]_B) = \begin{bmatrix} c+f \\ 2(a+d)+b+e \\ b+e-3(c+f) \end{bmatrix}$

$T([u]_B) + T([v]_B) = \begin{bmatrix} c+f \\ 2a+b+2d+e \\ b-3c+e-3f \end{bmatrix} = \begin{bmatrix} c+f \\ 2(a+d)+b+e \\ b+e-3(c+f) \end{bmatrix}$

$\underline{\underline{T(u+v) = T(u) + T(v) \text{ for alle } v \text{ og } u \text{ i } V}}$

$$c_1 \cdot [v]_B = \begin{bmatrix} c_1 a \\ c_1 b \\ c_1 c \end{bmatrix}$$

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$$T(c_1 [v]_B) = \begin{bmatrix} c_1 c \\ 2c_1 a + c_1 b \\ c_1 b - 3c_1 c \end{bmatrix} = \begin{bmatrix} c_1 c \\ c_1 (2a+b) \\ c_1 (b-3c) \end{bmatrix}$$

$$c_1 \cdot T([v]_B) = c_1 \cdot \begin{bmatrix} c \\ 2a+b \\ b-3c \end{bmatrix}$$

$\underline{\underline{T(c_1 v) = c_1 \cdot T(v) \text{ for alle } v \text{ og } c_1 \text{ i } V}}$

c)

d) $L: V \to \mathbb{R}$

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$$L(f) = \int_{-\pi}^{\pi} f(x)\cos(x)\,dx$$

$$L(\cos x) = \int_{-\pi}^{\pi} \cos^2 x\,dx = \int_{-\pi}^{\pi} \frac{1+\cos 2x}{2}\,dx$$

$$= \frac{1}{2}\int_{-\pi}^{\pi} 1+\cos 2x\,dx = \frac{1}{2}\left(\int_{-\pi}^{\pi} 1\,dx + \int_{-\pi}^{\pi} \cos 2x\,dx\right)$$

$$\int_{-\pi}^{\pi} 1\,dx = \Big[x\Big]_{-\pi}^{\pi} = \pi - (-\pi) = \underline{2\pi}$$

$$\int_{-\pi}^{\pi} \cos 2x\,dx = \int_{-\pi}^{\pi} \cos u\,\frac{du}{2} \qquad u = 2x \quad \to \quad dx = \frac{du}{2}$$

$$= \frac{1}{2}\int_{-\pi}^{\pi} \cos u\,du = \frac{1}{2}\Big[\sin 2x\Big]_{-\pi}^{\pi} = \frac{1}{2}\left(\sin 2\pi - \sin(-2\pi)\right) = \underline{\frac{1}{2}(0+0)}$$

$$= \frac{1}{2}\,2\pi = \underline{\pi}$$

$$\underline{\underline{\int_{-\pi}^{\pi} \cos^2 x\,dx = \pi}}$$

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e) $f = a + b\cos x + c\cos 2x$

$$L(f) = L(a + b\cos x + c\cos 2x) = \int_{-\pi}^{\pi} (a + b\cos x + c\cos 2x) \cdot \cos x \, dx$$

$$= \int_{-\pi}^{\pi} a\cos x + b\cos^2 x + c\cos(x) \cdot \cos(2x) \, dx$$

$$= \int_{-\pi}^{\pi} a\cos x \, dx + \int_{-\pi}^{\pi} b\cos^2 x \, dx + \int_{-\pi}^{\pi} c\cos x \cdot \cos 2x \, dx$$

$$\int_{-\pi}^{\pi} a\cos x \, dx = a\int_{-\pi}^{\pi} \cos x \, dx = a\left[\sin x\right]_{-\pi}^{\pi} = a(0 - 0) = \underline{0}$$

$$\int_{-\pi}^{\pi} b\cos^2 x \, dx = b\int_{-\pi}^{\pi} \cos^2 x \, dx = \underline{b\pi}$$

$$\int_{-\pi}^{\pi} c\cos x \cdot \cos 2x \, dx = c\int_{-\pi}^{\pi} \cos x \cdot \cos 2x \, dx$$

$u = \cos 2x$
$v' = \cos x$
$v = \sin x$
$u' = -2\sin 2x$

$$\left[\cos 2x \sin x\right]_{-\pi}^{\pi} - \underbrace{\int_{-\pi}^{\pi} -2\sin 2x \sin x \, dx}$$

$$\triangleright -2\int_{-\pi}^{\pi} \sin 2x \sin x \, dx = -2\int_{-\pi}^{\pi} \cos x \, 2\sin^2 x \, dx$$

$u = \sin x$

$dx = \frac{1}{\cos x}$

$$= -4\int_{-\pi}^{\pi} u^2 \, du = -4\left[\frac{u^3}{3}\right]_{-\pi}^{\pi} = -4\left[\frac{\sin x}{3}\right]_{-\pi}^{\pi} = -4\left(\frac{\sin \pi}{3} - \frac{\sin(\pi)}{3}\right)$$

$$= \underline{0}$$

$$\left[\cos 2x \sin x\right]_{-\pi}^{\pi} = \cos 2\pi \cdot \sin\pi - \cos(-2\pi)\sin(-\pi) = \underline{0}$$

$$\underline{L(a + b\cos x + c\cos 2x) = \pi b}$$

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For at $L(f)=0$ må $b=0$

ker L = alle f i V hvor $f = a + c\cos 2x$
for alle reelle a og c

dim (ker L) = 2

14) a) $S(T(v)) = v$ gir injektivitet
$T(S(w)) = w$ gir surjektivitet

b) $\operatorname{im} T = W$

c) Ja, siden S er en invers av T

$T(v) = 0 \rightarrow v = 0$

$T(S(w)) = 0 \rightarrow S(w) = 0 \rightarrow w = 0$