Martin Skatvedt, Øving 2

1) $p \lor (p \land q) \equiv p$

p	q	$p \land q$	$p \lor (p \land q)$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

2)

p	q	r	$q \lor r$	$p \land (q \lor r)$	$p \land q$	$p \land r$	$(p \land q) \lor (p \land r)$
0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
	1	1	1	1	1	1	1

p	q	r	$q \land r$	$p \lor (q \land r)$	$p \lor q$	$p \lor r$	$(p \lor q) \land (p \lor r)$
0	0	0	0	0	0	0	0
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	1	1	1	1	1	1

3) $[p \Rightarrow (q \vee r)] \equiv [(p \wedge \neg q) \Rightarrow r]$

$p \Rightarrow (q \vee r)$

$\equiv \neg p \vee (q \vee r)$, Conditional law

$\equiv (\neg p \vee q) \vee r$, Associative law

$\equiv \neg(\neg p \vee q) \Rightarrow r$, conditional law

$\equiv (p \wedge \neg q) \Rightarrow r$, D.M.G

4) $[(q \wedge p) \vee q] \wedge \neg(\neg q \vee p) \equiv q \wedge \neg p$

$[(q \wedge p) \vee q] \wedge \neg(\neg q \vee p)$

$\equiv (q) \wedge \neg(\neg q \vee p)$, Absorption law

$\equiv q \wedge (q \wedge \neg p)$, DMG

$\equiv (q \wedge q) \wedge \neg p$, Associativ law

$\equiv q \wedge \neg p$, $(q \wedge q) \equiv q$

5) a) $\forall_x S(x) \Rightarrow H(x)$

b) $\exists_x S(x) \Rightarrow H(x)$

c) $\neg\forall_x S(x) \Rightarrow H(x)$

d) $\exists_y \exists_x S(x)(\neg H(y) \wedge \neg H(x) \wedge \forall_z(\neg H(z) \Rightarrow (z=x) \vee (z=y)))$

6) $\exists_x \exists_y \exists_z (P(x,y) \wedge P(z,y) \wedge P(x,z) \wedge \neg P(z,x))$.

a) $R = \{(x,y) : x < y\}$ $U = \mathbb{N}$

$x = 1$ $P(1,3) = T$

$z = 2$ $P(2,3) = T$

$y = 3$ $P(1,2) = T$

$\neg P(2,1) = T$

<u>True</u>

b) $R = \{(x, x+1) : x \geq 0\}$ $U = \mathbb{N}$

$y = x+1 \wedge y = z+1 \wedge z = x+1 \wedge x \neq z+1$

siden $y = x+1$ og $y = z+1$ må $x = z$
da kan ikke $z = x+1$ og uttrykket blir <u>False</u>

c) $R = \{(A,B) : A \subseteq B\}$ $U = P(\mathbb{N})$

$y := P(\mathbb{N})$ $P(x,y) : \{\emptyset\} \subseteq P(\mathbb{N}) = T$

$x := \{\emptyset\}$ $P(z,y) = \{\{0\}\} \subseteq P(\mathbb{N}) = T$

$z := \{\{0\}\}$ $P(x,z) = \{\emptyset\} \subseteq \{\{0\}\} = T$

$\neg P(z,x) = \{\{0\}\} \not\subseteq \{\emptyset\} = T$

<u>True</u>

7) $\forall_x [P(x) \land q(x)]$

$\neg(\forall_x [P(x) \land q(x)])$

$\equiv \exists_x [\neg(P(x) \land q(x))]$

$\equiv \exists_x [\neg P(x) \lor \neg q(x)]$ D.M.G

8) $\exists_x \forall_y [P(y) \lor \neg q(x,y)]$

$\neg(\exists_x \forall_y [P(y) \lor \neg q(x,y)])$

$\equiv \forall_x \neg(\forall_y [P(y) \lor \neg q(x,y)])$

$\equiv \forall_x \exists_y \neg[P(y) \lor \neg q(x,y)]$

$\equiv \forall_x \exists_y [\neg P(y) \land q(x,y)]$ D.M.G