Martin Skatvedt - Øving 11 - MA0001

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$\boxed{7.4.4}$

b) $\displaystyle\int \frac{x^3}{(7x^2-5)^5}\,dx = \int \frac{x^3}{u^5}\,\frac{du}{28x^3}$

$u = 7x^4-5$

$\dfrac{du}{dx} = 28x^3$

$\dfrac{du}{28x^3} = dx$

$\displaystyle = \frac{1}{28}\int \frac{1}{u^5}\,du = \frac{1}{28}\cdot -\frac{1}{4}u^{-4} + C$

$\displaystyle = -\frac{1}{112}(7x^2-5)^{-4} + C = -\frac{1}{112(7x^4-5)^4} + C$

e) $\displaystyle\int_0^1 x^2(x^3+1)^9\,dx = \int_0^1 x^2 u^9\,\frac{du}{3x^2}$

$u = x^3+1$

$\dfrac{du}{dx} = 3x^2$

$\dfrac{du}{3x^2} = dx$

$\displaystyle = \frac{1}{3}\int_0^1 u^9\,du = \frac{1}{3}\left[\frac{1}{10}(x^3+1)^{10}\right]_0^1$

$\displaystyle = \frac{1}{3}\left[\frac{1}{10}\left(2^{10}-1^{10}\right)\right] = \frac{341}{10} = 34{,}1$

h) $\displaystyle\int t\sqrt{t^2+1}\,dt = \int t\,u^{\frac{1}{2}}\,\frac{du}{2t}$

$u = t^2+1$

$\dfrac{du}{dt} = 2t$

$\dfrac{du}{2t} = dt$

$\displaystyle = \frac{1}{2}\int u^{\frac{1}{2}}\,du = \frac{1}{3}(t^2+1)^{\frac{3}{2}} + C$

$\displaystyle = \frac{1}{3}\left(\sqrt{t^2+1}\right)^3 + C$

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$\boxed{7.4.12}$

$$\int \frac{1}{(1-x)^2}\,dx = \int \frac{1}{u^2}\cdot -du$$

$u = 1-x$

$\frac{du}{dx} = -1$

$-du = dx$

$$= -\int u^{-2}\,du = -1\cdot -\frac{1}{1}u^{-1} + C$$

$$= \underline{\underline{\frac{1}{1-x} + C}}$$

$$\left(\frac{x}{1-x} + C_2\right)' = \frac{1\cdot(1-x) + x\cdot(-1)}{(1-x)^2} = \frac{1-x-x}{(1-x)^2} = \underline{\underline{\frac{1}{(1-x)^2}}}$$

$\boxed{7.5.1}$

a) $$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$$

$u = x$

$v' = \cos x$

$u' = 1$

$v = \sin x$

$$= \underline{\underline{x\sin x + \cos x + C}}$$

e) $$\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \int \frac{1}{x}\cdot\frac{1}{2}x^2\,dx$$

$$= \frac{1}{2}\int x\,dx$$

$$= \frac{1}{4}x^2 + C$$

$u = \ln x$

$v' = x$

$u' = \frac{1}{x}$

$v = \frac{1}{2}x^2$

$$= \underline{\underline{\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C}}$$

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$\boxed{7.5.1}$

g) $\int_0^1 x e^{-x}\,dx = -xe^{-x} - \int -e^{-x}\,dx$

$= \int e^{-x}\,dx$

$= -e^{-x} + C$

$u = x$

$v' = e^{-x}$

$u' = 1$

$v = -e^{-x}$

$= \left[-xe^{-x} - e^{-x}\right]_0^1 = \left[-e^{-x}(x+1)\right]_0^1$

$\left(-e^{-1}(1+1)\right) - \left(-e^{0}(0+1)\right) = -2e^{-1} + 1$

$\underline{\underline{= 1 - 2e^{-1}}}$

$\boxed{7.5.2}$

a) $\int e^{x} \sin x \, dx$

$= -e^{x}\cos x - \int e^{x} \cdot -\cos x\,dx$

$\underline{\underline{= -e^{x}\cos x + \int e^{x}\cos x\,dx}}$

$u = e^{x}$

$v' = \sin x$

$u' = e^{x}$

$v = -\cos x$

b) $\int e^{x}\cos x\,dx$

$\underline{= e^{x}\sin x - \int e^{x}\sin x\,dx}$

$u = e^{x}$

$v' = \cos x$

$u' = e^{x}$

$v = \sin x$

Hele integral

$\underline{\underline{I = e^{x}\sin x - e^{x}\cos x - I}}$

$\boxed{7.5.2}$

c) $2I = e^x(\sin x - \cos x)$

$I = \frac{1}{2}e^x(\sin x - \cos x)$

$$\int e^x \sin x \, dx = \frac{1}{2}e^x(\sin x - \cos x) + C$$

$\boxed{7.7.2.a}$

$$\int_1^2 \frac{1}{x}\,dx = [\ln x]_1^2 = \underline{\ln 2} \approx \underline{0{,}6931}$$

$$\Delta x = \frac{2-1}{10} = \frac{1}{10}$$

$$T_{100} = \frac{\Delta x}{2}\big(f(1) + 2f(1{,}1) + 2f(1{,}2) + 2f(1{,}3) + 2f(1{,}4) + 2f(1{,}5) + 2f(1{,}6) + 2f(1{,}7) + 2f(1{,}8) + 2f(1{,}9) + f(2)\big) =$$

$$0{,}05(1 + 1{,}8181 + 1{,}6666 + 1{,}5384 + 1{,}4285 + 1{,}3333 + 1{,}25 + 1{,}1764 + 1{,}1111 + 1{,}0526 + 0{,}5) = \underline{\underline{0{,}6937}}$$