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Skriftlig innlevering 2

1 Forklar hvorfor $\cosh(x) = \frac{e^x + e^{-x}}{2}$, $D_f : [0, \infty)$ har en inversfunksjon $\cosh^{-1}(x)$

$\cosh(0) = \frac{e^0 + e^{-0}}{2} = \frac{2}{2} = \underline{1}$

* $\cosh(\infty) = \frac{e^{\infty} + e^{-\infty}}{2} = \frac{\infty + 0}{2} = \underline{\infty}$

$V_f = D_{f^{-1}} \rightarrow \underline{\underline{D_{f^{-1}} = [1, \infty)}}$

* Siden alle verdier av e^x eller e^{-x} er alltid positiv vil alle $\cosh(x)$ være positive Dvs
<u>$\cosh(x)$ er injektiv og har en invers</u>

* $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$

$\cosh^{-1}(y) = \frac{e^y + e^{-y}}{2} = x \rightarrow e^y + e^{-y} = 2x \quad | \cdot e^y$

$\rightarrow e^{2y} + e^0 = 2xe^y \rightarrow e^{2y} + 1 = 2xe^y$

$e^{2y} - 2xe^y + 1 = 0 \quad = u^2 - 2xu + 1 = 0$

<u>abc-formel</u>

$\frac{+2x \pm \sqrt{(2x)^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} \rightarrow \frac{2x \pm \sqrt{4x^2 - 4}}{2} = x \pm \sqrt{x^2 - 1}$

$u_1 = x + \sqrt{x^2 - 1} \qquad u_2 = x - \sqrt{x^2 - 1}$

$\underline{y = \ln(x + \sqrt{x^2 - 1})}$

Siden e^y alltid er positivt kan ikke $e^y = u_2$ være et svar

$$\cosh^{-1}(x) = \ln(x + \sqrt{x^2-1})$$

$$\cosh'(x) = \sinh(x)$$

$$(\cosh^{-1})'(x) = \frac{1}{\sinh(\ln(x+\sqrt{x^2-1}))}$$

$$(\cosh^{-1})'(\sqrt{170}) = \frac{1}{\sinh(\ln(\sqrt{170}+\sqrt{170-1}))}$$

$$= \frac{1}{\sinh(\ln(\sqrt{170}+13))}$$

nevner:

$$\frac{e^{\ln(\sqrt{170}+13)} - e^{-(\ln(\sqrt{170}+13))}}{2}$$

$$= \frac{\sqrt{170}+13 - \frac{1}{\sqrt{170}+13}}{2} =$$

teller:

$$= \frac{\sqrt{170}(\sqrt{170}+13) + 13(\sqrt{170}+13) - 1}{\sqrt{170}+13}$$

$$= \frac{170 + 13\sqrt{170} + 169 + 13\sqrt{170} - 1}{\sqrt{170}+13}$$

$$= \frac{338 + 26(\sqrt{170})}{\sqrt{170}+13} \qquad \frac{26(13+\sqrt{170})}{\sqrt{170}+13}$$

hele utrykket:

$$\frac{26}{2} = 13 \qquad (\cosh^{-1})'(\sqrt{170}) = \frac{1}{13}$$

$\boxed{2}$ $\bullet f(-x) = -f(x)$ $\bullet p = 2\pi \Rightarrow f(x+2\pi) = f(x)$

$\bullet f(x) \geq 0$ for $0 \leq x \leq \pi$, $f(\frac{\pi}{2}) = 1$

$\bullet \int_0^{\pi} f(x)\,dx = 2$

a) $\int_{17\pi}^{19\pi} |f(x)|\,dx$ $\quad \begin{cases} f(x), & 0 \leq x \leq \pi \\ -f(x), & x < 0 \vee x > \pi \end{cases}$

siden $p = 2\pi$ kan man ta integralet

$\int_{-\pi}^{\pi} |f(x)|\,dx = \int_{-\pi}^{0} -f(x)\,dx + \int_0^{\pi} f(x)\,dx$

setter $F(x)$ som $\int f(x)\,dx$

$= -F(x)\Big]_{-\pi}^{0} + F(x)\Big]_0^{\pi}$

siden $\int_0^{\pi} f(x)\,dx = 2$ er gitt vil

integralet $\int_{-\pi}^{\pi} f(x)\,dx = \underline{\underline{4}}$

b) $\int_0^{\pi/4} e^{f(2x)} f'(2x)\,dx$ $\quad u = f(2x)$

$\frac{du}{dx} = 2f'(2x)$

$dx = \frac{du}{2f'(2x)}$

$\int_{x=0}^{x=\frac{\pi}{4}} e^u f'(2x) \frac{du}{2f'(2x)} = \frac{1}{2}\int_{x=0}^{x=\frac{\pi}{4}} e^u\,du$

setter inn for u

$= \frac{1}{2}\Big[e^{f(2x)}\Big]_0^{\pi/4} = \frac{1}{2}\left(e^{f(\frac{\pi}{2})} - e^{f(0)}\right)$

$f(\frac{\pi}{2}) = 1$ er gitt og siden $f(x) \geq 0$ for $0 \leq x \leq \pi$ vil $f(0) = 0$

$\frac{1}{2}(e^1 - e^0) = \underline{\underline{\frac{1}{2}(e-1)}}$

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c) $\int_{-\pi}^{\pi} e^{(f(x))^2} \sin(f(x))\,dx$

Deler opp i to uttrykk

$\sin(x)$ er odde

$f(x)$ er odde

dvs. $\sin(f(x))$ er odde

$e^{(f(x))^2}$ $\quad f(\frac{\pi}{2}) = 1$

$e^{(f(\frac{\pi}{2}))^2} = e^1 = \underline{e}$

$f(-\frac{\pi}{2}) = -f(\frac{\pi}{2})$

$e^{(-1)^2} \to e^1 = \underline{e}$

siden $g(x) = g(-x)$ er funksjonen like

Produktet av en odde og like funksjon er odde.

siden integralet går fra $-a$ til a og funksjonen er odde, vil

$$\int_{-\pi}^{\pi} e^{(f(x))^2} \sin(f(x))\,dx = 0$$

$\boxed{3}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{3n+4i} = \int_A^B f(x_i)\,\Delta x_i$$

$$f(x) \qquad \Delta x_i$$

$$\frac{4}{3n+4i} \cdot \frac{n}{n} = \frac{4n}{3n^2+4in} = \left(\frac{4n}{3n+4i}\right)\cdot\left(\frac{1}{n}\right)$$

$$\frac{4n}{3n+4i} \;\Big|:n = \frac{\frac{4n}{n}}{\frac{3n}{n}\;\frac{4i}{n}} = \frac{4}{3+4\frac{i}{n}} \qquad \underline{x_i = \frac{i}{n}}$$

$$\underline{f(x_i) = \frac{4}{3+4x}}$$

$$\Delta x = \frac{B-A}{n} = \frac{1}{n}, \quad \underline{A=0} \qquad \underline{B=1}$$

$$\int_0^1 \frac{4}{3+4x}\,dx \qquad u = 3+4x \qquad \frac{du}{dx} = 4 \Rightarrow dx = \frac{du}{4}$$

$$\int_{x=0}^{x=1} \frac{4}{u}\frac{du}{4} = \int_{x=0}^{x=1} \frac{1}{u}\,du = \ln u\Big]_{x=0}^{x=1} = \ln(|4x+3|)\Big]_0^1$$

$$\ln(4+3) - \ln(3) = \underline{\ln(7) - \ln(3)}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{3n+4i} = \ln(7) - \ln(3)$$

$\boxed{4}$

$$\int_0^\infty e^{-ax}\cdot\cos(x)\,dx$$

$u = \cos(x)$ $\qquad u' = -\sin(x)$

$v' = e^{-ax}$ $\qquad v = -\frac{1}{a}e^{-ax}$

Delvis integrasjon

$$\int_0^\infty e^{-ax}\cdot\cos(x)\,dx = -\frac{1}{a}e^{-ax}\cdot\cos(x) - \underbrace{\int\frac{1}{a}e^{-ax}\sin(x)\,dx}$$

$u = -\sin(x)$ $\qquad u' = -\cos(x)$ Delvis integrasjon

$v' = -\frac{1}{a}e^{-ax}$ $\qquad v = \frac{1}{a^2}e^{-ax}$

$$= -\frac{1}{a}e^{-ax}\cdot\cos(x) - \left(-\frac{1}{a^2}e^{-ax}\cdot\sin(x) + \frac{1}{a^2}\underbrace{\int e^{-ax}\cdot\cos(x)\,dx}\right)$$

Dette er vårt første integral

$$\frac{a^2+1}{a^2}\int_0^\infty e^{-ax}\cdot\cos(x)\,dx = \frac{1}{a^2}e^{-ax}\sin(x) - \frac{1}{a}e^{-ax}\cos(x)$$

$$= \left.\frac{e^{-ax}\sin(x) - ae^{-ax}\cos(x)}{a^2+1}\right]_0^\infty = \left.\frac{e^{-ax}(\sin(x) - a\cos(x))}{a^2+1}\right]_0^\infty$$

Denne går mot 0

$$\frac{e^{-a\infty}(\sin(\infty) - a\cos(\infty))}{a^2+1} - \frac{e^{-0a}(\sin(0) - a\cos(0))}{a^2+1}$$

$$= -\frac{1(0-a)}{a^2+1} = \underline{\frac{a}{a^2+1}}$$

For å finne størst $a>0$ kan man derivere og sette lik 0 for å finne ekstremalverdier

$$\frac{1(a^2+1) - a\cdot 2a}{(a^2+1)^2} = \frac{a^2+1-2a^2}{(a^2+1)^2} = \frac{-a^2+1}{(a^2+1)^2}$$

$$\frac{-a^2+1}{(a^2+1)^2} = 0 \to a^2 = -1 \to a = \pm\sqrt{1}$$

$a = \pm 1$ siden $a>0$ må $\underline{a=1}$

Setter inn $a = 1$ i $\frac{a}{a^2+1}$

$$\frac{1}{1^2+1} = \underline{\frac{1}{2}}$$

$$\int_0^\infty e^{-ax} \cdot \cos(x)\, dx = \frac{1}{2} \quad \text{for } a = 1$$