Martin Skatvedt, Øving 1

1) If $p \Rightarrow q$ is false then
p = true and q = false

a) $p \wedge q \equiv$ true $\wedge$ false $\equiv$ $\underline{\underline{\text{false}}}$

b) $\neg p \vee q \equiv$ false $\vee$ false $\equiv$ $\underline{\underline{\text{false}}}$

c) $q \Rightarrow p \equiv$ false $\Rightarrow$ true $\equiv$ $\underline{\underline{\text{true}}}$

d) $\neg q \Rightarrow \neg p \equiv$ true $\Rightarrow$ false $\equiv$ $\underline{\underline{\text{false}}}$

2) a) $q \Rightarrow p$: If a the triangle ABC is equilateral, then the triangle must also be iscosceles

b) $\neg p \Rightarrow \neg q$: If the traingle ABC is not isosceles, then the triangle wont be equilateral

c) $q \Leftrightarrow r$: The triangle ABC is equilateral if and only if the triangle is equiangular

d) $p \wedge \neg q$: The triangle ABC is isoscles and not equilateral

e) $r \Rightarrow p$: If the triangle ABC is equiangluar then the triangle is isosceles

3) a) $\neg(p \wedge \neg q) \Rightarrow \neg p$

Conditional law $\equiv (p \wedge \neg q) \vee \neg p$

p	q	$p \wedge \neg q$	$(p \wedge \neg q) \vee \neg p$
0	0	0	1
0	1	0	1
1	0	1	1
1	1	0	0

b) $p \Rightarrow (q \Rightarrow r)$

p	q	r	$q \Rightarrow r$	$p \Rightarrow (q \Rightarrow r)$
0	0	0	1	1
0	0	1	1	1
0	1	0	0	1
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	0	0
1	1	1	1	1

4) a) $q \Leftrightarrow (\neg p \vee \neg q)$

Statement is a contradiction

q	p	$\neg p \vee \neg q$	$q \Leftrightarrow (\neg p \vee \neg q)$
0	0	1	0
0	1	1	0
1	0	1	1
1	1	0	0

b) $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$p \Rightarrow r$	$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	0	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	1
1	0	1	0	1	0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

The statement is a tautology

5) $(T_0 \Rightarrow [(\neg p \vee r) \wedge \neg s]) \wedge [\neg s \Rightarrow (\neg r \wedge T_0)]$

$\equiv (F_0 \vee [(\neg p \vee r) \wedge \neg s]) \wedge [s \vee (\neg r \wedge T_0)]$ D.M.G

Domination law → $\equiv (\neg p \vee r) \wedge \neg s$

$\equiv (s \vee \neg r) \wedge (s \vee T_0)$ ← Distributive law

$\equiv (s \vee \neg r) \wedge T_0$ ← Domination law

$\equiv (s \vee \neg r)$ ← Identity law

$\equiv [(\neg p \vee r) \wedge \neg s] \wedge (s \vee \neg r)$

p	r	s	$\neg p \vee r$	$(\neg p \vee r) \wedge \neg s$	$s \vee \neg r$	$[(\neg p \vee r) \wedge \neg s] \wedge (s \vee \neg r)$
0	0	0	1	1	1	1
0	0	1	1	0	1	0
0	1	0	1	1	0	0
0	1	1	1	0	1	0
1	0	0	0	0	1	0
1	0	1	0	0	1	0
1	1	0	1	1	0	0

1	0	1	0	0	1	0
1	1	0	1	1	0	0
1	1	1	1	0	1	0

6) a) $[p \Rightarrow (q \vee r)] \equiv [(p \Rightarrow q) \wedge (p \Rightarrow r)]$

p	q	r	$q \wedge r$	$p \Rightarrow (q \vee r)$	$p \Rightarrow q$	$p \Rightarrow r$	$(p \Rightarrow q) \wedge (p \Rightarrow r)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	0	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	0	1	0
1	1	0	0	0	1	0	0
1	1	1	1	1	1	1	1

They are logical equivalent

b) $[p \Rightarrow (q \vee r)] \equiv [\neg r \Rightarrow (p \Rightarrow q)]$

p	q	r	$q \vee r$	$p \Rightarrow (q \vee r)$	$p \Rightarrow q$	$\neg r \Rightarrow (p \Rightarrow q)$
0	0	0	0	1	1	1
0	0	1	1	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	0	0	0	0
1	0	1	1	1	0	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

They are logial equivalent

7) a) $(p \wedge q) \Rightarrow r$

$\neg[(p \wedge q) \Rightarrow r]$

$\equiv \neg[\neg(p \wedge q) \vee r]$ ← conditional law

$\equiv \neg\neg(p \wedge q) \wedge r$ ← D.M.G

$\equiv p \wedge (q) \wedge r$

b) $p \Rightarrow (\neg q \vee r)$

$\neg[p \Rightarrow (\neg q \vee r)]$

$\equiv \neg[\neg p \vee (\neg q \vee r)]$ ← conditional law

$\equiv p \wedge \neg(\neg q \vee r)$ ← D.M.G

$\equiv p \wedge (q \wedge r)$ ← D.M.G

8) AL1: $(p \vee q) \vee r \equiv (q \vee r) \vee p$

p	q	r	$p \vee q$	$(p \vee q) \vee r$	$q \vee r$	$(q \vee r) \vee p$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

AL2: $(p \wedge q) \wedge r \equiv (q \wedge r) \wedge p$

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$	$q \wedge r$	$(q \wedge r) \wedge p$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	1	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1

9) a) $p \Rightarrow (p \vee q)$

Tautology

p	q	$p \vee q$	$p \Rightarrow (p \vee q)$
0	0	0	1
0	1	1	1
1	0	1	1
1	1	1	1

b) $\neg[p \Rightarrow (p \vee q)]$

Contradiction

p	q	$p \vee q$	$p \Rightarrow (p \vee q)$	$\neg[p \Rightarrow (p \vee q)]$
0	0	0	1	0
0	1	1	1	0
1	0	1	1	0
1	1	1	1	0

c) $p \Rightarrow (p \Rightarrow q)$

Satisfiable

p	q	$p \Rightarrow q$	$p \Rightarrow (p \Rightarrow q)$
0	0	1	1
0	1	1	1
1	0	0	0
1	1	1	1

10) 1) $(\neg p \wedge q) \Leftrightarrow r$

2) $r \Leftrightarrow \neg p$

3) $(\neg r \wedge p) \Rightarrow q$

4) $p \Leftrightarrow (r \wedge q)$

5) $(r \wedge \neg q)$