Innlevering 3 Martin Skatvedt

$\boxed{1}$ $g(x) = b\sqrt{1-(x/a)^2} \quad 0 \le x \le a$

a) $V_1 = \pi \int_0^a g(x)^2 dx = \pi \int_0^a b^2(1-(x/a)^2)\,dx$

$= \pi \int_0^a b^2 - \frac{b^2x^2}{a^2}\,dx = \pi\left(b^2\int_0^a 1\,dx - \frac{b^2}{a^2}\int_0^a x^2\,dx\right)$

$\pi\left(b^2[x]_0^a - \frac{b^2}{a^2}\left[\frac{1}{3}x^3\right]_0^a\right) =$

$\pi\left(b^2a - \frac{b^2}{a^2}\frac{1}{3}a^3\right) = \pi\left(ab^2 - \frac{ab^2}{3}\right) = \underline{\frac{2ab^2\pi}{3}}$

$\underline{\underline{V_1 = \frac{2\pi ab^2}{3}}}$

b) $g(x) = b\sqrt{1-(x/a)^2}$

$V_2 = 2\pi \int_0^a g(x)\,x\,dx = 2\pi\int_0^a x\,b\sqrt{1-(x/a)^2}\,dx$

$u = 1-\left(\frac{x}{a}\right)^2 \quad \frac{du}{dx} = -\frac{2x}{a^2} \quad dx = -\frac{a^2\,du}{2x}$

$2\pi b\int_0^a -x\sqrt{u}\,a^2\frac{du}{2x} = -\pi ba^2\int_0^a \sqrt{u}\,du = -\pi ba^2\int_0^a u^{\frac{1}{2}}\,du$

$-\pi ba^2\left[\frac{2}{3}u^{\frac{3}{2}}\right]_0^a = -\pi ba^2\left[\frac{2}{3}\left(1-\left(\frac{x}{a}\right)^2\right)^{\frac{3}{2}}\right]_0^a$

$= \pi ba^2\left(\underbrace{\frac{2}{3}\left(1-\left(\frac{a}{a}\right)^2\right)^{\frac{3}{2}}}_{0} - \underbrace{\frac{2}{3}\left(1-\left(\frac{0}{a}\right)^2\right)^{\frac{3}{2}}}_{\frac{2}{3}}\right)$

$= -\pi ba^2\left(0 - \frac{2}{3}\right) = \underline{\frac{2}{3}\pi ba^2}$

$\underline{\underline{V_2 = \frac{2}{3}\pi a^2 b}}$

c) $V_1 = V_2 \rightarrow \frac{2}{3}\pi a b^2 = \frac{2}{3}\pi a^2 b$

$ab^2 = a^2 b \rightarrow \underline{\underline{a = b}}$

x - akse kule

y - akse halvkule

$\boxed{2}$ a) $\displaystyle\int_1^{\infty} \frac{dx}{x^{\frac{5}{3}} + x^{\frac{2}{3}}} = \frac{3}{2}\int_0^1 \frac{dt}{1+t^{\frac{3}{2}}}$

$\displaystyle\int_1^{\infty} \frac{dx}{x^{\frac{5}{3}} + x^{\frac{2}{3}}}$ $\quad t = x^{-\frac{2}{3}} \quad \frac{dt}{dx} = -\frac{2}{3x^{\frac{5}{3}}} \quad dx = -\frac{3}{2}x^{\frac{5}{3}}\,dt$

$\displaystyle\frac{3}{2}\int_0^1 \frac{x^{\frac{5}{3}}}{\left(x^{5/3} + t^{-1}\right)}\,dt$ $\quad$ Øvre grense: $\lim\limits_{x\to\infty} f(x) = 0$

Nedre grense $f(1) = 1$

$\displaystyle = \frac{3}{2}\int_0^1 \frac{x^{\frac{5}{3}}\,dt}{x^{\frac{5}{3}}\left(1 + \frac{t^{-1}}{x^{\frac{5}{3}}}\right)} = \frac{3}{2}\int_0^1 \frac{dt}{\left(1 + \frac{t^{-1}}{x^{\frac{5}{3}}}\right)}$

siden $t = x^{-\frac{2}{3}}$ vil $x^{\frac{5}{3}} = t^{-\frac{5}{2}}$.

$\displaystyle = \frac{3}{2}\int_0^1 \frac{dt}{\left(1 + \frac{t^{-1}}{t^{-\frac{5}{2}}}\right)} = \underline{\underline{\frac{3}{2}\int_0^1 \frac{dt}{1+t^{\frac{3}{2}}}}}$

b) S_{2n} hvor $2n = 4$ og $h = \frac{1-0}{4} = \frac{1}{4}$

$x_i = j \cdot \frac{1}{4} \quad j = 0, 1, 2, 3, 4$

$y_i = \frac{1}{1 + x_i^{3/2}} = \frac{1}{1 + \left(\frac{1}{4}i\right)^{\frac{3}{2}}}$

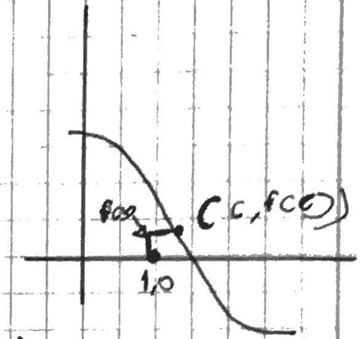
$S_4 = \frac{3}{2}\cdot\frac{h}{3}\left(y_0 + 4y_1 + y_2 + y_2 + 4y_3 + y_4\right)$

$= \frac{1}{8}\left(1 + \frac{4}{1+\left(\frac{1}{4}\right)^{\frac{3}{2}}} + \frac{1}{1+\left(\frac{1}{2}\right)^{\frac{3}{2}}} + \frac{1}{1+\left(\frac{1}{2}\right)^{\frac{3}{2}}} + \frac{4}{1+\left(\frac{3}{4}\right)^{\frac{3}{2}}} + \frac{1}{2}\right)$

$\approx \frac{1}{8}\left(1 + 3{,}56 + 0{,}74 + 0{,}74 + 2{,}42 + 0{,}5\right)$

$\underline{\underline{S_4 \approx 1{,}11976 \approx 1.12}}$

$\boxed{3}$ $f(x) = \cos(x) \qquad 0 \leq x \leq \pi$



a) $c \in (1, \pi/2)$

$g(x) = (x-1)^2 + \cos^2(x)$

$g(x) = x^2 - 2x + 1 + \cos^2(x)$

$g(x) = x^2 - 2x + 1 + \frac{1}{2} + \frac{1}{2}\cos(2x)$

$g'(x) = 2x - 2 - \sin(2x)$

$\underline{g'(1) = -\sin(2)}$

$\underline{g'(\frac{\pi}{2}) = \pi - 2}$

Dvs at $g'(x)$ går fra $-y$ til y
og at det finns en verdi for $g'(x) = 0$
<u>os at det finnes et tall c</u>

b) $h(x) = g'(x) = 2x - 2 - \sin(2x)$

$h'(x) = 2 - 2\cos(2x)$

$$x_{n+1} = x_n - \frac{h(x_n)}{h'(x_n)}$$

$x_0 = \frac{\pi}{2} \approx 1,57079$

$$x_1 = \frac{\pi}{2} - \frac{h(\frac{\pi}{2})}{h'(\frac{\pi}{2})} \approx 1,28539$$

$$x_2 = x_1 - \frac{h(x_1)}{h'(x_1)} \approx 1,27711$$

$$x_3 = x_2 - \frac{h(x_2)}{h'(x_2)} \approx 1,27709$$

$$x_4 = x_3 - \frac{h(x_3)}{h'(x_3)} \approx \underline{1,27709}$$

$f(c) = 0,28949$

$\underline{C = (1,27709, 0,28949)}$

$\boxed{4}$ $\lim_{x \to 1} \dfrac{\ln x - (x-1) + \frac{(x-1)^2}{2}}{(x-1)^3}$

Fra boka: $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{(n-1)} \frac{x^n}{n} + O(x^{n+1})$

$\ln x = x-1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 + O((x-1)^4)$ $x \to 4$

$\lim_{x \to 1} \dfrac{\cancel{(x-1)} - \cancel{\frac{1}{2}(x-1)^2} + \cancel{\frac{1}{3}}(x-1)^3 + O((\cancel{x-1})^{\cancel{4}}) - \cancel{(x-1)} + \cancel{\frac{(x-1)^2}{2}}}{\cancel{(x-1)^3}}$ $O((x-1)^3)$

$\lim_{x \to 1} \frac{1}{3} O((x-1)^3)$ når $x \to 4$, $O((x-1)^3) = 0$

$\underline{\underline{= \frac{4}{3}}}$