Martin Skatvedt, Øving 4

1) $w = 1 \quad y = 0$
$x = 1 \quad z = 0$

a) $\overline{xy} + \bar{x}\bar{y} = \overline{(1\cdot 1)} + \bar{1}\cdot\bar{0} = \underline{\underline{0}}$

b) $w + \bar{x}y = 1 + \bar{1}\cdot 0 = \underline{\underline{1}}$

c) $wx + \bar{y} + yz = 1\cdot 1 + \bar{0} + 0\cdot 0 = \underline{\underline{1}}$

d) $(wx + y\bar{z}) + w\bar{y} + \overline{(w+y)(\bar{x}+y)}$

$= (wx + y\bar{z}) + w\bar{y} + (\bar{w}\cdot\bar{y}) + (x\cdot\bar{y})$ ← D.M.G

$= (wx + y\bar{z}) + \bar{y}\cdot(w + \bar{w}) + (x\cdot\bar{y})$

$= (wx + y\bar{z}) + \bar{y}\cdot 1 + (x\cdot\bar{y})$

$= (wx + y\bar{z}) + \bar{y}$

$= (1\cdot 1 + 0\cdot 1) + 1 = \underline{\underline{1}}$

2) a) $xy + (x+y)\bar{z} + y = y + xy + (x+y)\bar{z}$

$= y + (x+y)\bar{z} = y + (\bar{z}\cdot x) + (\bar{z}\cdot y)$

$= y + (y\cdot\bar{z}) + (\bar{z}\cdot x) = \underline{\underline{y + (\bar{z}x)}}$

b) $x + y + \overline{(\bar{x} + y + z)} = x + y + (x\bar{y}\bar{z})$

$= y + x + (x\cdot(\bar{y}\bar{z})) = \underline{\underline{y + x}}$

c) $yz + wx + z + [wz(xy + wz)]$

$= wx + z + [zw(xy + wz)] \qquad A + AB = A$

$= \underline{\underline{wx + z}}$

3) for any $n \geq 0$

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Base case $n = 1$

$$\frac{1(1+1)(2+1)}{6} = \frac{6}{6} = \underline{1} \qquad \sum_{i=0}^{1} i^2 = \underline{1}$$

$$\frac{k(1+k)(2k+1)}{6} + (k+1)^2 = \frac{k(1+k)(2k+1) + 6(k+1)^2}{6} = \frac{2k^3 + 3k^2 + k + 6k^2 + 12k + 6}{6}$$

$$= \frac{2k^3 + 9k^2 + 13k + 6}{6}$$

$n = k+1$

$$\frac{(k+1)(k+2)(2k+3)}{6} = \frac{(k^2 + 3k + 2)(2k+3)}{6} = \underline{\underline{\frac{2k^3 + 9k^2 + 13k + 6}{6}}}$$

$$\frac{k(k+1)(2k+1)}{6}+(k+1)^2=\frac{n(n+1)(2n+1)}{6}$$ q.e.d.

4) for any $n \geq 0$ $\quad S(n)=\sum_{i=0}^{n} 2^{-i}$

a) $S(0)=\underline{1}$ $\quad S(1)=1+\frac{1}{2}=\underline{\frac{3}{2}}$

$S(2)=\frac{3}{2}+\frac{1}{2^2}=\frac{6}{4}+\frac{1}{4}=\underline{\frac{7}{4}}$

$S(3)=\frac{7}{4}+\frac{1}{2^3}=\frac{7}{4}+\frac{1}{8}=\frac{14}{8}+\frac{1}{8}=\underline{\underline{\frac{15}{8}}}$

b) $S(n)=2-\frac{1}{2^n}$

c) Base case:

$S(1)=2-\frac{1}{2}=\underline{\frac{3}{2}}$

$$\frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^3}+\dots+\frac{1}{2^k}=2-\frac{1}{2^k}$$

Goal

$n=k+1$ $\quad \frac{1}{2^0}+\frac{1}{2^1}+\dots+\frac{1}{2^{k+1}}=2-\frac{1}{2^{k+1}}$

$$2-\frac{1}{2^k}+\frac{1}{2^{k+1}}=2-\frac{1}{2^{k+1}}$$

d) $2-\varepsilon \leq S(n) \rightarrow \varepsilon \geq \frac{1}{2^n} \rightarrow \varepsilon \geq 2^{-n}$

$\rightarrow \varepsilon \geq \frac{1}{2^n} \rightarrow 2^n \geq \frac{1}{\varepsilon} \rightarrow \underline{\underline{n \geq \log_2\left(\frac{1}{\varepsilon}\right)}}$

5) $2^0 \cdot 1 + 2^1 \cdot 2 + 2^2 \cdot 3 + \cdots + 2^{n-1} \cdot n = 2^n(n-1) + 1)$

Base case

$S(1) : \underline{1}$

$2^1(1 - 1) + 1 = 0 + 1 = \underline{1}$

$2^0 \cdot 1 + 2^1 \cdot 2 + \cdots + 2^{k-1} \cdot k = 2^k(k-1) + 1$

$n = k+1 \quad 2^0 \cdot 1 + 2^1 \cdot 2 + \cdots + 2^k \cdot (k+1) = 2^{(k+1)}(k) + 1$

$S(n) = 2^{(k+1)}(k) + 1$

$2^k(k-1) + 1 + 2^k(k+1) = 2^k((k-1) + (k+1)) + 1$