Martin Skatvedt - Øving 10 - MA0001

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$\boxed{7.1.6}$

a) $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{1}{1-2}x^{1-2} + C = \underline{\underline{-\frac{1}{x} + C}}$

b) $\int (x+1)^3\,dx = \int u^3\,du = \frac{1}{4}u^4 + C$

$u = x+1$

$\frac{du}{dx} = 1$

$du = dx$

$= \underline{\underline{\frac{1}{4}(x+1)^4 + C}}$

c) $\int ax+b\,dx = \underline{\underline{\frac{1}{2}ax^2 + bx + C}}$

$\boxed{7.1.7}$

$a(t) = -9{,}8\,m/s^2$

a) $v(t) = \int a(t)\,dt = \underline{-9{,}8t + C}$

siden $v(0) = v_0$

må $\underline{v(t) = -9{,}8t + v_0}$

$s(t) = \int v(t)\,dt = \int -9{,}8t + v_0\,dt = \underline{-\frac{9{,}8}{2}t^2 + v_0 t + C}$

siden $s(0) = 0$ må

$\underline{\underline{s(t) = \frac{-9{,}8}{2}t^2 + v_0 t}}$

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$\boxed{7.17}$

b) $v(t)=0 \rightarrow -9{,}8t+v_0=0$

$= t = \dfrac{v_0}{9{,}8}$ s tid til høyeste punkt

$$S\left(\frac{v_0}{9{,}8}\right) = -\frac{9{,}8}{2}\left(\frac{v_0}{9{,}8}\right)^2 + v_0\left(\frac{v_0}{9{,}8}\right)$$

$$= -\frac{9{,}8}{2}\cdot\frac{v_0^2}{(9{,}8)^2} + \frac{v_0^2}{9{,}8} = \frac{v_0^2}{9{,}8} - \frac{v_0^2}{19{,}6}$$

$= \dfrac{v_0}{19{,}6}$ m høgde ved høgeste punkt

$S(t)=0 \rightarrow -\dfrac{9{,}8}{2}t^2 + v_0 t = 0$

$= t\left(-\dfrac{9{,}8}{2}t + v_0\right) = 0$

$t=0$ og $t = \dfrac{2v_0}{9{,}8}$ tid når høgde=0

$t = \dfrac{2v_0}{9{,}8} - \dfrac{v_0}{9{,}8} = \dfrac{v_0}{9{,}8}$ Tid til h=0

c) $S\left(\dfrac{100}{9{,}8}\right) = \dfrac{100}{19{,}6}$ m $= 5{,}10$ m

$\boxed{7.2.4}$

a) $\int_1^4 2\,dx = \left[2x\right]_1^4 = 8-2 = \underline{\underline{6}}$

c) $\int_1^k x^4\,dx = \left[\frac{1}{5}x^5\right]_1^k = \frac{1}{5}k^5 - \frac{1}{5} = \underline{\underline{\frac{k^5-1}{5}}}$

d) $\int_0^6 2-3t\,dt = \left[2t - \frac{3}{2}t^2\right]_0^6 = \left(2\cdot 6 - \frac{3}{2}6^2\right) - \left(2\cdot 0 - \frac{3}{2}0^2\right)$

$\underline{\underline{= 2\cdot 6 - \frac{3}{2}6^2}}$

e) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos v\,dv = \left[\sin v\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \sin\frac{\pi}{4} - \sin\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$

$\underline{\underline{= 0}}$

$\boxed{7.2.8}$ $y = 4-x^2$ og $y = (x-2)^2 - 6$

$y = y \rightarrow 4 - x^2 = x^2 - 4x + 4 - 6 \rightarrow 0 = 2x^2 - 4x - 6$

$x = \frac{4 \pm \sqrt{(-4)^2 - 4\cdot 2\cdot -6}}{4} = \frac{4 \pm 8}{4}$ $\underline{x = 3}$ og $\underline{x = -1}$

skjæringspunkter $: (3, y(3))$ og $(-1, y(-1))$

$\underline{\underline{= (3, -5) \text{ og } (-1, 3)}}$

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$\boxed{7.2.8}$

$$\text{Areal} = \int_{-1}^{3} 4 - x^2 \, dx - \int_{-1}^{3} x^2 - 4x - 2 \, dx$$

$$\int_{-1}^{3} 4 - x^2 \, dx = \left[4x - \frac{1}{3}x^3 \right]_{-1}^{3} = \left(4 \cdot 3 - \frac{1}{3}3^3\right) - \left(4(-1) - \frac{1}{3}(-1)^3\right)$$

$$= (12 - 9) - \left(-4 + \frac{1}{3}\right) = 3 + 4 - \frac{1}{3} = 7 - \frac{1}{3} = \underline{\frac{20}{3}}$$

$$\int_{-1}^{3} x^2 - 4x - 2 \, dx = \left[\frac{1}{3}x^3 - 2x^2 - 2x \right]_{-1}^{3}$$

$$= \left(\frac{1}{3}3^3 - 2 \cdot 3^2 - 2 \cdot 3\right) - \left(\frac{1}{3}(-1)^3 - 2 \cdot (-1)^2 - 2 \cdot (-1)\right)$$

$$= (9 - 18 - 6) - \left(-\frac{1}{3} - 2 + 2\right) = -15 + \frac{1}{3} = -\frac{46}{3}$$

$$\text{Areal} = \frac{20}{3} - \left(-\frac{44}{3}\right) = \frac{64}{3} = \underline{\underline{21 + \frac{1}{3}}}$$

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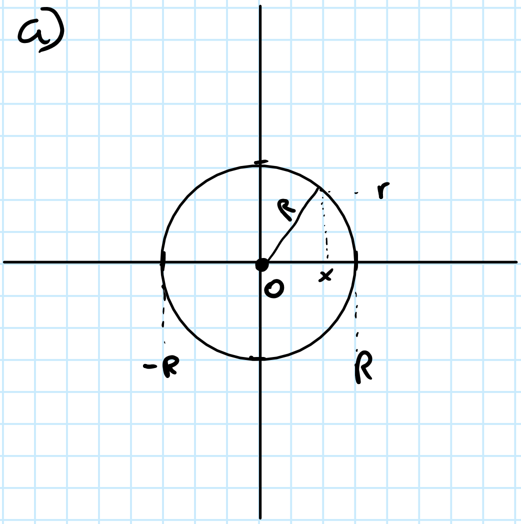
$\boxed{5}$ $V(r) = \frac{4}{3}\pi r^3$

a)



$R^2 = r^2 + x^2 \rightarrow r = \sqrt{R^2 - x^2}$

$A(r) = \pi r^2 = \pi(R^2 - x^2)$

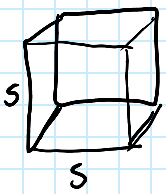
$\int_{-R}^{R} \pi(R^2 - x^2)\,dx = \pi\left[R^2 x - \frac{1}{3}x^3\right]_{-R}^{R} = \underline{\underline{\frac{4}{3}\pi R^3}}$

b) $V'(r) = A(r)$

$V'(r) = \frac{4}{3}\cdot\pi\cdot r^3 = \frac{4}{3}\cdot\pi\cdot 3r^2 = \underline{\underline{4\pi r^2}}$

c) $V(r) = 8r^3$ $\qquad V'(r) = 24r^2$

$r = \frac{s}{2}$ $\qquad V'(s) = 24\left(\frac{s}{2}\right)^2 = \frac{24s^2}{4} = \underline{6s^2}$



dvs 1 side $= s^2$ siden vi har 6 sider

blir det $6s^2 = V'(s)$

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$\boxed{7.2.4}$ $v(t) = V_0 e^{-kt}$ $v_0 = 3$

$k = 2$

$v(t) = 3e^{-2t}$

$\int_0^5 v(t)\,dt = \int_0^5 3e^{-2t}\,dt = -\frac{3}{2}\int_0^5 e^{u}\,du$

$u = -2t$

$\frac{du}{dt} = -2$

$\frac{du}{-2} = dt$

$= -\frac{3}{2}\left[e^{-2t}\right]_0^5 = -\frac{3}{2}\left(e^{-10} - e^{0}\right)$

$= -\frac{3}{2}\left(e^{-10} - 1\right) = 1{,}4999 \approx \underline{\underline{1{,}5 \text{ liter}}}$