TMA4110 - Øving 4 - Martin Skatvedt

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1) $\begin{bmatrix}1\\4\\2\end{bmatrix}, \begin{bmatrix}-2\\0\\1\end{bmatrix}, \begin{bmatrix}4\\-5\\8\end{bmatrix}$

$$\begin{bmatrix}1\\4\\2\end{bmatrix} \cdot \begin{bmatrix}-2\\0\\1\end{bmatrix} = 1\cdot(-2) + 4\cdot 0 + 2\cdot 1 = \underline{0}$$

$$\begin{bmatrix}1\\4\\2\end{bmatrix} \cdot \begin{bmatrix}4\\-5\\8\end{bmatrix} = 1\cdot 4 + 4\cdot(-5) + 2\cdot 8 = \underline{0}$$

$$\begin{bmatrix}-2\\0\\1\end{bmatrix} \cdot \begin{bmatrix}4\\-5\\8\end{bmatrix} = -2\cdot 4 + 0\cdot 5 + 1\cdot 8 = \underline{0}$$

skalarprodukt = 0 mellom hver vektor, så de står ortogonalt på hverandre

$$\left[\begin{array}{ccc|c}1 & -2 & 4 & 0\\4 & 0 & -5 & 0\\2 & 1 & 8 & 0\end{array}\right]$$

$$\left[\begin{array}{ccc|c}1 & -2 & 4 & 0\\0 & 8 & -21 & 0\\0 & 5 & 0 & 0\end{array}\right] \begin{array}{l} \\ +\mathrm{I}\cdot -4\\ +\mathrm{I}\cdot -2\end{array}$$

$$\left[\begin{array}{ccc|c}1 & -2 & 4 & 0\\0 & 5 & 0 & 0\\0 & 8 & -21 & 0\end{array}\right] \updownarrow$$

$$\left[\begin{array}{ccc|c}1 & -2 & 4 & 0\\0 & 1 & 0 & 0\\0 & 8 & -21 & 0\end{array}\right] \mathrm{II}\cdot\frac{1}{5}$$

$$\left[\begin{array}{ccc|c}1 & 0 & 4 & 0\\0 & 1 & 0 & 0\\0 & 0 & -21 & 0\end{array}\right] \begin{array}{l}+\mathrm{II}\cdot 2\\ \\ +\mathrm{II}\cdot -8\end{array}$$

$$\left[\begin{array}{ccc|c}1 & 0 & 4 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\end{array}\right] \mathrm{III}\cdot -\frac{1}{21}$$

$$\left[\begin{array}{ccc|c}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\end{array}\right] +\mathrm{III}\cdot -4$$

$x = y = z = 0$, de er lineart uavhengige

2) $v = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$ $sp\{e_1, e_2\}$

$P_{e_i}(v) = v_i e_i$

$P_{e_1}(v) = 3e_1$

$P_{e_2}(v) = 4e_2$

Dermed får vi $3e_1$ og $4e_2$

3) $u = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

a) $P_v(u) = \frac{v \cdot u}{v \cdot v} v = \frac{2 \cdot 1 + 0 \cdot (-1) + 1 \cdot 0}{1 \cdot 1 + (-1) \cdot (-1) + 0 \cdot 0} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$= \frac{2}{2} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \underline{\underline{\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}}}$

b) $u - P_v(u) = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\| u - P_v(u) \| = \sqrt{1^2 + 1^2 + 1^2} = \underline{\underline{\sqrt{3}}}$

c) $\dim(sp\{v\})^{\perp} = \dim R^3 - \dim(sp\{v\})$

$= 3 - 1 = \underline{\underline{2}}$

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4)

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

null A^T ortogonal på col A

col A^T = row A

dvs

$(\text{null } A)^\perp = \text{row } A$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 \\ 0 & -4 & 0 & 1 & 0 \end{array}\right] + I \cdot -2$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{4} & 0 \end{array}\right] \; III \cdot -\frac{1}{4}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & \frac{3}{4} & 0 \\ 0 & 1 & 0 & -\frac{1}{4} & 0 \end{array}\right] \begin{array}{l} + III \cdot -2 \\ + III \cdot -3 \end{array}$$

5) $v_1 \begin{bmatrix}0\\-1\\0\\1\end{bmatrix}$ og $v_2 = \begin{bmatrix}1\\0\\1\\2\end{bmatrix}$

a) $u_1 = v_1$

$$u_2 = v_2 - P_{u_1}(v_2) = v_2 - \frac{\langle u_1, v_2 \rangle}{\langle u_1, u_1 \rangle} u_1$$

$$= \begin{bmatrix}1\\0\\1\\2\end{bmatrix} - \frac{\cancel{0\cdot 1} + \cancel{(-1)\cdot 0} + \cancel{0\cdot 1} + 1\cdot 2}{\cancel{0\cdot 0} + (-1)\cdot(-1) + \cancel{0\cdot 0} + 1\cdot 1} \begin{bmatrix}0\\-1\\0\\1\end{bmatrix} = \begin{bmatrix}1\\0\\1\\2\end{bmatrix} - \frac{2}{2}\begin{bmatrix}0\\-1\\0\\1\end{bmatrix}$$

$$= \begin{bmatrix}1\\0\\1\\2\end{bmatrix} - \begin{bmatrix}0\\-1\\0\\1\end{bmatrix} = \underline{\begin{bmatrix}1\\1\\1\\1\end{bmatrix}}$$

Vi får da v_1 og $[1,1,1,1]^T$ som ortogonale basiser

b) $P_U(e_1) = P_{u_1}(e_1) + P_{u_2}(e_2)$

$$P_{u_1}(e_1) = \frac{u \cdot e_1}{u_1 \cdot u_1} u_1 = \frac{\cancel{0\cdot 1} + \cancel{(-1)\cdot 0} + \cancel{0\cdot 0} + \cancel{1\cdot 0}}{\cancel{0\cdot 0} + -1\cdot -1 + \cancel{0\cdot 0} + 1\cdot 1} \begin{bmatrix}0\\-1\\0\\1\end{bmatrix}$$

$$= \frac{0}{2}\begin{bmatrix}0\\-1\\0\\1\end{bmatrix} = \underline{\begin{bmatrix}0\\0\\0\\0\end{bmatrix}}$$

$$P_{u_2}(e_1) = \frac{1\cdot 1 + 0\cdot 1 + 0\cdot 1 + 0\cdot 1}{1\cdot 1 + 1\cdot 1 + 1\cdot 1 + 1\cdot 1} \begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \underline{\frac{1}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}}$$

$$P_U(e_1) = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} + \frac{1}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \underline{\begin{bmatrix}\frac14\\\frac14\\\frac14\\\frac14\end{bmatrix}}$$

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$$P_U(e_2) = \frac{\begin{bmatrix}0\\-1\\0\\1\end{bmatrix}\cdot\begin{bmatrix}0\\1\\0\\0\end{bmatrix}}{\begin{bmatrix}0\\-1\\0\\1\end{bmatrix}\cdot\begin{bmatrix}0\\-1\\0\\1\end{bmatrix}}\begin{bmatrix}0\\-1\\0\\1\end{bmatrix} + \frac{\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\cdot\begin{bmatrix}0\\1\\0\\0\end{bmatrix}}{\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\cdot\begin{bmatrix}1\\1\\1\\1\end{bmatrix}}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$= \frac{-1}{2}\begin{bmatrix}0\\-1\\0\\1\end{bmatrix} + \frac{1}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\\frac{1}{2}\\0\\-\frac{1}{2}\end{bmatrix} + \begin{bmatrix}\frac{1}{4}\\\frac{1}{4}\\\frac{1}{4}\\\frac{1}{4}\end{bmatrix} = \underline{\begin{bmatrix}\frac{1}{4}\\\frac{3}{4}\\\frac{1}{4}\\-\frac{1}{4}\end{bmatrix}}$$

$$P_U(e_3) = \frac{\begin{bmatrix}0\\-1\\0\\1\end{bmatrix}\cdot\begin{bmatrix}0\\0\\1\\0\end{bmatrix}}{\begin{bmatrix}0\\-1\\0\\1\end{bmatrix}\cdot\begin{bmatrix}0\\-1\\0\\1\end{bmatrix}}\begin{bmatrix}0\\-1\\0\\1\end{bmatrix} + \frac{\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\cdot\begin{bmatrix}0\\0\\1\\0\end{bmatrix}}{\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\cdot\begin{bmatrix}1\\1\\1\\1\end{bmatrix}}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$= \frac{0}{2}\begin{bmatrix}0\\-1\\0\\1\end{bmatrix} + \frac{1}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{4}\\\frac{1}{4}\\\frac{1}{4}\\\frac{1}{4}\end{bmatrix}$$

$$P_U(e_4) = \frac{\begin{bmatrix}0\\-1\\0\\1\end{bmatrix}\cdot\begin{bmatrix}0\\0\\0\\1\end{bmatrix}}{\begin{bmatrix}0\\-1\\0\\1\end{bmatrix}\cdot\begin{bmatrix}0\\-1\\0\\1\end{bmatrix}}\begin{bmatrix}0\\-1\\0\\1\end{bmatrix} + \frac{\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\cdot\begin{bmatrix}0\\0\\0\\1\end{bmatrix}}{\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\cdot\begin{bmatrix}1\\1\\1\\1\end{bmatrix}}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}0\\-1\\0\\1\end{bmatrix} + \frac{1}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\-\frac{1}{2}\\0\\\frac{1}{2}\end{bmatrix} + \begin{bmatrix}\frac{1}{4}\\\frac{1}{4}\\\frac{1}{4}\\\frac{1}{4}\end{bmatrix} = \underline{\begin{bmatrix}\frac{1}{4}\\-\frac{1}{4}\\\frac{1}{4}\\\frac{3}{4}\end{bmatrix}}$$

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$$[P_U] = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{1}{4} & \frac{3}{4} \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & -1 \\ 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 3 \end{bmatrix}$$

c) $V = \mathbb{R}^4$

$$U = \operatorname{sp}\left\{ \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix} \right\}$$

$\dim U^\perp = \dim \mathbb{R}^4 - \dim U$

$\dim U^\perp = 4 - 2 = \underline{\underline{2}}$

d)

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} = 0$$

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e)

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & 3 & 1 & -1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 1 & 3 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & 0 & -2 & 0 \\ 0 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \text{II} \cdot \frac{1}{2}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} +\text{II} \cdot -1 \\ \\ +\text{II} \cdot 2 \end{array}$$

$$\text{null } A = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix} s$$

$\ker([P_c])$: Alle vektorer på formen $\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} t + \begin{bmatrix} -2 \\ 1 \\ 0 \\ 1 \end{bmatrix} s$

$$\text{col } A = \text{sp}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 3 \end{bmatrix} \right\}$$

6) $\langle f, g \rangle = \int_0^1 f(x)\, g(x)\, dx$

a) $\langle x, \frac{1}{x^2+1} \rangle = \int_0^1 x \cdot \frac{1}{x^2+1}\, dx$

$= \int_0^1 \frac{x}{x^2+1}\, dx$ $\qquad u = x^2+1$

$\qquad \frac{du}{dx} = 2x$

$= \int_0^1 \frac{1}{2u}\, du = \frac{1}{2}\int_0^1 \frac{1}{u}\, du$ $\qquad dx = \frac{du}{2x}$

$= \frac{1}{2}\left[\ln u\right]_0^1 = \frac{1}{2}\left[\ln(x^2+1)\right]_0^1$

$= \frac{1}{2}(\ln 2 - \ln 1) = \underline{\frac{1}{2}\ln 2}$

Ikke ortogonale

b) $u_1 = 1$

$u_2 = v_2 - \frac{\langle u_1, v_2 \rangle}{\langle u_1, u_1 \rangle} u_1 = x^2 - \frac{\langle 1, x \rangle}{\langle 1, 1 \rangle} 1$

$\langle 1, x \rangle = \int_0^1 1 \cdot x\, dx = \left[\frac{1}{2}x^2\right]_0^1 = \frac{1}{2}\cdot 1^2 - \frac{1}{2}\cdot 0^2 = \underline{\frac{1}{2}}$

$\langle 1, 1 \rangle = \int_0^1 1 \cdot 1\, dx = \left[x\right]_0^1 = 1 - 0 = \underline{1}$

$u_2 = x^2 - \frac{\frac{1}{2}}{\frac{1}{1}} \cdot 1 = \underline{x^2 - \frac{1}{2}}$

$$u_3 = v_3 - \frac{\langle u_1, v_3 \rangle}{\langle u_1, u_1 \rangle} u_1 - \frac{\langle u_2, v_3 \rangle}{\langle u_2, u_2 \rangle} u_2$$

$$= x^4 - \frac{\langle 1, x^4 \rangle}{\langle 1, 1 \rangle} \cdot 1 - \frac{\langle x^2 - \frac{1}{2}, x^4 \rangle}{\langle x^2 - \frac{1}{2}, x^2 - \frac{1}{2} \rangle} \cdot x^2 - \frac{1}{2}$$

$$\langle 1, x^4 \rangle = \int_0^1 x^4 \, dx = \left[\frac{1}{5} x^5\right]_0^1 = \frac{1}{5} \cdot 1 - \frac{1}{5} \cdot 0 = \underline{\frac{1}{5}}$$

$$\langle 1, 1 \rangle = \underline{1}$$

$$\langle x^2 - \frac{1}{2}, x^4 \rangle = \int_0^1 \left(x^2 - \frac{1}{2}\right)\left(x^4\right) dx = \int_0^1 x^6 - \frac{1}{2} x^4 \, dx$$

$$= \int_0^1 x^6 \, dx - \frac{1}{2} \int_0^1 x^4 \, dx = \left[\frac{1}{7} x^7\right]_0^1 - \frac{1}{2} \left[\frac{1}{5} x^5\right]_0^1$$

$$= \left(\frac{1}{7} \cdot 1 - \frac{1}{7} \cdot 0\right) - \frac{1}{2} \left(\frac{1}{5} \cdot 1 - \frac{1}{5} \cdot 0\right) = \frac{1}{7} - \frac{1}{10} = \frac{10}{70} - \frac{7}{10}$$

$$= \underline{\frac{3}{70}}$$

$$\langle x^2 - \frac{1}{2}, x^2 - \frac{1}{2} \rangle = \int_0^1 \left(x^2 - \frac{1}{2}\right)\left(x^2 - \frac{1}{2}\right) dx = \int_0^1 x^4 - x^2 + \frac{1}{4} \, dx$$

$$= \int_0^1 x^4 \, dx - \int_0^1 x^2 \, dx + \frac{1}{4} \int_0^1 1 \, dx = \left[\frac{1}{5} x^5\right]_0^1 - \left[\frac{1}{3} x^3\right]_0^1 + \frac{1}{4} \left[x\right]_0^1$$

$$= \frac{1}{5} - \frac{1}{3} + \frac{1}{4} = \frac{12}{60} - \frac{20}{60} + \frac{15}{60} = \underline{\frac{7}{60}}$$

$$u_3 = x^4 - \frac{\frac{1}{5}}{1} - \frac{\frac{3}{70}}{\frac{7}{60}} \cdot \left(x^2 - \frac{1}{2}\right) = x^4 - \frac{1}{5} - \frac{18}{49} \cdot \left(x^2 - \frac{1}{2}\right)$$

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$u_3 = x^2 - \frac{1}{5} - \frac{18}{49}x^2 + \frac{18}{98} = \underline{\frac{31}{49}x^2 - \frac{8}{490}}$

vi får $\underline{\underline{1,\ 2x^2-1 \text{ og } 310x^2-8}}$

c) $\left\| \frac{1}{\sqrt{x^2+1}} \right\|$ $\qquad \|v\| = \sqrt{\langle v, v \rangle}$

$\left\langle \frac{1}{\sqrt{x^2+1}}, \frac{1}{\sqrt{x^2+1}} \right\rangle = \int_0^1 \frac{1}{\sqrt{x^2+1}} \cdot \frac{1}{\sqrt{x^2+1}}\, dx$

$= \int_0^1 \frac{1}{x^2+1}\, dx = \Big[\arctan x\Big]_0^1 = \arctan 1 - \arctan 0$

$= \frac{\pi}{4} - 0 = \underline{\frac{\pi}{4}}$

$\left\| \frac{1}{\sqrt{x^2+1}} \right\| = \sqrt{\frac{\pi}{4}} = \underline{\underline{\frac{\sqrt{\pi}}{2}}}$

7) $U = \text{sp}\{\sin x\}$ av $C[0, 2\pi])$

$$\langle f, g \rangle = \frac{1}{2\pi}\int_0^{2\pi} f(x)g(x)\,dx$$

a) $$P_U(\cos^2 x) = \frac{\langle \sin x, \cos^2 x\rangle}{\langle \sin x, \sin x\rangle}\sin x$$

$$\langle \sin x, \cos^2 x\rangle = \frac{1}{2\pi}\int_0^{2\pi} \sin x \cos^2 x\,dx$$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$dx = -\frac{du}{\sin x}$

$$= \frac{1}{2\pi}\int_0^{2\pi} \sin x \cdot u^2 \cdot -\frac{du}{\sin x} = -\frac{1}{2\pi}\int_0^{2\pi} u^2\,du$$

$$= -\frac{1}{2\pi}\left[\frac{1}{3}u^3\right]_0^{2\pi} = \frac{1}{2\pi}\left[\frac{1}{3}\cos^3 x\right]_0^{2\pi} = \frac{1}{3}\cos^3(2\pi) - \frac{1}{3}\cos^3 0$$

$$= -\frac{1}{2\pi}\left(\frac{1}{3} - \frac{1}{3}\right) = \underline{0}$$

siden $\langle \sin x, \cos^2 x\rangle = 0$ vil

$$\underline{\underline{P_U(\cos^2 x) = 0}}$$

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b) $\| \sin x - \cos^2 x \| = \frac{1}{2\pi}\int_0^{2\pi} \sin x - \cos^2 x \, dx$

$$= \frac{1}{2\pi}\int_0^{2\pi} \sin x \, dx - \frac{1}{2\pi}\int_0^{2\pi} \cos^2 x \, dx$$

$$\frac{1}{2\pi}\int_0^{2\pi} \sin x \, dx = \frac{1}{2\pi}\Big[-\cos x\Big]_0^{2\pi} = \frac{1}{2\pi}\left(-\cos 2\pi + \cos 0\right) = \underline{0}$$

$$-\frac{1}{2\pi}\int_0^{2\pi} \cos^2 x \, dx = \frac{\cos x \sin x}{2} + \frac{1}{2}\int 1 \, dx$$

$$= -\frac{1}{2\pi}\left[\frac{\cos x \sin x}{2} + \frac{1}{2}x\right]_0^{2\pi} = \left[-\frac{\cos x \sin x + x}{4\pi}\right]_0^{2\pi}$$

$$= -\frac{\cos 2\pi \cdot \sin 2\pi + 2\pi}{4\pi} + \frac{\cos 0 \sin 0 + 0}{4\pi}$$

$$= -\frac{1 \cdot 0 + 2\pi}{4\pi} + \frac{1 \cdot 0 + 0}{4\pi} = \underline{-\frac{1}{2}}$$

$$\underline{\underline{\| \sin x - \cos^2 x \| = \frac{1}{\sqrt{2}}}}$$

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8)

a)

9)

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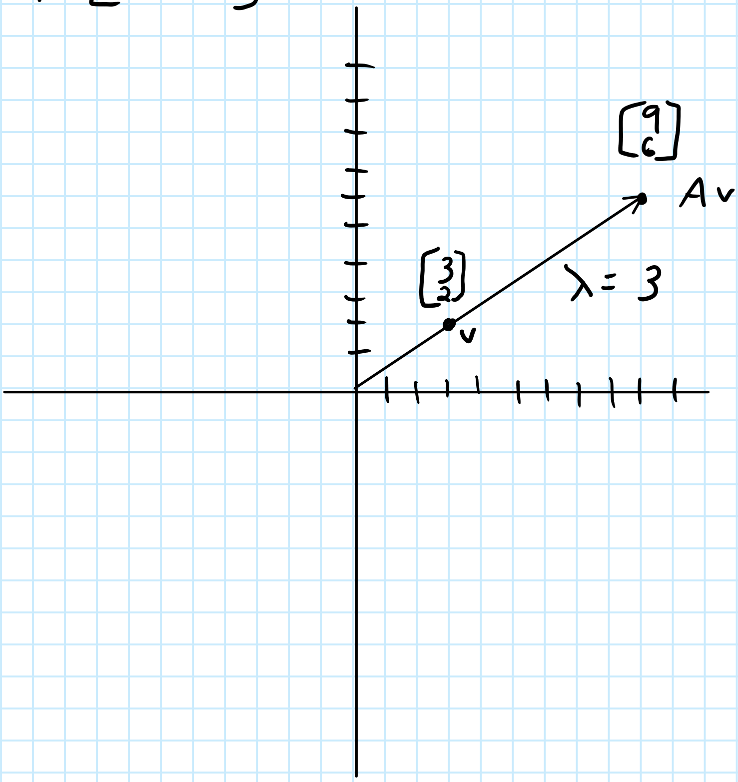
a) En vektor som er lik seg selv eller ganget med et tall etter en lineærtransformasjon

b) Tallet du må gange med en vektor v, får å få egenvektoren

$A = \begin{bmatrix} 1 & 3 \\ 4 & -3 \end{bmatrix}$ $v = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ $Av = \begin{bmatrix} 9 \\ 6 \end{bmatrix}$



10) $A = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$ $x = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$$Ax = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \cdot -1 + 1 \cdot 2 \\ -2 \cdot -1 + -2 \cdot 2 \end{bmatrix} = \underline{\begin{bmatrix} 1 \\ -2 \end{bmatrix}}$$

Hvis $\lambda = -1$ vil $\lambda x = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

x er en egenvektor for A med egenverdi: $\lambda = -1$

11) a) Sann, definisjon av egenvektor

b) sann, λ kan bli representert som λI

$Ax = \lambda I x \rightarrow Ax - \lambda I x = 0 \rightarrow (A - \lambda I)x = 0$

c) sann, $\det(A - \lambda I) = 0$ for at

$(A - \lambda I)x = \vec{0}$, når $x \neq \vec{0}$

d) Usann

e) Sann, $Ax = \lambda x$

f) Usann, $Ax = \vec{0}$ hvis A ikke er inverterbar

g) Usann, $\lambda = 0$ hvis A ikke er inverterbar

h) Sann, for da vil alle R bli egenverdier

12)

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a) 1. Ja

2. Ax ligger på $sp\{x\}$

3. $\lambda = 2$

b) 1. Ja

2. Ax ligger på $sp\{x\}$

3. $\lambda = -1$

c) 1. Ja

2. $Ax = x$

3. $\lambda = 1$

d) 1. Nei

2. Ax er rotert vekk

e) 1. Ja

2. Nullvektor ligger i $sp\{x\}$

3. $\lambda = 0$

f) 1. Ja

2. Ax ligger på $sp\{x\}$

3. $\lambda = \frac{1}{3}$