Martin Skatvedt - Øving 9 - MA0001

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$\boxed{1}$

a) $f(x) = \sin^2 x = \sin x \cdot \sin x$

$f'(x) = \sin' x \cdot \sin x + \sin x \cdot \sin' x$

$= \cos x \cdot \sin x + \sin x \cdot \cos x$

$= 2 \cos x \sin x = \underline{\underline{\sin(2x)}}$

b) $f(x) = \sin x \cos x$

$f'(x) = \sin' x \cos x + \sin x \cos' x$

$= \cos^2 x - \sin^2 x$

$\underline{\underline{= \cos(2x)}}$

c) $f(x) = x \ln x$

$f'(x) = x' \cdot \ln x + x \cdot \ln' x$

$= 1 \cdot \ln x + x \cdot \frac{1}{x}$

$\underline{\underline{= \ln x + 1}}$

d) $f(x) = e^{x \ln x}$

$f'(x) = e^{x \ln x} \cdot (x \ln x)'$

$\underline{\underline{= e^{x \ln x} \cdot (\ln x + 1)}}$

$\boxed{1}$

e) $f(x) = x^x$

$$\ln f(x) = x \ln x$$

$$f(x) = e^{x \ln x}$$

$$f'(x) = e^{x \ln x} \cdot (\ln x + 1)$$

$$= x^x (\ln x + 1)$$

$\boxed{6.7.9}$

$$E(v) = \frac{1}{v}\left[(v-35)^2 + 297\right]$$

$v =$ fart : km/t

Deriverer $(v-35)^2 + 297$

$$= 2(v-35)$$

$$E'(v) = \left(\frac{1}{v}\right)' \cdot \left[(v-35)^2 + 297\right] + \frac{1}{v} \cdot \left[(v-35)^2 + 297\right]'$$

$$= -\frac{1}{v^2} \cdot \left[(v-35)^2 + 297\right] + \frac{1}{v} \cdot 2(v-35)$$

$$= \frac{2v - 70}{v} - \frac{v^2 - 70v + 1225 + 297}{v^2}$$

$$= \frac{2v^2 - 70v - v^2 + 70v - 1225 - 297}{v^2}$$

$$= \frac{v^2 - 1522}{v^2}$$

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$\boxed{6.7.9}$

$E(v) = 0$

$\frac{v^2 - 1522}{v^2} = 0$

$= v^2 - 1522 = 0$

$= v = \sqrt{1522}$

$= v = \pm 39.01$

Siden fart ikke kan være negativ

$\underline{\underline{v = 39.01 \text{ km/t}}}$

$\boxed{6.D.30+31}$

x og $y \in \mathbb{R}$

$x + y = 10 \rightarrow y = 10 - x$

$P(x, y) = x \cdot y = x \cdot (10 - x) = 10x - x^2$

$P'(x, y) = 10 - 2x$

$P'(x, y) = 0$

$10 - 2x = 0$

$2x = 10$

$\underline{x = 5}$

$y = 10 - 5 = \underline{5}$

$\underline{P(5, 5) = 25}$

<u>Det største produktet er 25</u>

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$\boxed{6.13.30+31}$

$x + y = k \qquad y = k - x$

$P(x,y) = x \cdot y = x(k-x) = kx - x^2$

$f'(x,y) = k - 2x$

$f'(x,y) = 0$

$k - 2x = 0$

$\underline{x = \frac{k}{2}}$

$\underline{y = k - \frac{k}{2} = \frac{k}{2}}$

$x \cdot y = \frac{k}{2} \cdot \frac{k}{2} = \underline{\frac{k^2}{4}}$

Maks produkt blir $\frac{k^2}{4}$ ved sum k

$\boxed{4}$ $f(x) = (1+x)^{-1}$ i $x = 0$

$f(0) = 1$

$f'(x) = -(1+x)^{-2} = (x-1)^{-2}$

$f'(0) = -1$

$f''(x) = 2(1+x)^{-3}$

$f''(0) = 2$

$f'''(x) = -6(1+x)^{-4} = 6(x-1)^{-4}$

$f'''(0) = -6$

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$\boxed{4}$

$T_3(x) = 1 - 1x + \frac{2}{2} x^2 - \frac{6}{6} x^3$

$$T_3(x) = 1 - x + x^2 - x^3$$

$f^{(n)}(x) = \frac{n!}{(x+1)^{(n+1)}} \cdot (-1)^n$

Taylorrekke:

$$f(x) = 1 - x + x^2 - x^3 + \dots + (-1)^n \cdot (x)^n$$

$\boxed{5}$

$v_x = 2 \, m/s$ $\quad y = x^2 + 1$

$x(t) = 2t$

$y(t) = (2t)^2 + 1 = 4t^2 + 1$

$y'(t) = 8t$

$$y'(3) = 8 \cdot 3 = 24 \, m/s$$

6) $y - y_0 = a(x - x_0)$

$y(3) = 4 \cdot 3^2 + 1 = \underline{37}$

$y - 37 = 24(x - 3)$

$y = 24x - 72 + 37$

$$y = 2x - 35$$

$\boxed{6}$ $f(x) = \cos x \quad g(x) = x$

$x_0 = 1$ og $n = 3$

$h(x) = 6\sin x - 3x^2$

$\cos x = x \rightarrow \cos x - x = 0$

$w(x) = \cos x - x$

$w'(x) = -\sin x - 1$

$$x_1 = x_0 - \frac{w(x_0)}{w'(x_0)} = 1 - \frac{\cos 1 - 1}{-\sin 1 - 1} = 0{,}750364$$

$$x_2 = x_1 - \frac{w(x_1)}{w'(x_1)} = 0{,}739113$$

$$\underline{\underline{x_3 = x_2 - \frac{w(x_2)}{w'(x_2)} = 0{,}739085}}$$

$h(x) = 6\sin x - 3x^2$

$h'(x) = 6\cos x - 6x = 6(\cos x - x)$

Toppunkt er

$h'(x) = 0 \rightarrow 6(\cos x - x) = 0$

vi har $\cos x - x = 0$ ved $x \approx 0{,}739085$

derfor vil $h(x)$ ha toppunkt i

$\underline{\underline{x \approx 0{,}739085}}$