TMA4110 - Martin Skatvedt
Innlevering 1

1 a) $(1+i)^2 = 1 + 2i - 1 = \underline{2i}$

b) $\frac{1+3i}{2-i+3+2i} = \frac{1+3i}{2-i+3-2i} = \frac{1+3i}{2+3+(-1-2)i} = \frac{1+3i}{5-3i}$

$= \frac{(1+3i)(5+3i)}{34} = \frac{5+3i+15i+9}{34} = \frac{18i-4}{34} = \underline{-\frac{2}{17} + \frac{9i}{17}}$

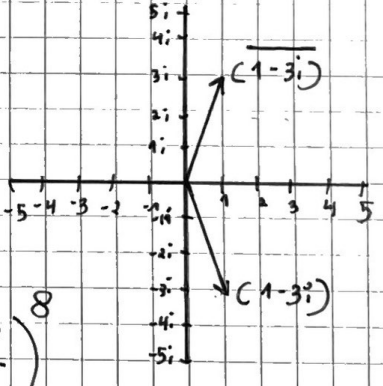
c) $\frac{1}{i^5} = \frac{1}{i \cdot i \cdot i \cdot i \cdot i} = \frac{1}{(-1)\cdot(-1)\cdot i} = \frac{1}{i} = j^{-1} = \underline{-i}$

2 a) $-1 + \overline{2+3i} = -1 + 2 - 3i = 1 - 3i$

$\Theta = \arctan \frac{-3}{1} = \arctan -3 = -\arctan(3)$

$r = \sqrt{1^2 + (-3)^2} = \sqrt{10}$

$= \underline{\sqrt{10}\, e^{-i\arctan(3)}}$



b) $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^8 = \left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i} \cdot \frac{1+\sqrt{3}i}{1+\sqrt{3}i}\right)^8$

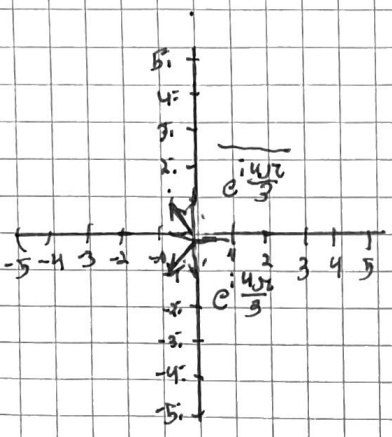
$= \left(\frac{1+2\sqrt{3}i+3i^2}{1-3i^2}\right)^8 = \left(\frac{2\sqrt{3}i-2}{4}\right)^8 = \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)^8$

$r = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{1} = \underline{1}$

$\Theta = \arctan \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} + \pi, \quad a < 0$

$= -\arctan\sqrt{3} + \pi = -\frac{\pi}{3} + \pi = \frac{3\pi - \pi}{3} = \underline{\frac{2\pi}{3}}$

Polarform: $\left(e^{i\frac{2\pi}{3}}\right)^8 = e^{i8\frac{2}{3}\pi} = \underline{e^{i\frac{4\pi}{3}}}$



3

$z = re^{i\theta}$ og $w = se^{i\alpha}$

$= r(\cos\theta + i\sin\theta) \cdot s(\cos\alpha + i\sin\alpha)$

$= (r\cos\theta + ri\sin\theta) \cdot (s\cos\alpha + si\sin\alpha)$

$= r-$

4

$\mathrm{Re}\ z = \frac{1}{2}(z + \bar{z})$ og $\mathrm{Im}\ z = \frac{1}{2}(z - \bar{z})$

$= \frac{1}{2}(a+bi + a - bi)$ $= \frac{1}{2}(a+bi - (a-bi))$

$= \frac{1}{2}(2a) = \underline{a}$ $= \frac{1}{2}(a+bi - a + bi)$

$= \frac{1}{2}(2bi) = \underline{bi}$

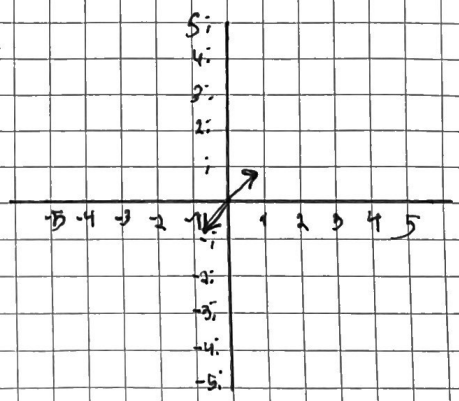
$\underline{a = \text{reell komponent}}$ $\underline{bi = \text{imagi@r komponent}}$

5

$z = \sqrt[4]{-1} = -1^{\frac{1}{4}} = \left(-1^{\frac{1}{2}}\right)^{\frac{1}{2}} = i^{\frac{1}{2}} = \underline{\sqrt{i}}$

$\sqrt{i} = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$ og $= \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$

$\underline{= \frac{1}{\sqrt{2}}(1 \pm i) \text{ og } \frac{1}{\sqrt{2}}(-1 \pm i)}$



6 $z = i^3 + i^i$ $\qquad i^3 = -i$

$i = e^{i\frac{\pi}{2}}$

$i^i = \left(e^{i\frac{\pi}{2}}\right)^i = e^{i^2\frac{\pi}{2}} = \underline{e^{-\frac{\pi}{2}}}$

$z = e^{-\frac{\pi}{2}} - i$

$\underline{\underline{Re = e^{-\frac{\pi}{2}}}} \qquad \underline{\underline{Im = -1}}$

7 a) $z^3 = \sqrt{5} + 2i$

$r = \sqrt{(\sqrt{5})^2 + 2^2} = \sqrt{5+4} = \underline{3}$

$\theta = \arctan\frac{2}{\sqrt{5}}$, $a > 0$

$z^3 = 3e^{i\arctan\left(\frac{2}{\sqrt{5}}\right)}$

$w_0 = \sqrt[3]{3}\, e^{i\left(\frac{\arctan\left(\frac{2}{\sqrt{5}}\right)}{3}\right)}$

$w_1 = e^{i\left(\frac{2\pi}{3}\right)}$

$w_1 = \sqrt[3]{3}\, e^{i\frac{1}{3}\left(\arctan\left(\frac{2}{\sqrt{5}}\right) + 2\pi\right)}$

$w_2 = \sqrt[3]{3}\, e^{i\frac{1}{3}\left(\arctan\left(\frac{2}{\sqrt{5}}\right) + 4\pi\right)}$

$w_3 = \sqrt[3]{3}\, e^{i\frac{1}{3}\left(\arctan\left(\frac{2}{\sqrt{5}}\right) + 6\pi\right)}$

$\underline{w_n = \sqrt[3]{3}\, e^{i\frac{1}{3}\left(\arctan\left(\frac{2}{\sqrt{5}}\right) + 2\pi n\right)} \quad \text{for} \quad n = 0,1,2}$

b) $z^2 + 2iz - 1 - i = 0$ $\qquad a = 1,\ b = 2i,\ c = (-1-i)$

$z = \frac{-2i \pm \sqrt{(2i)^2 - 4 \cdot 1 \cdot (-1-i)}}{2}$

$= \frac{-2i \pm \sqrt{-4 + 4 + 4i}}{2} = \frac{-2i \pm \sqrt{4i}}{2}$

$r = \sqrt{0^2 + 4^2} = \underline{4} \qquad \theta = \frac{\pi}{2}$, $a = 0$ og $b > 0$

$w_0 = 2e^{i\frac{\pi}{4}} = 2\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right) = \underline{\sqrt{2} + i\sqrt{2}}$

$w_1 = 2e^{i\frac{5\pi}{4}} = 2\left(\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right) = \underline{-\sqrt{2} - i\sqrt{2}}$

$$z = \frac{-2i \pm \left(\frac{2}{\sqrt{2}} + i\frac{2}{\sqrt{2}}\right)}{2} = \frac{-2i \pm \frac{2}{\sqrt{2}}(1+i)}{2}$$

$$\underline{z = -i \pm \frac{1}{\sqrt{2}}(1+i)}$$

c) $(z+1)^4 = (z-1)^4$

$(z+1)^2 = (z-1)^2$

$\Downarrow$

$z^2 + 2z + 1 = z^2 - 2z + 1 \rightarrow 4z = 0 \rightarrow \underline{z = 0}$

$(z+1)^2 = -(z-1)^2$

$\Downarrow$

$z^2 + 2z + 1 = -z^2 + 2z - 1 \rightarrow 2z^2 = -2$

$z^2 = -1 \rightarrow \underline{z = \pm i}$

$$\underline{z = 0 \quad \text{og} \quad z = \pm i}$$

9 a)

$$\begin{bmatrix} \underline{2} & 7 & -1 & 0 \\ 0 & 0 & \underline{1} & 0 \\ \underline{3} & 0 & 1 & 0 \end{bmatrix}$$

2, 1 og 3 er pivotelementer

$$\downarrow$$

$$\begin{bmatrix} 2 & 7 & -1 & 0 \\ 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & \frac{7}{2} & -\frac{1}{2} & 0 \\ 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{I} \cdot \left(\frac{1}{2}\right)$$

$$\downarrow$$

$$\begin{bmatrix} 1 & \frac{7}{2} & -\frac{1}{2} & 0 \\ 0 & -\frac{21}{2} & \frac{5}{2} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{II} + \text{I} \cdot (-3)$$

$$\downarrow$$

$$\begin{bmatrix} 1 & \frac{7}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{5}{21} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{II} \cdot \left(-\frac{2}{21}\right)$$

$$\downarrow$$

$$\begin{bmatrix} 1 & \frac{7}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{II} + \text{III} \cdot \left(\frac{5}{21}\right)$$

$$\downarrow$$

$$\begin{bmatrix} 1 & \frac{7}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{I} + \text{III} \cdot \left(\frac{1}{2}\right)$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \text{I} + \text{II} \cdot \left(-\frac{7}{2}\right)$$

b)

9, 6 og 9 er pivotelementer

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 9 & 1 & 0 & 1 \\ 6 & 5 & -1 & 0 & 5 \\ 0 & 9 & 0 & 0 & 0 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 9 & 1 & 0 & 1 \\ 1 & \frac{5}{6} & -\frac{1}{6} & 0 & \frac{5}{6} \\ 0 & 9 & 0 & 0 & 0 \end{bmatrix} \quad \text{III} \cdot \left(\frac{1}{6}\right)$$

$$\downarrow$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & \frac{5}{6} & -\frac{1}{6} & 0 & \frac{5}{6} \\ 0 & 9 & 0 & 0 & 0 \end{bmatrix} \quad \text{II} + \text{III} \cdot (-1)$$

$$\downarrow$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & \frac{5}{6} & -\frac{1}{6} & 0 & \frac{5}{6} \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad \text{III} \cdot \left(\frac{1}{9}\right)$$

$$\downarrow$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & -\frac{1}{6} & 0 & \frac{5}{6} \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad \text{II} + \text{III} \cdot \left(-\frac{5}{6}\right)$$

$$\downarrow$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad \text{I} + \text{II} \cdot \left(\frac{1}{6}\right)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$\left[\begin{array}{cccc|cc} 2 & 1 & 0 & 3 & 5 & 7 \\ -1 & 0 & 1 & 2 & -2 & -1 \\ 1 & 1 & 0 & 1 & 1 & 6 \end{array}\right]$$

$\downarrow$

$$\left[\begin{array}{cccc|cc} 2 & 1 & 0 & 3 & 5 & 7 \\ 0 & 1 & 1 & 3 & -1 & 5 \\ 1 & 1 & 0 & 1 & 1 & 6 \end{array}\right] \quad \text{II} + \text{III}$$

$\downarrow$

$$\left[\begin{array}{cccc|cc} 0 & -1 & 0 & 1 & 3 & -5 \\ 0 & 1 & 1 & 3 & -1 & 5 \\ 1 & 1 & 0 & 1 & 1 & 6 \end{array}\right] \quad \text{I} + \text{II} \cdot (-2)$$

$\downarrow$

$$\left[\begin{array}{cccc|cc} 0 & 0 & 1 & 4 & 2 & 0 \\ 0 & 1 & 1 & 3 & -1 & 5 \\ 1 & 1 & 0 & 1 & 1 & 6 \end{array}\right] \quad \text{I} + \text{II}$$

$\downarrow$

$$\left[\begin{array}{cccc|cc} 0 & 0 & 1 & 4 & 2 & 0 \\ 0 & 1 & 0 & -1 & -3 & 5 \\ 1 & 1 & 0 & 1 & 1 & 6 \end{array}\right] \quad \text{II} + \text{I} \cdot (-1)$$

$\downarrow$

$$\left[\begin{array}{cccc|cc} 0 & 0 & 1 & 4 & 2 & 0 \\ 0 & 1 & 0 & -1 & -3 & 5 \\ 1 & 0 & 0 & 2 & 4 & 1 \end{array}\right] \quad \text{III} + \text{II} \cdot (-1)$$

$\downarrow$

$$\left[\begin{array}{cccc|cc} 1 & 0 & 0 & 2 & 4 & 1 \\ 0 & 1 & 0 & -1 & -3 & 5 \\ 0 & 0 & 1 & 4 & 2 & 0 \end{array}\right] \quad \text{bytter rekkefølge}$$

a) $x = 4 - 2t$
$y = t - 3$
$z = 2 - 4t$

b) $a = 1 - 2t$
$b = 5 + t$
$c = -4t$

11

a)

12 a)

$$\left[\begin{array}{ccc|c} 3 & 8 & -1 & -10 \\ 8 & 6 & -5 & -23 \\ -6 & 1 & 5 & 18 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 3 & 8 & -1 & -10 \\ 2 & 7 & 0 & -5 \\ -6 & 1 & 5 & 18 \end{array}\right] \text{ II + III}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -5 \\ 2 & 7 & 0 & -5 \\ -6 & 1 & 5 & 18 \end{array}\right] \text{ I + II} \cdot (-1) \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & -5 \\ 2 & 7 & 0 & -5 \\ 0 & 22 & 5 & 3 \end{array}\right] \text{ III + II} \cdot (-3)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -5 \\ 0 & 5 & 2 & 5 \\ 0 & 22 & 5 & 3 \end{array}\right] \text{ II + I} \cdot (-2) \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & -5 \\ 0 & 1 & \frac{2}{5} & 1 \\ 0 & 22 & 5 & 3 \end{array}\right] \text{ II} \cdot \frac{1}{5}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -5 \\ 0 & 1 & \frac{2}{5} & 1 \\ 0 & 0 & -\frac{19}{5} & -19 \end{array}\right] \quad \text{III} + \text{II} \cdot (-22)$$

$$\frac{25}{5} - \frac{44}{5} = -\frac{19}{5}$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -5 \\ 0 & 1 & \frac{2}{5} & 1 \\ 0 & 0 & 1 & 5 \end{array}\right] \quad \text{III} \cdot \left(-\frac{5}{19}\right)$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{7}{5} & -6 \\ 0 & 1 & \frac{2}{5} & 1 \\ 0 & 0 & 1 & 5 \end{array}\right] \quad \text{I} + \text{III} \cdot -1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 5 \end{array}\right] \quad \begin{array}{l} \text{I} + \text{III} \cdot \left(\frac{7}{5}\right) \\ \text{II} + \text{III} \cdot \left(-\frac{2}{5}\right) \end{array}$$

$$x_1 = 1$$

$$x_2 = -1$$

$$x_3 = 5$$

b)

$$\left[\begin{array}{ccc|c} i & 3 & -1 & 1+i \\ 2 & -4i & 0 & 2 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} i & 3 & -1 & 1+i \\ 0 & 2i & -2i & 2i \end{array}\right] \quad II + I \cdot (2i)$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -3i & i & 1-i \\ 0 & 2i & -2i & 2i \end{array}\right] \quad I \cdot (-i)$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & -3i & i & 1-i \\ 0 & i & -i & i \end{array}\right] \quad II \cdot \left(\frac{1}{2}\right)$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2i & 1+2i \\ 0 & i & -1 & i \end{array}\right] \quad I + II \cdot (3)$$

$$\downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -2i & 1+2i \\ 0 & 1 & -1 & 1 \end{array}\right] \quad II \cdot \left(\frac{1}{i}\right)$$

$z = 1 + 2i + 2it$

$w = 1 + 1t$