2) $\ln(1+y) + \sin(xy) = \ln 5$

Bruker implisitt derivasjon

$$\frac{1}{1+y} \cdot \frac{dy}{dx} + \cos(xy) \cdot \left(1 \cdot y + x \cdot \frac{dy}{dx}\right) = 0$$

$$\Rightarrow \frac{\frac{dy}{dx}}{1+y} + \cos(xy) \cdot \left(y + x\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} \cdot \frac{1}{1+y} + y\cos(xy) + \frac{dy}{dx} x\cos(xy) = 0$$

$$\frac{dy}{dx}\left(\frac{1}{1+y} + x\cos(xy)\right) + y\cos(xy) = 0$$

$$\frac{dy}{dx} = \frac{-y\cos(xy)}{\frac{1}{1+y} + x\cos(xy)} = \frac{\frac{-y\cos(xy)}{1}}{\frac{1+ \quad 1+y(x\cos(xy))}{1+y}}$$

$$= \frac{-y\cos(xy) - y^2\cos(xy)}{1 + x\cos(xy) + xy\cos(xy)} = \frac{dy}{dx}$$

Bruker implisitt derivasjon igjen

$$f(x,y) = -y\cos(xy) - y^2\cos(xy)$$

$$f'(x,y) = \left(-\frac{dy}{dx}\cos(xy) + y\sin(xy)\left(y + x\frac{dy}{dx}\right)\right) +$$

$$\left(-2y\frac{dy}{dx}\cos(xy) + y^2\cos(xy)\left(y + x\frac{dy}{dx}\right)\right)$$

$$= -\frac{dy}{dx}\cos(xy) + y^2\sin(xy) + \frac{dy}{dx}xy\sin(xy)$$

$$-\frac{dy}{dx}2y\cos(xy) + y^3\cos(xy) + \frac{dy}{dx}xy^2x\cos(xy)$$

$g(x,y) = 1 + x\cos(xy) + xy\cos(xy)$

$g'(x,y) = (\cos(xy - x\sin(xy)(y + x\frac{dy}{dx})$

$+((y + x\frac{dy}{dx})\cos(xy) - xy\sin(xy)(y + x\frac{dy}{dx}))$

$= \cos(xy - xy\sin(xy) - \frac{dy}{dx}x^2\sin(xy$

$+ y\cos(xy + \frac{dy}{dx}x\cos(xy) - xy^2\sin(xy) - \frac{dy}{dx}x^2y\sin(xy)$

$$\frac{d^2y}{dx^2} = \frac{f'(x,y)\cdot g(x,y) - f(x,y)\cdot g'(x,y)}{g(x,y)^2}$$

Her ga jeg opp :)

$\boxed{3}$ $\lim_{x \to -2} \frac{3x^3+x^2-12x-4}{x^2-4} = -5$

$0 < |x+2| < \delta \rightarrow |f(x)+5| < \varepsilon$

Forenkler utrykket

$3x^3+x^2-12x-4 \rightarrow 3x^3+x^2-4 \cdot 3x-4$

$\rightarrow 3x(x^2-4)+x^2-4 \rightarrow 3x(x^2-4)+1(x^2-4)$

$\rightarrow (x^2+4)(3x+1)$

....

Forenklet $\Rightarrow$ $\frac{\cancel{(x^2+4)}(3x+1)}{\cancel{(x^2-4)}} = \frac{3x+1}{}$

$|3x+1+5| < \varepsilon = |3x+6| < \varepsilon$

$-\varepsilon < 3x+6 < \varepsilon = -6-\varepsilon < 3x < \varepsilon-6$

$= -2-\frac{\varepsilon}{3} < x < \frac{\varepsilon}{3}-2$

$|x+2| < \delta$

$\rightarrow -2-\frac{\varepsilon}{3}+2 < x+2 < \frac{\varepsilon}{3}-2+2 = -\frac{\varepsilon}{3} < x+2 < \frac{\varepsilon}{3}$

$= \underline{|x+2| < \frac{\varepsilon}{3}}$ δ

For alle $\varepsilon > 0$ vil $\delta = \frac{\varepsilon}{3}$

$|x+2| < \frac{\varepsilon}{3} \Rightarrow -\frac{\varepsilon}{3} < x+2 < \frac{\varepsilon}{3}$

$\Rightarrow -\varepsilon < 3x+6 < \varepsilon \Rightarrow \underline{|3x+6| < \varepsilon}$

<u>Grensen finnes</u>

$\boxed{4}$ a) $\lim_{x \to -1} \dfrac{(1+x)^2}{\sqrt{1+(x+1)^2-1}} = \dfrac{1+2x+x^2}{\sqrt{1+x^2+2x+1-1}}$

$= \dfrac{x^2+2x+1}{\sqrt{x^2+2x+1}} = x^2+2x+1 \quad \lim_{x \to -1} x^2+2x+1$

$= (-1)^2 + 2\cdot(-1) + 1 = 1 - 2 + 1 = \underline{0}$

$$\lim_{x \to -1} \frac{(1+x)^2}{\sqrt{1+(x+1)^2-1}} = 0$$

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b) $\lim_{x \to 0} \dfrac{19\pi x \sin(7x)}{14(1-\cos(19x))}$

Må bruke L'Hôpitals regel

$f(x) = 19\pi x \sin(7x)$

$f'(x) = 19\pi\,(\overbrace{1 \cdot \sin(7x) + x \cdot \cos(7x) \cdot 7}^{\text{Produktregel}})$ Kjerneregel

$\underline{f'(x) = 19\pi(\sin(7x) + 7x\cos(7x))}$

$g(x) = 14(1-\cos(19x))$

$g'(x) = 14(0 + \sin(19x) \cdot 19)$

$g'(x) = 14(19\sin(19x))$

$\underline{g'(x) = 266\sin(19x)}$

$\dfrac{f'(x)}{g'(x)} = \dfrac{19\pi(\sin(7x) + 7x\cos(7x))}{266\sin(19x)}$

$= \dfrac{\pi(\sin(7x) + 7x\cos(7x))}{14\sin(19x)} = 0$

$\quad = 0$

Bruke L'Hôpitals igjen

$h(x) = \pi(\sin(7x) + 7x\cos(7x))$

$h'(x) = \pi(7\cos(7x) + 7(1\cdot\cos(7x) + x\cdot -\sin(7x)7))$

$h'(x) = \pi(7\cos(7x) + 7(\cos(7x) - 7x\sin(7x))$

$\underline{h'(x) = 7\pi(2\cos(7x) - 7x\sin(7x))}$

$L(x) = 14\sin(19x)$

$L'(x) = 14\cos(19x)\cdot 19$

$\underline{L'(x) = 266\cos(19x)}$

$$\frac{h'(x)}{L'(x)} = \frac{14\cos(7x) - 49x\sin(7x)}{266\cos(19x)}$$

$$\lim_{x \to 0} \frac{\pi(14\cos(7\cdot 0) - 49x\sin(7x))}{266\cos(19\cdot 0)}$$

$$= \frac{14\pi \cdot 1}{266} = \frac{14\pi}{266} = \underline{\frac{\pi}{19}}$$

$$\underline{\underline{\lim_{x \to 0} \frac{19\pi x\sin(7x)}{14(1-\cos(19x))} = \frac{\pi}{19}}}$$