NTNU
Kunnskap for en bedre verden

KANDIDAT

10208

PRØVE

MA0301 1 Elementær diskret matematikk

Emnekode	MA0301
Vurderingsform	Hjemmeeksamen
Starttid	18.05.2021 07:00
Sluttid	18.05.2021 11:00
Sensurfrist	09.06.2021 21:59
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Cover Letter

Oppgave	Status	Poeng	Oppgavetype
□			Dokument

Logic - Multiple choice (5 points)

Oppgave	Status	Poeng	Oppgavetype
1	Riktig	5/5	Flervalg

Compute negation (3 points)

Oppgave	Status	Poeng	Oppgavetype
2	Besvart	Rettes manuelt	Filopplasting

New Logic Connective (12 points)

Oppgave	Status	Poeng	Oppgavetype
3	Besvart	Rettes manuelt	Filopplasting

Multiple Choice (Relation) (5 points)

Oppgave	Status	Poeng	Oppgavetype
4	Feil	0/5	Flervalg

Equivalence relation (5 points)

Oppgave	Status	Poeng	Oppgavetype
5	Besvart	Rettes manuelt	Filopplasting

Partial order (9 points)

Oppgave	Status	Poeng	Oppgavetype
6	Besvart	Rettes manuelt	Filopplasting

Induction Part I (6 points)

Oppgave	Status	Poeng	Oppgavetype
7	Besvart	Rettes manuelt	Filopplasting

Induction Part II (6 points)

Oppgave	Status	Poeng	Oppgavetype
8	Ubesvart	Rettes manuelt	Filopplasting

Functions Part I (3 points)

Oppgave	Status	Poeng	Oppgavetype
9	Riktig	3/3	Flervalg

Functions Part II (5 points)

Oppgave	Status	Poeng	Oppgavetype
10	Besvart	Rettes manuelt	Filopplasting

Combinatorics I (5 points)

Oppgave	Status	Poeng	Oppgavetype
11	Besvart	Rettes manuelt	Filopplasting

Combinatorics II (6 points)

Oppgave	Status	Poeng	Oppgavetype
12	Besvart	Rettes manuelt	Filopplasting

Automata I (3 points)

Oppgave	Status	Poeng	Oppgavetype
13	Riktig	3/3	Flervalg

Automata II (6 points)

Oppgave	Status	Poeng	Oppgavetype
14	Besvart	Rettes manuelt	Filopplasting

Automata III (6 points)

Oppgave	Status	Poeng	Oppgavetype
15	Besvart	Rettes manuelt	Filopplasting

Graph Theory I (5 points)

Oppgave	Status	Poeng	Oppgavetype
16	Besvart	Rettes manuelt	Filopplasting

Graph Theory II (10 points)

Oppgave	Status	Poeng	Oppgavetype
17	Besvart	Rettes manuelt	Filopplasting

1 $(p \square (p \square q)) \square q$ is

Select one alternative:

- neither a tautology nor a contradiction
- a tautology ✓
- a contradiction

2 Compute the negation of $\square x \square y(\neg P(x, y) \square Q(x, y))$

Din fil ble lastet opp og lagret i besvarelsen din.

˜ Last ned | ⎦ Fjern | Erstatt

Filnavn:	Oppgave 2.pdf
Filtype:	application/pdf
Filstørrelse:	108.54 KB
Opplastingstidspunkt:	18.05.2021 09:18
Status:	**Lagret**

3 Consider the new logic connective $\downarrow$, where $p \downarrow q$ is logically equivalent to $\neg(p \square q)$.

1. (3 points) Write down the truth table of this new connective $\downarrow$.
2. (3 points) Is $\neg p$ logically equivalent to $p \downarrow p$? Give an argument about your answer. (Either use a truth table or laws of logic).
3. (6 points) Is $p \square q$ logically equivalent to $(p \downarrow q) \downarrow (p \downarrow q)$? Give an argument about your answer **using laws of logic**. No point will be given if you use a truth table.

Din fil ble lastet opp og lagret i besvarelsen din.

˜ Last ned	⎮ Fjern	Erstatt

Filnavn:	Oppgave 3.pdf
Filtype:	application/pdf
Filstørrelse:	263.14 KB
Opplastingstidspunkt:	18.05.2021 07:27
Status:	**Lagret**

4 (5 points) Consider the relation $R := \{(x, y) \in Z \times Z \mid x + y = 6\}$. Which of the following statements about R is true?

Select one alternative:

- None of the choices here ✔
- It is reflexive, transitive, and not antisymmetric
- It is transitive, not reflexive, and antisymmetric
- It is symmetric, reflexive, and not antisymmetric ✖

5 (5 points) Let $A := \{0, 1, 2, 3\}$. Define the relation

$$R := \{(0, 0), (1, 1), (2, 2), (2, 3), (3, 2)\}.$$

Is R an equivalence relation? If it is not, how to turn R into an equivalence relation by adding the minimum amount of elements?

Din fil ble lastet opp og lagret i besvarelsen din.

Last ned | Fjern | Erstatt

Filnavn:	Oppgave 5.pdf
Filtype:	application/pdf
Filstørrelse:	139.02 KB
Opplastingstidspunkt:	18.05.2021 09:18
Status:	**Lagret**

6 (9 points) Let $A := \{3, 4, 6, 12, 20\}$ be ordered by divisibility. Compute the minimal and maximal elements. Moreover, draw the Hasse diagram.

Din fil ble lastet opp og lagret i besvarelsen din.

˜ Last ned | Fjern | Erstatt

Filnavn:	Oppgave 6.pdf
Filtype:	application/pdf
Filstørrelse:	71.91 KB
Opplastingstidspunkt:	18.05.2021 09:19
Status:	**Lagret**

7 (6 points) Use induction to show that for all integers $m > 0$

$$\sum_{i=1}^{m} 2i - 1 = m(m+1) - 1$$

Din fil ble lastet opp og lagret i besvarelsen din.

Last ned	Fjern	Erstatt

Filnavn:	Oppgave 7.pdf
Filtype:	application/pdf
Filstørrelse:	170.89 KB
Opplastingstidspunkt:	18.05.2021 10:52
Status:	**Lagret**

8 (6 points) Consider the sequence $\{a_n\}_{n>0}$, where $a_1 = 3, a_2 = 6$ and, for integers $k > 2, a_k = a_{k-1} + a_{k-2}$.

Use induction to show that, for all integers $n > 0$, a_n can be divided by 3.

Upload your file here. Maximum one file.

Alle filtyper er tillatt. Maksimal filstørrelse er **50 GB**.

Velg fil for opplasting

9 (3 points) Which of the following statements is true?

Select one alternative:

- If the functions $f : A \to B$ and $g : B \to C$ are injective, then the composite $g \circ f : A \to C$ is injective. ✓
- None of them
- If the functions $f : A \to B$ and $g : B \to C$ are injective, then the composite $f \circ g : B \to C$ is injective.
- If the functions $f : A \to B$ and $g : A \to B$ are injective, then the composite $g \circ f : A \to B$ is injective.

10 (5 points) Consider the function $f(x) = x - 7$ with integers as domain and codomain (namely, $f : Z \to Z$). Is it injective, surjective, or bijective? Give an argument to justify your answer.

Din fil ble lastet opp og lagret i besvarelsen din.

Filnavn:	Oppgave 10.pdf
Filtype:	application/pdf
Filstørrelse:	78.89 KB
Opplastingstidspunkt:	18.05.2021 10:53
Status:	**Lagret**

11 (5 points) According to the binomial formula, what is the coefficient of x^6y^6 in the expansion of $(3x^3 + 2y)^8$? What is the coefficient of x^4y^3?

Din fil ble lastet opp og lagret i besvarelsen din.

Last ned	Fjern	Erstatt

Filnavn:	Oppgave 11.pdf
Filtype:	application/pdf
Filstørrelse:	121.47 KB
Opplastingstidspunkt:	18.05.2021 10:17
Status:	**Lagret**

12 (6 points) What is the number of permutations formed from the letters of "ALLTALK"? How about with the restriction that the two A's must be next to each other?

Din fil ble lastet opp og lagret i besvarelsen din.

Last ned | Fjern | Erstatt

Filnavn:	Oppgave 12.pdf
Filtype:	application/pdf
Filstørrelse:	202.66 KB
Opplastingstidspunkt:	18.05.2021 10:17
Status:	**Lagret**

13 (3 points) Let $\Sigma := \{0, 1, 2\}$. Which of the following words is ***not*** in the language $L(r)$ for the regular expression $r = 011^{\square}2^{\square}$?

Select one alternative:

- None of the choices here ✓
- 0111
- 012
- 0112

14 (6 points) See the given PDF file.

Din fil ble lastet opp og lagret i besvarelsen din.

Last ned Fjern Erstatt

Filnavn:	Oppgave 14.pdf
Filtype:	application/pdf
Filstørrelse:	237.03 KB
Opplastingstidspunkt:	18.05.2021 08:21
Status:	**Lagret**

15 (6 points) See the given PDF file.

Din fil ble lastet opp og lagret i besvarelsen din.

Last ned	Fjern	Erstatt

Filnavn:	Oppgave 15.pdf
Filtype:	application/pdf
Filstørrelse:	113.12 KB
Opplastingstidspunkt:	18.05.2021 09:19
Status:	**Lagret**

16 (5 points) See the given PDF file.

Din fil ble lastet opp og lagret i besvarelsen din.

Last ned | Fjern | Erstatt

Filnavn:	Oppgave 16.pdf
Filtype:	application/pdf
Filstørrelse:	78.55 KB
Opplastingstidspunkt:	18.05.2021 08:21
Status:	**Lagret**

17 (10 points) See the given PDF file.

Din fil ble lastet opp og lagret i besvarelsen din.

Filnavn:	Oppgave 17.pdf
Filtype:	application/pdf
Filstørrelse:	339.39 KB
Opplastingstidspunkt:	18.05.2021 08:20
Status:	**Lagret**

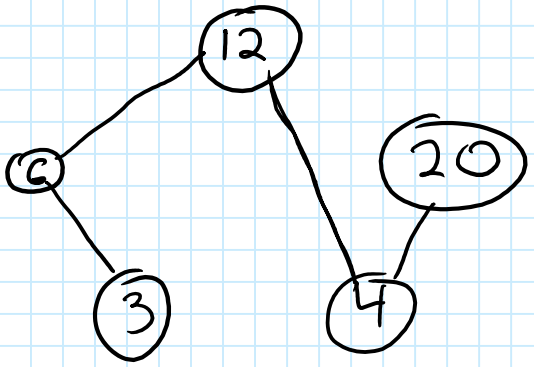
5

- It is symmetric
- It is transitive
- It is not reflexive
  : This means that it isnt a equivalence relation

If we add (3,3) it becomes reflexive and it becomes an equivalence relation

6

A: {3, 4, 6, 12, 20}



Maximum : 12
Minimum : 3, 4

7

$\sum_{i=1}^{m} 2i - 1 = m(m+1) - 1 \quad m > 0$

Base case

$m = 1$

$m(m+1) - 1 \rightarrow 1(2) - 1 = \underline{1}$

$\sum_{i=1}^{1} 2i - 1 = 2 \cdot 1 - 1 = \underline{1}$

True

this implies ↓

$1 + 3 + \cdots + 2k - 1 = k(k+1) - 1$

$k(k+1) - 1 + 2k + 1 \neq (k+1)(k+2) - 1$

It is not equal, which means it doesnt hold for all $m > 0$

10

$f(x) = x - 7$

As the function is linear, for each y, there is only one x.

This means that it is

bijective

2

$$\neg\left[\exists_x \forall_y \left(\neg P(x,y) \wedge Q(x,y)\right)\right]$$

$$\equiv \forall_x \left[\neg \forall_y \left(\neg P(x,y) \wedge Q(x,y)\right)\right]$$

$$\equiv \forall_x \exists_y \neg\left(\neg P(x,y) \wedge Q(x,y)\right)$$

$$\equiv \forall x \exists y \left(P(x,y) \vee \neg Q(x,y)\right)$$

11

$$\binom{8}{6}(3x^2)^2(2y)^6$$

$$\frac{8!}{6! \cdot 2!} = \frac{8 \cdot 7}{2} = 28$$

$$28 \cdot 3^2 \cdot x^6 \cdot 2^6 \cdot y^6 = 28 \cdot 9 \cdot 64 \cdot x^6 \cdot y^6$$

$$= 16128 x^6 y^6$$

There is no coefficient for $x^4 y^3$

12

Word : ALLTALK

n Letters = 7

$$\text{Permutations} = \frac{n!}{m_A! \, m_L! \, m_T! \, m_K!}$$

$$= \frac{7!}{2! \cdot 3!} = \frac{5040}{12} = \underline{420}$$

With two A's next to each other:

- Put the two A's "into" one letter

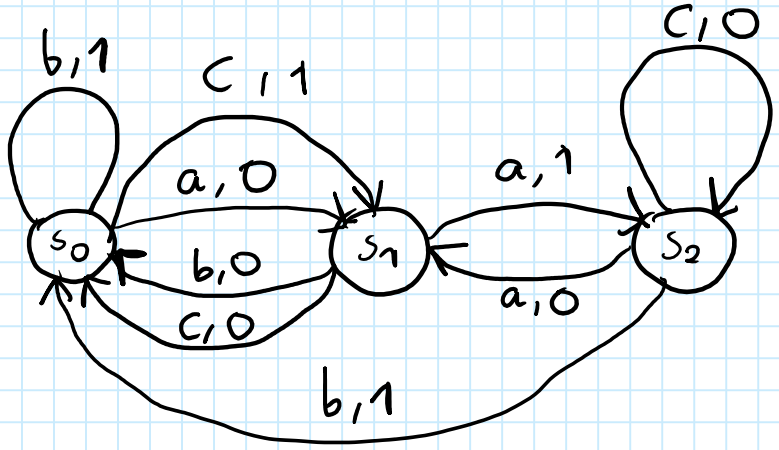
$$\frac{6!}{3!} = 6 \cdot 5 \cdot 4 = \underline{120}$$

- There are 2! permutatinsos of "aa"

Which gives us nPermutations : $120 \cdot 2 = \underline{240}$

14

M	V			W		
	a	b	c	a	b	c
s_0	s_1	s_0	s_1	0	1	1
s_1	s_2	s_0	s_0	1	0	0
s_2	s_1	s_0	s_2	0	1	0



word: acabacab

state	s_0	s_1	s_0	s_1	s_0	s_1	s_0	s_1	s_0
Input	a	c	a	b	a	c	a	b	
output	0	0	0	0	0	0	0	0	

output is 000000006

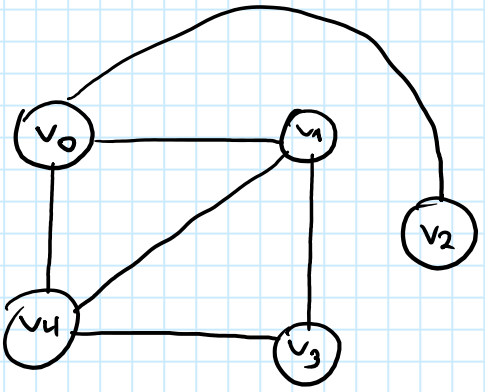
15

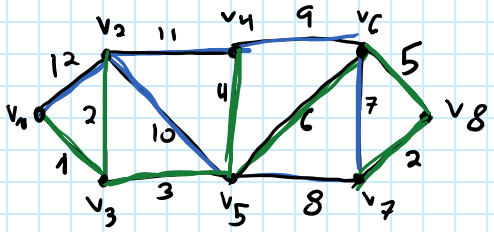
S	a	b
S_0	S_0	S_1
S_1	S_1	S_2
S_2	S_2	S_1

The accepted language are words consisting of an even nuber of "b". The letter "a" wont have any impact.

16

	v_0	v_1	v_2	v_3	v_4
v_0	0	1	1	0	1
v_1	1	0	0	1	1
v_2	1	0	0	0	0
v_3	0	1	0	0	1
v_4	1	1	0	1	0





1) Choose v_1v_3

2) Choose v_7v_8

3) Choose v_2v_3 $\rightarrow$ v_1v_2 would now form Cycle

4) -11- v_3v_5 $\rightarrow$ v_2v_5 would now form Cycle

5) -11- v_4v_5 $\rightarrow$ v_2v_4 would now form Cycle

6) -11- v_6v_8 $\rightarrow$ v_6v_7 would now form Cycle

7) -11- v_5v_6 $\rightarrow$ There are already paths for v_5v_7 and v_4v_6 so we are finished

$$\{v_1v_3, v_7v_8, v_2v_3, v_3v_5, v_4v_5, v_6v_8, v_5v_6\}$$

1)

p	q	p∨q	p↓q
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

2) $p \downarrow p \equiv \neg(p \vee p)$

$\equiv \neg p \wedge \neg p$ D.M.G

$\equiv \neg p$

p	¬p	¬p∧¬p
0	1	1
1	0	0

They are logically equivalent

3) $(p \downarrow q) \downarrow (p \downarrow q)$

$\equiv \neg([\neg(p \vee q)] \vee [\neg(p \vee q)])$

$\equiv \neg[\neg(p \vee q)] \wedge \neg[\neg(p \vee q)]$ D.M.G

$\equiv (p \vee q) \wedge (p \vee q)$ Double negation

$\equiv p \vee q$ As $r \wedge r \equiv r$

$p \vee q \equiv (p \downarrow q) \downarrow (p \downarrow q)$