Optional Study for ITP 2019 – Martin Skilleter

- At the start of semester, Isabel Longbottom and I read the first chapter of *Homotopy Type Theory* as an introduction to dependent type theory. I then read *Theorem Proving in Lean* and attempted to complete the exercises there.
- Isabel and I defined an arithmetic function (a function $f: \mathbb{N} \to \mathbb{C}$) and proved that the arithmetic functions were an additive commutative group under pointwise addition. We defined the Dirichlet convolution ($(f \star g)(n) = \sum_{d|n} f(d) g\left(\frac{n}{d}\right)$) as a multiplication on this collection of functions and made progress towards proving that the arithmetic functions form a commutative ring. We also stated the theorem that an arithmetic function f is a unit in this ring if and only if $f(1) \neq 0$, although we had to modify it slightly because the natural numbers in Lean are indexed starting from 0. The code for this is in my repository, in the folder *Problems_for_Fun*.
- After Scott Morrison mentioned me in the chat about the 100 theorems challenge (because I had proven the Cauchy-Schwarz Inequality), Isabel and I attempted to prove that the square root of 2 was irrational, as well as the Arithmetic-Geometric Mean Inequality. We managed to state both of these theorems, but were unable to prove them. We then read through the completed solutions for these two problems that were already implemented in Lean to try to understand how they were proven.
- I am currently getting ready for a PR to MathLib for my project on inner product spaces. I will probably end up coordinating with Andreas so that some of both his and my work makes it into the final product. I am still hoping to generalize the things I've proven away from just the real numbers, and hopefully he can help.
- I have been following several threads on the Zulip chat so that I can learn new mathematics. For example, I followed the conversation about the Perfectoid spaces project and listened to Kevin Buzzard's Junior Number Theory seminar (available at http://wwwf.imperial.ac.uk/~buzzard/docs/Junior_Number_Theory_Seminar_09_05_2019.mp4). I also attempted to state and prove Sharkovsky's theorem (I managed to state the theorem in Lean and understand the proof in real life, though I never attempted to translate the proof into Lean), which I discovered through the Zulip chat https://leanprover.zulipchat.com/#narrow/stream/113488-general/topic/Sharkovsky's.20Theorem.