a)

$$\nabla^T \underline{q} = 0$$

Multiply with weight function v & integrate over domain

0.25p

$$\int_A v \nabla^T \underline{q} \cancel{t} \, dA = \int_A 0 \, dA = 0$$

0.5p

$$\int_A v \nabla^T \underline{q} \, dA = \{ \text{integr. by parts} \} =$$

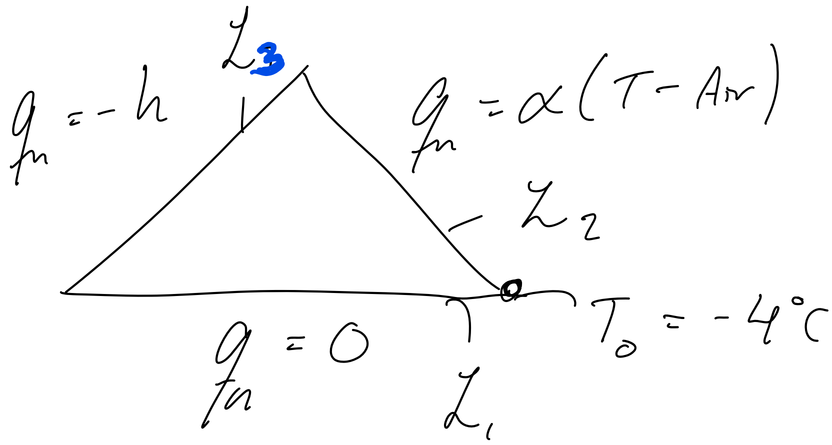
$$= \underbrace{\int_A \nabla^T (v \underline{q}) \, dA}_{\int_{\mathcal{L}} v q_n \, dL} - \int_A (\nabla v)^T \underline{q} \, dA = 0$$

0.25p

0.25p

$$\underline{q} = -\underline{D} \nabla T \implies$$

$$\int_A (\nabla v)^T \underline{D} \nabla T \, dA = -\int_{\mathcal{L}_g} v q_n \, dL - \int_{\mathcal{L}_h} v q_n \, dL$$



0.5p

$$\Rightarrow \begin{cases} \displaystyle\int_A (\nabla v)^T D \, \nabla T \, dA = -\int_{\mathcal{L}_2} v\alpha(T - T_{air})\, d\mathcal{L} + \int_{\mathcal{L}_3} v h \, d\mathcal{L} \\ T = T_0 \text{ at lower right corner} \end{cases}$$

b) Doing the FE-form yields

$$f_b = -\int_{\mathcal{L}_2} N^T \alpha (N a - T_{\infty})\, d\mathcal{L}$$

$$+ \int_{\mathcal{L}_3} N^T h \, d\mathcal{L}$$

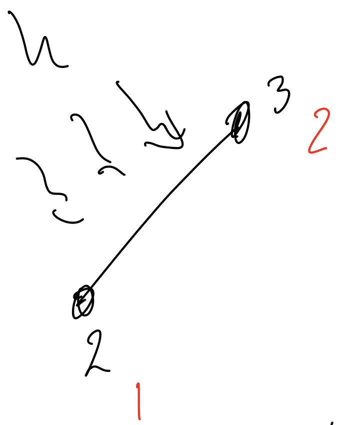
Specifically, the contribution from element 2 comes as a contribution

$$f_b^{e2} = \int_{L^e} N^{eT} h \, d\mathcal{L}$$

Identify: 0.5p

where L^e is the element boundary of element 2 coinciding with a part of $\mathcal{L}_3$.

For simplicity, we are only considering the contribution to the nodes along the boundary with local numbering 1 and 2, whereby f_b^{e2} becomes



$$\int_{L^e} \begin{bmatrix} N_1^e \\ N_2^e \end{bmatrix} h \, dL = \frac{L^e \cdot h}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{0.5p}$$

with

$$L^e = \sqrt{H^2 + (W/2)^2}$$

$$= \sqrt{5^2 + 5^2} = 5\sqrt{2} \text{ m} \quad \text{0.5p}$$

$$\Rightarrow f_b^{e2} = \frac{5 \cdot \sqrt{2} \cdot 10}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

which are assembled into $f_b = \begin{bmatrix} f_{b1} \\ f_{b2} \\ \vdots \\ f_{b6} \end{bmatrix}$

as

$$f_b^{e2} = \frac{5\sqrt{2} \cdot 10}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{array}{l} \rightarrow f_{b2} \\ \rightarrow f_{b3} \end{array} \quad \text{0.5p}$$

c) Divide a into free & prescribed degrees of freedom such that

$$a = \begin{bmatrix} a_f \\ a_p \end{bmatrix} \quad \text{with} \quad a_f = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_6 \end{bmatrix}, \quad a_p = \begin{bmatrix} a_5 \end{bmatrix}$$

0.25p

Thereby, we can write the system of equations as

$$\begin{bmatrix} K_{ff} & K_{fp} \\ K_{pf} & K_{pp} \end{bmatrix} \begin{bmatrix} a_f \\ a_p \end{bmatrix} = \begin{bmatrix} f_f \\ f_p \end{bmatrix}$$

0.25p

from which the free degrees of freedom are computed as:

$$K_{ff}\, a_f = f_f - K_{fp}\, a_p$$

$$\iff \quad a_f = K_{ff}^{-1}\left(f_f - K_{fp}\, a_p\right)$$

0.5p

where

$$\mathbb{K}_{ff} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{26} \\ \vdots & & & & \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{46} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{66} \end{bmatrix}$$

0.25p

$$\mathbb{K}_{fp} = \begin{bmatrix} K_{15} \\ K_{25} \\ K_{35} \\ K_{45} \\ K_{65} \end{bmatrix}, \quad a_p = \begin{bmatrix} a_5 \end{bmatrix} = \begin{bmatrix} -4 \end{bmatrix}$$

0.25p

0.25p

$$\mathbb{f}_f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_6 \end{bmatrix}$$

0.25p