1 Exercise 1: Derivatives

(a)

$$
\begin{aligned}
\frac{df(x)}{x} &= cos(x^3)\frac{dln(\frac{1}{11}x^4)}{dx} + (-1)sin(x^3)3x^2 ln(\frac{1}{11}x^4) \\
&= cos(x^3)\frac{4}{11}x^3\frac{1}{\frac{1}{11}x^4} - 3x^2 sin(x^3)ln(\frac{1}{11}x^4) \\
&= 11x^{-4}\frac{4}{11}x^3 cos(x^3) - 3x^2 sin(x^3)ln(\frac{1}{11}x^4) \\
&= \frac{44}{11}x^{-1}cos(x^3) - 3x^2 sin(x^3)ln(\frac{1}{11}x^4) \\
&= 4x^{-1}cos(x^3) - 3x^2 sin(x^3)ln(\frac{1}{11}x^4)
\end{aligned}
$$

(b)

$$
\begin{aligned}
\frac{d}{dx}e^{-\frac{(x-\mu)^2}{2\sigma^2}} &= e^{-\frac{(x-\mu)^2}{2\sigma^2}}\frac{d}{dx}(-\frac{(x-\mu)^2}{2\sigma^2}) \\
&= e^{-\frac{(x-\mu)^2}{2\sigma^2}}(-\frac{1}{2\sigma^2}\frac{d}{dx}(x-\mu)^2) \\
&= \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}2(x-\mu)\frac{d}{dx}(x-\mu)}{2\sigma^2} \\
&= \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}(x-\mu)(\frac{d}{dx}(x)+\frac{d}{dx}(-\mu))}{\sigma^2} \\
&= -\frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}(x-\mu)(1+0)}{\sigma^2} \qquad = -\frac{(x-\mu)e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sigma^2}
\end{aligned}
$$

(c)

$$
\begin{aligned}
\frac{d}{dx}\frac{1}{1+e^{-x}} &= -\frac{\frac{d}{dx}(e^{-x}+1)}{(e^{-x}+1)^2} \\
&= -\frac{\frac{d}{dx}(e^{-x})+\frac{d}{dx}(1)}{(e^{-x}+1)^2} \\
&= -\frac{e^{-x}\frac{d}{dx}(-x)+0}{(e^{-x}+1)^2} \\
&= \frac{e^{-x}}{(e^{-x}+1)^2}
\end{aligned}
$$

(d)

Let

$$f(\mathbf{x}) = \begin{pmatrix} \sqrt{x_1^2 - x_3} \\ e^{x_2}\cos(4x_3) \end{pmatrix}, \quad f : \mathbb{R}^3 \to \mathbb{R}^2.$$

The Jacobian matrix $\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}}$ is given by:

$$\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial}{\partial x_1}\sqrt{x_1^2 - x_3} & \frac{\partial}{\partial x_2}\sqrt{x_1^2 - x_3} & \frac{\partial}{\partial x_3}\sqrt{x_1^2 - x_3} \\ \frac{\partial}{\partial x_1}e^{x_2}\cos(4x_3) & \frac{\partial}{\partial x_2}e^{x_2}\cos(4x_3) & \frac{\partial}{\partial x_3}e^{x_2}\cos(4x_3) \end{pmatrix}.$$

Computing the partial derivatives:

$$\begin{aligned}
\frac{\partial}{\partial x_1}\sqrt{x_1^2 - x_3} &= \frac{1}{2\sqrt{x_1^2 - x_3}} \cdot 2x_1 = \frac{x_1}{\sqrt{x_1^2 - x_3}}, \\
\frac{\partial}{\partial x_2}\sqrt{x_1^2 - x_3} &= 0, \\
\frac{\partial}{\partial x_3}\sqrt{x_1^2 - x_3} &= \frac{-1}{2\sqrt{x_1^2 - x_3}}, \\
\frac{\partial}{\partial x_1}e^{x_2}\cos(4x_3) &= 0, \\
\frac{\partial}{\partial x_2}e^{x_2}\cos(4x_3) &= e^{x_2}\cos(4x_3), \\
\frac{\partial}{\partial x_3}e^{x_2}\cos(4x_3) &= e^{x_2} \cdot (-\sin(4x_3)) \cdot 4 = -4e^{x_2}\sin(4x_3).
\end{aligned}$$

Thus, the Jacobian is:

$$\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \begin{pmatrix} \frac{x_1}{\sqrt{x_1^2 - x_3}} & 0 & \frac{-1}{2\sqrt{x_1^2 - x_3}} \\ 0 & e^{x_2}\cos(4x_3) & -4e^{x_2}\sin(4x_3) \end{pmatrix}.$$

(e)

Let

$$f(\mathbf{x}) = (a(x_1), \ldots, a(x_n)), \quad f : \mathbb{R}^n \to \mathbb{R}^n.$$

The Jacobian matrix $\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}}$ is:

$$\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \begin{pmatrix} \frac{\partial a(x_1)}{\partial x_1} & 0 & \cdots & 0 \\ 0 & \frac{\partial a(x_2)}{\partial x_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial a(x_n)}{\partial x_n} \end{pmatrix}.$$

The resulting Jacobian matrix is diagonal, where each diagonal entry is $a'(x_i)$. The characteristic feature of this matrix is that it is a **diagonal matrix**.

(f)

Consider the following functions:

$$f_1(\mathbf{x}) = \sin(x_1)\cos(x_2), \quad \mathbf{x} \in \mathbb{R}^2,$$

$$f_2(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T\mathbf{y}, \quad \mathbf{x}, \mathbf{y} \in \mathbb{R}^2,$$

$$f_3(\mathbf{x}) = \sin(\log(\mathbf{x}^T\mathbf{x})), \quad \mathbf{x} \in \mathbb{R}^n.$$

Compute the gradient $\nabla_{\mathbf{x}} f_i(\cdot)$ with respect to $\mathbf{x}$ for all three functions.

Gradient of $f_1(\mathbf{x})$

$$f_1(\mathbf{x}) = \sin(x_1)\cos(x_2),$$

$$\nabla_{\mathbf{x}} f_1(\mathbf{x}) = \begin{pmatrix} \frac{\partial}{\partial x_1}\sin(x_1)\cos(x_2) \\ \frac{\partial}{\partial x_2}\sin(x_1)\cos(x_2) \end{pmatrix}.$$

Compute the partial derivatives:

$$\frac{\partial}{\partial x_1}\sin(x_1)\cos(x_2) = \cos(x_1)\cos(x_2),$$

$$\frac{\partial}{\partial x_2}\sin(x_1)\cos(x_2) = -\sin(x_1)\sin(x_2).$$

Thus, the gradient is:

$$\nabla_{\mathbf{x}} f_1(\mathbf{x}) = \begin{pmatrix} \cos(x_1)\cos(x_2) \\ -\sin(x_1)\sin(x_2) \end{pmatrix}.$$

The dimension of the gradient is $\mathbb{R}^2$.

Gradient of $f_2(\mathbf{x}, \mathbf{y})$

$$f_2(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T\mathbf{y} = \sum_{i=1}^{2} x_i y_i.$$

The gradient with respect to $\mathbf{x}$ is:

$$\nabla_{\mathbf{x}} f_2(\mathbf{x}, \mathbf{y}) = \begin{pmatrix} \frac{\partial}{\partial x_1}(\mathbf{x}^T\mathbf{y}) \\ \frac{\partial}{\partial x_2}(\mathbf{x}^T\mathbf{y}) \end{pmatrix}.$$

Compute the partial derivatives:

$$\frac{\partial}{\partial x_i}(\mathbf{x}^T\mathbf{y}) = y_i, \quad i = 1, 2.$$

Thus, the gradient is:

$$\nabla_{\mathbf{x}} f_2(\mathbf{x}, \mathbf{y}) = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \mathbf{y}.$$

The dimension of the gradient is $\mathbb{R}^2$.

Gradient of $f_3(\mathbf{x})$

$$f_3(\mathbf{x}) = \sin(\log(\mathbf{x}^T\mathbf{x})).$$

Let $g(\mathbf{x}) = \log(\mathbf{x}^T\mathbf{x})$, so:

$$f_3(\mathbf{x}) = \sin(g(\mathbf{x})).$$

Using the chain rule:

$$\nabla_{\mathbf{x}} f_3(\mathbf{x}) = \cos(g(\mathbf{x})) \cdot \nabla_{\mathbf{x}} g(\mathbf{x}).$$

Now compute $\nabla_{\mathbf{x}} g(\mathbf{x})$:

$$g(\mathbf{x}) = \log(\mathbf{x}^T\mathbf{x}),$$

$$\nabla_{\mathbf{x}} g(\mathbf{x}) = \frac{1}{\mathbf{x}^T\mathbf{x}} \cdot \nabla_{\mathbf{x}}(\mathbf{x}^T\mathbf{x}).$$

$$\nabla_{\mathbf{x}}(\mathbf{x}^T\mathbf{x}) = 2\mathbf{x}.$$

Thus:

$$\nabla_{\mathbf{x}} g(\mathbf{x}) = \frac{2\mathbf{x}}{\mathbf{x}^T\mathbf{x}}.$$

Finally:

$$\nabla_{\mathbf{x}} f_3(\mathbf{x}) = \cos(\log(\mathbf{x}^T\mathbf{x})) \cdot \frac{2\mathbf{x}}{\mathbf{x}^T\mathbf{x}}.$$

The dimension of the gradient is $\mathbb{R}^n$.

Summary of Gradients

$$\begin{aligned}
\nabla_{\mathbf{x}} f_1(\mathbf{x}) &\in \mathbb{R}^2, \\
\nabla_{\mathbf{x}} f_2(\mathbf{x}, \mathbf{y}) &\in \mathbb{R}^2, \\
\nabla_{\mathbf{x}} f_3(\mathbf{x}) &\in \mathbb{R}^n.
\end{aligned}$$

(g)

We define:

$$f(\mathbf{z}, \mathbf{y}) := \log p(\mathbf{x}, \mathbf{z}) - \log q(\mathbf{z}, \mathbf{y}),$$

$$\mathbf{z} := t(\mathbf{w}, \mathbf{y}),$$

where the functions p, q, t are differentiable and:

$$\mathbf{x} \in \mathbb{R}^D, \quad \mathbf{z} \in \mathbb{R}^E, \quad \mathbf{y} \in \mathbb{R}^F, \quad \mathbf{w} \in \mathbb{R}^F.$$

We compute the gradient $\nabla_{\mathbf{y}} f(\mathbf{z}, \mathbf{y})$ using the chain rule.

Step 1: Expand $\nabla_{\mathbf{y}} f(\mathbf{z}, \mathbf{y})$ By the chain rule:

$$\nabla_{\mathbf{y}} f(\mathbf{z}, \mathbf{y}) = \nabla_{\mathbf{z}} f(\mathbf{z}, \mathbf{y}) \cdot \frac{\partial \mathbf{z}}{\partial \mathbf{y}} + \frac{\partial f(\mathbf{z}, \mathbf{y})}{\partial \mathbf{y}}.$$

Step 2: Compute $\nabla_{\mathbf{z}} f(\mathbf{z}, \mathbf{y})$ From the definition of $f(\mathbf{z}, \mathbf{y})$:

$$f(\mathbf{z}, \mathbf{y}) = \log p(\mathbf{x}, \mathbf{z}) - \log q(\mathbf{z}, \mathbf{y}),$$

we compute:

$$\nabla_{\mathbf{z}} f(\mathbf{z}, \mathbf{y}) = \nabla_{\mathbf{z}} \log p(\mathbf{x}, \mathbf{z}) - \nabla_{\mathbf{z}} \log q(\mathbf{z}, \mathbf{y}).$$

For each term:

$$\nabla_{\mathbf{z}} \log p(\mathbf{x}, \mathbf{z}) = \frac{1}{p(\mathbf{x}, \mathbf{z})} \nabla_{\mathbf{z}} p(\mathbf{x}, \mathbf{z}),$$

$$\nabla_{\mathbf{z}} \log q(\mathbf{z}, \mathbf{y}) = \frac{1}{q(\mathbf{z}, \mathbf{y})} \nabla_{\mathbf{z}} q(\mathbf{z}, \mathbf{y}).$$

Thus:

$$\nabla_{\mathbf{z}} f(\mathbf{z}, \mathbf{y}) = \frac{\nabla_{\mathbf{z}} p(\mathbf{x}, \mathbf{z})}{p(\mathbf{x}, \mathbf{z})} - \frac{\nabla_{\mathbf{z}} q(\mathbf{z}, \mathbf{y})}{q(\mathbf{z}, \mathbf{y})}.$$

Step 3: Compute $\frac{\partial \mathbf{z}}{\partial \mathbf{y}}$ From $\mathbf{z} = t(\mathbf{w}, \mathbf{y})$, we compute:

$$\frac{\partial \mathbf{z}}{\partial \mathbf{y}} = \nabla_{\mathbf{y}} t(\mathbf{w}, \mathbf{y}),$$

which is the Jacobian matrix of $t(\mathbf{w}, \mathbf{y})$ with respect to $\mathbf{y}$.

Step 4: Compute $\frac{\partial f(\mathbf{z}, \mathbf{y})}{\partial \mathbf{y}}$ For the second term in $f(\mathbf{z}, \mathbf{y})$, only $\log q(\mathbf{z}, \mathbf{y})$ depends explicitly on $\mathbf{y}$. Thus:

$$\frac{\partial f(\mathbf{z}, \mathbf{y})}{\partial \mathbf{y}} = -\nabla_{\mathbf{y}} \log q(\mathbf{z}, \mathbf{y}),$$

and:

$$\nabla_{\mathbf{y}} \log q(\mathbf{z}, \mathbf{y}) = \frac{1}{q(\mathbf{z}, \mathbf{y})} \nabla_{\mathbf{y}} q(\mathbf{z}, \mathbf{y}).$$

Step 5: Combine Results Substituting all terms into the chain rule expansion:

$$\nabla_{\mathbf{y}} f(\mathbf{z}, \mathbf{y}) = \left(\frac{\nabla_{\mathbf{z}} p(\mathbf{x}, \mathbf{z})}{p(\mathbf{x}, \mathbf{z})} - \frac{\nabla_{\mathbf{z}} q(\mathbf{z}, \mathbf{y})}{q(\mathbf{z}, \mathbf{y})} \right) \nabla_{\mathbf{y}} t(\mathbf{w}, \mathbf{y}) - \frac{\nabla_{\mathbf{y}} q(\mathbf{z}, \mathbf{y})}{q(\mathbf{z}, \mathbf{y})}.$$

This is the gradient $\nabla_{\mathbf{y}} f(\mathbf{z}, \mathbf{y})$.

(h)

Let:

$$f(\mathbf{y}) = \sin(\mathbf{y}), \quad \mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{b},$$

where $\mathbf{A} \in \mathbb{R}^{M \times N}$, $\mathbf{x} \in \mathbb{R}^N$, and $\mathbf{b} \in \mathbb{R}^M$. The sine function, $\sin(\cdot)$, is applied element-wise. Compute the derivative $\frac{df}{d\mathbf{x}}$ using the chain rule.

Step 1: Apply the Chain Rule Using the chain rule:

$$\frac{df}{d\mathbf{x}} = \frac{\partial f}{\partial \mathbf{y}} \cdot \frac{\partial \mathbf{y}}{\partial \mathbf{x}}.$$

Step 2: Compute $\frac{\partial f}{\partial \mathbf{y}}$ The function $f(\mathbf{y}) = \sin(\mathbf{y})$ is element-wise. Its derivative with respect to $\mathbf{y}$ is also element-wise:

$$\frac{\partial f}{\partial \mathbf{y}} = \cos(\mathbf{y}).$$

This is a diagonal matrix, as the derivatives are taken element-wise:

$$\frac{\partial f}{\partial \mathbf{y}} = \mathrm{diag}(\cos(y_1), \cos(y_2), \ldots, \cos(y_M)),$$

where $\mathbf{y} = (y_1, y_2, \ldots, y_M)^T$.

Step 3: Compute $\frac{\partial \mathbf{y}}{\partial \mathbf{x}}$ From $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{b}$, the derivative is:

$$\frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \mathbf{A}.$$

Step 4: Combine Results Substitute into the chain rule:

$$\frac{df}{d\mathbf{x}} = \frac{\partial f}{\partial \mathbf{y}} \cdot \frac{\partial \mathbf{y}}{\partial \mathbf{x}}.$$

Substituting the values:

$$\frac{df}{d\mathbf{x}} = \mathrm{diag}(\cos(y_1), \cos(y_2), \ldots, \cos(y_M)) \cdot \mathbf{A}.$$

Since $\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{b}$, we can write this as:

$$\frac{df}{d\mathbf{x}} = \mathrm{diag}(\cos(\mathbf{A}\mathbf{x} + \mathbf{b})) \cdot \mathbf{A}.$$

Step 5: Dimension of $\frac{df}{d\mathbf{x}}$ The resulting matrix $\frac{df}{d\mathbf{x}}$ has the same dimensions as $\mathbf{A}$, i.e., $M \times N$.

Final Answer

$$\frac{df}{d\mathbf{x}} = \mathrm{diag}(\cos(\mathbf{A}\mathbf{x} + \mathbf{b})) \cdot \mathbf{A}, \quad \text{dimension: } M \times N.$$

2. Computational Graph and Backpropagation

(a) Introducing Intermediate Variables

For the equations:

$$A = \sqrt{a + b + c^2} + \log(a + b + c^2) + \frac{a + b + c^2}{bc^2},$$

$$B = \sum_{i=1}^{3} (w_0 + w_1 x_i - y_i)^2,$$

we introduce the following intermediate variables:

For A:

$$
\begin{aligned}
t_1 &= a + b + c^2, \\
t_2 &= \sqrt{t_1}, \\
t_3 &= \log(t_1), \\
t_4 &= bc^2, \\
t_5 &= \frac{t_1}{t_4}, \\
A &= t_2 + t_3 + t_5.
\end{aligned}
$$

For B:

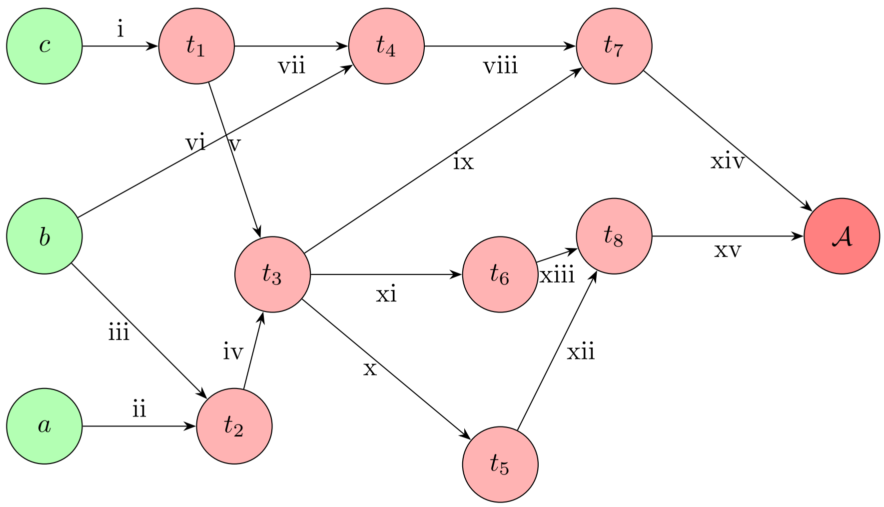
$$
\begin{aligned}
t_6^i &= w_0 + w_1 x_i, \quad i = 1, 2, 3, \\
t_7^i &= t_6^i - y_i, \quad i = 1, 2, 3, \\
t_8^i &= (t_7^i)^2, \quad i = 1, 2, 3, \\
B &= \sum_{i=1}^{3} t_8^i.
\end{aligned}
$$

(b) Drawing the Computational Graph

The computational graph consists of the following steps:

1. Nodes for the inputs: $a, b, c, w_0, w_1, x_i, y_i$. 2. Nodes for intermediate variables $t_1, t_2, \ldots, t_8^i$ as defined above. 3. Final nodes for A and B.

Computational Graph for A and B



(a)

$$
\begin{aligned}
t_1 &= c^2, \\
t_2 &= a + b, \\
t_3 &= t_1 + t_2, \\
t_4 &= b \cdot t_1, \\
t_5 &= \sqrt{t_3}, \\
t_6 &= \log(t_3), \\
t_7 &= \frac{t_3}{t_4}, \\
t_8 &= t_5 + t_6, \\
\mathcal{A} &= t_7 + t_8.
\end{aligned}
$$

$$\mathrm{xv} = \frac{\partial \mathcal{A}}{\partial \mathcal{A}} \frac{\partial \mathcal{A}}{\partial t_8}$$

$$\mathrm{xiv} = \frac{\partial \mathcal{A}}{\partial \mathcal{A}} \frac{\partial \mathcal{A}}{\partial t_7}$$

$$\mathrm{xiii} = \frac{\partial \mathcal{A}}{\partial t_8} \frac{\partial t_8}{\partial t_6}$$

$$\mathrm{xii} = \frac{\partial \mathcal{A}}{\partial t_8} \frac{\partial t_8}{\partial t_5}$$

$$\mathrm{xi} = \frac{\partial \mathcal{A}}{\partial t_6} \frac{\partial t_6}{\partial t_3}$$

$$\mathrm{x} = \frac{\partial \mathcal{A}}{\partial t_5} \frac{\partial t_5}{\partial t_3}$$

$$\mathrm{ix} = \frac{\partial \mathcal{A}}{\partial t_7} \frac{\partial t_7}{\partial t_3}$$

$$\mathrm{viii} = \frac{\partial \mathcal{A}}{\partial t_7} \frac{\partial t_7}{\partial t_4}$$

$$\mathrm{vii} = \frac{\partial \mathcal{A}}{\partial t_3} \frac{\partial t_3}{\partial t_2}$$

$$\mathrm{vi} = \frac{\partial \mathcal{A}}{\partial t_3} \frac{\partial t_3}{\partial t_1}$$

$$\mathrm{v} = \frac{\partial \mathcal{A}}{\partial t_4} \frac{\partial t_4}{\partial t_1}$$

$$\mathrm{iv} = \frac{\partial \mathcal{A}}{\partial t_4} \frac{\partial t_4}{\partial b}$$

$$\mathrm{iii} = \frac{\partial \mathcal{A}}{\partial t_2} \frac{\partial t_2}{\partial b}$$

$$\mathrm{ii} = \frac{\partial \mathcal{A}}{\partial t_2} \frac{\partial t_2}{\partial a}$$

$$\mathrm{i} = \frac{\partial \mathcal{A}}{\partial t_1} \frac{\partial t_1}{\partial c}$$

$$\mathcal{A} : \frac{\partial \mathcal{A}}{\partial \mathcal{A}}$$

$$t_8 : \frac{\partial \mathcal{A}}{\partial t_8}$$

$$t_7 : \frac{\partial \mathcal{A}}{\partial t_7}$$

$$t_6 : \frac{\partial \mathcal{A}}{\partial t_6}$$

$$t_5 : \frac{\partial \mathcal{A}}{\partial t_5}$$

$$t_4 : \frac{\partial \mathcal{A}}{\partial t_4}$$

$$t_3 : \frac{\partial \mathcal{A}}{\partial t_7}\frac{\partial t_7}{\partial t_3} + \frac{\partial \mathcal{A}}{\partial t_6}\frac{\partial t_6}{\partial t_3} + \frac{\partial \mathcal{A}}{\partial t_5}\frac{\partial t_5}{\partial t_3}$$

$$t2 : \frac{\partial \mathcal{A}}{\partial t_2}$$

$$t1 : \frac{\partial \mathcal{A}}{\partial t_4}\frac{\partial t_4}{\partial t_1} + \frac{\partial \mathcal{A}}{\partial t_3}\frac{\partial t_3}{\partial t_1}$$

$$c : \frac{\partial \mathcal{A}}{\partial c}$$

$$b : \frac{\partial \mathcal{A}}{\partial t_4}\frac{\partial t_4}{\partial b} + \frac{\partial \mathcal{A}}{\partial t_2}\frac{\partial t_2}{\partial b}$$

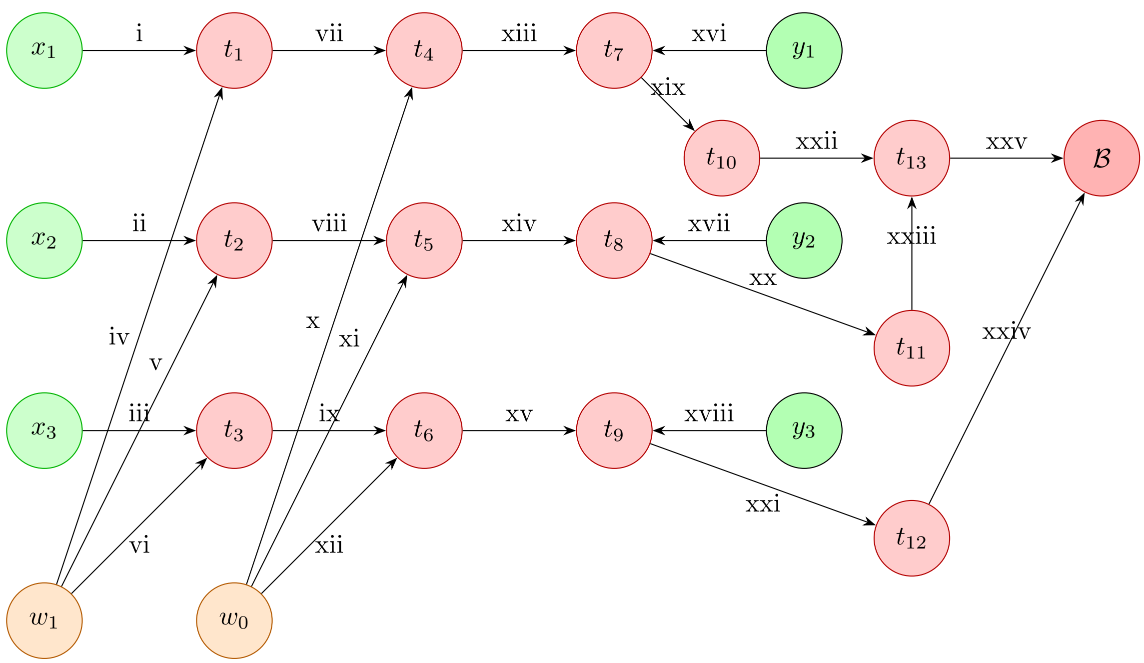
$$a : \frac{\partial \mathcal{A}}{\partial a}$$

(a) Computations

$$\mathcal{B} = \sum_{i=1}^{3}(w_0 + w_1 x_i - y_i)^2$$

$$
\begin{aligned}
t_1 &= w_1 \cdot x_1, \\
t_2 &= w_1 \cdot x_2, \\
t_3 &= w_1 \cdot x_3, \\
t_4 &= w_0 + t_1, \\
t_5 &= w_0 + t_2, \\
t_6 &= w_0 + t_3, \\
t_7 &= t_4 - y_1, \\
t_8 &= t_5 - y_2, \\
t_9 &= t_6 - y_3, \\
t_{10} &= t_7^2, \\
t_{11} &= t_8^2, \\
t_{12} &= t_9^2, \\
t_{13} &= t_{10} + t_{11}, \\
\mathcal{B} &= t_{13} + t_{12}.
\end{aligned}
$$

(b) Computational Graph



$$\mathrm{xxv} := \frac{\partial \mathcal{B}}{\mathcal{B}} \frac{\partial B}{\partial t_{13}}, \quad \mathrm{xxiv} := \frac{\partial \mathcal{B}}{\partial \mathcal{B}} \frac{\partial \mathcal{B}}{\partial t_{12}}, \quad \mathrm{xxiii} := \frac{\partial \mathcal{B}}{\partial \sqcup_{\infty \ni}} \frac{\partial t_{13}}{\partial t_{11}},$$

$$\mathrm{xxii} := \frac{\partial \mathcal{B}}{\partial t_{13}} \frac{\partial t_{13}}{\partial t_{10}}, \quad \mathrm{xxi} := \frac{\partial \mathcal{B}}{\partial t_{12}} \frac{\partial t_{12}}{\partial t_{9}}, \quad \mathrm{xx} := \frac{\partial \mathcal{B}}{\partial t_{11}} \frac{\partial t_{11}}{\partial t_{8}},$$

$$\mathrm{xix} := \frac{\partial \mathcal{B}}{\partial t_{10}} \frac{\partial t_{10}}{\partial t_{7}}, \quad \mathrm{xviii} := \frac{\partial \mathcal{B}}{\partial t_{9}} \frac{\partial t_{9}}{\partial t_{3}}, \quad \mathrm{xvii} := \frac{\partial \mathcal{B}}{\partial t_{8}} \frac{\partial t_{8}}{\partial y_{2}},$$

$$\mathrm{xvi} := \frac{\partial \mathcal{B}}{\partial t_{7}} \frac{\partial t_{7}}{\partial y_{1}}, \quad \mathrm{xv} := \frac{\partial \mathcal{B}}{\partial t_{9}} \frac{\partial t_{9}}{\partial t_{6}}, \quad \mathrm{xiv} := \frac{\partial \mathcal{B}}{\partial t_{8}} \frac{\partial t_{8}}{\partial t_{5}},$$

$$\mathrm{xiii} := \frac{\partial \mathcal{B}}{\partial t_{7}} \frac{\partial t_{7}}{\partial t_{4}}, \quad \mathrm{xii} := \frac{\partial \mathcal{B}}{\partial t_{6}} \frac{\partial t_{6}}{\partial w_{0}}, \quad \mathrm{xi} := \frac{\partial \mathcal{B}}{\partial t_{5}} \frac{\partial t_{5}}{\partial w_{0}},$$

$$\mathrm{x} := \frac{\partial \mathcal{B}}{\partial t_{4}} \frac{\partial t_{4}}{\partial w_{0}}$$

$$\mathrm{ix} := \frac{\partial \mathcal{B}}{\partial t_{6}} \frac{\partial t_{6}}{\partial t_{3}}$$

$$\mathrm{viii} := \frac{\partial \mathcal{B}}{\partial t_{5}} \frac{\partial t_{5}}{\partial t_{2}}$$

$$\mathrm{vii} := \frac{\partial \mathcal{B}}{\partial t_{4}} \frac{\partial t_{4}}{\partial t_{1}}$$

$$\mathrm{vi} := \frac{\partial \mathcal{B}}{\partial t_{3}} \frac{\partial t_{3}}{\partial w_{1}}$$

$$\mathrm{v} := \frac{\partial \mathcal{B}}{\partial t_{2}} \frac{\partial t_{2}}{\partial w_{1}}$$

$$\mathrm{iv} := \frac{\partial \mathcal{B}}{\partial t_{1}} \frac{\partial t_{1}}{\partial w_{1}}$$

$$\mathrm{iii} := \frac{\partial \mathcal{B}}{\partial t_{3}} \frac{\partial t_{3}}{\partial x_{3}}$$

$$\mathrm{ii} := \frac{\partial \mathcal{B}}{\partial t_{2}} \frac{\partial t_{2}}{\partial x_{2}}$$

$$\mathrm{i} := \frac{\partial \mathcal{B}}{\partial t_{1}} \frac{\partial t_{1}}{\partial x_{1}}$$

$$
\begin{aligned}
\mathcal{B} &: \frac{\partial \mathcal{B}}{\partial \mathcal{B}} \\
t_{13} &: \frac{\partial \mathcal{B}}{\partial t_{13}}, \quad t_{12} : \frac{\partial \mathcal{B}}{\partial t_{12}}, \\
t_{11} &: \frac{\partial \mathcal{B}}{\partial t_{11}}, \quad t_{10} : \frac{\partial \mathcal{B}}{\partial t_{10}}, \\
t_{9} &: \frac{\partial \mathcal{B}}{\partial t_{9}}, \quad t_{8} : \frac{\partial \mathcal{B}}{\partial t_{8}}, \\
t_{7} &: \frac{\partial \mathcal{B}}{\partial t_{7}}, \quad t_{6} : \frac{\partial \mathcal{B}}{\partial t_{6}}, \\
t_{5} &: \frac{\partial \mathcal{B}}{\partial t_{5}}, \quad t_{4} : \frac{\partial \mathcal{B}}{\partial t_{4}}, \\
t_{3} &: \frac{\partial \mathcal{B}}{\partial t_{3}}, \quad t_{2} : \frac{\partial \mathcal{B}}{\partial t_{2}}, \\
t_{1} &: \frac{\partial \mathcal{B}}{\partial t_{1}} \\
x_{3} &: \frac{\partial \mathcal{B}}{x_{3}} \\
x_{2} &: \frac{\partial \mathcal{B}}{x_{2}} \\
x_{1} &: \frac{\partial \mathcal{B}}{x_{1}} \\
y_{3} &: \frac{\partial \mathcal{B}}{y_{3}} \\
y_{2} &: \frac{\partial \mathcal{B}}{y_{2}} \\
y_{1} &: \frac{\partial \mathcal{B}}{y_{1}} \\
w_{1} &: \frac{\partial \mathcal{B}}{\partial t_{3}} \frac{\partial t_{3}}{\partial w_{1}} + \frac{\partial \mathcal{B}}{\partial t_{2}} \frac{\partial t_{2}}{w_{1}} + \frac{\partial \mathcal{B}}{\partial t_{1}} \frac{\partial t_{1}}{\partial w_{1}} \\
w_{0} &: \frac{\partial \mathcal{B}}{\partial t_{6}} \frac{\partial t_{6}}{\partial w_{0}} + \frac{\partial \mathcal{B}}{\partial t_{5}} \frac{\partial t_{5}}{w_{0}} + \frac{\partial \mathcal{B}}{\partial t_{4}} \frac{\partial t_{4}}{\partial w_{0}}
\end{aligned}
$$