The multi capacitated clustering problem

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Abstract

Clustering problems are combinatorial optimization problems with several industrial applications. The variant in which the individuals have distinct types and the clusters have multiple capacities for each type was not reported in the available literature. This paper aims at presenting the Multi Capacitated Clustering Problem (MCCP), a new variant of the capacitated clustering problem with the above mentioned characteristics. A 0-1 programming model is presented for the proposed variant. Preliminary experiments lead to the development of heuristics for the resolution of the MCCP in admissible computational times.

Keywords: Combinatorial Optimization; Location Theory; Integer Programming.

1. Introduction

The clustering problem is a difficult and widely studied combinatorial optimization problem, with several real world applications. It consists of defining a set of clusters with a maximum homogeneity of elements in a same group, and, concomitantly, a maximum heterogeneity between clusters. A Capacitated Clustering Problem (CCP) appears when each cluster has a bounded capacity for receive individuals.

Concerning to capacitated clustering problems, several approaches have been reported in the literature. Models and algorithms for the CCP are presented by Mulvey & Beck (1984), França et. al. (1999), Ibrahim & Christofides (1999), Ahmadi & Osman (2005), Negreiros & Palhano (2006), Chaves & Lorena (2010), Scheurer & Wendolsky (2006), Deng & Bard (2011) and Yang et al. (2011). In the reviewed literature was not found a capacitated clustering problem in which the clusters had multiple capacities.

The Multi Capacitated Clustering Problem (MCCP) consists in a capacitated clustering problem in which each cluster has a given capacity for each type of individual. If all the individuals are of the same type, the problem is reduced to the classical capacitated clustering problem. The existence of multiple types of individuals, as well as multiple capacities in the facilities, increases the complexity of this new variant of the clustering problem.

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An example of a practical application for the MCCP is the clustering of students in schools. Each student lives in a certain address and studies in a given grade level, and the schools have capacities for each type of grade level. The problem consists in allocating all the students into the multi capacitated schools, aiming the minimization of the distance between the students and the schools. Other practical application for the MCCP is the clustering of products in facilities, like, for example, a refinery. The products of several types need to be processed in the plants, since a given plant has a specific capacity for each type of product.

In Figure 1, we have a visualization of an instance for the MCCP. This instance considers eight individuals, being five individuals of type 1 (in blue) and three individuals of type 2 (in red), and also two clusters. The first cluster has a capacity of 3 individuals of type 1 and 1 individual of type 2, and the second cluster has a capacity of 2 individuals of type 1 and 2 individuals of type 2. A feasible solution for the instance is presented in Figure 1.

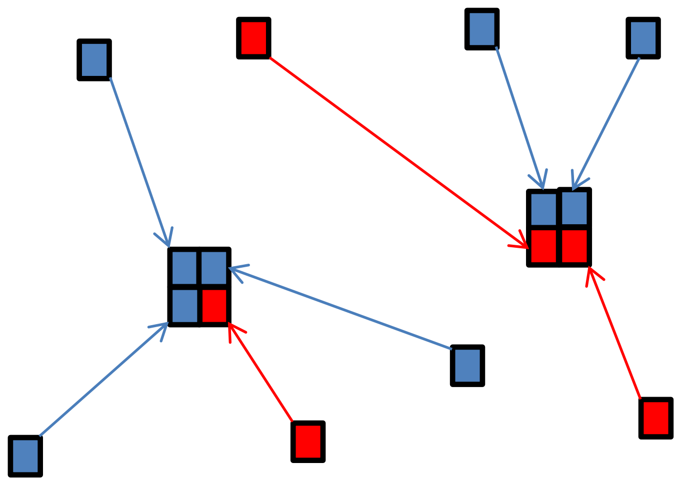

Figure 1. MCCP solution for an instance with 8 individuals and 2 clusters.

This paper aims at presenting the Multi Capacitated Clustering Problem. A mixed integer programming formulation is proposed for the above mentioned problem.

2. Problem statement

In our approach, the individuals are considered with multiple characteristics and the clusters have capacities for each type of individual. In what follows, some notation is introduced for the MCCP proposed model. Let *D* be a matrix representing the set of distance between individuals and candidates to receive a cluster.

Sets

M – individuals

N – candidates for cluster

K – types of clients

P – maximum number of clusters

Parameters

d_{ij} – distance between the i^{th} client and the j^{th} cluster.

c_{jk} – capacity of the j^{th} cluster for the k^{th} type of client.

q_{ik} – amount of the i^{th} client for the k^{th} type of client.

In this problem we want to allocate an individual for the nearest cluster, satisfying the constraint of the capacity for each cluster, for each type of client. The variables of the model are as follows.

Binary decision variables

$$x_{ij} = \begin{cases} 1, & \text{if the } i^{\text{th}} \text{ client is allocated to the } j^{\text{th}} \text{ cluster} \\ 0, & \text{otherwise} \end{cases}$$

$\forall\, i \in M, \forall\, j \in N$

$$y_j = \begin{cases} 1, & \text{if the } j^{\text{th}} \text{ potential cluster is selected} \\ 0, & \text{otherwise} \end{cases}$$

$\forall\, j \in N$

Objective: minimize the total dissimilarity between each individual in the multi capacitated clusters.

The MCCP can now be formulated as follows.

[MCCP]

minimize $$z = \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} x_{ij} \qquad (1)$$

subject to:

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad \forall\, i = 1, \ldots, m. \qquad (2)$$

$$\sum_{i=1}^{m} q_{ik} x_{ij} \leq c_{jk} \qquad \forall\, j = 1, \ldots, n\,,\ \forall\, k = 1, \ldots, p. \qquad (3)$$

$$x_{ij} - y_j \leq 0 \qquad \forall\, i = 1, \ldots, m,\ \forall\, j = 1, \ldots, n \qquad (4)$$

$$\sum_{j=1}^{n} y_i = p \qquad (5)$$

$$x_{ij} \in \{0,1\} \qquad \forall\, i = 1, \ldots, m\,,\ \forall\, j = 1, \ldots, n \qquad (6)$$

$$y_j \in \{0,1\} \qquad \forall\, j = 1, \ldots, n. \qquad (7)$$

The objective function (1) minimizes the distance between the individuals and the multi capacitated clusters. Constraints (2) impose that an individual is allocated to one cluster. Constraints (3) impose the amount of individuals, for each type, allocated for a given cluster. Constraint (4) prohibits that an individual is allocated in a cluster which is not selected. Constraint (5) bounds the maximum number of clusters. Finally constraints (6) and (7) define the scope of the model variables.

3. Conclusions

This paper introduces a new variant of the capacitated clustering problems named multi capacitated clustering problem. To the best knowledge of the author, there is in the literature no other formulation for the MCCP. Preliminary experiments, which are not reported here, lead to the development of heuristics for the resolution of the MCCP in admissible computational times.

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