

On-line recognition of handwritten mathematical symbols

Bachelor's Thesis of

Martin Thoma

At the Department of Informatics
Institute for Anthropomatics and Robotics (IAR)
Karlsruhe Institute of Technology (KIT)
Karlsruhe, Germany

School of Computer Science
Language Technologies Institute (LTI)
Carnegie Mellon University (CMU)
Pittsburgh, United States

Reviewer:	Dr. Stücker
Second reviewer:	?
Advisor:	Prof. Dr. Waibel
Second advisor:	Prof. Dr. Metze

Duration: XX. Monat 2014 – XX. Monat 2014

I declare that I have developed and written the enclosed thesis completely by myself, and have not used sources or means without declaration in the text.

Pittsburgh, DD. MM. 2014

.....
(**Martin Thoma**)

Acknowledgement

TODO



interACT



Studienstiftung
des deutschen Volkes



Baden-Württemberg
STIPENDIUM®

Ein Programm der

**Baden-
Württemberg
Stiftung**

WIR STIFTEN ZUKUNFT



Contents

1. Introduction	1
1.1. Mathematical notation	1
2. Baseline system	3
3. Data, Preprocessing and Features	5
3.1. Preprocessing	5
3.1.1. Scaling and shifting	5
3.2. Features	5
3.2.1. Local features	6
3.2.2. Global features	6
4. Artificial Neural Nets	7
4.1. Artificial neurons	7
4.2. Multilayer Perceptron	8
4.2.1. Supervised Training with Backpropagation	9
4.3. Activation functions	10
4.3.1. Unit step function	10
4.3.2. Sigmoid function	10
4.3.3. Hyperbolic tangent	10
5. Evaluation	11
5.1. Baseline system: Greedy matching	11
6. Conclusion	13
Bibliography	15
Appendix	19
A. Algorithms	19
B. Figures	19

1. Introduction

Handwriting recognition is the problem of finding a proper textual representation given a handwritten symbol or sequence of symbols.

In off-line handwriting recognition, all algorithms have to work on pixel image information of the handwriting. On-line handwriting recognition on the other hand can use the information how symbols were written. This thesis is about on-line handwriting recognition.

1.1. Mathematical notation

I will use the notation $v^{(i)}$ when I want to write about the i -th element of a vector v . Vectors will always be denoted by lowercase latin letters.

Matrices will be denoted by uppercase latin letters.

2. Baseline system

A system for symbol recognition was already written and is described in [Kir10]. It uses an algorithm called greedy matching which is similar to Dynamic time warping (DTW).

The Greedy Matching algorithm takes two series of points (A, B) and matches the first points (a, b) of A to B . The distance that point had to be moved is measured. Afterwards, it tries to match the next point a' of A to b as well as the next point b' of B and b' to a . It takes the matching in which the points had to be moved the shortest distance and continues like that.

Pseudocode is on page 19.

3. Data, Preprocessing and Features

The data that was used for all experiments was collected with write-math.com, a website designed solely for this purpose. This website makes use of HTML5 canvas elements. Those elements can be used to track fingers or a mouse cursor touching the canvas, moving and lifting. The origin is at the upper left corner and get bigger to the right (x -coordinate) and to the bottom (y -coordinate).

The data is stored and shared in JSON format. Each handdrawing is stored as a list of lines, where each line consists of tuples (x, y, t) , where x and y are canvas coordinates and t is a timestamp in seconds. This timestamp gives the time in milliseconds from 1970.

The time resolution between points as well as the resolution of the image depends on the device that was used. However, most symbols have time resolution of about 20 ms and are within a bounding box of a 250px \times 250px.

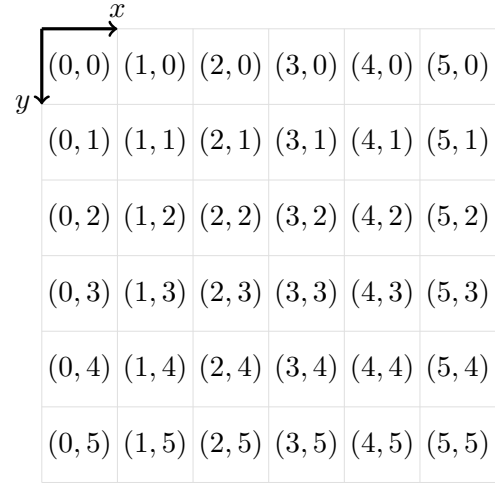


Figure 3.1.: HTML5 canvas plane. Each step is one pixel. There cannot be non-integer coordinates.

3.1. Preprocessing

3.1.1. Scaling and shifting

In many experiments the datapoints were scaled to fit into a unit square while keeping their aspect ratio. Afterwards, the points were shifted to the $(0, 1) \times (0, 1)$ unit square. The algorithm is given in pseudocode on page 20. It was shown in [Kir10, HZK09] that this kind of preprocessing boosts classification accuracy significantly.

3.2. Features

A number of different features have been suggested so far for on-line handwriting recognition. They can be grouped into local features and global features. Local features apply to a given point on the drawing plane and sometimes even only to point on the drawn curve whereas global features apply to a complete line or even the complete image.

3.2.1. Local features

- Curvature[MFW95]
- Speed[HZK09]
- Binary pen pressure[KR98, KRLP99]
- Direction[MFW95, HK06]
- Bitmap-environment[MFW95]

[KR98, KRLP99] suggest that speed is a bad feature, because it “highly inconsistent”.

3.2.2. Global features

- Re-curvature[HK06] (TODO: Explain (all))
- Center point[HK06]
- Stroke length[HK06]
- Number of strokes[HZK09]
- Pseudo-Zernike Features (TODO: Really global? What is it?)[KC]
- Shadow Code Features[KC]

4. Artificial Neural Nets

Artificial neural networks (ANNs) are models for classification that were inspired by the brain. They consist of artificial neurons and have a lot of different subtypes like Feed Forward Neural Nets.

4.1. Artificial neurons

Artificial neurons are inspired by biological neurons. Signals are sent within the cell by charged particles, so called *ions*. But before a biological neuron sends a signal, a threshold charge has to be reached at the axon hillock. This threshold charge is called *action potential*. The action potential can be reached by multiple factors, but the one I want to focus on are charges sent by other neurons. Depending on where the other axon terminals are located and how long the distance to the axon hillock is, the signal contributes more or less to reaching the action potential. After that, it simply sends a signal.

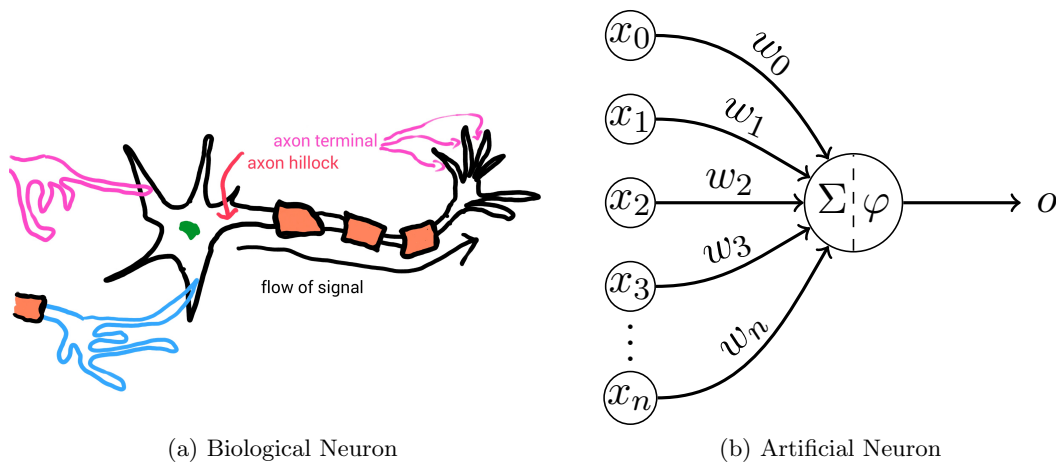


Figure 4.1.: Both neurons receive weighted input, apply a function to that and give output

Artificial neurons are similar. They receive at least one input and give at least one output. Those inputs might get weighted as well as the output.

The neurons apply a function to the sum of all weighted inputs. This function is also called *activation function*.

An artificial neuron using the unit step function (see section 4.3.1) is called a *perceptron*.

The artificial neuron sums all weighted inputs $x_i \cdot w_i$ up and applies its activation function f to it.

4.2. Multilayer Perceptron

Multilayer perceptrons (MLPs) are neural nets which neurons are structured in layers. Each layer is fully connected with the next layer, but there are no other connections between neurons.

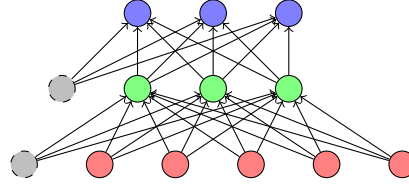


Figure 4.2.: Feedforward artificial neural network

The red neurons in figure 4.2 are input neurons, the green ones are hidden neurons and the blue one is an output node. The gray neurons are bias neurons. Bias neurons have a fixed output of 1.

Usually, you have as many output neurons as you have classes. So in the case of symbol recognition that would be about 1076 neurons.

The number of input neurons is equal to the number of features.

Two neighboring layers of neurons are fully connected and have weights between two layers. This means you can store those weights in form of matrices. Assuming you have n neurons in layer i and m neurons in layer $i + 1$, you would have a matrix

$$W_i = \begin{pmatrix} w_{1,1} & w_{1,2} & \dots & w_{1,m} \\ w_{2,1} & w_{2,2} & \dots & w_{2,m} \\ w_{3,1} & w_{3,2} & \dots & w_{3,m} \\ \vdots & & \ddots & \vdots \\ w_{n,1} & w_{n,2} & \dots & w_{n,m} \end{pmatrix}$$

This means you can easily get the input for layer $i + 1$. As the input neurons output is $x \in \mathbb{R}^{1 \times n}$ you can simply multiply $x \cdot W_1$ to get the output $x \cdot W_1 := \text{net}_j \in \mathbb{R}^{1 \times m}$.

After that, you can apply the activation function φ to i . As I assume that the activation function will be the same foreach node in the same layer l , I will simply write $\varphi_l(o)$. This means that φ_l is applied componentwise to each o_i with $i \in 1, \dots, m$.

So the output vector o of a 3-layer (input, hidden, output) neural net can be computed by

$$o = \varphi_2(\varphi_1(x \cdot W_{1,2}) \cdot W_{2,3})$$

The output of a general MLP can be computed by

$$\begin{aligned} \Phi(x, 1) &:= \varphi_1(x \cdot W_{1,2}) \\ \Phi(x, n) &:= \varphi_{n-1}(\Phi(x, n-1) \cdot W_{n-1,n}) \end{aligned}$$

4.2.1. Supervised Training with Backpropagation

The backpropagation algorithm is a supervised algorithm for training an MLP. This means the trainigset T consists of tuples (x, t) where x is input and t_x is the desired output.

The backpropagation algorithm first makes a forward pass to get to know what the current output o_x would be for each $(x, t_x) \in T$. To evaluate how good the current MLP is, an error function can be defined:

$$E : \mathbb{R}^{n_1 \times n_2} \times \mathbb{R}^{n_2 \times n_3} \times \dots \times \mathbb{R}^{n_{m-1}, m} \rightarrow \mathbb{R}_{\geq 0}$$

$$E_T(W) = \frac{1}{2} \sum_{(x, t_x) \in T} \sum_{k \in out} \left(t_x^{(k)} - o_x^{(k)} \right)^2$$

where n_i is the numer of neurons in the i -th layer and m is the number of layers and out is the set of all output neurons.

This function is isomorphic to

$$E : \mathbb{R}^{\sum_{i=2}^m n_{i-1} \cdot n_i} \rightarrow \mathbb{R}_{\geq 0}$$

This error should be minimized. As the error is the sum of non-negative values, we will get a lower error by minimizing the error for a single training example. However, note that those minimizations are not independant. This means, we could get trapped in a local minimum.

The idea is to “go” into the direction in which the error E decreases most. This is the gradient and the process is thus called *gradient descent*.

At this point we need to decide how far we want to go. If we make too big steps in the direction of the gradient, we might overshoot. If we make too small steps, the algorithm will take too long to get to the minimum. As reducing the error is basically learning the number is called learning rate $\eta \in \mathbb{R}_{>0}$.

So the algorithm is

Algorithm 1 Backpropagate

```

function BACKPROPAGATE( $T, W$ )
  while True do
    for all  $(x, t_x) \in T$  do
      for all node  $i$  do
        for all nodes  $j$  following  $i$  do
           $w_{ij} \leftarrow w_{ij} - \eta \frac{\partial E_{\{x\}}}{\partial w_{ij}}(W)$ 
```

Computing the partial derivatives $\frac{\partial E_{\{x\}}}{\partial w_{ij}}$ is not a trivial task. To do that, we have to take a closer look at the error function:

Let n be the number of layers of the MLP.

$$\frac{\partial E_{\{x\}}}{\partial w_{ij}} = \frac{\partial}{\partial w_{ij}} \frac{1}{2} \sum_{(x,t_x) \in T} \sum_{k \in \text{out}} \left(t_x^{(k)} - o_x^{(k)} \right)^2 \quad (4.1)$$

$$= \frac{1}{2} \sum_{(x,t_x) \in T} \sum_{k \in \text{out}} \frac{\partial}{\partial w_{ij}} \left(t_x^{(k)} - o_x^{(k)} \right)^2 \quad (4.2)$$

$$= \frac{1}{2} \sum_{(x,t_x) \in T} \sum_{k \in \text{out}} 2 \left(t_x^{(k)} - o_x^{(k)} \right) \cdot \frac{\partial}{\partial w_{ij}} \left(t_x^{(k)} - o_x^{(k)} \right) \quad (4.3)$$

$$= - \sum_{(x,t_x) \in T} \sum_{k \in \text{out}} \left(t_x^{(k)} - o_x^{(k)} \right) \cdot \frac{\partial}{\partial w_{ij}} \left(\varphi_{n-1}(\Phi(x, n-1) \cdot W_{n-1,n})^{(k)} \right) \quad (4.4)$$

At this point, it doesn't get simpler except if you know where w_{ij} is. If w_{ij} is for example a weight for an output node, you can apply the chain rule¹ once:

$$\frac{\partial E_{\{x\}}}{\partial w_{ij}} = - \sum_{(x,t_x) \in T} \sum_{k \in \text{out}} \left(t_x^{(k)} - o_x^{(k)} \right) \cdot \left(\frac{\partial}{\partial w_{ij}} \varphi_{n-1} \right) (\Phi(x, n-1) \cdot W_{n-1,n})^{(k)} \quad (4.5)$$

$$(4.6)$$

4.3. Activation functions

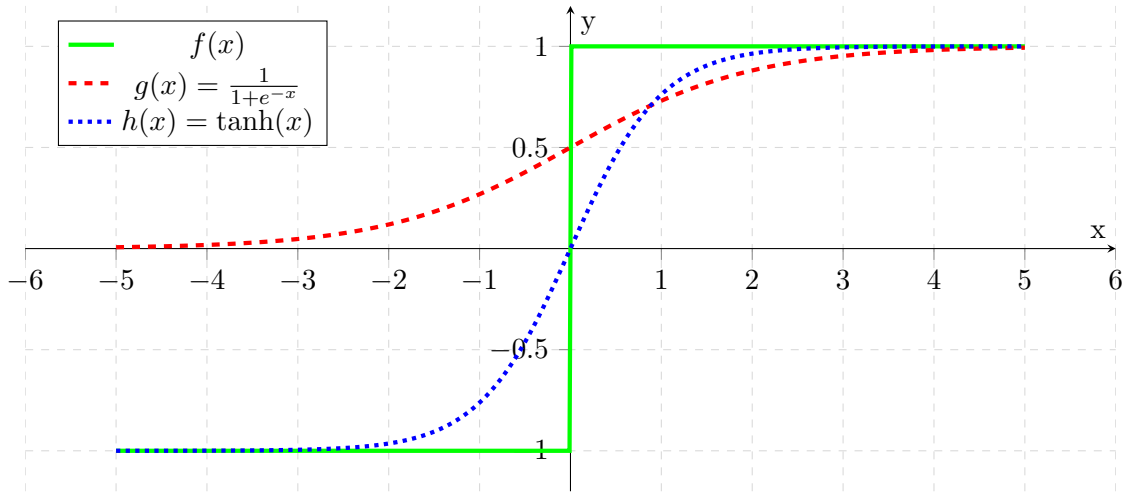


Figure 4.3.: A variation of the sign function f with $f(0) = -1$, the sigmoid function g and the hyperbolic tangent h .

4.3.1. Unit step function

Not so good, because it's not differentiable. Therefore, the backpropagation algorithm cannot be used.

4.3.2. Sigmoid function

Is great because it is infinitely often differentiable.

4.3.3. Hyperbolic tangent

Also differentiable, but gradient descent converges faster (sometimes?)

¹ $(f(g))' = f'(g) \cdot g'$

5. Evaluation

A recognition system has two important characteristics: Its recognition accuracy and the time it needs to recognize a new symbol. Recognition accuracy and time are measured with new data which was not seen before.

Tests can be divided into two groups: Tests where some examples of the handwriting of the writer were known at training time and tests where that's not the case.

Known-writer tests are created this way:

Algorithm 2 Creation of k bins of datasets

```
data  $\leftarrow$   $k$ -dimensional array of Lists  
 $i \leftarrow 0$   
Group labeled datasets by symbol  
Filter all symbols that have less than  $k$  datasets  
for all Group  $g$  in datasets do  
  for all Dataset  $(x, t)$  in  $g$  do  
    data[ $i$ ].APPEND( $(x, t)$ )  
     $i \leftarrow (i + 1) \bmod k$ 
```

After that, a k -fold cross validation is run:

I will call result of such a 10-fold cross-validation *classification accuracy*. The first part of the tuple is called *Top-1 accuracy* and the second one is called *Top-10 accuracy*.

5.1. Baseline system: Greedy matching

The greedy matching algorithm got with scaling and shifting (see page 20) a Top-1 accuracy of 83.11% and a Top-10 accuracy of 97.66%.

Algorithm 3 k -fold cross-validation

```

function CROSSVALIDATION( $k$ , grouped dataset  $d$ , classifier  $c$ )
  correct, wrong  $\leftarrow 0, 0$ 
  c10, w10  $\leftarrow 0, 0$ 
  for  $i \in 0, \dots, k-1$  do
    for  $j \in 0, \dots, k-1$  do
      if  $i \neq j$  then
         $c$ .TRAIN( $d[i]$ )
    for all  $(x, t) \in d[i]$  do  $\triangleright$  List of possible classifications descending by probability
       $L \leftarrow c$ .CLASSIFY( $x$ )
      if  $L[0] == t$  then
        correct  $\leftarrow$  correct + 1
        c10  $\leftarrow$  c10 + 1
      else if  $t \in L$  then
        c10  $\leftarrow$  c10 + 1
        wrong  $\leftarrow$  wrong + 1
      else
        w10  $\leftarrow$  wrong + 1
        wrong  $\leftarrow$  wrong + 1
  return ( $\frac{\text{correct}}{\text{correct}+\text{wrong}}, \frac{\text{c10}}{\text{c10}+\text{w10}}$ )

```

6. Conclusion

...

Bibliography

- [HK06] B. Huang and M.-T. Kechadi, “An HMM-SNN method for online handwriting symbol recognition,” in *Image Analysis and Recognition*, ser. Lecture Notes in Computer Science, A. Campilho and M. Kamel, Eds. Springer Berlin Heidelberg, 2006, vol. 4142, pp. 897–905. [Online]. Available: http://dx.doi.org/10.1007/11867661_81
- [HZK09] B. Q. Huang, Y. Zhang, and M.-T. Kechadi, “Preprocessing techniques for online handwriting recognition,” in *Intelligent Text Categorization and Clustering*, ser. Studies in Computational Intelligence, N. Nedjah, L. de Macedo Mourelle, J. Kacprzyk, F. França, and A. de De Souza, Eds. Springer Berlin Heidelberg, 2009, vol. 164, ch. Preprocessing Techniques for Online Handwriting Recognition, pp. 25–45. [Online]. Available: http://dx.doi.org/10.1007/978-3-540-85644-3_2
- [KC] A. Khotanzad and C. Chung, “Hand written digit recognition using combination of neural network classifiers.” [Online]. Available: <http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=666880>
- [Kir10] D. Kirsch, “Detexify: Erkennung handgemalter LaTeX-symbole,” Diploma thesis, Westfälische Wilhelms-Universität Münster, 10 2010. [Online]. Available: <http://danielkirs.ch/thesis.pdf>
- [KR98] A. Kosmala and G. Rigoll, “Recognition of on-line handwritten formulas,” in *In Proceedings of the Sixth International Workshop on Frontiers in Handwriting Recognition*, 1998, pp. 219–228. [Online]. Available: <http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.41.9056>
- [KRLP99] A. Kosmala, G. Rigoll, S. Lavirotte, and L. Pottier, “On-line handwritten formula recognition using hidden markov models and context dependent graph grammars,” in *Proceedings of the Fifth International Conference on Document Analysis and Recognition (ICDAR)*, 1999, pp. 107–110. [Online]. Available: http://hal.inria.fr/docs/00/56/46/45/PDF/kosmala-rigoll-etal_1999.pdf
- [MFW95] S. Manke, M. Finke, and A. Waibel, “The use of dynamic writing information in a connectionist on-line cursive handwriting recognition system,” in *Advances in Neural Information Processing Systems* 7, G. Tesauro, D. Touretzky, and T. Leen, Eds. MIT Press, 1995, pp. 1093–1100. [Online]. Available: <http://papers.nips.cc/paper/908-the-use-of-dynamic-writing-information-in-a-connectionist-on-line-cursive-handwriting-recognition-system.pdf>

Glossary

ANN artificial neural network. 7

DTW dynamic time warping. 3

MLP multilayer perceptron. 8, 9

Appendix

A. Algorithms

Algorithm 4 Greedy matching as described in [Kir10]

```
 $a \leftarrow \text{next from } A$   
 $b \leftarrow \text{next from } B$   
 $d \leftarrow \delta(a, b)$   
 $a' \leftarrow \text{next from } A$   
 $b' \leftarrow \text{next from } B$   
while points left in  $A \wedge$  points left in  $B$  do  
   $l, m, r \leftarrow \delta(a', b), \delta(a', b'), \delta(a, b')$   
   $\mu \leftarrow \min \{l, m, r\}$   
   $d \leftarrow d + \mu$   
  if  $l = \mu$  then  
     $a \leftarrow a'$   
     $a' \leftarrow \text{next from } A$   
  else if  $r = \mu$  then  
     $b \leftarrow b'$   
     $b' \leftarrow \text{next from } B$   
  else  
     $a \leftarrow a'$   
     $b \leftarrow b'$   
     $a' \leftarrow \text{next from } A$   
     $b' \leftarrow \text{next from } B$   
if no points left in  $A$  then  
  for all points  $p$  in  $B$  do  
     $d \leftarrow d + \delta(a', p)$   
else if no points left in  $B$  then  
  for all points  $p$  in  $A$  do  
     $d \leftarrow d + \delta(b', p)$ 
```

B. Figures

ein Bild

Figure B.1.: A figure

Algorithm 5 Scale and shift a list of lines to the $(0, 1) \times (0, 1)$ unit square

```

function SCALE_AND_SHIFT(pointlist)
   $min_x, min_y = pointlist[0][x'], pointlist[0][y']$ 
   $max_x, max_y = pointlist[0][x'], pointlist[0][y']$ 
  for all lines in pointlist do
    for all p in lines do
      if  $p[x'] < min_x$  then
         $min_x \leftarrow p[x']$ 
      else if  $p[x'] > max_x$  then
         $max_x \leftarrow p[x']$ 
      if  $p[y'] < min_y$  then
         $min_y \leftarrow p[y']$ 
      else if  $p[y'] > max_y$  then
         $max_y \leftarrow p[y']$ 
   $width, height \leftarrow max_x - min_x, max_y - min_y$ 
   $factor = 1$ 
  if  $width \neq 0$  then
     $factor_x = \frac{1}{width}$ 
  if  $height \neq 0$  then
     $factor_y = \frac{1}{height}$ 
   $factor = \min(factor_x, factor_y)$ 
   $add_x, add_y = 0, 0$ 
  for all lines in pointlist do
    for all p in lines do
       $p[x'] \leftarrow (p[x''] - min_x) \cdot factor$ 
       $p[y'] \leftarrow (p[y''] - min_y) \cdot factor$ 
  return pointlist

```
