Design of a Self-Tuning Hierarchical Fuzzy Logic Controller for Nonlinear Swing Up and Stabilizing Control of Inverted Pendulum

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***Abstract*—Fuzzy logic controllers suffer from rule explosion problem as the number of rules increases exponentially with the number of input variables. Although several methods have been proposed for eliminating the combinatorial rule explosion problem, none of them is fully satisfactory. In this paper, we describe a new adaptive method for the design of cascaded layer based hierarchical fuzzy system with high input dimensions. This new adaptive hierarchical architecture could be applied to dimensionality reduction in fuzzy modeling. An evolutionary algorithm based off-line leaning algorithm is employed to generate the fuzzy rules and their corresponding membership functions. The evolutionary learning paradigm is a powerful tool to tune the fuzzy logic controllers since it requires no prior knowledge about the system's behavior in order to formulate a set of functional control rules through adaptive learning. The resulting hierarchical fuzzy system has not only an equivalent approximation capability, but less number of fuzzy rules compared with the conventional fuzzy logic system. Simulation studies exhibit competing results with high accuracy that illustrating the effectiveness of the approach.**

Keywords—*Fuzzy control, Adaptive hierarchical fuzzy logic system, Evolutionary Algorithms, Optimization, Cart Pole type Inverted Pendulum*

I. INTRODUCTION

Fuzzy logic systems (FLS) are fundamental methodologies to represent and process linguistic information, with mechanisms to deal with uncertainty and imprecision. With such remarkable attributes, FLS have been widely and successfully applied to control [1], classification [2] and modeling problem and in a considerable number of applications [3]-[5]. During the design of fuzzy logic controllers (FLCs) one of the most important factors is how to reduce the number of involved fuzzy control rules and their corresponding computation requirements. As the total number of input and output variables increase, the size of the decision table and thereby the number of elemental fuzzy rules of the system grows exponentially [6]. In the fuzzy logic controllers there is a direct relationship between the number of membership functions of input variables of the system and the number of fuzzy rules. The number of rules is defined by $r = m^n$ where m is the number of fuzzy sets for each input and n is the number of inputs into the fuzzy system. Because of this exponential growth, the problem is referred to as the rule explosion problem.

To overcome the problem, various approaches have been investigated. One approach is to reduce the number of membership functions (MFs) that each input has. However, this may reduce the accuracy of the system [7]. As an alternative, the number of rules in the rule base can also be trimmed if it is known that some rules are never used [8]. In certain circumstances, the rule reduction process to design Mamdani FLCs yields unsatisfactory results [9]. The most obvious is to limit the number of inputs that the system is using. Again, this may reduce the accuracy of the system, and in many cases, render the system being modeled unusable. Another most important idea of using hierarchical structure in designing a FLCs has been investigated [6][10]-[11].

In this paper, to solve the rule explosion problem in multi input FLS; a method is described for automating the design of an hierarchical fuzzy logic controller (HFLC) using an evolutionary algorithm. Evolutionary algorithms are employed as an adaptive method for the selection of fuzzy rules and their corresponding MFs that can provide effective control of the systems. We describe how a control system evolves through the self-adaptive learning of FLCs. In order to increase the system reliability and adaptability, each layer of hierarchy achieves a new class of goals. The proposed system has been evaluated to the control of cart pole type inverted pendulum non-linear system.

The remaining of the paper is organized as follows: In section II, previous methods of HFLC are given. In Section III, the dynamic model of the cart pole type inverted pendulum is described. In Section IV, the new hierarchical architecture is demonstrated and learning algorithms for hierarchical fuzzy systems are discussed in this section as well. In Section V, Experiment results that demonstrate the efficiency of the proposed methodology are given. Finally, in Section VI, some concluding remarks and some future directions are stated.

II. HIERARCHICAL FUZZY SYSTEMS: BRIEF OVERVIEW

In this section, we describe the related works in the field of hierarchical fuzzy system. Raju and Zhou, in [6][10], suggested using a hierarchical fuzzy logic structure for such fuzzy logic systems to overcome rule explosion problem. In their proposed approach, input variables are put into a collection of low dimensional fuzzy logic units (FLUs) and the outputs of the FLUs are used as the input variables for the FLUs in the next layer. In their hierarchical structure, the most influential system variables are applied to the first layer, the next most important variables at the next level, and so on. Yager [11] proposed a hierarchy called hierarchical prioritized structure (HPS) where specific fuzzy rules override the more general ones. Adhering to fuzzy rule explosion problem, Chung el al. [12] has presented a hierarchical fuzzy system where fuzzification and defuzzification are performed at every level.

Many researchers have explored the use of evolutionary algorithms to tune the key hierarchical FLC factors. The MFs is a key in designing the FLC. These researches differ mostly in the sequence or selection of different shapes, widths and distributions of MFs in the HFLC performance. Moreover, differences between the previous approaches lie mainly in the type of EA coding and how rule sets and MF shapes and widths are optimized.

III. PROBLEM STATEMENT

Being an under-actuated mechanical system and inherently open loop unstable with highly non-linear dynamics, the inverted pendulum system is a perfect test-bed for the design of a wide range of classical and contemporary control techniques. The cart pole balancing problem is a popular demonstration of using feedback control to stabilize an open-loop unstable system with fewer control inputs than the degrees of freedom. The cart-pole task involves a balancing pole hinged to a motion less cart that travels left or right along a straight bounded track as shown in Fig. 1. The pole is free to rotate only in the vertical plane of the cart and track. There are no sidelong resultant forces on the pole and it remains balanced as shown in Fig. 2.

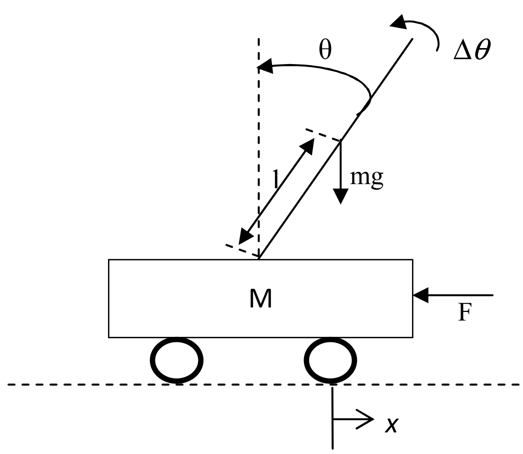


Figure 1. Cart-pole typed inverted pendulum system

The control objective is to apply a sequence of left or right forces of fixed magnitude to the wheeled cart so that it swings up the pendulum from its natural pendant position and stabilizes in the inverted position, once it reaches the upright equilibrium point (Fig.2). The cart must also be homed to a reference position on the rail. Here, the system state is specified by four real-valued variables: x-the horizontal position of the cart; Δx -the velocity of the cart (rate at which the error of position changes); θ, the angle of the pole/shaft with respect to the vertical line; $\Delta\theta$, the angular velocity of the pole/shaft. The force $F \in$ [-10, 10] newton's is applied to the cart and a zero magnitude force is not permitted. The dynamics of the cart-pole system are modeled by the following non-linear differential equations:

$$\dot{x} = \Delta x$$

$$\Delta\dot{x} = F + ml(Sin(\theta)\Delta\theta^2 - \Delta\dot{\theta}\, Cos(\theta))/(M + n)$$

$$\dot{\theta} = \Delta\theta$$

$$\ddot{\theta} = \frac{g \sin\theta + \cos\theta \left(\dfrac{-F - ml\theta^2 Sin\theta}{M + m} \right)}{l \left(\dfrac{4}{3} - \dfrac{m \cos^2\theta}{M + m} \right)}$$

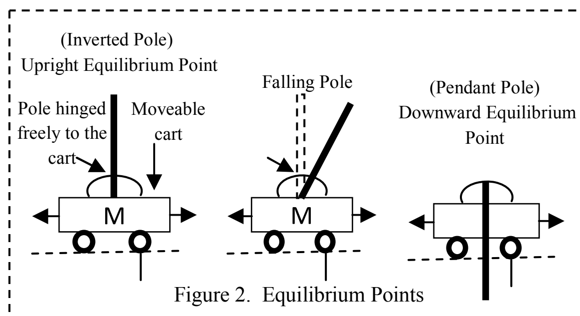


Figure 2. Equilibrium Points

IV. PROPOSED HFLC: DESIGN AND METHODOLOGY

In this section, we first describe why hierarchical architecture is used for designing a fuzzy control system in short then briefly describe our proposed approach. A conventional fuzzy logic controller would typically consist of one fuzzy block with an input for each state variable (x_i) and an output for each control action (Y_i), as shown in Fig. 3. Each input is explicitly associated to the other inputs through rule base in the knowledgebase. As mentioned before, this conventional approach is impractical due to the rule explosion problem for complex control system with large input and output variables. For the cart-pole type inverted pendulum system if each of the n=4 variables partition into five MFs then there are 625 rules in the rule base (5^4 = 625). An initial analysis of the learning of fuzzy rules in a single block fuzzy system was given in [13]. The hierarchical structure of a fuzzy logic controller results from the desire to achieve a system goal for a complex process using a divide and conquer strategy. The goal of the paper is to develop a HFLC controller that is aggressively effective, computationally efficient, and easily adaptable to unknown environments.

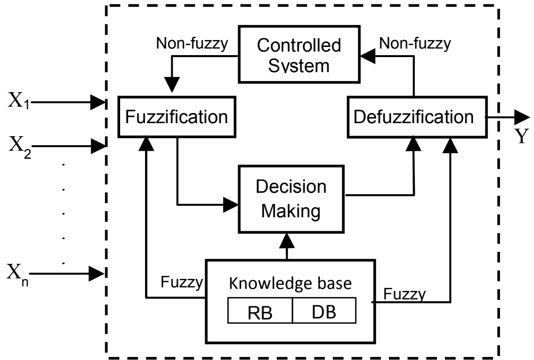


Figure 3. Single layer FLC

The proposed HFLC shown in Fig. 4 and Fig. 6, are an alternative to the conventional, single-block fuzzy controller. Three and two layered hierarchical fuzzy logic topology is used to create a fuzzy rule base for the control system. In this two layer architecture (Fig.3), lower layer FLCs control closely related to the system variables (such as the angle and angular velocity of a particular system variable) while higher layer FLCs incorporate the outputs of pairs of lower level sub-controllers into an intermediate or final control output. This hierarchical architecture uses the hierarchical knowledge base (Fig.5) for the fuzzification, defuzzification and fuzzy inference systems. In this way hierarchical fuzzy inference system not only provides a more complex and flexible architecture for modeling nonlinear systems, but can also reduce the size of rule base to a considerable extent. Analyzing the dynamics of cart pole type inverted pendulum control system, and dividing the control tasks allows adequate control with much fewer rules than required by the conventional fuzzy controller. If the inputs are not tightly coupled, the hierarchical controller can achieve adequate performance with fewer rules. HFLCs are used to solve the actual problems of multiple variable inputs and to make hierarchical control such as the global decision making (high level) and local control action (lower level).

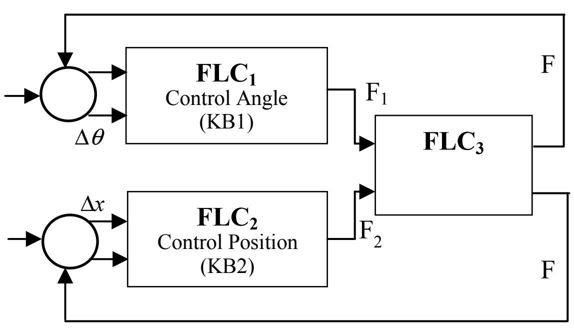


Figure 4. Two Layer Hierarchical Fuzzy System

Three different FLCs are designed to build an HFLC for the swing up and stabilizing control of cart pole type inverted pendulum. First, FLC is designed for controlling the pole, second FLC for controlling the cart and a third one for combining the outputs of the first and second FLCs. Figure 4 shows the schematic of the inputs and outputs of angle and position control FLC. The total Force is a determined through another fuzzy logic controller named FLC3.

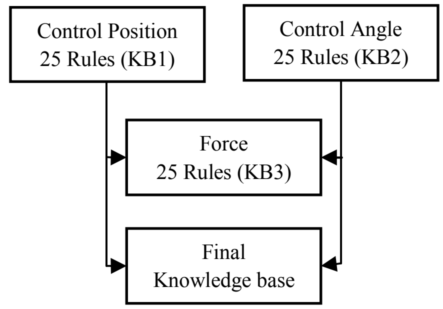


Figure 5. Hierarchical Knowledge base

Another three layer hierarchical control architecture shown in Fig. 6 consists of three FLCs: control the angle, position and angular displacement of the pendulum. The input configuration of this hierarchical control architecture is as follows: two input variables in the first layer control the pendulum angle and one input variable with intermediate control output (F1 and F2) in layer 2 and 3.

B. Evolutionary Algorithm

Evolutionary algorithms have been widely used to derive fuzzy rules and their associates MFs for efficient design of FLCs [14]. In our study, we will consider a specific GA [15] which evolves with a good trade-off between exploration and exploitation, and thus, being a good choice in problems with complex search spaces. The selection mechanism is that N parents and theirs generated offspring are combined to select the best N chromosomes to take part in the population of next generation. In order to create the diversity in the population, the proposed GA used a incest prevention mechanism and a restarting approach, instead of the well-known mutation operator.

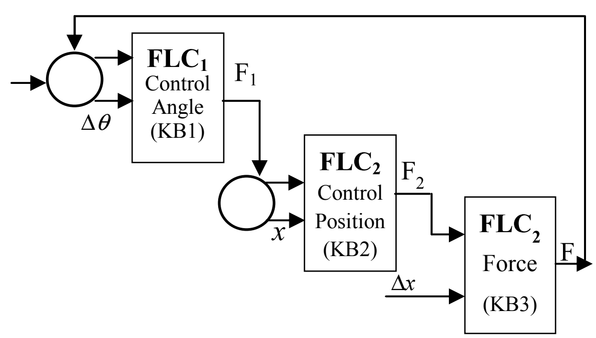


Figure 6. Three Layer Hierarchical Fuzzy System

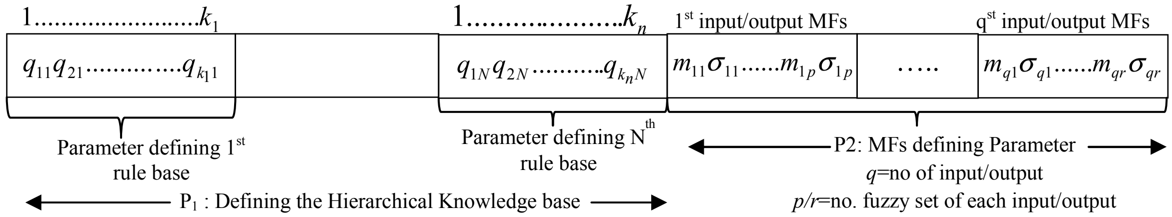


Figure 7. GA coding schema of HFLC

C. Coding of Rule base and their Associated MFs

The coding scheme allows a flexible representation of fuzzy rules and their associated MFs in a genetic string. The consequent parameters of each rule base and their corresponding MFs are coded as real numbers. Our FLCs has both input and output variables, where the universe of discourse of each of the variables is covered by fixed fuzzy sets given by the designer in advance. Let us consider a cart pole type inverted pendulum control problem with four inputs and one output variable and a database defined from expert knowledge determining the MFs for the following labels:

Inputs	MFs
x	NH: Negative High,
Δx	NL: Negative Low, ZE: Zero,
θ	PL: Positive Low
$\Delta\theta$	PH: Positive High

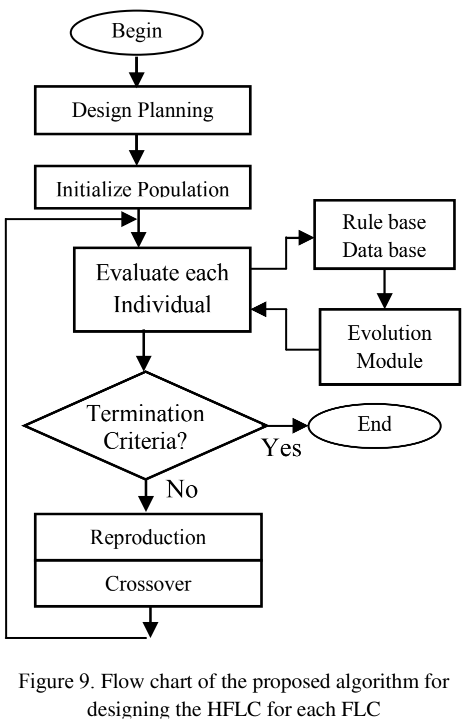


Figure 9. Flow chart of the proposed algorithm for designing the HFLC for each FLC

For FLC_1 the fuzzy control rules can be represented as a linguistic matrix shown in table I where the input variables θ and $\Delta\theta$ with the linguistic level sets {NH, NL, ZE, PL, PH} and {NH, NL, ZE, PL, PH} respectively. Each cell of the linguistic matrix denotes the linguistic values of the consequent part of a rule.

TABLE I. REPRESENTATION OF FUZZY CONTROL RULE MATRIX

F_1		θ				
		NH	NL	ZE	PL	PH
$\Delta\theta$	NH	$q_{11}(NH,NH)$				
	NL					
	ZE					
	PL					
	PH					$q_{k_1 1}(PH,PH)$

For the hierarchical fuzzy control system if it needs $N = \{1, 2, 3, ...n\}$ rule bases and their corresponding maximum number of rules is given by the set, $K = \{k_1, k_2, k_3........k_n\}$. For example, our proposed hierarchical architecture for cart pole type inverted pendulum system requires $N = 3$ rule base and $k_1 = 25$ for FLC_1, $k_2 = 25$ for FLC_2 and $k_3 = 25$ for FLC_3 respectively. The coding schema of GA chromosome can be written as shown in Fig.7. Each chromosome has two parts $(P = P_1 + P_2)$, P_1, P_2: vector of real numbers with size S_1 and S_2 respectively that depends on the problem being solved. P_1 is used to define the hierarchical knowledge base. The symbol $q_{k_n N}$ is an identifier corresponding to the consequent of the rules. The parameter of P_2 is used to define the MFs of input and output variables. The symbol $m_{qr}\sigma_{qr}$ is an identifier corresponding to the MFs where m_{qr} and σ_{qr} represent the center of r^{th} linguistic term fuzzy set of q^{th} input/output variables and width of the corresponding MFs respectively. The Gaussian MFs (Fig. 8) was chosen because of its simplicity and popularity.

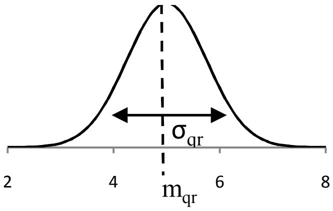


Figure 8. A Gaussian MFs type fuzzy set

The firing strength of a fuzzy rule is calculated by the mechanism which is used to implement the **And** operation in the antecedent part of the rules. In this paper, we propose the product to measure the degrees of each variable to its corresponding membership functions. The firing strength is then used to shape the output fuzzy set that represents the consequent part of the rule. Therefore, the firing strength of each rule can be defined as:

$$\phi = \prod_{i=1}^{j} \exp\{-\frac{(x_i - m_i)^2}{\sigma_i^2}\}$$
$$= \exp\{-\sum_{i=1}^{j} \frac{(x_i - m_i)^2}{\sigma_i^2}\}$$

TABLE II (A). FUZZY RULE MATRIX FOR FLC_1

F_1		θ				
		NH	NL	ZE	PL	PH
$\Delta\theta$	NH	[1]NH	[2]NL	[3]PL	[4]NL	[5]ZE
	NL	[6]NH	[7]NL	[8]NH	[9]ZE	[10]PL
	ZE	[11]NL	[12]NH	[13]ZE	[14]NH	[15]NL
	PL	[16]NL	[17]ZE	[18]PL	[19]PH	[20]PL
	PH	[21]ZE	[22]PH	[23]PL	[24]NH	[25]PL

TABLE II (B). FUZZY RULE MATRIX FOR FLC2

F_2		x				
		NH	NL	ZE	PL	PH
Δx	NH	[1]PH	[2]NL	[3]NH	[4]ZE	[5]NL
	NL	[6]PL	[7]NL	[8]PL	[9]ZE	[10]NH
	ZE	[11]NH	[12]PH	[13]ZE	[14]NL	[15]NL
	PL	[16]ZE	[17]NH	[18]PH	[19]PL	[20]PH
	PH	[21]PH	[22]NL	[23]PH	[24]NL	[25]NH

TABLE II (C). FUZZY RULE MATRIX FOR FLC3

F		F_1				
		NH	NL	ZE	PL	PH
F_2	NH	[1]NL	[2]NH	[3]ZE	[4]PH	[5]NH
	NL	[6]ZE	[7]PH	[8]NL	[9]NH	[10]NL
	ZE	[11]ZE	[12]PH	[13]PH	[14]ZE	[15]ZE
	PL	[16]NH	[17]ZE	[18]PH	[19]NH	[20]PH
	PH	[21]PL	[22]ZE	[23]NL	[24]NL	[25]PL

D. Evolutionary Learning Paradigm

The functional schema of genetic learning module is shown in Fig.9. At first design the fuzzy control system and the design includes: identifying controller inputs and outputs, determining number of fuzzy partitions, dividing the inputs outputs space into fuzzy regions, choosing type of MFs, fuzzifier and defuzzifier and defining inference engine. Then the first generations of chromosomes are generated randomly. Each generated chromosome is a potential solution to the given problem and represents a fuzzy rule set and their corresponding MFs. After the initialization, each potential solution (decoded chromosome) is evaluated and assigned a fitness value according to the solving performance of the problem. The next step is to sort the individuals according to their fitness values. The final step is checking termination criterion. If satisfied algorithm stops and returns the best fitted chromosome (set of fuzzy rules and shape/width of MFs), otherwise, generate the new chromosome through reproduction as well as a new generation.

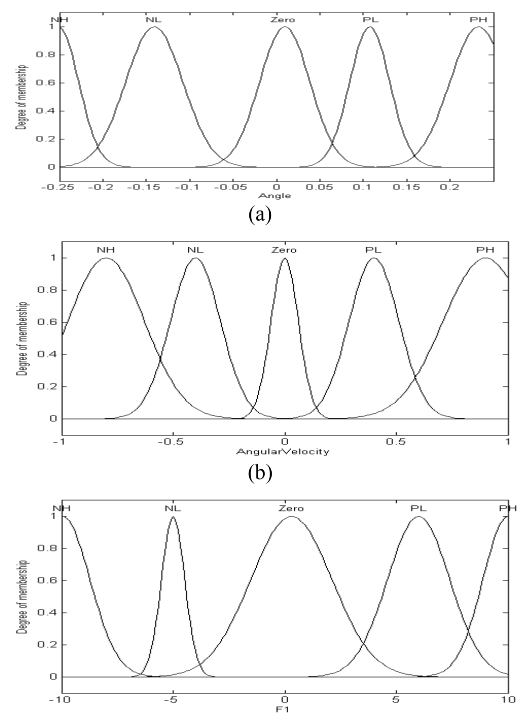
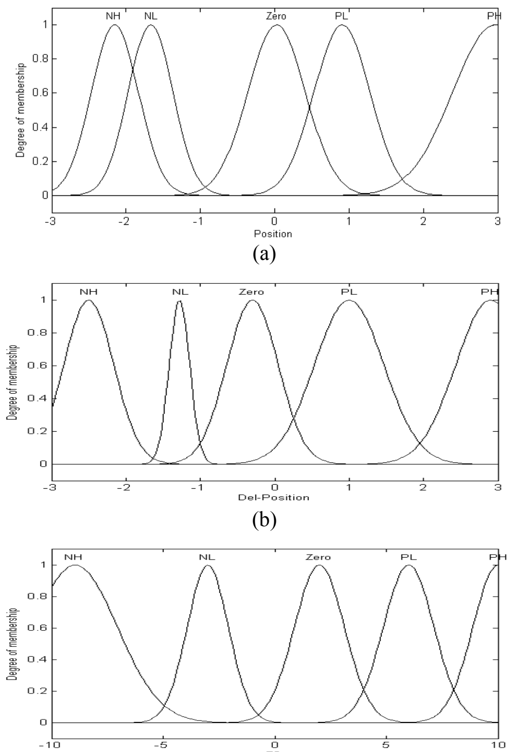


Figure 10. Tuned Membership function for FLC_1 (a) Angle (□) (b) Angular Velocity ($\Delta\theta$) (c) Force F_1

V. SIMULATION RESULTS AND EVALUATION

This section describes our simulation results for the cart pole type inverted pendulum system described earlier. The proposed HFLC has been tested for inverted pendulum system on the following initial angles: 20^0, -20^0, 15^0, and -15^0 degrees. At the same time cart position controller has been simulated on the following distances from the centre of the track: 2.5, -2, 1.5, and -1.5 meters. The rule sets are discovered by GA Learning module without any guidance from human experts as shown in table. 1I(A), II(B), and II(C).

In our proposed system, we tune/adjust the 7 MFs sets instead of 9 because of the output of FLC_1 and output of FLC_2 and the inputs of FLC_3 use the same MFs. The MFs found by EA for the HFLC are shown in Fig. 10 and Fig.11. Figure 12 shows the MFs for the variables quantifying the external final force (F) applied to the cart. The generated control surfaces define the degrees of nonlinearities between the input and output variables are shown in Fig. 15.



(a) (b) (c)

Figure 11. Tuned Membership function for FLC_2 (a) Position (x) (b) Angular displacement (Δx) (c) Force F_2

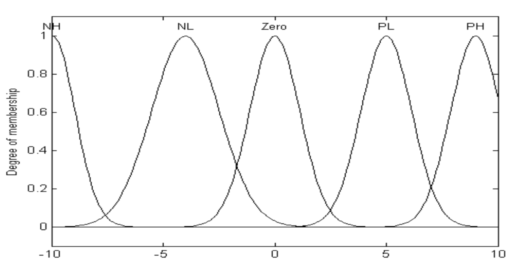
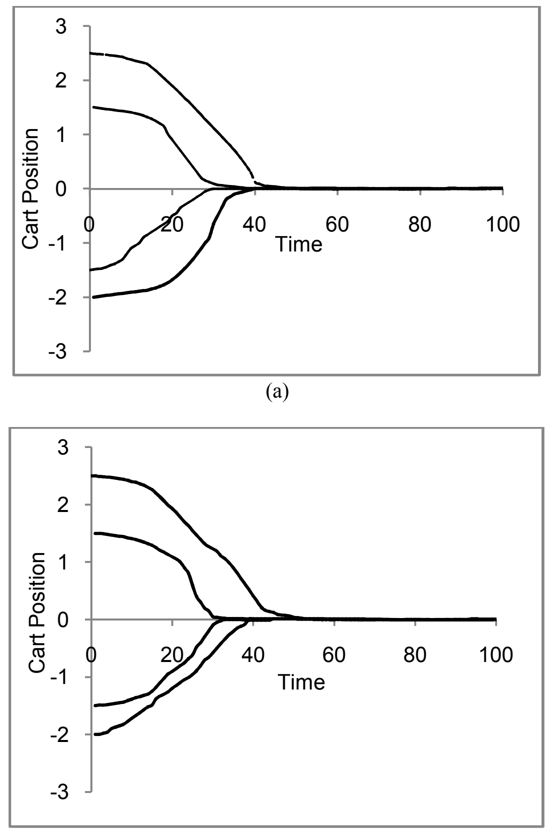


Figure 12. Final Force (F) Membership function for FLC_3

Figure 14 shows a simulation plot of θ, during stabilization with non-zero angle initial conditions and momentary pole external disturbances. Initially the pendulum is in the pendant position. It swings-up gradually, responding to the bounded oscillations of the cart. It is clear that the pendulum stabilized quickly, (less than a second) from an initial angle of -15^0 degrees and stabilizing the cart to the reference point on the rail. Then, the pole was physically disturbed and each time regaining stability in the inverted position in less than a second. This shows the fast response and robust nature of the controller. Figure 13 shows a simulation plot of swing up and stabilization of cart displacement.

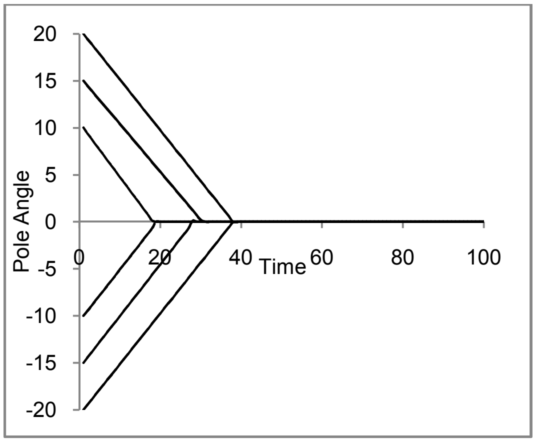


(a) (b)

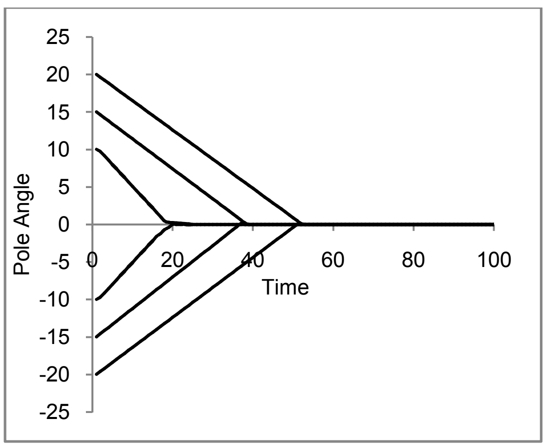
Figure 13. Simulation of Swing-Up & Stabilization of cart displacement (a) Two Layer HFLC (b) Three Layer HFLC

It can be seen that the controllers were perfectly symmetrical, and moreover, they were capable to generalize the control of the pole and the cart from initial positions not used during the optimization process. From the simulation it can be clearly seen that all the simulated responses tend to zero which is a stationary stable point regardless the initial conditions are.

In each generation, all chromosomes are evaluated by the FLC to generate the fitness error. Chromosomes are sorted according to the fitness error and the best one among all chromosomes is selected in each generation. The corresponding plots of the best, average and worst fitness against generation count are shown in Figs. 16. The performance of the two layers and three layers HFLC for controlling the pole and the cart separately are shown in Fig.13 and Fig. 14.



(a)

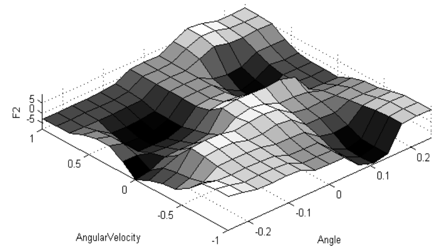


(b)

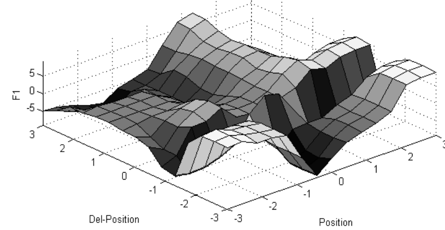
Figure 14. Simulation of Swing-Up & Stabilization of pole Angle for different initial conditions (a) Two Layer HFS (b) Three Layer HFS

A. Discussion and Evaluation

In order to evaluate our new EA-based hierarchical approach, we compare it against well-known reinforcement learning method called SANE (Symbiotic, Adaptive Neuro-Evolution)[16]. SANE is an evolutionary neural networks based cooperative model that has been applied to cart pole type inverted pendulum system with very promising results [16]. SANE algorithm works on the basis of group interaction-based evolutionary algorithm for improving the symbiotic GA. The group interaction based evolutionary algorithm is developed from a symbiotic evolution. In SANE each chromosome refers to a single neuron, and the neural networks formed are the results of the evolution of a population of these chromosomes through coevolution and speciation.



(a)



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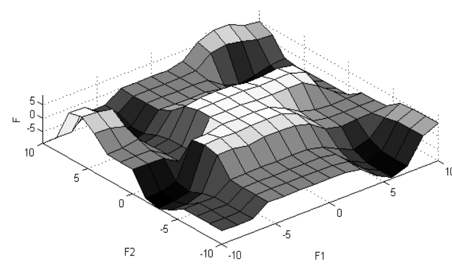

(c)

Figure 15. Overall input output curves (Control Surface) for two layer HFLC

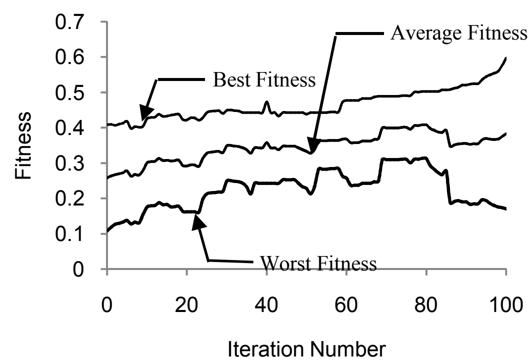

Figure 16: Plot of best, average and worst fitness against iteration count for two layer HFLC

The inverted pendulum is a classical and contemporary open loop control problem. It is an inherently open loop unstable with highly non-linear dynamics and an under actuated mechanical system and used as a well-known test bed for the evaluation of new control algorithms. The control system for cart-pole inverted system is stabilized within a few seconds or time steps in several studies including [15-16]. The closer inspection of the study in [15] reveals that the system is stabilized within 46 time steps for the best case and 4461 for the worst cases from initial pole angles $\pm 15^0$ degrees and random cart positions ±2.4 meters. For the same initial condition, the number of time steps required to stabilize the cart-pole system is 33 for best case and 3123 is for the worst case while using our proposed approach. The time steps required for cart and pole stabilization were computed over 20 simulations for each method. The advantage of using this hierarchical fuzzy logic structure is that the number of rules used in the fuzzy knowledge base has been reduced substantially also.

VI. CONCLUSIONS

In this paper we have presented a novel tuning, adaptation and control algorithm based on a hierarchical fuzzy–evolutionary system. We have shown that by defining the cart-pole system as a two-objective search problem, the GA is able to learn a set of fuzzy rules and their corresponding MFs to swing up and stabilizing the inverted pendulum. We have shown that global control structure is divided into sub control structures and we devise a control strategy for coordinating the sub-controllers to achieve the overall objective of the system. By using this hierarchical control strategy for the fuzzy controller the number of rules is reduced in comparison to non-hierarchical control strategy, thereby reducing the computational time while maintaining system robustness and efficiency. The result of simulation and experiment demonstrates that the proposed adaptive hierarchical fuzzy controller has satisfactory behaviour and a comparison with an existing controller was made to show the effectiveness of the proposed method.

Suggestions for follow-up works that may come after this study are as follows: The research work is to be extended by embedding rules reduction technique in the proposed hierarchical architecture. In this case statistical least square method is used to determine the importance order of rules and cut the unimportant rules. This study also extended through the parallel implementation of the proposed algorithm that may be used to improve the convergence speed of the proposed method. Finally, apply the proposed method to more complex real world applications.

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