Einführung in Wissensbasierte Systeme WS 2018/19, 3.0 VU, 184.737

Exercise Sheet 1 – Logic, Part 1

Exercise 1.1: Check using semantics whether the following statements hold (do *not* use truth tables). If a statement does not hold, provide a counterexample.

1. $\neg(p \rightarrow q) \models p \rightarrow \neg q$
2. $p \rightarrow q \models (p \rightarrow \neg q) \rightarrow \neg p$
3. $p \rightarrow q \models q \rightarrow \neg p$

Exercise 1.2: Let ϕ be a tautology, ψ a contradiction, and χ a contingency (i.e., χ is satisfiable as well as falsifiable). Which of the following formulas are (i) tautological, (ii) contradictory, (iii) contingent, or (iv) logically equivalent to χ? Justify your answers.

1. $\phi \wedge \chi$.
2. $\phi \vee \chi$.
3. $\psi \wedge \chi$.
4. $\psi \vee \chi$.
5. $\phi \vee \psi$.
6. $\chi \to \psi$.

Exercise 1.5: Prove or refute whether the following formulas are tautologies. Establish your claim by purely semantic means (i.e., without applying a calculus like **TC1**).

(a) $\exists x\,(P(x) \to P(f(x)))$

(b) $\forall x \exists y\, R(x,y) \to \exists y \forall x\, R(x,y)$

Hint: Truth tables do not work.

Exercise 1.6: Translate the following arguments into entailment problems of first-order logic and check the validity of each statement. If the argument is valid, provide a proof in TC1. Otherwise, prove that there exists no closed tableau for the (negation of the) corresponding formula in TC1.

(a) Influenza is caused by a virus. Pseudomonas is a bacterium and not a virus, therefore it doesn't cause influenza.

(b) Every chicken eats corn. Berta eats grass. Therefore Berta is not a chicken.

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Exercise Sheet 2 – Nonmonotonic Reasoning

Exercise 2.1: Consider the following theory:

$$T = \{\forall x(\neg P(x) \to Q(x)), \neg P(b), R(a), \forall x(R(x) \to P(x))\}.$$

(a) Determine $\mathrm{CWA}(T)$ as well as $\mathrm{CWA}^Q(T)$.

(b) Prove or refute whether $\mathrm{CWA}(T)$ is consistent. What about $\mathrm{CWA}^Q(T)$?

Exercise 2.3: Consider the open default theory $T = (W, \Delta)$, where

$$W = \{\exists x(P(x) \vee Q(x)), \forall x(P(x) \vee R(x)), \forall x(R(x) \to Q(x))\}$$
$$\Delta = \left\{ \frac{P(x) : \neg Q(x)}{\neg Q(x)}, \frac{Q(x) : \neg P(x)}{\neg P(x)}, \frac{\top : R(x)}{R(x)} \right\}.$$

Compute the closure of T and determine the possible candidates for being an extension. Then compute the classical reduct Δ_E for each candidate E, and determine all extensions of T.

Exercise 2.5: The property *semi-monotonicity* is a weaker form of monotonicity and is defined as follows:

> Let $T = (W, \Delta)$ and $T' = (W, \Delta')$ be default theories with $\Delta \subseteq \Delta'$, and let E be an extension of T. Then T' has an extension E' such that $E \subseteq E'$.

Prove or disprove whether semi-monotonicity holds in default logic.

Exercise 2.6: You are given the propositional default theory $T = (W, \Delta)$, where

$$\begin{aligned} W =& \{a\}, \\ \Delta =& \left\{ \frac{a : b}{b}, \frac{b : c, \neg d}{c}, \frac{b : d, \neg c}{d}, \frac{c : e}{\neg e} \right\}. \end{aligned}$$

Now consider the following (erroneous) reasoning trying to find an extension E of T:

1. Starting with the certain knowledge $a \in W$, we can apply the default $a : b/b$ to derive b.
2. Having derived b, we can now apply the second default $b : c, \neg d/c$ to derive c.
3. Due to c, the third default is blocked, but we can apply the fourth default $c : e/\neg e$ and derive $\neg e$.
4. No more defaults are applicable, so we arrive at $E = Cn(\{a, b, c, \neg e\})$.

However, E is *not* an extension of T.

Your task is to analyse the above reasoning and explain in detail why this approach does not work. Furthermore, find the actual extensions of T.

Exercise 2.7: Consider the following information:

1. People who live in a city usually rent a flat.
2. People who live in a city and rent a flat are usually not rich.
3. Rich people usually do not rent a flat.
4. Emily lives in a city.
5. Emily is rich.

Formalise the given information in terms of an open default theory and compute all of its extensions. Use

- $city(x)$ for "x lives in a city",
- $flat(x)$ for "x rents a flat",
- $rich(x)$ for "x is rich", and
- the constant symbol e for "Emily".

Einführung in Wissensbasierte Systeme WS 2018/19, 3.0 VU, 184.737

Exercise Sheet 3 – Answer-Set Programming and Probabilistic Reasoning

Exercise 3.1: Let P be a Horn logic program and $M_1, M_2 \subseteq HB(P)$ classical models of P. Prove that $M_1 \cap M_2$ is also a classical model of P. What about $M_1 \cup M_2$?

Exercise 3.2: Consider the following disjunctive logic program P:

$$P = \left\{ \begin{array}{r} a \vee b \leftarrow \text{not}\, c. \\ c \leftarrow b, \text{not}\, d. \\ d \leftarrow \text{not}\, c. \\ a \leftarrow d. \end{array} \right\}.$$

(i) Determine all answer sets of P. For each proposed answer set S, argue formally that it is indeed an answer set (using the Gelfond-Lifschitz reduct P^S).

(ii) Is it possible to add new rules Q to P such that $S' = \{b, c\}$ is an answer set of $P \cup Q$? If yes, give the new rules Q, if not, argue why.

(iii) The same as in (**??**), but for $S' = \{a, b\}$.

Exercise 3.3: For a program P, we denote by $AS(P)$ the set of all answer sets of P. Let P, Q be programs. We say that P, Q are

(i) *equivalent* if $AS(P) = AS(Q)$ and

(ii) *strongly equivalent* if $AS(P \cup R) = AS(Q \cup R)$ for every program R.

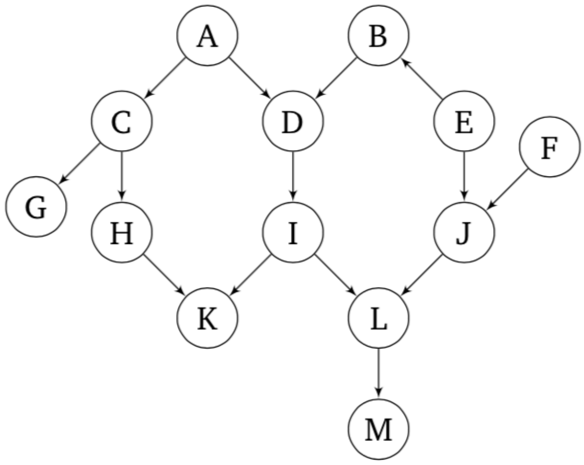
Prove or refute whether

(a) (i) implies (ii),

(b) (ii) implies (i).

Exercise 3.4: Let S_1, S_2 be answer sets of a program P. Show that $S_1 \subseteq S_2$ implies $S_1 = S_2$.

Exercise 3.6: Consider the following graph of a Bayesian network:



Answer the following questions and give an explanation of your answer.

(i) Is B conditionally independent of G?

(ii) Is B conditionally independent of G given evidence M?

(iii) Which evidences are needed such that G is conditionally independent of M?

(iv) Which evidences are needed such that A is conditionally independent of L?

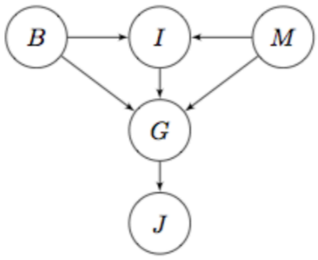
Exercise 3.7: Given the following Bayesian network with the Boolean variables

- $B = \mathit{BrokeElectionLaw}$,
- $I = \mathit{Indicted}$,
- $M = \mathit{PoliticallyMotivatedProsecutor}$,
- $G = \mathit{FoundGuilty}$, and
- $J = \mathit{Jailed}$,

where the probabilities for B and M are $P(B) = 0.9$ and $P(M) = 0.1$, respectively. The conditional probabilities for $P(G|B, I, M)$, $P(J|G)$, and $P(I|B, M)$ are as follows:

B	M	$P(I\|B, M)$
0	0	0.1
0	1	0.5
1	0	0.5
1	1	0.9

G	$P(J\|G)$
1	0.9
0	0.0



B	I	M	$P(G\|B, I, M)$
0	0	0	0.0
0	0	1	0.0
0	1	0	0.1
0	1	1	0.2
1	0	0	0.0
1	0	1	0.0
1	1	0	0.8
1	1	1	0.9

(i) Which, if any, of the following relations hold in view of the *network structure alone* (ignoring the conditional probability tables for now)?

(a) $P(B, I, M) = P(B)P(I)P(M)$.

(b) $P(J|G) = P(J|G, I)$.

(c) $P(M|G, B, I) = P(M|G, B, I, J)$.

(ii) Calculate $P(b, i, \neg m, g, j)$.

(iii) Calculate the probability that someone goes to jail given that he or she broke the law, has been indicted, and faces a politically motivated prosecutor.