

# Semantics



# Definition of Security Properties

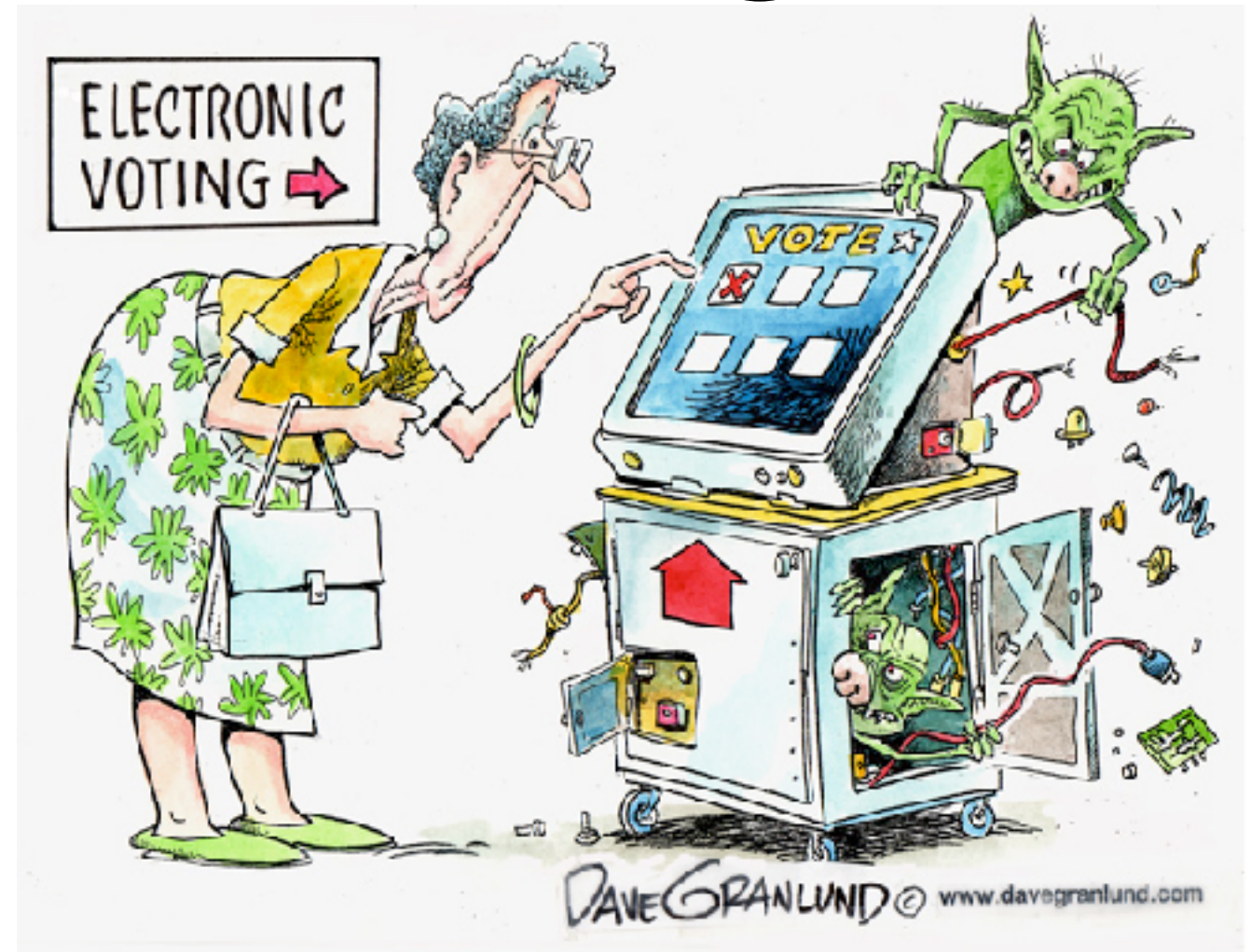
Ana Matos

Miguel Correia

Pedro Adão

# Example: E-Voting


- Functionality
  - Cast vote, audit vote, determine winner, ...
- Assets
  - votes, voting service,
- Threat model
  - Goal: the adversary desires to influence the election.
  - Methods: Voting coercion, disabling or controlling voting booths, communications, servers...
- (...)



- (...)
- Security properties
  - Every citizen can cast a vote. (availability)
  - The identity of who placed each vote is anonymous, and the choice of each voter is known only to the voter. (confidentiality)
  - Only the voter can determine its vote. (integrity)
  - The voter can verify that its vote was casted correctly. (auditability)...
- Enforcement mechanism
  - Authentication, cryptographic protocols ensure (up to cryptographic strength) anonymity, integrity and auditability, ...

# How can we get strong guarantees?

We need to be precise.

- 
- “Is the program secure?”
  - “Is the program secure against this threat?”
  - “Is the program secure against attacker model A?”
  - “Does the program meet security property  $P$  in the execution context  $C$ ?”
  - “Can we prove that the program meets security property  $P$  in the execution context  $C$ ?”

# “Secure” programs

- We want to ensure that programs are “secure”.
- First, we must be clear about what we are protecting, what is the power of the attacker, and what being secure actually means.

# Class Outline

- Noninterference, intuitively
- Formal semantics
  - Big-step operational semantics
- Formalization of Noninterference

# Class Outline

- Noninterference, intuitively
- Formal semantics
  - Big-step operational semantics
- Formalization of Noninterference



# “no illegal flows”

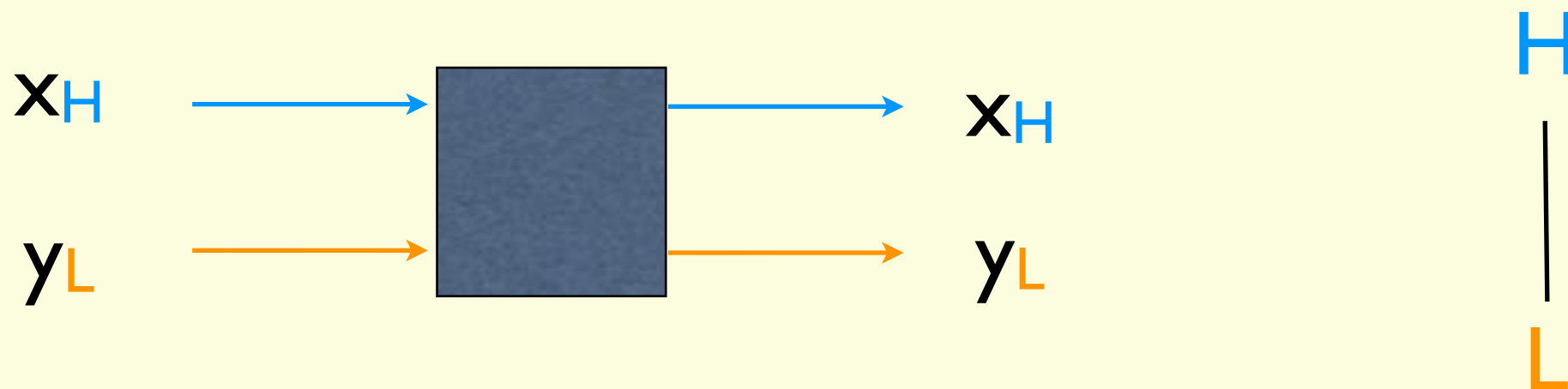
- We want to ensure that propagation of information by programs respects information flow policies, i.e. there are no illegal flows
- “no illegal flows” = an attacker cannot infer secret input or affect critical output by inserting inputs into the system and observing its outputs.
- How can we express the property of whether a program respects information flow policies?

# Challenge for next class

- Secure program = the program does not encode illegal information flows
- Can you formulate a security property that defines when a program is secure?

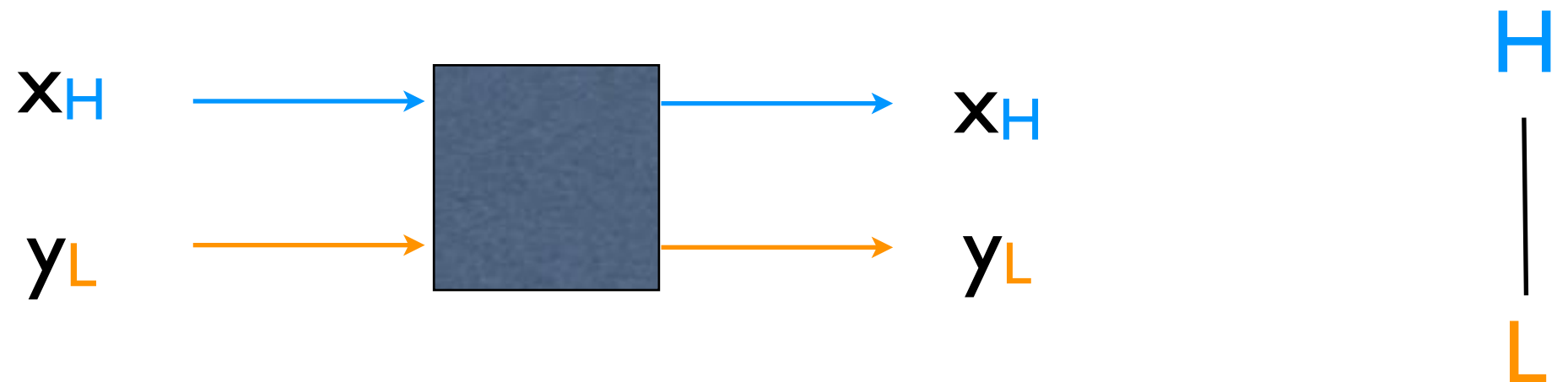
# Tips

- Suppose you are the attacker. You are given a black box program, and you can control the “low” inputs of the program, and read its “low” outputs.
- How many executions would you need to find out if the program leaks information?
- What inputs would you give it in order to find out whether the program leaks information?



- Which of the following experiments allows you to conclude whether the program encodes leaks?
- Input:  $y_L=0$ , Output:  $y_L=0$
- Input:  $y_L=0$ , Output:  $y_L=1$
- Input:  $y_L=0$ , Output:  $y_L=0$  and Input:  $y_L=0$ , Output:  $y_L=0$
- Input:  $y_L=0$ , Output:  $y_L=0$  and Input:  $y_L=1$ , Output:  $y_L=1$
- Input:  $y_L=0$ , Output:  $y_L=0$  and Input:  $y_L=0$ , Output:  $y_L=1$

# Noninterference (for H-L)













- The program is secure if, for any two runs of the program that are given the same *low* (L) inputs, if the program terminates, it produces the same low (L) outputs.

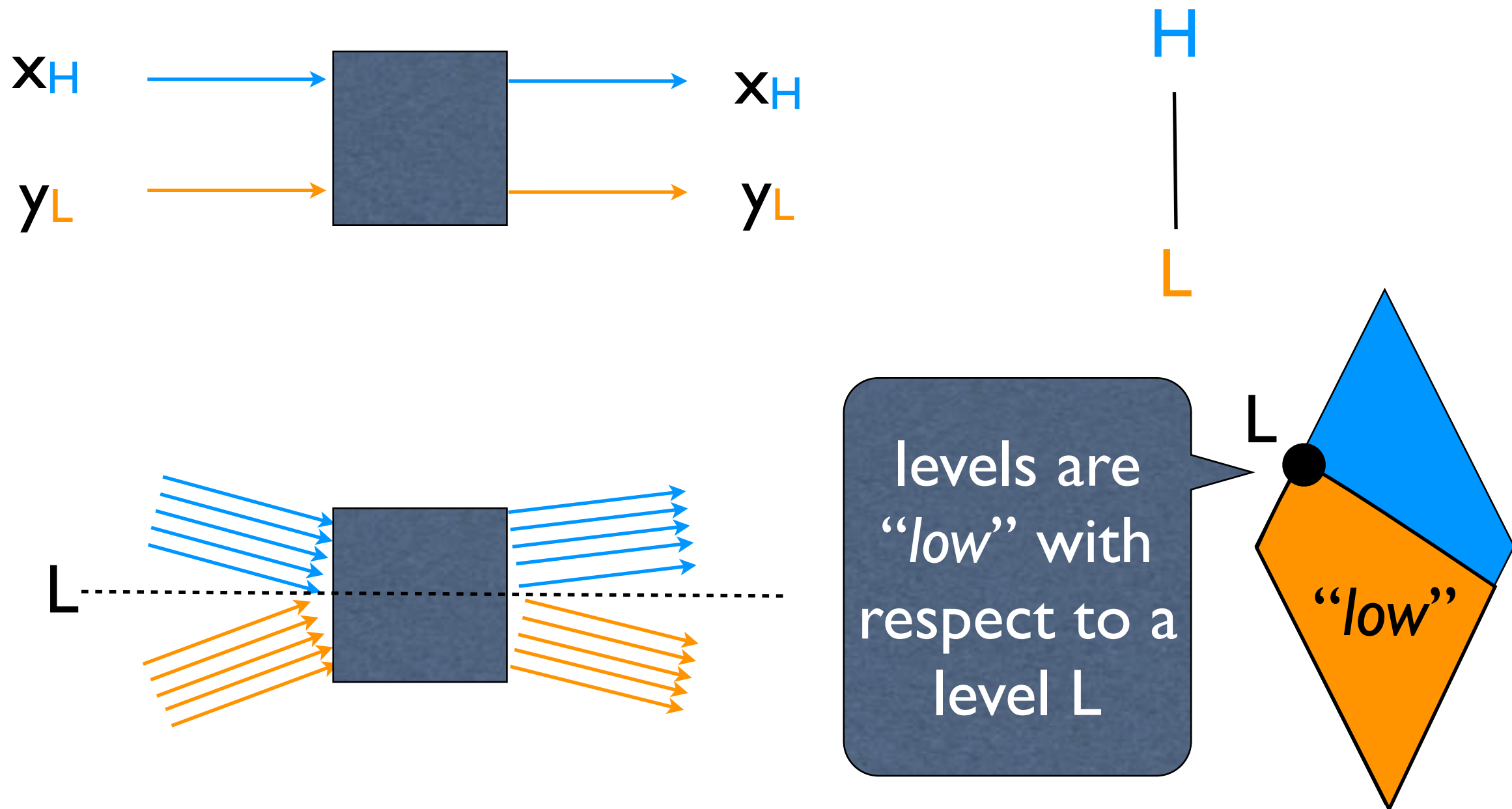
# Preserves confidentiality?

- $y_{\text{secret}} := x_{\text{you}}$  ✓
  - $x_{\text{you}} := y_{\text{secret}}$  ✗
  - if  $y_{\text{secret}}$  then  $x_{\text{you}} := 0$  else  $x_{\text{you}} := 0$  ✓
  - if  $y_{\text{secret}}$  then  $x_{\text{you}} := 0$  else  $x_{\text{you}} := 1$  ✗
  - if  $x_{\text{you}}$  then  $y_{\text{secret}} := 0$  else  $y_{\text{secret}} := 1$  ✓
  - $x_{\text{you}} := 0$  ; while  $x_{\text{you}} < y_{\text{secret}}$  do  $x_{\text{you}} := x_{\text{you}} + 1$  ✗
  - $x_{\text{you}} := 1$  ; while  $y_{\text{secret}}$  do  $x_{\text{you}} := 0$  ✓
  - while  $x_{\text{you}}$  do skip ;  $y_{\text{secret}} := 0$  ✓
- Explicit leak
- Implicit leak

# Preserves integrity?

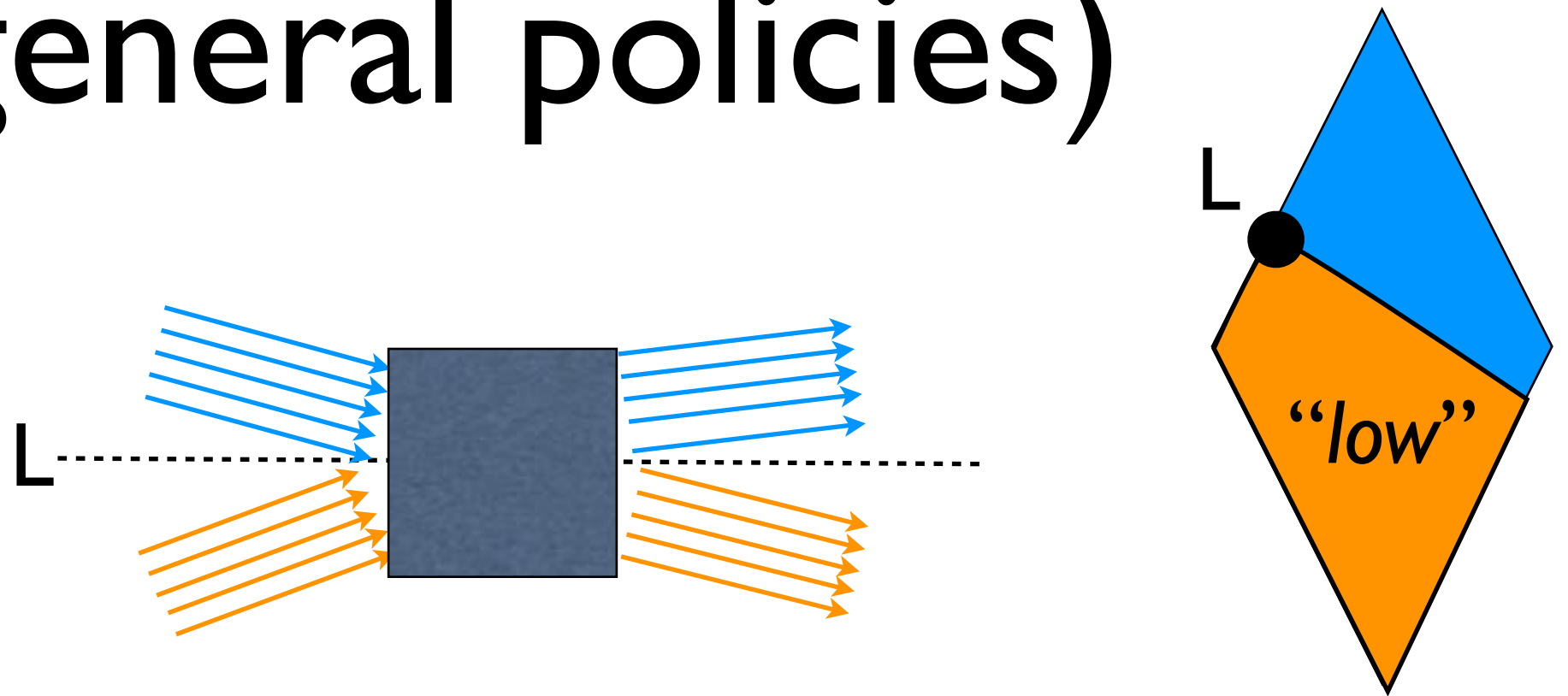
- $y_{\text{untainted}} := x_{\text{you}}$    Explicit leak
- $x_{\text{you}} := y_{\text{untainted}}$  
- if  $y_{\text{untainted}}$  then  $x_{\text{you}} := 0$  else  $x_{\text{you}} := 0$  
- if  $y_{\text{untainted}}$  then  $x_{\text{you}} := 0$  else  $x_{\text{you}} := 1$  
- if  $x_{\text{you}}$  then  $y_{\text{untainted}} := 0$  else  $y_{\text{untainted}} := 1$    Implicit leak
- $y_{\text{untainted}} := 0$  ; while  $x_{\text{you}} < y_{\text{untainted}}$  do  $y_{\text{untainted}} := y_{\text{untainted}} + 1$  
- $x_{\text{you}} := 1$  ; while  $y_{\text{secret}}$  do  $x_{\text{you}} := 0$  
- while  $x_{\text{you}}$  do skip ;  $y_{\text{untainted}} := 0$  

# Noninterference, for general policies





# Noninterference (general policies)



- A program is secure if, for every observational level  $L$ , for any two runs of the program that are given the same *low* inputs, if the program terminates, it produces the same *low* outputs.

# What is the power of the attacker?

- The attacker we are assuming cannot observe
  - non-terminating computations
  - intermediate steps of computations
  - time that it takes to produce the output
  - likelihood of each possible output
- This can be inadequate for different
  - execution contexts
  - language expressivity

# Deterministic Input-Output attacker

- Execution context: sequential (deterministic)
- Intuition: an attacker is a program that is executed sequentially after the observed program, and has access to “low” outputs.
- Deterministic Input-Output Noninterference:  
Is only sensitive to outputs of terminating computations.

# Concurrent execution context

- Is our notion of Noninterference adequate in a concurrent context?
  - $x_L := 1 \parallel (x_L := 2; x_L := x_L + 2)$
- The above program is considered insecure because of its non-deterministic behavior
  - $\langle x_L := 1 \parallel (x_L := 2; x_L := x_L + 2), \rho \rangle \rightarrow \rho[x_L := 4]$
  - $\langle x_L := 1 \parallel (x_L := 2; x_L := x_L + 2), \rho \rangle \rightarrow \rho[x_L := 1]$
  - $\langle x_L := 1 \parallel (x_L := 2; x_L := x_L + 2), \rho \rangle \rightarrow \rho[x_L := 3]$

# Concurrent execution context

- The following programs are considered secure with respect to our notion of Noninterference
  - if  $PIN_H$  then  $y_H := \text{false}$  else  $z_H := \text{false}$
  - while  $y_H$  do skip ;  $x_L := 1$  ;  $z_H := \text{false}$
  - while  $z_H$  do skip ;  $x_L := 0$  ;  $y_H := \text{false}$
- But when composed concurrently, the program is insecure!

# Concurrent attacker

- Execution context: concurrent
- Intuition: an attacker program that is concurrently composed with the observed program does not depend on its termination. It has access to “low” outputs, and possibly non-termination (or even intermediate steps).
- Possibilistic Input-Output Noninterference:  
Is sensitive to whether the program is capable of terminating and producing certain final outputs.

# Secure? (w.r.t. ...)

- $y_H := x_L$  ✓
- $x_L := y_H$  ✗
- if  $y_H$  then  $x_L := 0$  else  $x_L := 1$  ✗
- while  $y_H$  do skip ;  $x_L := 0$  ✗

Termination leak

Counter-example:

$\rho_1(x_L) = \rho_2(x_L) = 1$  and  $\rho_1(y_H) = \text{false}$  and  $\rho_2(y_H) = \text{true}$   
(the program cannot terminate on  $\rho_2$ )

# Intermediate-step attacker

$x_L := y_H ; x_L := 1$

Possible low outcomes do not depend on  $y_H$ .  
However, the intermediate steps differ.

- Intermediate-step-sensitive Noninterference:  
Is sensitive to intermediate steps of computations.

(To define an intermediate-step-sensitive property  
we would use a small-step semantics.)



# Time-sensitive attacker

$x_L := 0$  ; if  $y_H$  then skip else skip;skip;skip;skip ;  $x_L := 1$

Possible outcomes and intermediate steps do not depend on  $y_H$ . However, the time it takes to change the value of  $x_L$  is different.

- Temporal Noninterference:  
Is sensitive to the time it takes to produce outputs.

# Probabilistic attacker

$x_L := y_H \parallel x_L := \text{random}(100)$

Possible outcomes do not depend on  $y_H$ . However, the probability of the value of  $x_L$  revealing that of  $y_H$  is higher.

- Probabilistic Noninterference:  
Is sensitive to the likelihood of outputs.

# Limits of Noninterference

- As sensitive as the attacker model.
  - Covert channels?
- As flexible as the security property permits.
  - Too restrictive?

# What is the power of the attacker?

- Can the attacker observe:
  - That a program does not terminate?
  - Intermediate steps of the computation?
  - The possibility of producing certain states?
  - The likelihood of producing certain states?

# What is the power of the attacker?

- Execution context: sequential (deterministic)
- Intuition: an attacker is a program that is sequentially composed with  $S$ , and has access to “low” outputs.
- Deterministic Input-Output attacker:  
Can only observe final outputs of terminating computations.

# Covert channels

- Information can be transferred via other side-channels that are not intended for that purpose:
  - computation time
  - memory allocation
  - power consumption
  - ...

# More flexibility

- Noninterference is simple and provides strong security guarantees. But sometimes we need to **leak information in a controlled way**.
- Need to reveal a bit of a secret (for confidentiality)  
if (password<sub>H</sub> == attempt<sub>L</sub>)  
  then print<sub>L</sub> “Right!”  
  else print<sub>L</sub> “Wrong!”
- Need to trust certain user input (for integrity)  
\$filename<sub>L</sub>=<STDIN>;  
open<sub>H</sub>(FOO,"> \$filename<sub>L</sub>");

# Downgrading

- **Declassification** (for confidentiality)

Example: flow declarations locally enable more flows

```
declassify password:L in
  if (passwordH == attemptL) {
    then printL "Right!";
    else printL "Wrong!";
  }
```

- **Endorsement** (for integrity)

Example: pattern matching in Perl's taint mode

```
if ($filenameL =~ /^([-\\@\\w.]+)$/) {
  $filenameH = $1H;
  openH(F00, "> $filenameH");
}
```



# Conclusion

- The framework for the analysis should reflect the assumptions we want to study:
  - Language expressivity?
  - Execution context?
  - Attacker power?
  - Strength/flexibility of the security requirements?

# In this course

- We will focus primarily on Input-Output Deterministic Noninterference.
- We will study different enforcement mechanisms for this security property.
- We want to have strong guarantees of that our mechanisms work.
- We will therefore use formal methods.

# To start with, we need

- A precise way to talk about:
  - what are the possible programs and inputs
  - what is the (final) result (output) of executing a (deterministic) program with a certain input.
- In other words, we need:
  - a formal language (syntax and semantics)
  - notation to reason about terminating computations.

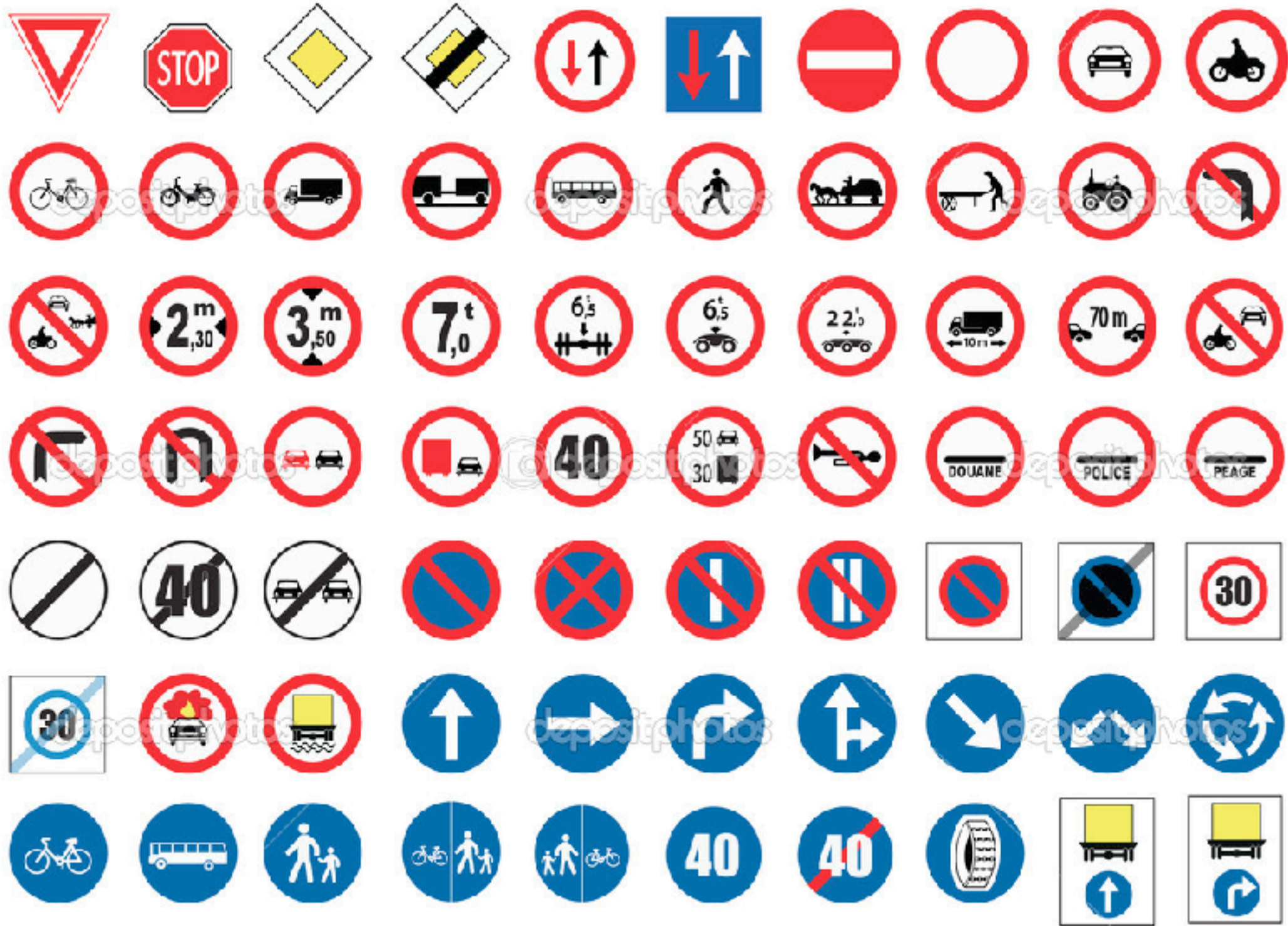
# Formal security property

- Why use a **formal** security property?
  - For clarity (preventing ambiguities in the specification)
  - For conciseness
  - For correctness (the basis for implementation, analysis and verification)
- What do we need?
  - Techniques for reasoning about the syntax and the semantics of programs.

# Class Outline

- Noninterference, intuitively
- Formal semantics
  - Big-step operational semantics
- Formalization of Noninterference

# Semantics?



# Programming language semantics

specification of the meaning of programs in  
that programming language



“Semantics with Applications: A Formal Introduction”, H. Nielson,  
F. Nielson, 1992 (Chapter 1 -- Section 2.1.).

# Defining the semantics of a programming language

We will use two techniques:

- **Denotational** semantics for **expressions**: defines mathematically **what** is the result of a computation.
- **Operational** semantics for **instructions**: describes **how** the effect of a computation is produced when executed on a machine.



# A WHILE language

- Note:
  - The next slides present a WHILE language as defined in the tutorial G. Barthe et al. (2011).
  - The language is a (simpler) variation of the one in Nielson & Nielson (1999).(\*)

(\*) Differences are:

- We refer to the natural operational semantics as “big-step semantics”, and to the structural operational semantics as “small-step semantics”.
- Constants are simply integers (and not numerals).
- Arithmetic expressions use operations that are used directly in the semantics.
- Boolean expressions are reduced to comparisons between arithmetic expressions (tests).

# Syntax of WHILE

- Syntactic categories:  
c - constants (integers)  
x - variables  
a - arithmetic expressions  
t - tests  
S - statements

- Grammar (BNF notation)

$op ::= + \mid - \mid \times \mid /$

$cmp ::= < \mid \leq \mid = \mid \neq \mid \geq \mid >$

$a ::= c \mid x \mid a_1 \ op \ a_2$

$t ::= a_1 \ cmp \ a_2$

$S ::= x:=a \mid skip \mid S_1 ; S_2 \mid \text{if } t \text{ then } S \text{ else } S \mid \text{while } t \text{ do } S$

operations

expressions

comparisons

tests

statements

# State (a.k.a. memory): $\rho$

What is a state/memory?

- $\rho$  - a function that maps variables to integers

Example:  $\rho(x) = 1$ ,  $\rho(y) = 42 \dots$

# Functions that give meaning

Let's make use of the following semantic functions:

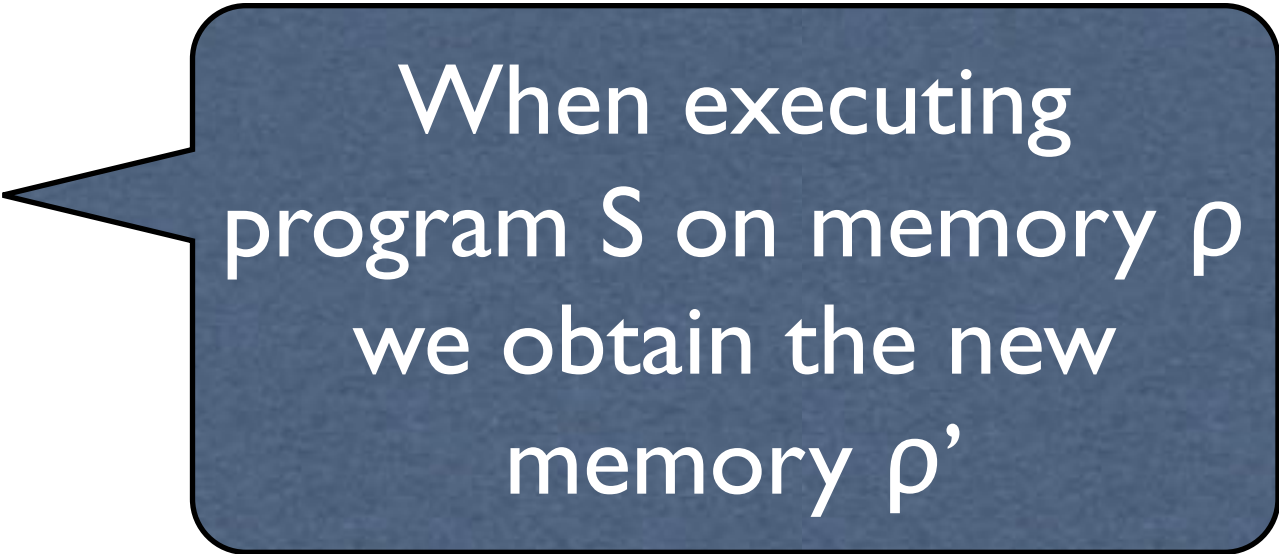
- $\mathcal{A}$  - function that maps pairs of arithmetic expression and state to integers (assume defined) ✓
- $\mathcal{B}$  - function that maps pairs of test and state, to booleans (assume defined) ✓
- $\mathcal{S}$  - partial function that maps pairs of statement and state to state (to define next).



# Meaning of statements: $S$

- $S$  - partial function that maps pairs (statement, state) to state.
- We will define it using **big-step transitions** that speak about how the overall result of executions is obtained.

$$\langle S, \rho \rangle \rightarrow \rho'$$



When executing  
program  $S$  on memory  $\rho$   
we obtain the new  
memory  $\rho'$

- We could also use **small-step transitions** that speak about how the individual steps of the computations take place.

$$\langle S, \rho \rangle \Rightarrow \langle S', \rho' \rangle$$

Performing one step of program  $S$  on memory  $\rho$  leaves the continuation  $S'$  and produces new memory  $\rho'$

- (We will use small step semantics later in the course.)

# Big-step Operational Semantics

- **Skip:**  $\langle \text{skip}, \rho \rangle \rightarrow \rho$

Does nothing!

- **Assignment:**  $\langle x := a, \rho \rangle \rightarrow \rho[x \mapsto \mathcal{A}[a]\rho]$

Updates the value of  $x$  in  $\rho$  with the result of evaluating  $a$

the update of state  $\rho$   
is defined as:

$$\begin{cases} (\rho[y \mapsto c])(x) = c, & \text{if } x=y \\ \rho(x), & \text{otherwise} \end{cases}$$

Axioms - do not depend on any hypothesis in order to give the final result of the **entire computation**



When the first program  
starting on  $\rho$   
produces  $\rho'$ ...

... and the second program  
starting on  $\rho'$   
produces  $\rho''$ ...

- **Sequential composition:**

$\langle S_1, \rho \rangle \rightarrow \rho' \quad \langle S_2, \rho' \rangle \rightarrow \rho''$

$\langle S_1; S_2, \rho \rangle \rightarrow \rho''$

... then the entire sequential  
composition starting on  $\rho$   
produces  $\rho''$ .

Rules - the final result of the **entire computation** below the  
line, depends on the hypothesis above the line



When  $t$  evaluates  
to true...

... and the first branch  
starting on  $\rho$  produces  
 $\rho'$ ...

- Conditional test:

$$\frac{\langle S_1, \rho \rangle \rightarrow \rho'}{\langle \text{if } t \text{ then } S_1 \text{ else } S_2, \rho \rangle \rightarrow \rho'}$$

if  $\mathcal{B}\llbracket t \rrbracket_\rho = \text{true}$

then the conditional  
starting on  $\rho$  produces  
 $\rho'$ .

$$\frac{\langle S_2, \rho \rangle \rightarrow \rho'}{\langle \text{if } t \text{ then } S_1 \text{ else } S_2, \rho \rangle \rightarrow \rho'}$$

if  $\mathcal{B}\llbracket t \rrbracket_\rho = \text{false}$

When  $t$  evaluates to true...

- **While loop:**

$\langle S, \rho \rangle \rightarrow \rho' \quad \langle \text{while } t \text{ do } S, \rho' \rangle \rightarrow \rho''$     **if**  $\mathcal{B}\llbracket t \rrbracket_{\rho} = \text{true}$

$\langle \text{while } t \text{ do } S, \rho \rangle \rightarrow \rho''$

... the body starting on  $\rho$  produces  $\rho'$ ...

... and the continuation of the cycle on  $\rho'$  produces  $\rho''$ ...

... then the cycle on  $\rho$  produces  $\rho''$ .

$\langle \text{while } t \text{ do } S, \rho \rangle \rightarrow \rho$     **if**  $\mathcal{B}\llbracket t \rrbracket_{\rho} = \text{false}$

When  $t$  evaluates to false the cycle does nothing.

# (All) Big-step axioms & rules

**Assignment:**  $\langle x := a, \rho \rangle \rightarrow \rho[x \mapsto \mathcal{A}[a]_\rho]$

**Skip:**  $\langle \text{skip}, \rho \rangle \rightarrow \rho$

**Sequential composition:** 
$$\frac{\langle S_1, \rho \rangle \rightarrow \rho' \quad \langle S_2, \rho' \rangle \rightarrow \rho''}{\langle S_1; S_2, \rho \rangle \rightarrow \rho''}$$

**Conditional test:** 
$$\frac{\langle S_1, \rho \rangle \rightarrow \rho'}{\langle \text{if } t \text{ then } S_1 \text{ else } S_2, \rho \rangle \rightarrow \rho'} \quad \text{if } \mathcal{B}[t]_\rho = \text{true}$$
$$\frac{\langle S_2, \rho \rangle \rightarrow \rho'}{\langle \text{if } t \text{ then } S_1 \text{ else } S_2, \rho \rangle \rightarrow \rho'} \quad \text{if } \mathcal{B}[t]_\rho = \text{false}$$

**While loop:** 
$$\frac{\langle S, \rho \rangle \rightarrow \rho' \quad \langle \text{while } t \text{ do } S, \rho' \rangle \rightarrow \rho''}{\langle \text{while } t \text{ do } S, \rho \rangle \rightarrow \rho''} \quad \text{if } \mathcal{B}[t]_\rho = \text{true}$$
$$\langle \text{while } t \text{ do } S, \rho \rangle \rightarrow \rho \quad \text{if } \mathcal{B}[t]_\rho = \text{false}$$

# Example - Evaluation

- Evaluate  $(z:=x ; x:=y) ; y:=z$ , starting from a state  $\rho_0$  that maps all variables except  $x$  and  $y$  to 0, and has  $\rho_0(x) = 5$  and  $\rho_0(y) = 7$ .

$$\langle z:=x, \rho_0 \rangle \rightarrow \rho_1$$

$$\langle x:=y, \rho_1 \rangle \rightarrow \rho_2$$

derivation tree

---

$$\langle (z:=x ; x:=y), \rho_0 \rangle \rightarrow \rho_2$$

$$\langle y:=z, \rho_2 \rangle \rightarrow \rho_3$$

---

$$\langle (z:=x ; x:=y) ; y:=z, \rho_0 \rangle \rightarrow \rho_3$$

$$\rho_1 = \rho_0[z \mapsto 5] \quad \rho_2 = \rho_1[x \mapsto 7] \quad \rho_3 = \rho_2[y \mapsto 5]$$

# Evaluation - derivation tree

To evaluate a statement  $S$ , starting from a state  $\rho$ , a **derivation tree** must be constructed:

1. Construct the tree from root upwards.
2. Try to find an axiom or rule whose left side matches  $\langle S, \rho \rangle$  and whose side conditions are satisfied.
  - If it is an axiom - determine the final state and terminate.
  - If it is a rule - try to construct the derivation tree for the premisses of the rule in order to determine the final state.

# The semantic function $S$

- The meaning of statements is given as a (partial) function from the set of states to the set of states.

$$S \llbracket S \rrbracket_{\rho} = \begin{cases} \rho' & \text{if } \langle S, \rho \rangle \rightarrow \rho' \\ \text{undefined,} & \text{otherwise} \end{cases}$$

no meaning is given to  
non-terminating  
computations

# Summarizing: Big-step transition system

- Configurations:
  - intermediate  $\langle \text{Statement } S, \text{state } \rho \rangle$
  - terminal  $\rho$
- Transitions:  $\langle S, \rho \rangle \rightarrow \rho'$
- Rules: 
$$\frac{\langle S_1, \rho_1 \rangle \rightarrow \rho_1' \dots \langle S_n, \rho_n \rangle \rightarrow \rho_n'}{\langle S, \rho \rangle \rightarrow \rho'} \quad \text{if } \dots$$

# Conclusions

- We have defined formally the operational semantics for a simple language WHILE.
- The same can be done **for any language in a similar** manner.
- This allows us to **reason rigorously about the behavior** of programs in the language.
- We will now use it to **formalize and prove security properties**, which are often semantic.



# Class Outline

- Noninterference, intuitively
- Formal semantics
  - Big-step operational semantics
- Formalization of Noninterference

# Formalization of Noninterference



“Language-Based Information-Flow Security”, A. Sabelfeld and A. Myers , 2002.

# Noninterference (informally)

## **Noninterference -**

A program is secure if, for any observational level  $L$ , for any two runs of the program that are given the same *low* inputs, if the program terminates, it produces the same *low* outputs.

# Noninterference (informally)

Program from a given programming language.

Given an information flow policy, all security levels that are lower or equal to  $L$ .

## **Noninterference -**

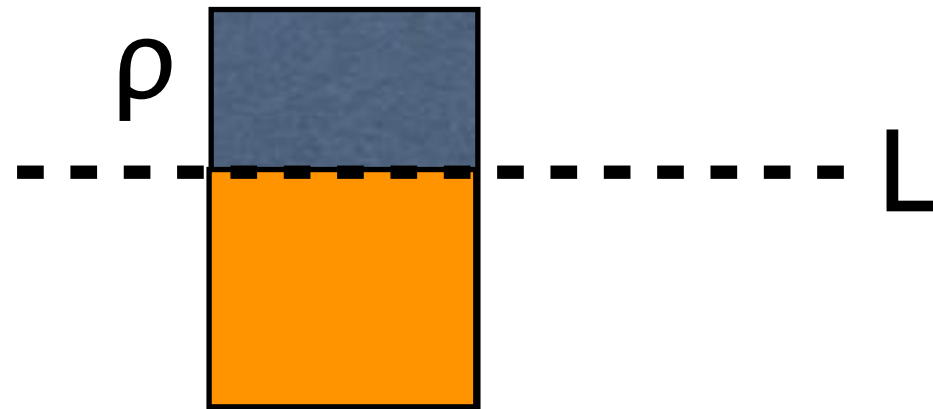
A **program** is secure if, for any **observational level  $L$** , for any two **runs** of the program that are given the **same low inputs**, if the program terminates, it produces the **same low outputs**.

Execution of the program according to specified semantics.

Security labeling and indistinguishability relation between states.

# Observable at level L

The part of a memory  $\rho$  that is **observable at level L** corresponds to the set of variables that are lower or equal to L.

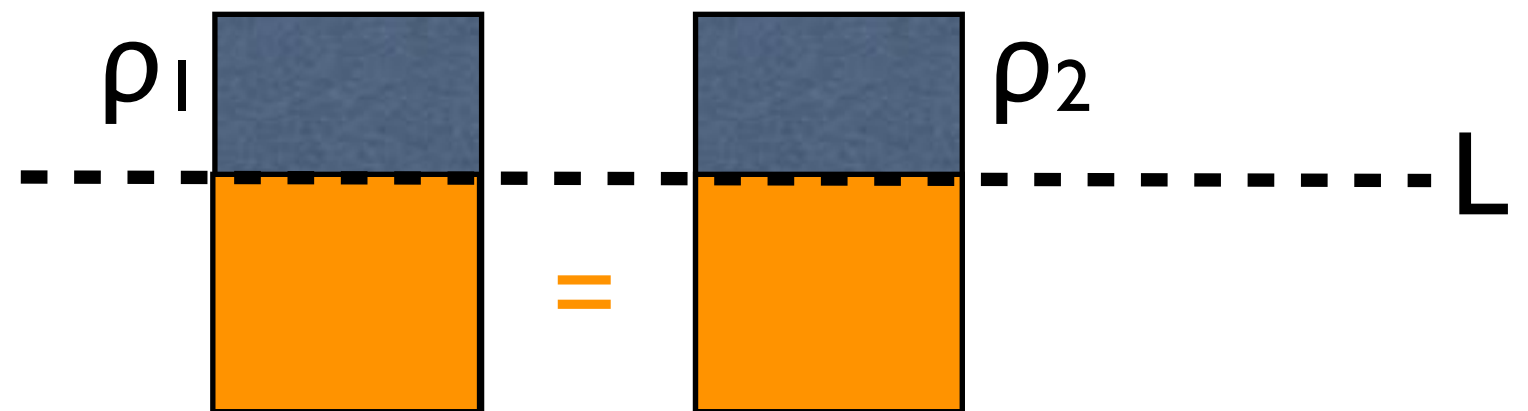


(Omitting the parameter  $\Gamma$  for simplicity.)

# Indistinguishable at level L

Two memories  $\rho_1$  and  $\rho_2$  are **indistinguishable** with respect to a security level **L**, if  $\rho_1$  and  $\rho_2$  agree on the values of variables that are observable at level L.

I.e.: For all  $x$  such that  $\Gamma(x) \leq L$ , then  $\rho_1(x) = \rho_2(x)$ .



We then write  $\rho_1 \sim_L \rho_2$ .

(Omitting the parameter  $\Gamma$  for simplicity.)

# Noninterference (formally)

## **Deterministic Input-Output Noninterference -**

A program  $S$  is secure if for every security level  $L$  and for all pairs of memories  $\rho_1$  and  $\rho_2$  such that

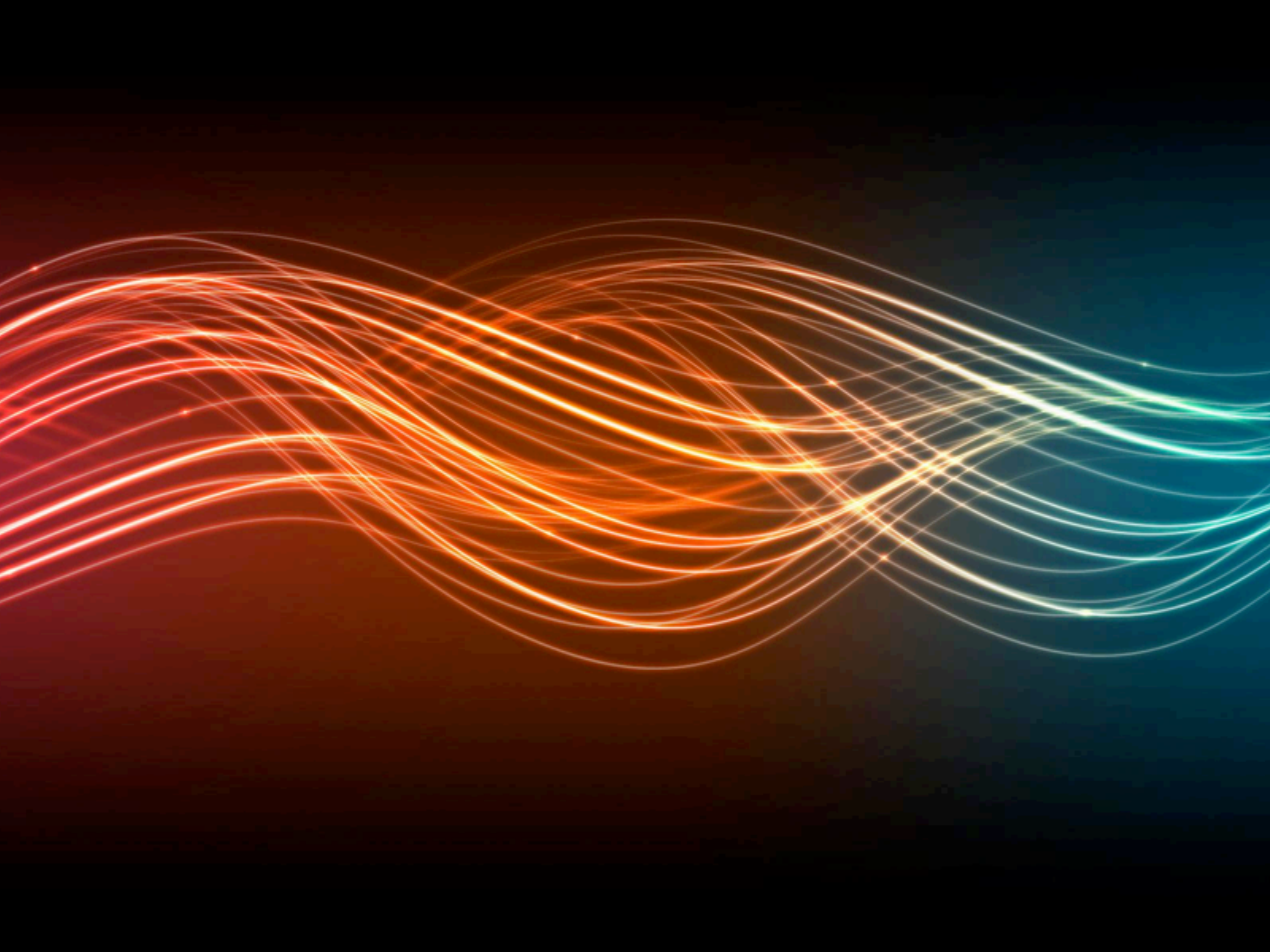
$\rho_1 \sim_L \rho_2$ , we have that

$\langle S, \rho_1 \rangle \rightarrow \rho_1'$  and  $\langle S, \rho_2 \rangle \rightarrow \rho_2'$  implies  $\rho_1' \sim_L \rho_2'$ .

# Conclusions

- In order to have strong guarantees about security, we need to **be precise** about the security property we want to enforce.
- Defining a security property requires defining what is the **threat model**, including the execution context and the power of the attacker.
- To be precise about a (semantic) security property requires a **formal semantics** for expressing it.





# Semantics

