Week 2

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Ex 4

Ex 5

Ex 6 Empirical mean and variance

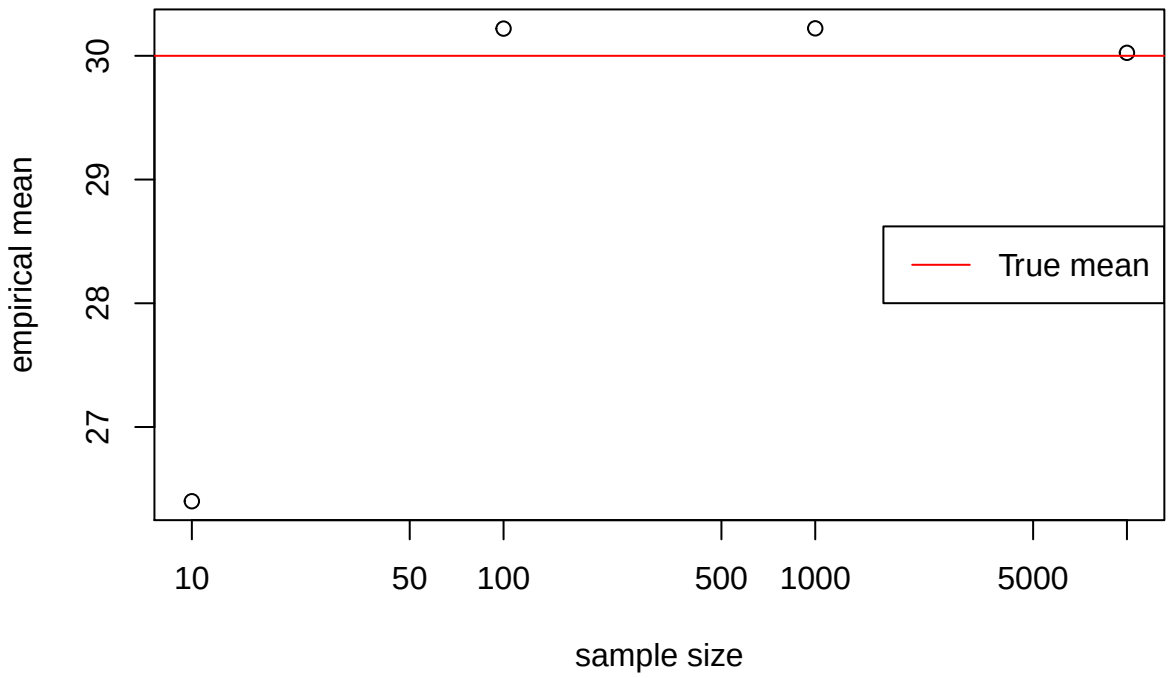
6.1

```
experiment <- function(n){
  S <- rbinom(n, size = 100, prob = 0.3)
  m <- mean(S)
  v <- var(S)
  sd <- sqrt(var(S))
  return(c(m, v, sd))
}
experiment(100)
```

```
## [1] 30.140000 25.778182  5.077222
```

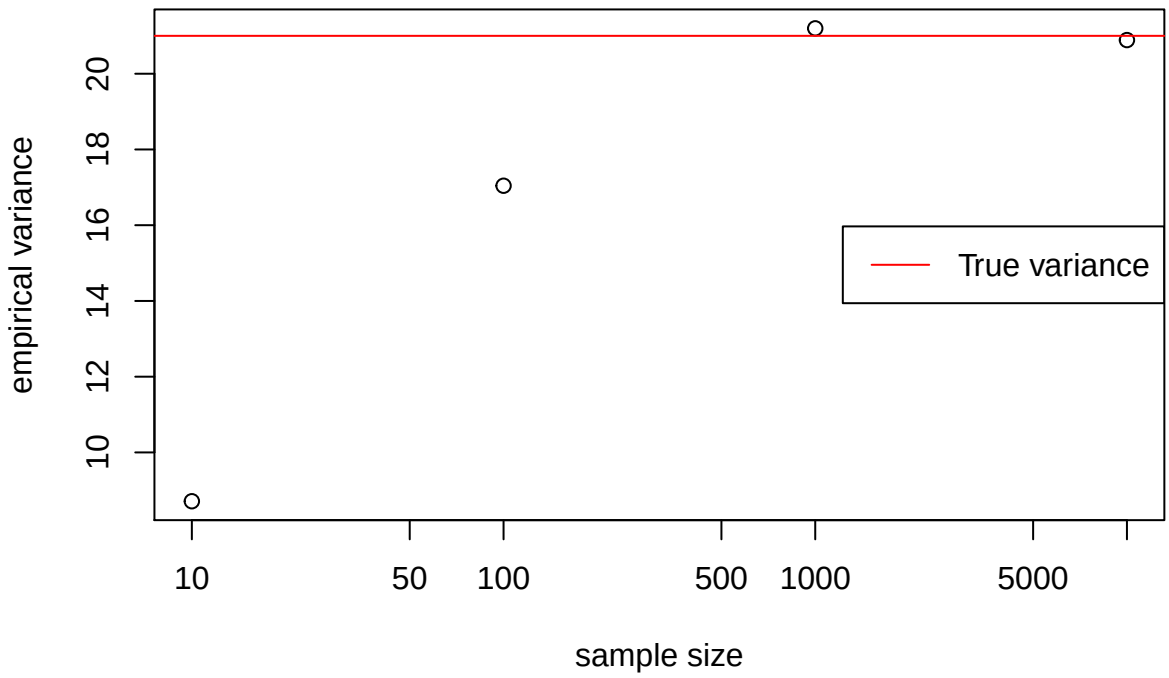
Comparing true mean $\mathbb{E}(X) = 100 \times 0.3 = 30$ and empirical mean,

```
D <- sapply(c(10, 100, 1000, 10000), experiment)
plot(c(10, 100, 1000, 10000), D[1,],
     xlab = "sample size", ylab = "empirical mean",
     log = "x")
abline(h = 30, col = "red")
legend("right", legend = "True mean", col = "red",
       lty = 1 )
```



Comparing the true variance $\mathbb{V}(X) = 100 \times 0.3 \times (1 - 0.3) = 21$,

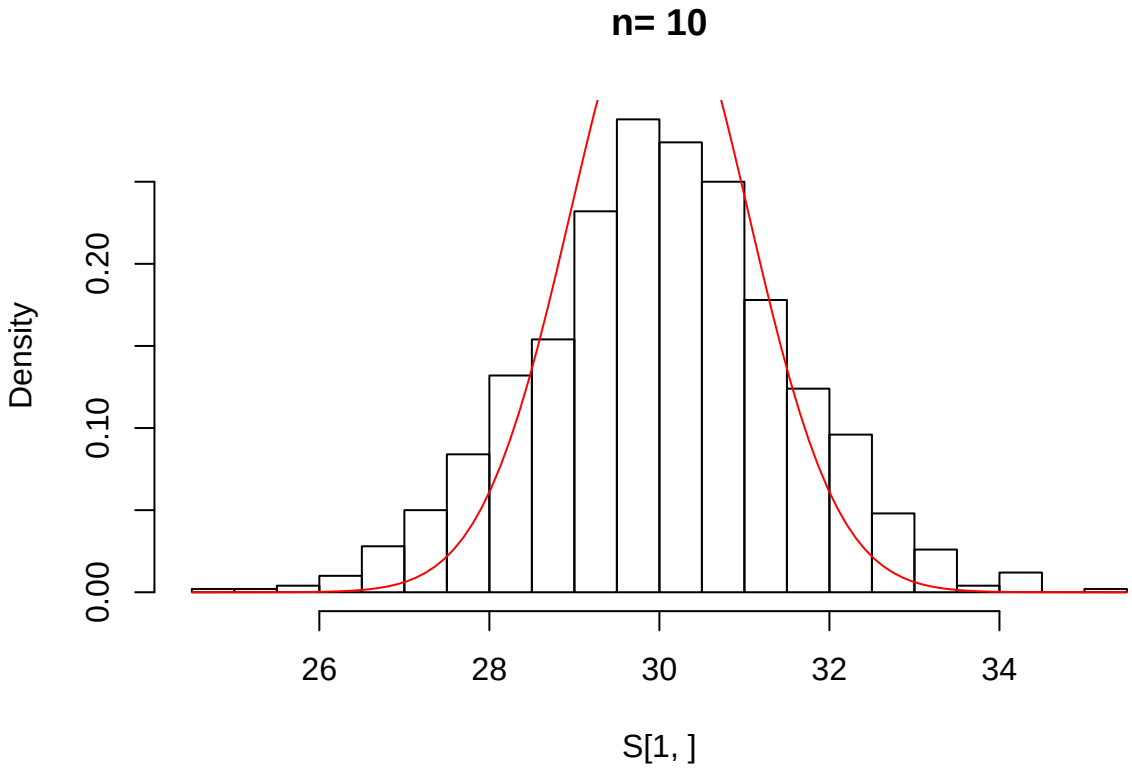
```
plot(c(10, 100, 1000, 10000), D[2,],
     xlab = "sample size", ylab = "empirical variance"
     , log = "x")
abline(h = 21, col = "red")
legend("right", legend = "True variance",
       col = "red",
       lty = 1 )
```

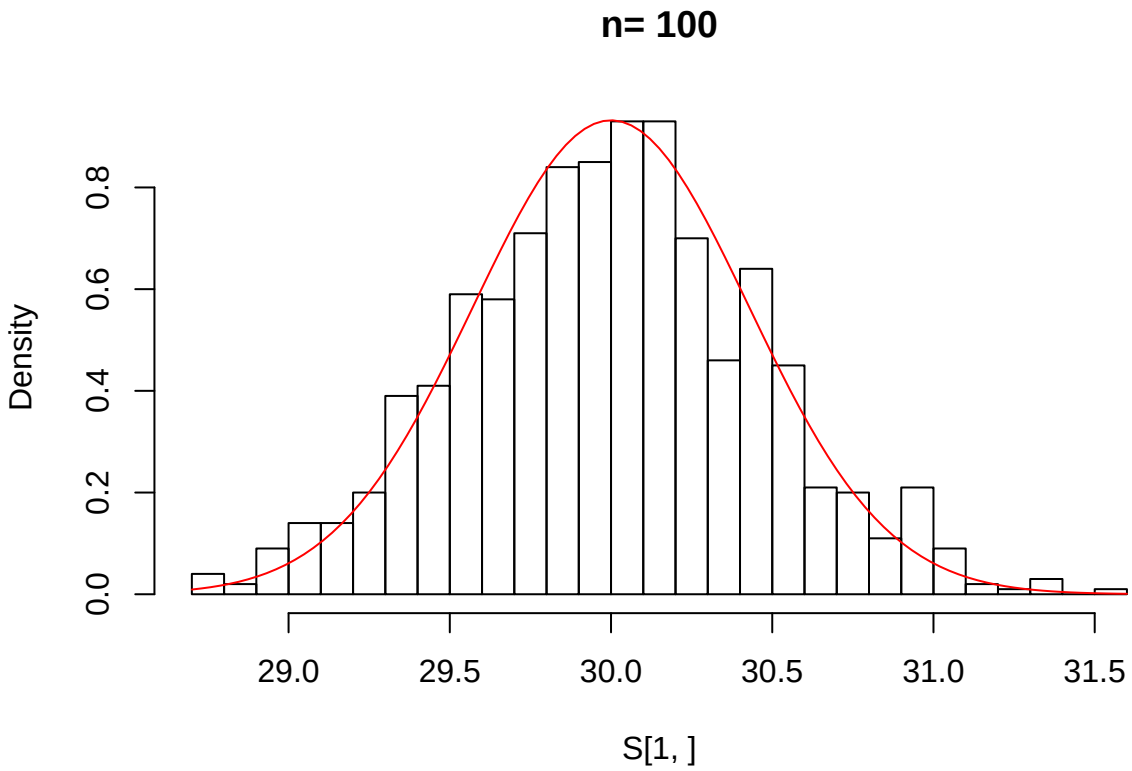


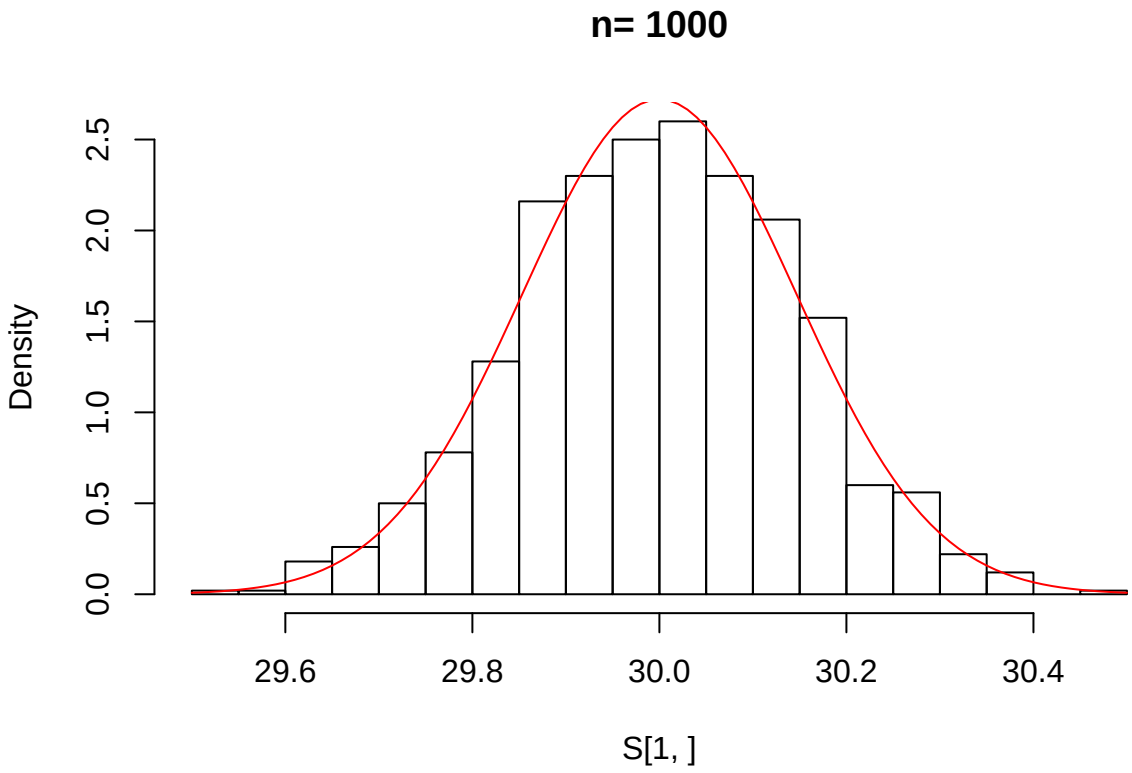
6.2

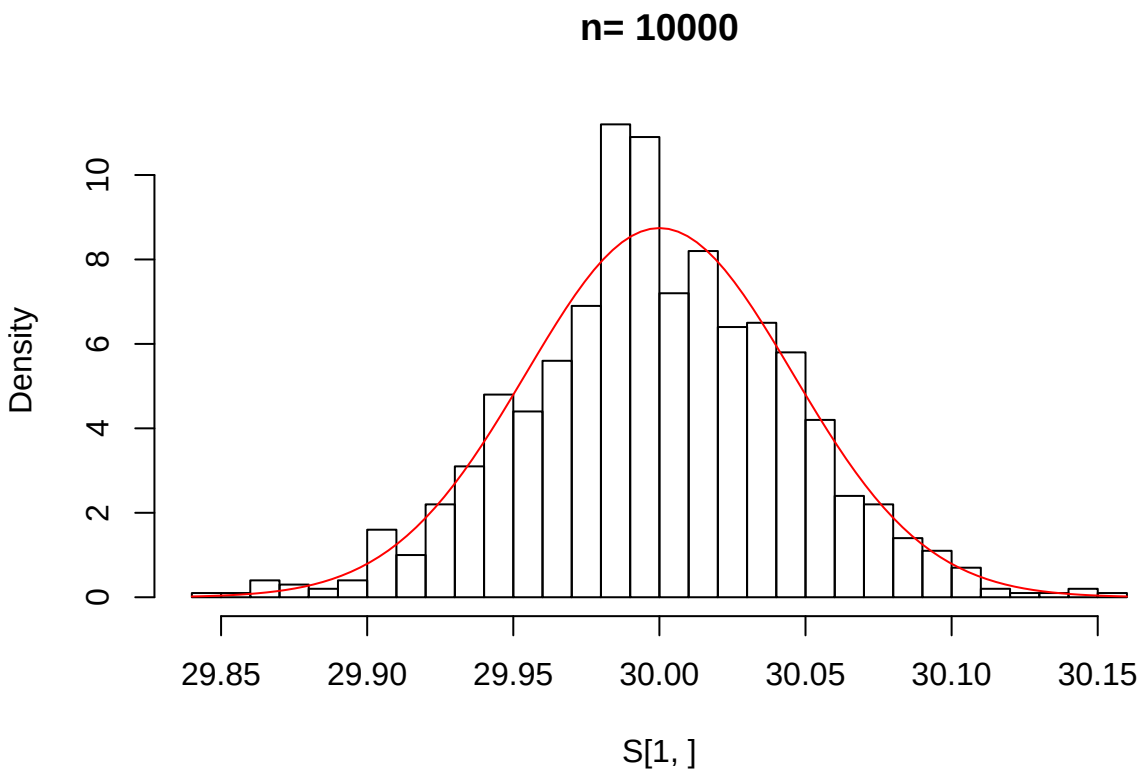
```r
big_experiment <- function(n){
  S <- replicate(n = 1000, experiment(n))
  hist(S[1,], breaks = "FD", probability = TRUE,
       main = paste("n=", n))
  v <- var(S[1,])
  se <- S[3,1] / sqrt(n)
  curve( dnorm(x, mean = 30, sd = se), add = TRUE,
         col = "red")
  return( c(v, sqrt(v), se))
}

sapply( 10^(1:4), big_experiment)
```


n= 10
Density
0.00
0.10
0.20
26
28
30
32
34
S[1,]


n= 100
Density
0.0
0.2
0.4
0.6
0.8
29.0
29.5
30.0
30.5
31.0
31.5
S[1,]


n= 1000
Density
0.0
0.5
1.0
1.5
2.0
2.5
29.6
29.8
30.0
30.2
30.4
S[1,]




```
##          [,1]      [,2]       [,3]        [,4]
## [1,] 2.211836 0.2137823 0.02154109 0.002089555
## [2,] 1.487224 0.4623659 0.14676882 0.045711656
## [3,] 1.044031 0.4280045 0.14661576 0.045625334
```