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EEG signal denoising and artifact removal using machine learning techniques

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dedication

Abstract

This thesis aims to investigate the use of machine learning techniques for EEG signal denoising. The first part of the thesis is dedicated to the introduction of the problem, the statement of the research questions and an overview of the basic concepts. The second part is dedicated to history and the state of the art of the signal analysis, and more specifically of the EEG signal analysis and denoising methods. The third part is dedicated to the machine learning techniques. The fourth part is dedicated to the experimental results and the last part is dedicated to the conclusions and future work.

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Chapter 1

Introduction

The chapter is built as follows: in the first section there is a brief introduction of the problem, the reasons that led to the choice of the topic and the outline of the thesis. In the second section there is the statement of the research questions. In the third section there is a brief description of the basic concepts.

1.1 Problem statement

What is EEG in the first place and why is it important? Electroencephalography (EEG) is a non-invasive technique used to measure the electrical activity of the brain. EEG signals are still among the less explored ones in the field of signal processing, despite their widespread use in clinical practice and research. In recent years, there has been a growing interest in developing denoising methods for EEG data to improve their quality and reliability.

The main objective of this thesis is to investigate and compare different EEG denoising methods, and to evaluate their performance in terms of signal quality, artifact removal, and preservation of underlying brain activity. Specifically, we will focus on the most recent machine learning techniques, such as generative adversarial networks (GANs), autoencoders (AEs), and deep learning (DL) models. We will also investigate the potential of combining different denoising methods to improve the performance of EEG denoising.

Overall, this thesis aims to provide a better understanding of the strengths and limitations of different EEG denoising methods, and to help researchers and clinicians make informed decisions when selecting the most appropriate denoising method for their EEG data analysis. By improving the quality of EEG signals, we can enhance our understanding of brain function and ultimately contribute to the development of more effective diagnostic and therapeutic tools for neurological disorders.

1.2 Research questions

The research questions are the following:

- How can we remove artifacts from EEG signals?

- How can we exploit the most recent machine learning techniques for EEG denoising?
- What are the strengths and limitations of these new methods?
- What are the performance of these new methods?

1.3 Basic concepts

In this section we will introduce the basic concepts that will be used in the thesis. The first part is dedicated to the EEG signal, the second part is dedicated to the EEG denoising.

1.3.1 EEG

When talking about EEG, in this thesis, we are referring to the electroencephalogram, in particular we are interested in the EEG waves. Electroencephalogram (EEG) waves are the patterns of electrical activity that are recorded by EEG measurements. These waves have different frequencies and amplitudes, and they reveal different states of brain activity. We divide the EEG waves in 5 main categories depending on their frequency: Alpha, Beta, Theta, Delta and Gamma waves.

Alpha waves are typically observed when someone is relaxed and awake, with a frequency of 8-13 Hz. Beta waves, on the other hand, are associated with active cognitive processing and have a higher frequency of 14-30 Hz. Theta waves are usually observed during drowsiness or light sleep and have a frequency of 4-7 Hz, while delta waves are typically observed during deep sleep and have a frequency of less than 4 Hz.

Gamma waves have a frequency of 30-100 Hz and are associated with higher cognitive functions such as attention and memory. Mu waves, with a frequency of 8-13 Hz, are observed in the sensorimotor cortex during movement and motor planning.

It's important to remember that EEG waves are not distinct entities but represent a continuous spectrum of activity that can be influenced by various factors such as task demands, attention, and emotion. Interpreting EEG waves requires expertise and context since different patterns of EEG activity may reflect different states of brain activity depending on the individual and the experimental conditions. Furthermore, research has shown that EEG waves can be useful in clinical diagnosis and prognosis, as well as in the assessment of cognitive function and brain injury. Therefore, understanding the various EEG waves and their characteristics can provide valuable insights into brain function and activity.

1.3.2 EEG denoising

EEG signals can be influenced by various factor that alter the real waves originated from neural activities, those factors are defined as artifacts. The artifacts can be classified as[1]:

- intrinsic artifacts: artifacts that depend on physiological sources, such as ocular artifacts (EOG) that come from eye movement and blinking, muscle artifacts (EMG) and cardiac artifacts (ECG)
- extrinsic artifacts: artifacts generated from external electromagnetic such as power line noise sources.

Denoising of EEG data is an essential task to be able to work on data and to extract meaningful information from it. The denoising process is complex and it does lead to different level of quality of the data depending on the method used, the quality of the data and the type of artifact.

There are several challenges[2] related both to single methods characteristic and general artifact removal. For example, some methods are computationally expensive and require a lot of time to be applied, some methods are not able to remove all kind of artifacts, some methods require a lot of data to be applied. On a general level, there is the problem of the lack of a standard method to evaluate the quality of the denoised data and the EEG applications are not yet fully commercial, so there hasn't been a sufficient investment in hardware and software to make the denoising process easier.

However the main goal of latest studies is to find a method that can denoise from all kind of artifacts and that can be used in a flexible and fast way, to accomodate the needs of all the different EEG applications.

Chapter 2

Literature review

In this chapter we will present the main methods used for the denoising of EEG data. In the first part we will focus on the methods used for the removal of artifacts from signals in general, in the second part we will present the ones specifically developed for the denoising of EEG data and in the third part we will explore the latest, machine learning related, methods.

2.1 Signal analysis

In this section we will present the basic techniques used for the analysis of signals that constituted a base for the ones developed for the analysis of EEG data.

2.1.1 Fourier transform

The first method used for the removal of artifacts from signals is the Fourier transform[3]. The Fourier transform is a mathematical tool used to decompose a signal into sine and cosine functions, that are used as basis functions for the original signal. It is defined as:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt \quad (2.1)$$

where $f(t)$ is the signal and ω is the frequency. The Fourier transform of a signal is a complex number, which can be decomposed into its real and imaginary parts. The real part of the Fourier transform is called the amplitude and the imaginary part is called the phase.

2.1.2 Fast Fourier transform

The Fourier transform is a very useful tool for the analysis of signals, but it is computationally expensive. The Fast Fourier transform (FFT)[4] is an algorithm that is used to calculate the Fourier transform of a signal in a shorter time: it reduces the time needed to $N \log(N)$, obtaining a speed-up of a factor of $N/\log(N)$. The FFT is used in many different fields of science such as signal processing, image processing, etc...

2.1.3 Short Time Fourier Transform

The Short Time Fourier Transform (STFT)[5][6] is a method used to calculate the Fourier transform of a signal. It is used with non stationary signals to find the frequency components of a signal over time. The STFT equation is defined as:

$$S(\tau) = s(t) \cdot h(t - \tau) \quad (2.2)$$

where $s(t)$ is the signal, $h(t)$ is the window function and τ is the time. The STFT can be used to find the frequency components of a signal:

$$S(\omega) = \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} s(\tau) \cdot h(t - \tau) e^{-i\omega t} d\tau \quad (2.3)$$

where $s(\tau)$ is the signal, $h(t)$ is the window function and ω is the frequency.

2.1.4 Wigner-Ville distribution

The Wigner-Ville distribution (WVD)[7] is a method used to provide the description of a signal in the time-frequency domain with higher resolution. It is described by the following equation:

$$W(\tau, \omega) = \int_{-\infty}^{\infty} s(t - \tau) \cdot s^*(t - \tau) e^{-2\pi i \omega \tau} d\tau \quad (2.4)$$

where $s(t)$ is the signal, τ is the time and ω is the frequency. The main problem of the WVD is that it introduces the so called cross terms. In fact, for a signal

$$x(t) = x_1(t) + x_2(t) \quad (2.5)$$

the corresponding WVD is:

$$WVD(t, f) = WVDx_1(t, f) + WVDx_2(t, f) + 2Re[WVDx_1x_2(t, f)] \quad (2.6)$$

where $2Re[WVDx_1x_2(t, f)]$ is the cross term which is removable at the expense of a loss of resolution.

2.1.5 Wavelet transform

The wavelet transform[8] is a method used to find the frequency components of a signal over time. It is used with non stationary signals and it provides a better time-frequency resolution than the STFT and the WVD since it gives a better simultaneous time-frequency localization.

It expresses the signal as a linear combination of mother wavelets. The wavelet transform is defined as:

$$W(\tau, \omega) = \int_{-\infty}^{\infty} s(t - \tau) \cdot \psi(t - \tau) e^{-2\pi i \omega \tau} d\tau \quad (2.7)$$

where $s(t)$ is the signal, τ is the time, ω is the frequency and $\psi(t)$ is the wavelet function.

A possible choice for the wavelet function is the Morlet wavelet:

$$\psi(t) = \frac{1}{\sqrt{2c\pi}} \cdot e^{-\frac{t^2}{2c^2}} \cdot e^{i\omega_0 t} \quad (2.8)$$

where ω_0 is the central frequency of the wavelet and c is the width of the wavelet.

2.1.6 Matching Pursuit

The Matching Pursuit (MP)[10] is a greedy algorithm used to approximate a signal with a linear combination of basis functions called time-frequency atoms, selected from a dictionary of functions.

A general family of time-frequency atoms can be generated by scaling, translating and modulating a single window function $g(t)$ as follows:

$$g_\gamma(t) = \frac{1}{\sqrt{s}} \cdot g\left(\frac{t-u}{s}\right) \cdot e^{i\xi t} \quad (2.9)$$

where γ is the tuple (s, u, ξ) s is the scale, u is the translation and ξ is the frequency modulation.

The algorithm aims to find a set of atoms $\gamma_1, \gamma_2, \dots, \gamma_N$ that best approximate the signal $f(t)$, i.e.:

$$f(t) \approx \sum_{i=1}^N a_i \cdot g_{\gamma_i}(t) \quad (2.10)$$

where a_i are the expansion coefficients.

The vector f can be decomposed into

$$f(t) = \langle f, g_\gamma \rangle g_\gamma + Rf(t) \quad (2.11)$$

where Rf is the residual vector after approximating f in the direction of g_γ .

MP iteratively decomposes the residue Rf by projecting it on a vector of D that matches Rf at best.

2.2 EEG denoising

The denoising of EEG signals is performed using techniques developed from the field of signal processing, but more suited to the specific characteristics of EEG signals and artifacts. Here we present the most common methods.

2.2.1 Regression

The regression method[11], considers the EEG signal to be a linear combination of the artifacts and the clean EEG signal. In particular, the EEG signal is modeled as:

$$EEG = \alpha \cdot EMG + \beta \cdot EOG + \gamma \cdot ECG + \delta \cdot CEEG \quad (2.12)$$

where *EEG* is the registered EEG signal, *EMG* is the muscle artifact, *EOG* is the ocular artifact, *ECG* is the cardiac artifact and *CEEG* is the clean EEG signal. The regression coefficients $\alpha, \beta, \gamma, \delta$ are estimated using the least squares method. The regression method is simple and fast, but it doesn't take into account the temporal correlation between the artifacts and the EEG signal, the so called cross terms. Furthermore it requires the knowledge of the artifacts signals.

2.2.2 Blind source separation

Blind source separation (BSS)[12] is a class of methods used to separate a mixture of signals into their individual components. There are several BSS algorithms, all based on an unsupervised component separation. The most common BSS algorithms are the Principal Component Analysis (PCA), the Principal Component Analysis (ICA) and the Canonical Correlation Analysis.

Principal Component Analysis

The PCA[13] is a method used to find a linear transformation of the mixture signals that maximizes the variance of the transformed signals. First, data are standardized by subtracting the mean and dividing by the standard deviation.

$$\mathbf{Z} = (\mathbf{X} - \mathbf{m})/\mathbf{s} \quad (2.13)$$

Then, the covariance matrix is computed.

$$\mathbf{C} = \frac{1}{n-1} \mathbf{Z}^T \mathbf{Z} \quad (2.14)$$

The eigenvectors of the covariance matrix are the principal components of the data while the eigenvalues are the variances of the principal components.

$$\mathbf{C}\mathbf{v}_i = \lambda_i \mathbf{v}_i \quad (2.15)$$

The principal components found by projecting \mathbf{x} onto those perpendicular basis vectors are uncorrelated, and their directions orthogonal. This scheme is very efficient in redundancy reduction, and can be said to maximize the amount of information spanned by a subset of dimensions of the initial vector.

Independent Component Analysis

The ICA[14] is a method used to find a linear transformation of the mixture signals that maximizes the non-Gaussianity of the transformed signals. It consists in decomposing the signal in Independent components and discarding the ones containing artifacts. In blind source separation, the original independent sources are assumed to be unknown, and we only have access to their weighted sum.[15]

The signal is modeled as:

$$x(t) = \sum_{i=1}^N a_i \cdot s_i(t) \quad (2.16)$$

where $x(t)$ is the mixture signal, $s_i(t)$ is the i -th source signal and a_i is the mixing coefficient.

The ICA algorithm aims to find the source signals $s_i(t)$ and the mixing coefficients a_i that maximize the non-Gaussianity of the mixture signal $x(t)$. It is based on the assumption that the sources are statistically independent.

Canonical Correlation Analysis

The CCA[16] is a method used to find a linear transformation of the mixture signals that maximizes the correlation between the transformed signals. More specifically, CCA identifies two sets of variables, X and Y, and finds linear combinations of X and Y that are maximally correlated. These linear combinations are called canonical variates. The first canonical variate is the linear combination of X and Y that has the highest correlation, and each subsequent canonical variate is the linear combination that has the highest correlation subject to being orthogonal to the previous canonical variates.

2.2.3 Empirical Mode Decomposition

The EMD[17] is a method used to decompose a signal into a set of intrinsic mode functions (IMFs). The EMD is a data-driven method that does not require any prior knowledge of the signal.

It decomposes a signal into a finite number of intrinsic mode functions (IMFs), which are functions that capture the local behavior of the signal. EMD is based on the concept of sifting, which involves iteratively extracting the local maxima and minima of the signal to generate IMFs. The basic steps of EMD are as follows:

- Given a signal $x(t)$, find all of its local maxima and minima.
- Interpolate between the local maxima and minima to create an upper and lower envelope for the signal. This step effectively eliminates the high-frequency components of the signal and captures its slowly varying behavior.
- Calculate the mean of the upper and lower envelopes to obtain a first IMF, $c_1(t)$.
- Subtract $c_1(t)$ from the original signal to obtain a new signal, $r_1(t) = x(t) - c_1(t)$.
- Repeat steps 1-4 on the residual signal $r_1(t)$ to obtain the second IMF, $c_2(t)$.
- Continue this process until a stopping criterion is met, such as the number of IMFs or the amplitude of the residual signal falling below a certain threshold.

The resulting IMFs are functions that oscillate around zero with a characteristic scale and capture the local behavior of the signal at different scales. The final residual signal is a monotonic function that represents the long-term trend of the signal.

2.2.4 Filtering techniques

Adaptive filtering

Adaptive filtering[18] is a method used to estimate the unknown input signal from the noisy output signal. It is based on the assumption that the input signal is a linear combination of the unknown input signal and the noise.

The basic steps of adaptive filtering are as follows:

- Given a noisy signal $y(t)$, estimate the unknown input signal $x(t)$.
- Initialize the filter coefficients w_0 .
- For each sample $y(t)$, compute the error signal $e(t)$.
- Update the filter coefficients $w(t)$.
- Repeat steps 2-4 until a stopping criterion is met.

The error signal is defined as the difference between the noisy signal and the estimated input signal. The filter coefficients are updated according to the following equation:

$$w(t+1) = w(t) + \mu \cdot e(t) \cdot x(t) \quad (2.17)$$

where μ is the step size.

Wiener filtering

Wiener filtering[19] is a method used to estimate the unknown input signal from the noisy output signal. It is based on the assumption that the input signal is a linear combination of the unknown input signal and the noise.

The basic steps of Wiener filtering are as follows:

- Given a noisy signal $y(t)$, estimate the unknown input signal $x(t)$.
- Compute the power spectral density (PSD) of the noise signal $n(t)$.
- Compute the PSD of the input signal $x(t)$.
- Compute the PSD of the output signal $y(t)$.
- Compute the Wiener filter coefficients $w(t)$.
- Apply the Wiener filter to the noisy signal $y(t)$ to obtain the estimated input signal $x(t)$.

The Wiener filter coefficients are computed according to the following equation:

$$w(t) = \frac{S_x(t)}{S_x(t) + S_n(t)} \quad (2.18)$$

where $S_x(t)$ is the PSD of the input signal $x(t)$ and $S_n(t)$ is the PSD of the noise signal $n(t)$.

2.3 Machine learning

The latest approaches involve the use of machine learning techniques to denoise EEG signals. Machine learning is a subfield of artificial intelligence that focuses on the development of computer programs that can learn from data. In the following chapter we will discuss the different architectures of neural networks that have been used in previous works and in this thesis.

2.3.1 Fully connected neural network

Fully Connected Neural Networks, also known as Feedforward Neural Networks or Multilayer Perceptrons, are a type of artificial neural network architecture commonly used in deep learning. In this type of neural network, each neuron in a layer is connected to all the neurons in the previous layer and all the neurons in the subsequent layer. These connections are weighted, and each neuron computes a weighted sum of its inputs, passes this sum through an activation function, and then forwards the result to the next layer.

The architecture of a fully connected neural network consists of one or more layers of neurons, including an input layer, one or more hidden layers, and an output layer. The input layer receives the data to be processed, and each neuron in this layer represents a feature of the input. The hidden layers perform complex computations on the input data, and the output layer produces the final output of the network. The number of neurons in the input and output layers is determined by the size of the input and output data.

The operation of a fully connected neural network can be described mathematically using the following equations:

Linear transformation: Each neuron in the hidden layers and output layer performs a linear transformation of its inputs, which is a weighted sum of the outputs of the neurons in the previous layer. The linear transformation of neuron i in layer l can be expressed as:

$$z_i^l = \sum_{j=1}^{n^{l-1}} w_{ij}^l a_j^{l-1} + b_i^l$$

where n^{l-1} is the number of neurons in the previous layer, w_{ij}^l is the weight of the connection between neuron j in layer $l-1$ and neuron i in layer l , a_j^{l-1} is the output of neuron j in layer $l-1$, b_i^l is the bias term of neuron i in layer l , and z_i^l is the pre-activation value of neuron i in layer l .

Activation function: The pre-activation values of each neuron are passed through an activation function to introduce non-linearity into the network. Commonly used activation functions include sigmoid, tanh, ReLU, and softmax. The activation function of neuron i in layer l can be expressed as:

$$a_i^l = \sigma(z_i^l)$$

where σ is the activation function.

Output computation: The output of the network is produced by the output layer, which applies a final activation function to the pre-activation values of its neurons. The activation function of the output layer depends on the task the network is designed to perform. The output of neuron i in the output layer can be expressed as:

$$\hat{y}_i = \sigma(z_i^L)$$

where L is the index of the output layer, σ is the output activation function, and \hat{y}_i is the predicted value of the i -th output.

The network is trained by adjusting the weights and biases using backpropagation [6.1.2] with respect to a loss function [6.1.1].

2.3.2 Convolutional neural network

Convolutional Neural Networks (CNNs) are a type of artificial neural network architecture commonly used in deep learning for image and video analysis. Unlike fully connected neural networks, CNNs take into account the spatial structure of input data, such as images, by using convolutional layers that apply filters to local regions of the input data.

The architecture of a CNN consists of one or more convolutional layers, followed by one or more fully connected layers. The convolutional layers perform feature extraction by applying filters to local regions of the input data, producing feature maps that highlight different aspects of the input data. The fully connected layers perform classification or regression on the feature maps produced by the convolutional layers.

The operation of a CNN can be described mathematically using the following equations:

Convolution: In the convolutional layers, a set of filters or kernels is applied to local regions of the input data to produce feature maps. The convolution operation can be expressed as:

$$z_{i,j,k} = \sum_{l=1}^{d_{in}} \sum_{m=1}^f \sum_{n=1}^f x_{i+m-1,j+n-1,l} w_{m,n,l,k} + b_k$$

where x is the input data, w is the filter, b is the bias, d_{in} is the number of input channels, f is the size of the filter, z is the output feature map, and i, j, k are the indices of the feature map.

Activation function: The pre-activation values of each neuron are passed through an activation function to introduce non-linearity into the network. Commonly used activation functions include ReLU and sigmoid. The activation function of neuron i in layer l can be expressed as:

$$a_{i,j,k}^l = \sigma(z_{i,j,k}^l)$$

where σ is the activation function.

Pooling: The pooling layer downsamples the output of the convolutional layer, reducing the size of the feature maps and introducing translation invariance. Commonly used pooling operations include max pooling and average pooling.

Flattening: The output of the pooling layer is flattened into a vector

Fully connected layers: The fully connected layers perform classification or regression on the feature maps produced by the convolutional and pooling layers. The operation of a fully connected layer is similar to that of a fully connected neural network.

2.3.3 Recurrent neural network

2.3.4 Long short-term memory

2.3.5 Generative adversarial network

2.3.6 Autoencoder

Chapter 3

Methodology

3.1 Data

The data used in this thesis are the EEG signals of the EEGdenoiseNet dataset[20]. It is comprehensive benchmark dataset that proves to be an exemplary resource for training and evaluating deep learning-based EEG denoising models. It is composed of 4514 clean EEG epochs, 3400 EOG epochs, and 5598 EMG epochs, making it an inclusive and extensive dataset that caters to the needs of researchers and practitioners alike.

The incorporation of these various types of epochs in the dataset enables to generate a considerable number of noisy EEG epochs with their corresponding ground truth data. As a result, this compilation has proven to be an invaluable resource for model training and testing, facilitating an enhanced understanding of EEG denoising.

3.2 Preprocessing

3.3 Denoising

3.4 Machine learning

Chapter 4

Experimental results

4.1 Results

4.2 Discussion

Chapter 5

Conclusions and future work

5.1 Conclusions

5.2 Future work

Chapter 6

Appendix

6.1 Neural network

6.1.1 Loss function

To train the network, a loss function is used to measure the difference between the predicted output and the true output. Commonly used loss functions include mean squared error, cross-entropy, and binary cross-entropy.

6.1.2 Backpropagation

The weights and biases of the network are updated during training using an algorithm called backpropagation, which computes the gradient of the loss function with respect to the network parameters. The gradient is then used to update the weights and biases of the network using an optimization algorithm such as stochastic gradient descent.

Bibliography

- [1] Jiang, X.; Bian, G.-B.; Tian, Z. Removal of Artifacts from EEG Signals: A Review. *Sensors* 2019, 19, 987.
- [2] Wajid Mumtaz, Suleman Rasheed, Alina Irfan, Review of challenges associated with the EEG artifact removal methods, *Biomedical Signal Processing and Control*, Volume 68, 2021, 102741, ISSN 1746-8094
- [3] Boashash, B., ed. (2003), *Time–Frequency Signal Analysis and Processing: A Comprehensive Reference*, Oxford: Elsevier Science, ISBN 978-0-08-044335-5.
- [4] Cooley, James W. (1987). The Re-Discovery of the Fast Fourier Transform Algorithm (PDF). *Microchimica Acta*. Vol. III. Vienna, Austria. pp. 33–45. Archived (PDF) from the original on 2016-08-20.
- [5] R. Álvarez, E. Borbor and F. Grijalva, "Comparison of methods for signal analysis in the time-frequency domain," 2019 IEEE Fourth Ecuador Technical Chapters Meeting (ETCM), 2019, pp. 1-6, doi: 10.1109/ETCM48019.2019.9014860.
- [6] W. -k. Lu and Q. Zhang, "Deconvolutive Short-Time Fourier Transform Spectrogram," in *IEEE Signal Processing Letters*, vol. 16, no. 7, pp. 576-579, July 2009, doi: 10.1109/LSP.2009.2020887.
- [7] E. Chassande-Mottin and A. Pai, "Discrete time and frequency Wigner-Ville distribution: Moyal's formula and aliasing," in *IEEE Signal Processing Letters*, vol. 12, no. 7, pp. 508-511, July 2005, doi: 10.1109/LSP.2005.849493
- [8] S. Zhou, B. Tang and R. Chen, "Comparison between Non-stationary Signals Fast Fourier Transform and Wavelet Analysis," 2009 International Asia Symposium on Intelligent Interaction and Affective Computing, Wuhan, China, 2009, pp. 128-129, doi: 10.1109/ASIA.2009.31.
- [9] V. M. Pukhova and M. S. Stepanova, "Up-Chirplet and Down-Chirplet Transforms of Non-Stationary Signals," 2019 IEEE Conference of Russian Young Researchers in Electrical and Electronic Engineering (EIConRus), Saint Petersburg and Moscow, Russia, 2019, pp. 1221-1225, doi: 10.1109/EIConRus.2019.8657179.
- [10] S. G. Mallat and Zhifeng Zhang, "Matching pursuits with time-frequency dictionaries," in *IEEE Transactions on Signal Processing*, vol. 41, no. 12, pp. 3397-3415, Dec. 1993, doi: 10.1109/78.258082.

- [11] Klados, M.A.; Papadelis, C.; Braun, C.; Bamidis, P.D. REG-ICA: A hybrid methodology combining blind source separation and regression techniques for the rejection of ocular artifacts. *Biomed. Signal Process Control*
- [12] J. A. Nascimento, J. M. S. Nascimento, and J. M. C. Nascimento, "Blind source separation: A review," in *IEEE Signal Processing Magazine*, vol. 26, no. 6, pp. 22-38, Nov. 2009, doi: 10.1109/MSP.2009.937000.
- [13] Casarotto, S.; Bianchi, A.M.; Cerutti, S.; Chiarenza, G.A. Principal component analysis for reduction of ocular artefacts in event-related potentials of normal and dyslexic children. *Clin. Neurophysiol.* 2004, 115, 609–619.
- [14] Jung, T.P.; Makie, S.; Bell, A.J.; Sejnowski, T.J. Independent component analysis of electroencephalographic and event-related potential data. *Cent. Audit. Process. Neural Model.* 1996, 2, 1548–1551.
- [15] Vigário, R. Extraction of ocular artifacts from EEG using independent component analysis. *Electroencephalogr. Clin. Neurophysiol.* 1997, 103, 395–404
- [16] N. Robinson, K. P. Thomas and A. P. Vinod, "Canonical correlation analysis of EEG for classification of motor imagery," 2017 IEEE International Conference on Systems, Man, and Cybernetics (SMC), Banff, AB, Canada, 2017, pp. 2317-2321, doi: 10.1109/SMC.2017.8122967.
- [17] V. Bajaj and R. B. Pachori, "Classification of Seizure and Nonseizure EEG Signals Using Empirical Mode Decomposition," in *IEEE Transactions on Information Technology in Biomedicine*, vol. 16, no. 6, pp. 1135-1142, Nov. 2012, doi: 10.1109/TITB.2011.2181403.
- [18] He, P., Wilson, G. Russell, C. Removal of ocular artifacts from electroencephalogram by adaptive filtering. *Med. Biol. Eng. Comput.* 42, 407–412 (2004). <https://doi.org/10.1007/BF02344717>
- [19] Somers, B., Francart, T., Bertrand, A. (2018). A generic EEG artifact removal algorithm based on the multi-channel Wiener filter. *Journal of neural engineering*, 15(3), 036007.
- [20] EEGdenoiseNet: A Comprehensive Benchmark Dataset for EEG Denoising, <https://arxiv.org/abs/2103.03894>