Problem Set #3

Perturbations Methods, Prof. Kerk Phillips
Martina Fraschini

Exercise 1

From the Lecture Notes we know that the second-order terms of $F(x,u)=0$ are:

$$\begin{aligned} &F_{xx}\{x(u),u\}x_u(u)x_u(u)+F_{xu}\{x(u),u\}x_u(u)+\\ &+F_x\{x(u),u\}x_{uu}(u)+F_{xu}\{x(u),u\}x_u(u)+F_{uu}\{x(u),u\}=0 \end{aligned}$$

Taking the derivative of this formula with respect to u, we obtain

$$x_{uuu}=-\frac{F_{xxx}x_u^3+3(F_{xxu}x_u^2+F_{xuu}x_u+F_{xx}x_ux_{uu}+F_{xu}x_{uu})+F_{uuu}}{F_x}.$$

The third-order Taylor approximation is given by

$$x(u)=x\left(u_0\right)+x_u\left(u_0\right)\left(u-u_0\right)+\frac{1}{2}x_{uu}\left(u_0\right)\left(u-u_0\right)^2+\frac{1}{6}x_{uuu}\left(u_0\right)\left(u-u_0\right)^3$$

Exercise 2

For the numerical solution of this exercise, I strictly follow the Jupyter Notebook "Perturb2_example2.ipynb" provided in the GitHub repository for the course.
Please, look at the computational part on the Jupyter Notebook "Perturb.ipynb".

Exercise 3

For the numerical solution of this exercise, I strictly follow the Jupyter Notebook "Perturb2_example2.ipynb" provided in the GitHub repository for the course.
Please, look at the computational part on the Jupyter Notebook "Perturb.ipynb".