Problem Set #2

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Exercise 1

For the Brock and Mirman model, using Uhlig's notation, we have that:

$$F = \beta \frac{\alpha \bar{K}^{\alpha-1}}{\bar{K}^{\alpha} - \bar{K}},$$

$$G = -\beta \frac{\alpha \bar{K}^{\alpha-1} \left(\alpha + \bar{K}^{\alpha-1} \right)}{\bar{K}^{\alpha} - \bar{K}},$$

$$H = \beta \frac{\alpha^2 \bar{K}^{2(\alpha-1)}}{\bar{K}^{\alpha} - \bar{K}},$$

$$L = -\beta \frac{\alpha \bar{K}^{2\alpha-1}}{\bar{K}^{\alpha} - \bar{K}},$$

$$M = \beta \frac{\alpha^2 \bar{K}^{2(\alpha-1)}}{\bar{K}^{\alpha} - \bar{K}},$$

$$N = \rho,$$

where $\bar{K} = A^{\frac{1}{1-\alpha}}$ and $A = \alpha\beta$. The policy function is given by

$$K_{t+1} = \bar{K} + P \left(K_t - \bar{K} \right) + Q z_t,$$

where

$$P = \frac{-G \pm \sqrt{G^2 - 4FH}}{2F},$$

$$Q = -\frac{LN + M}{FN + FP + G}.$$

From Exercise 1 in Problem Set # 1, we know that the algebraic solution is given by $K_{t+1} = \alpha\beta e^{z_t} K_t^{\alpha}$.
Please, look at the computational part on the Jupyter Notebook "Linear.ipynb".

Exercise 2

From the previous exercise we know that $K_{t+1} = \bar{K} + P \left(K_t - \bar{K} \right) + Q z_t$. Substituting $K \equiv \ln k$, we have that $k_{t+1} = \exp\{\bar{K} + P \left(\ln k_t - \bar{K} \right) + Q z_t\}$.
Please, look at the computational part on the Jupyter Notebook "Linear.ipynb".

Exercise 3

We know that

$$E_t \left\{ F\tilde{X}_{t+1} + G\tilde{X}_t + H\tilde{X}_{t-1} + L\tilde{Z}_{t+1} + M\tilde{Z}_t \right\} = 0,$$

$$\tilde{Z}_t = N\tilde{Z}_{t-1} + \varepsilon_t,$$
$$\tilde{X}_t = P\tilde{X}_{t-1} + Q\tilde{Z}_t.$$

With some tedious algebra we have that

$$E_t\left\{F\tilde{X}_{t+1} + G\tilde{X}_t + H\tilde{X}_{t-1} + L\tilde{Z}_{t+1} + M\tilde{Z}_t\right\} = 0,$$

$$\iff E_t\left\{F\left(P\tilde{X}_t + Q\tilde{Z}_{t+1}\right) + G\left(P\tilde{X}_{t-1} + Q\tilde{Z}_t\right) + H\tilde{X}_{t-1} + L\left(N\tilde{Z}_t + \varepsilon_{t+1}\right) + M\tilde{Z}_t\right\} = 0,$$

$$\iff FP\left(P\tilde{X}_{t-1} + Q\tilde{Z}_t\right) + FQN\tilde{Z}_t + GP\tilde{X}_{t-1} + GQ\tilde{Z}_t + H\tilde{X}_{t-1} + LN\tilde{Z}_t + M\tilde{Z}_t = 0,$$

$$\iff \Big[FPP + GP + H\Big]\tilde{X}_{t-1} + \Big[FPQ + FQN + GQ + LN + M\Big]\tilde{Z}_t = 0,$$

$$\iff \Big[(FP + G)P + H\Big]\tilde{X}_{t-1} + \Big[(FQ + L)N + (FP + G)Q + M\Big]\tilde{Z}_t = 0.$$

Exercise 4
This problem is the same presented in Problem Set #1, Exercise 6.
Please, look at the computational part on the Jupyter Notebook "DSGE.ipynb".

Exercise 5
This problem is the same presented in Problem Set #1, Exercise 7.
Please, look at the computational part on the Jupyter Notebook "DSGE.ipynb".

Exercise 6
For the numerical solution of this exercise, I strictly follow the Jupyter Notebook "DSGEexample_key.ipynb" provided in the GitHub repository for the course.
Please, look at the computational part on the Jupyter Notebook "Linear.ipynb".

Exercise 7
For the numerical solution of this exercise, I strictly follow the Jupyter Notebooks "DSGEexample_key.ipynb" and "DSGE_LinApp.ipynb"" provided in the GitHub repository for the course.
Please, look at the computational part on the Jupyter Notebook "Linear.ipynb".

Exercise 8
Please, look at the computational part on the Jupyter Notebook "Linear.ipynb".

Exercise 9
For the numerical solution of this exercise, I strictly follow the Jupyter Notebook "DSGEexample_key.ipynb" provided in the GitHub repository for the course.
Please, look at the computational part on the Jupyter Notebook "Linear.ipynb".