CBDC and QE/QT dynamics

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PREMISES

This project is the result of my time at the Bank of England as part of the PhD intern program. It is an extension of the work presented in the paper "CBDC and Banks: Threat or Opportunity?", co-authored with Luciano Somoza[1] and chapter of my dissertation "Three Essays on Innovation in Finance". I develop an economic framework that I calibrate on UK data to study the impact of a central bank digital currency (CBDC) in a quantitative easing (QE) environment. This documents only presents the model and its calibration, and it serves as part of the documentation to understand the code. The quantitative tightening (QT) part still needs to be developed. The motivation and results are discussed in the final presentation (slides are available) and will be integrated in the document soon.

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1 Baseline banking Model

Before introducing and analyzing the effect of CBDC, we develop a baseline dynamic partial equilibrium model of the banking sector. The setting is based on Corbae and D'Erasmo (2021).

We consider a representative commercial bank that operates over infinite time. Each period the bank intermediates between a unit mass of ex-ante identical entrepreneurs and a unit mass of households. The entrepreneurs borrow one unit from the bank and invest it in a technology that generates a stochastic return in the next period. The return depends on the economic shock. Households are risk-neutral and sufficiently patient to exercise their saving options. At the beginning of the period the bank observes the state of the economy and chooses the interest rates on loans and deposits. Finally, the central bank regulates the commercial bank and conducts monetary policy, intervening in the markets in times of crisis (with quantitative easing).

Figure 1 outlines the baseline structure, representating the relations between entrepreneurs, households, commercial bank, and central bank.

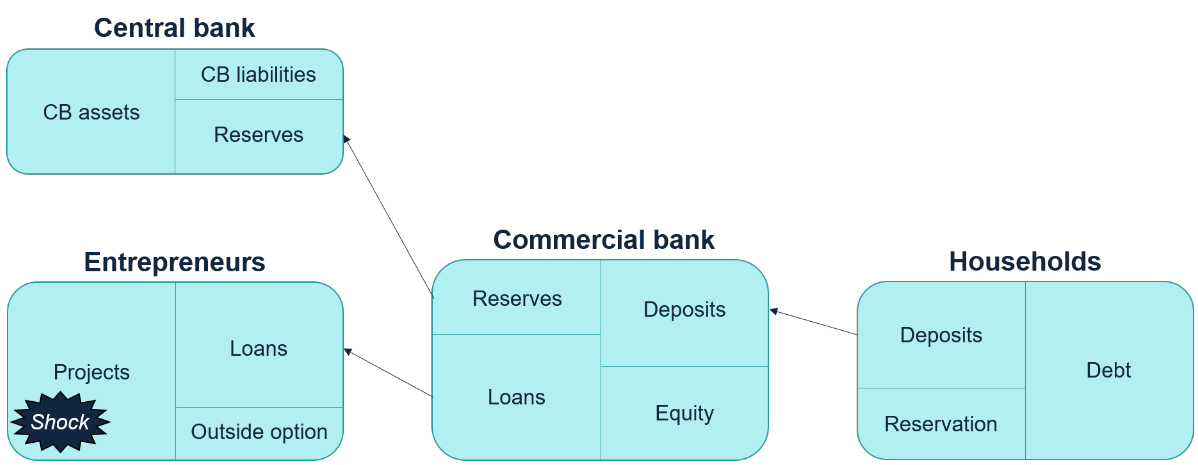

Figure 1. The figure illustrates the baseline model of the banking sector. The bank intermediates between entrepreneurs and households. Entrepreneurs borrow from the bank and invest in a technology that generates a return depending on the economic shock. Households put their savings in bank deposits. The bank decides the interest rates on loans and deposits. The central bank regulates the commercial bank and conducts monetary policy.

1.1 Entrepreneurs

The entrepreneurs are infinitely lived and risk-neutral. Each period, they need to borrow from the bank in order to fund a new project. The project requires one unit of investment and returns

$\mathcal{R}$, which is stochastic and can assume the following values:

$$\mathcal{R}_t = \begin{cases} 1 + z_t R_{t-1}, & \text{with prob. } p(R_{t-1}, z_t) \\ 1 - \lambda, & \text{with prob. } 1 - p(R_{t-1}, z_t) \end{cases}, \tag{1}$$

where z_t is the aggregate shock realized in the current period, R_{t-1} the entrepreneurs' choice of technology in the previous period, $p(R_{t-1}, z_t)$ the probability of success, λ the loss for project failure. The project's success is independent across entrepreneurs, but it depends on the entrepreneurs' choice of technology, $R_{t-1} \geqslant 0$, and the realized aggregate technology shock, z_t.

The aggregate shock represents the state of the economy and follows an log-AR(1) process:

$$\ln(z_t) = \rho \ln(z_{t-1}) + u_t, \tag{2}$$

where ρ is the autoregressive coefficient and u_t the innovation, that is i.i.d. and drawn from the normal distribution $\mathcal{N}(0, \sigma_u^2)$.

A project with higher returns has more risk of failure, and there is less failure in good times. Therefore, we parametrize the stochastic process for the entrepreneurs' project as $s_t = az_t - bR_{t-1} + \varepsilon_t$, where ε_t is i.i.d. across agents and time and drawn from $\mathcal{N}(0, \sigma_\varepsilon^2)$. As the success is defined by $s_t > 0$, the probability of success is given by:

$$\begin{aligned} p(R_{t-1}, z_t) &= \Pr(s_t > 0 \,|\, R_{t-1}, z_t) \\ &= 1 - \Pr(s_t \leqslant 0 \,|\, R_{t-1}, z_t) \\ &= 1 - \Pr(\varepsilon_t \leqslant -az_t + bR_{t-1}) \\ &= \Phi\left(\frac{az_t - bR_{t-1}}{\sigma_\varepsilon}\right), \end{aligned} \tag{3}$$

where $\Phi(.)$ is a standard normal cumulative distribution. The technology exhibits a risk-return trade-off because the probability of success of the project is increasing with z_t and decreasing with R_{t-1}.

Entrepreneurs have an outside option (or reservation utility) $\omega_t \in [\underline{\omega}, \overline{\omega}]$, i.i.d. over time and drawn from distribution function $\Omega(\omega_t)$, that for simplicity we assume to be the uniform distribution

$\mathcal{U}(\underline{\omega}, \overline{\omega})$. This outside option represents an alternative source of finance to the bank loan.

While entrepreneurs are ex-ante identical, they are ex-post heterogeneous due to the shocks' realizations to the return on their project. Both R_{t-1} and ω_t are private information to the entrepreneur, as well as his history of past borrowing and repayments.

When the entrepreneur asks for a loan from the bank, there is a limited liability on the borrower's part. Therefore, the entrepreneur will pay back $1 + r^L$ in case of success, where r^L is the interest rate charged by the bank on the loan, and $1 - \lambda$ in the unsuccessful state. The expected payoff will be:

$$\begin{aligned}\Pi_t &= p\left(R_{t-1}, z_t\right)\Big[\left[1 + z_t R_{t-1}\right] - \left[1 + r_t^L\right]\Big] + \Big[1 - p(R_{t-1}, z_t)\Big]\Big[[1-\lambda] - [1-\lambda]\Big] \\ &= p\left(R_{t-1}, z_t\right)\left[z_t R_{t-1} - r_t^L\right]. \end{aligned} \tag{4}$$

Entrepreneurs take the loan interest rate r^L as given and choose whether to demand a loan and, if so, the technology R. If they decide to participate and request a loan, the entrepreneurs choose the technology to solve the following problem:

$$v\left(r_t^L, z_{t-1}\right) = \max_{R_{t-1}} \ \mathbb{E}_{z_t|z_{t-1}}\Big[\ p\left(R_{t-1}, z_t\right)\left[z_t R_{t-1} - r_t^L\right]\Big] \tag{5}$$

$$\text{s.t.} \quad v\left(r_t^L, z_{t-1}\right) \geqslant \omega_t. \tag{6}$$

The first-order condition is given by:

$$\mathbb{E}_{z_t|z_{t-1}}\left[p\left(R_{t-1}, z_t\right) z_t + \frac{\partial p\left(R_{t-1}, z_t\right)}{\partial R_{t-1}}\left[z_t R_{t-1} - r_t^L\right]\right] = 0. \tag{7}$$

The first term is positive and represents the benefit of choosing a higher return project. The second term is negative and corresponds to the cost associated with the increased risk of failure.

The aggregate demand for loans is:

$$L_t\left(r_t^L, z_{t-1}\right) = \int_{\underline{\omega}}^{\overline{\omega}} \mathbb{1}_{\omega_t \leqslant v\left(r_t^L, z_{t-1}\right)} d\Omega(\omega_t). \tag{8}$$

In other words, the total demand is the sum of all entrepreneurs for which the optimal project value is higher than their outside option. Applying the envelope theorem, we can easily demonstrate that $\frac{\partial L_t\left(r_t^L, z_{t-1}\right)}{\partial r_t^L} < 0$, meaning that borrowers are worse off the higher the interest rate charged by the bank.

Finally, the optimal choice of technology R_{t-1} depends on the loan interest rate r_t^L and the economic shock z_{t-1}. Thus, we can express the project's probability of success as $p\left(r_t^L, z_{t-1}, z_t\right)$.

1.2 Households

Each period, risk-neutral households are endowed with one unit of good. They can choose to supply their endowment to a bank or an individual borrower. If households deposit their endowment with a bank, they receive r^D whether the bank succeeds or fails since we assume deposit insurance. Otherwise, if households directly fund the entrepreneurs' projects, then they must compete with bank loans. Hence, households could not expect to receive more than the bank lending rate r^L in successful states and must pay a monitoring cost. Since banks can minimize monitoring costs more efficiently, as in Diamond (1984), there is no benefit for households to fund entrepreneurs directly.

Households have also access to a storage technology that yields $1+\theta_t$, where the reservation value $\theta_t \in \left[\underline{\theta}, \overline{\theta}\right]$ is drawn from distribution function $\Theta(\theta_t)$ and is i.i.d. over time. The reservation value θ_t is private information to the household. For simplicity, we assume $\Theta(\theta_t)$ to be a uniform distribution of type $\mathcal{U}\left(\underline{\theta}, \overline{\theta}\right)$. We can interpret the alternative saving option as a deposit outside the banking sector that pays θ_t, or as either cash or consumption with the reservation value as convenience yield.

If $r_t^D = \theta_t$, then a household would be indifferent between matching with a bank and using the alternative storage technology so we assign such households to a bank. The total supply of deposits is the sum of all households for which the interest rate offered by the bank is higher than the reservation value:

$$D_t\left(r_t^D\right) = \int_{\underline{\theta}}^{\bar{\theta}} \mathbb{1}_{\theta_t \leqslant r_t^D} \; d\Theta(\theta_t). \tag{9}$$

We can easily prove that $\frac{\partial D_t\left(r_t^D\right)}{\partial r_t^D} > 0$, meaning that households are better off the higher the interest rate paid by the bank.

1.3 Central Bank

The central bank regulates the banking sector and conducts monetary policy. It set the liquidity and capital requirement for the representative bank. Since we do not distinguish between short-term and long-term maturities, we interpret the liquidity requirement as a constraint on reserves: the commercial bank has to store at least δ of its deposits in reserves held at the central bank. Moreover, because of a possible moral hazard problem, the central bank requires the commercial bank to finance at least κ of its loans with equity.

The commercial bank's reserves are liabilities on the central bank's balance sheet that can be remunerated. Under normal circumstances, the only reserves held at the central bank are the mandatory ones as liquidity buffer, and they are usually backed by safe assets (short-term government bonds, or gilts). For simplicity, we consider a partial equilibrium at the moment and do not model the market for government bonds. In times of crises, the central bank intervenes in the markets to save the economy from collapsing (Section 2).

Finally, the interest rate on reserves, also called the Bank Rate in UK, is a monetary policy tool that is set by the Monetary Policy Committee (MPC). In the model, we assume that it depends on the economic shock. For simplicity, in the calibration, we consider this monetary policy tool fixed in time, so that $r_t^M(z_{t-1}) = r^M$.

1.4 Commercial Bank

The representative bank intermediates between entrepreneurs, that need loans to fund their projects, and households, that hold their savings in the form of deposits. In its maximization problem, the bank chooses the interest rates on loans and deposits for the next period. These choices will determine the demand for loans and supply of deposits. At equilibrium, the demand and supply for loans and deposits meet because the market clears.

Each period, the bank receives payments on its loans from the entrepreneurs and on its reserves from the central bank, and it pays the interest on deposits to households. For each unit of lending, the commercial bank collect the agreed interest rate if the project succeeds, otherwise it looses λ.

The expected payoff the loans is:

$$\mathcal{P}_t\left(r_t^L, z_{t-1}, z_t\right) = p\left(r_t^L, z_{t-1}, z_t\right)\left[1 + r_t^L\right] + \left[1 - p\left(r_t^L, z_{t-1}, z_t\right)\right]\left[1 - \lambda\right]. \tag{10}$$

Thus, the total bank's profit is given by:

$$\pi_t\left(r_t^L, r_t^D, z_{t-1}, z_t\right) = \mathcal{P}_t\left(r_t^L, z_{t-1}, z_t\right) L_t\left(r_t^L, z_{t-1}\right) + \left[1 + r_t^M(z_{t-1})\right] M_t\left(r_t^D\right) - \left[1 + r_t^D\right] D_t\left(r_t^D\right), \tag{11}$$

where M_t is the amount of reserves held at the central bank:

$$M_t\left(r_t^D\right) = \delta D_t\left(r_t^D\right), \tag{12}$$

with δ representing the liquidity requirement set by the central bank.

We define the dividends paid to the shareholders taking into account their limited liability:

$$d_{t+1}\left(r_t^L, r_{t+1}^L, r_t^D, r_{t+1}^D, z_{t-1}, z_t\right) = \max\left\{\pi_t\left(r_t^L, r_t^D, z_{t-1}, z_t\right); 0\right\} - f_{t+1}\left(r_{t+1}^L, r_{t+1}^D, z_t\right), \tag{13}$$

where f_{t+1} is the bank's equity. The equity is given by the following accounting identity:

$$f_{t+1}\left(r_{t+1}^L, r_{t+1}^D, z_t\right) = L_{t+1}\left(r_{t+1}^L, z_t\right) + M_{t+1}\left(r_{t+1}^D\right) - D_{t+1}\left(r_{t+1}^D\right), \tag{14}$$

with the constraint given by the capital requirement κ set by the central bank:

$$f_{t+1}\left(r_{t+1}^L, r_{t+1}^D, z_t\right) \geqslant \kappa\, L_{t+1}\left(r_{t+1}^L, z_t\right). \tag{15}$$

The bank's objective is to maximize the discounted stream of dividends paid to the shareholders. We denote with β the discount factor. The maximization problem can be written in the form of Bellman equation as follows:

$$V\left(r_t^L, r_t^D, z_{t-1}, z_t\right) = \max_{r_{t+1}^L, r_{t+1}^D} d_{t+1}\left(r_t^L, r_{t+1}^L, r_t^D, r_{t+1}^D, z_{t-1}, z_t\right) + \beta \mathbb{E}_{z_{t+1}|z_t}\left[V\left(r_{t+1}^L, r_{t+1}^D, z_t, z_{t+1}\right)\right], \tag{16}$$

where the aggregate shock z_{t+1} follows the log-AR(1) process in equation (2).

1.5 Timeline

We use x_t to indicate a variable that is observable at the beginning of period t and x_{t+1} for a variable that is observable at the end of the period.

The sequence of events in each period t of the model can be summarized as follows:

1. At the end of period $t-1$, the interest rates for loans and deposits, r_t^L and r_t^D, are public information. The entrepreneurs make their choice of technology R_{t-1}. Accordingly, L_t entrepreneurs demand and obtain a loan, and D_t households deposit their savings at the bank.
2. At the beginning of period t, the aggregate shock z_t is realized.
3. Entrepreneurs make profit or loss based on the previous period's choice of technology, and, accordingly, they repay the bank for the loan obtained in period $t-1$.
4. The bank gets the return from the previous period loans and reserves, and it pays back the interest on deposits. It makes the profit π_t.
5. The bank chooses the new interest rates on loans and deposits, r_{t+1}^L and r_{t+1}^D.
6. The reservation values for entrepreneurs and households, ω_{t+1} and θ_{t+1}, are drawn, and the new amount of loans and deposits, L_{t+1} and D_{t+1}, are determined.
7. At the end of the period, the bank pays the dividends d_{t+1} to the shareholders.

Table 5 and Table 6 in Appendix A summarizes the notation we use throughout the model.

2 Banking Model with Quantitative Easing

After the Global Financial Crisis in 2008, the Bank of England and other central banks decided to implement a new type of monetary policy called Quantitative Easing (QE). The monetary policy has been implemented in a low interest rate environment by purchasing longer-term government bonds or corporate bonds from other financial institutions in exchange for newly created reserves. While purchasing these securities, the central bank increases their prices and lowers their interest rates, boosting spending in the economy.

The central bank intervention changes the dynamics of the commercial bank, but not the ones of entrepreneurs and households. Their description in Sections 1.1 and 1.2 also holds under QE.

2.1 Central Bank

The central bank continue to regulate the banking sector, setting the liquidity and capital requirement for the commercial bank.

In times of crises, when the banking sector is not solvent anymore, the central bank intervenes in the markets to save the economy from collapsing. As already mentioned, we do not model the bond market nor securities with different maturities. We introduce the possibility of conducting QE by allowing the central bank to purchase a part Q_t of the distressed loans (similarly to *corporate bond purchases*) when the representative bank is not solvable, i.e., with negative profit. The central bank repays the purchases with new reserves, injecting liquidity in the banking sector and relaxing the commercial bank's financial constraint (*bank lending channel*). When loans expire each period, as they have 1-period maturity, the central bank roll them back and repurchases the same amount from the banking sector, leaving the level of reserves unchanged.

Finally, we assume that the interest rate on reserves depends both on the economic shock and the central bank's monetary policy. We are in a quantitative easing environment when $Q_t > 0$, meaning that the bank has participated in asset-purchase programs. We define the Bank Rate in the model as:

$$r_t^M(z_{t-1}, Q_t) = \begin{cases} \overline{r}_t^M(z_{t-1}), & \text{when } Q_t = 0 \\ \underline{r}_t^M(z_{t-1}), & \text{when } Q_t > 0 \end{cases}, \tag{17}$$

meaning that the central bank set a lower interest rate while conducting quantitative easing (*signalling channel*). For simplicity, in the calibration, we consider this monetary policy tool fixed in time, so that $r_t^M(z_{t-1}, Q_t) = r_t^M(Q_t)$.

2.2 Commercial Bank

As in Section 1.4, each period, the bank receives payments on its loans from the entrepreneurs and on its reserves from the central bank, and it pays the interest on deposits to households. However, when the profits from these operations are negative, the banking sector is at risk and the central bank intervenes with quantitative easing, taking upon itself a part of the distressed loans.

The total bank's profit is given by:

$$\pi_t\left(r_t^L, r_t^D, z_{t-1}, z_t, Q_t\right) = \mathcal{P}_t\left(r_t^L, z_{t-1}, z_t\right)\Big[L_t\left(r_t^L, z_{t-1}\right) - Q_t\Big] + \Big[1 + r_t^M(z_{t-1}, Q_t)\Big] M_t\left(r_t^D, Q_t\right)$$
$$- \left[1 + r_t^D\right] D_t\left(r_t^D\right), \quad (18)$$

where Q_t is the amount of assets already purchased by the central bank. Since in reality the central bank purchases assets on the secondary market, we impose in the model that the loans are made by the commercial bank in the first place and then a part is handed over to the central bank in exchange for reserves. Each period it must hold that $L_t\left(r_t^L, z_{t-1}\right) - Q_t \geqslant 0$, meaning that the bank must ensure that at least Q_t loans are attractive to entrepreneurs, so they do not seek alternative sources of finance. Therefore, the way we implement quantitative easing in the model indirectly lowers the maximum interest rate the commercial banks can ask for a loans, capturing one of the main transmission channels of this monetary policy.

The amount of reserves held at the central bank becomes:

$$M_t\left(r_t^D, Q_t\right) = Q_t + \delta D_t\left(r_t^D\right). \quad (19)$$

The central bank intervenes only when the banking sector is in crisis, that in the model means the commercial bank's profits are negative. Otherwise, the amount of loans on the central bank's balance sheet remains constant. We write the law of motion of the asset purchases as:

$$Q_{t+1}\left(r_t^L, r_t^D, z_{t-1}, z_t, Q_t\right) = Q_t + \max\Big\{0; \Delta Q_t\left(r_t^L, r_t^D, z_{t-1}, z_t, Q_t\right)\Big\}, \quad (20)$$

where the increment is such that the central bank covers the loss or, in other words, the profits after the new QE round are zero:

$$\Delta Q_t\left(r_t^L, r_t^D, z_{t-1}, z_t, Q_t\right) = \frac{-\pi_t\left(r_t^L, r_t^D, z_{t-1}, z_t, Q_t\right)}{1 - \mathcal{P}_t\left(r_t^L, z_{t-1}, z_t\right)}. \quad (21)$$

We define the dividends paid to the shareholders taking into account their limited liability:

$$d_{t+1}\left(r_t^L, r_{t+1}^L, r_t^D, r_{t+1}^D, z_{t-1}, z_t, Q_t\right) = \max\left\{\pi_t\left(r_t^L, r_t^D, z_{t-1}, z_t, Q_t\right); 0\right\} \\ - f_{t+1}\left(r_t^L, r_{t+1}^L, r_t^D, r_{t+1}^D, z_{t-1}, z_t, Q_t\right), \quad (22)$$

where f_{t+1} is the bank's equity. The equity is given by the following accounting identity:

$$f_{t+1}\left(r_t^L, r_{t+1}^L, r_t^D, r_{t+1}^D, z_{t-1}, z_t, Q_t\right) = L_{t+1}\left(r_{t+1}^L, z_t\right) - Q_{t+1}\left(r_t^L, r_t^D, z_{t-1}, z_t, Q_t\right) \\ + M_{t+1}\left(r_t^L, r_t^D, r_{t+1}^D, z_{t-1}, z_t, Q_t\right) - D_{t+1}\left(r_{t+1}^D\right), \quad (23)$$

with the constraint given by the capital requirement κ set by the central bank:

$$f_{t+1}\left(r_t^L, r_{t+1}^L, r_t^D, r_{t+1}^D, z_{t-1}, z_t, Q_t\right) \geqslant \kappa\left[L_{t+1}\left(r_{t+1}^L, z_t\right) - Q_{t+1}\left(r_t^L, r_t^D, z_{t-1}, z_t, Q_t\right)\right]. \quad (24)$$

The bank's objective is to maximize the discounted stream of dividends paid to the shareholders. We denote with β the discount factor. The maximization problem can be written in the form of Bellman equation as follows:

$$V\left(r_t^L, r_t^D, z_{t-1}, z_t, Q_t\right) = \max_{r_{t+1}^L, r_{t+1}^D} d_{t+1}\left(r_t^L, r_{t+1}^L, r_t^D, r_{t+1}^D, z_{t-1}, z_t, Q_t\right) \\ + \beta \mathbb{E}_{z_{t+1}|z_t}\left[V\left(r_{t+1}^L, r_{t+1}^D, z_t, z_{t+1}, Q_{t+1}\right)\right], \quad (25)$$

where the aggregate shock z_{t+1} follows the log-AR(1) process in equation (2) and Q_{t+1} the law of motion in equation (20).

3 Calibration

To bring our model to the data, we divide our identification strategy into two steps. The first step consists in directly calibrating a set of parameters. They are either pinned down from the data, arbitrary set to zero, or obtained from the extant financial literature. The second step consists in estimating the remaining parameters by matching specific moments.

Since the representative bank in our model portrays the entire banking sector, we use aggregate data for the UK banking sector. We distinguish two time periods with different monetary policies. The first interval goes from 1995 to 2007 to avoid considering the extraordinary monetary policy measures adopted during and after the financial crisis. The second period considers data from 2008 to 2020 to calibrate the baking sector subject to the quantitative easing policies adopted after the financial crisis.

We take the "Total Factor Productivity (TFP) at constant national prices for the United Kingdom" from FRED to calibrate the parameters associated with the stochastic process of aggregate technology shocks. We estimate the parameters ρ and σ_u in equation (2) by fitting the TFP series in a log-AR(1) process. With the values obtained, we simulate the aggregate shocks with a discretized log-AR(1) process using the method presented in Tauchen (1986). For tractability, we set the number of grid points to five (low, medium-low, medium, medium-high, high shocks).[2]

Financial regulators arbitrarily set the reserve and capital requirements. We measure the reserve ratio as the amount of reserves over the total amount of deposits in the UK. For the first period, we keep the capital requirement enforced by Basel I and Basel II (introduced in 2004), which requires banks to hold capital equal to at least 8% of their risk-weighted assets. Our model simplifies this concept as the ratio of equity over loans. For the second period, we use Basel III that brings the total minimum requirement to 7 percent.[3] We measure the loss for project failure λ as the recovery rate on all loans for all commercial banks. We construct the series as the ratio between "charge-off rate on all loans, all commercial banks" and "delinquency rate on all loans, all commercial banks".[4] Finally, following Corbae and D'Erasmo (2021), we set the discount rate at 5%, and we establish the minimum household reservation value and the minimum entrepreneur outside option at zero by default. Table 1 reports the values for these parameters.

The remaining parameters cannot be pinned down from the data. Therefore we estimate them by matching specific moments that we can observe. Table 2 summarizes the identification strategy.

We estimate the success volatility of the projects, σ_ε, by targeting the return on equity (ROE). The project's probability of success is inversely correlated to volatility, meaning that entrepreneurs

[2] Results are not substantially different if we increase the number of points in the grid.

[3] `https://www.bis.org/bcbs/publications.htm?m=2566`

[4] The data used for this estimate are from the US. We make the strong assumption that they are somehow similar in the UK.

Table 1
The table shows the values of the model parameters that we either pin down from the data, arbitrary set to zero, or obtain from the extant financial literature.

Par.	Definition	Value 1995-2008	Value 2009-2020	Source
β	Discount factor	0.95	0.95	(default)
δ	Reserve ratio	0.029	0.244	Bank of England
κ	Capital requirement	0.08	0.07	Basel
λ	Loss for project failure	0.302	0.276	Bank of England
ρ	Aggregate shock persistence	0.844	0.614	FRED
σ_u	Aggregate shock distribution (%)	0.718	0.874	FRED
r^M	Average bank rate (annualized)	0.053	0.008	Bank of England
$\underline{\omega}$	Min entrepreneur outside option	0	0	(default)
$\underline{\theta}$	Min household reservation value	0	0	(default)

Table 2
The table shows the values of the model parameters that we calibrate targeting specific moments.

Par.	Definition	Value 1995-2008	Value 2009-2020	Target moment
σ_ε	Project success distribution	0.090	0.170	ROE
a	Success prob., weight shock	2.600	4.300	Default frequency
b	Success prob., weight risk	26.000	25.600	Borrower return
$\overline{\omega}$	Max entrepreneur outside option	0.315	0.295	Interest margin
$\overline{\theta}$	Max household reservation value	0.022	0.046	Leverage

can take up riskier projects and afford higher interest rates for low levels of volatility. In turn, higher interest rates translate into higher ROE for the bank.

We look at the loans' default frequency to calibrate a, the weight of the aggregate shock in the project's probability of success. When this weight increases, so does the probability of success of the entrepreneurs' projects. Thanks to this, the default frequency is lower for higher values of a.

The other parameter in the project's probability of success, b, is the weight of the entrepreneur's choice of project risk. We target the borrower return, that we proxy with the S&P500 annual return. The intuition is that, for higher b, entrepreneurs' probability of success of their projects decreases, leading to lower returns.

The maximum entrepreneur reservation value, $\overline{\omega}$, determines the demand for loans, given the interest rate and the shock on the economy. If entrepreneurs have a valid outside option, they reduce the demand for loans. With less loans, the bank needs less funding, thus offering a lower interest rate on deposits and increasing its intermediation margin.

Finally, we calibrate the maximum household reservation value $\overline{\theta}$ by matching the leverage, defined as equity over total assets. Increasing the maximum reservation value of households makes deposits more expensive for the bank, thus forcing it to increase interest rates on loans. In response, entrepreneurs choose riskier projects, making the overall bank riskier and increasing moral hazard. Therefore the bank chooses higher leverage to benefit more from limited liability and government bailouts.

Table 3

The table shows the targeted moments obtained from the data and the model estimate.

Moment	Target (%) 1995-2008	Estimate (%) 1995-2008	Target (%) 2009-2020	Estimate (%) 2009-2020
ROE	14.81	15.84	2.72	3.41
Default frequency	2.23	2.02	2.33	2.38
Borrower return	7.94	7.08	10.02	11.75
Interest rate margin	1.61	1.53	1.58	1.27
Leverage	9.17	8.92	9.32	9.19

Table 3 displays the values obtained from the data (target) and the ones from the calibration of our model (estimate). The parameters generate moments that are relatively close to the ones from the data. We report the main moments of the banking sector at equilibrium in Table 4.

Table 4

The table shows the main moments of the banking sector obtained from the model calibration.

Moment	Estimate (%) 1995-2008	Estimate (%) 2009-2020
Deposit interest rate	0.5	2.1
Lending interest rate	2.0	3.4
Bank deposits	23.1	46.5
Lending	22.5	39.8
Profits	2.6	0.8
Equity	2.6	4.7
Leverage	9.2	9.2
ROE	15.8	3.4
Dividends	0.3	0.2

4 Counterfactual: CBDC

We introduce a CBDC in the baseline model calibrated in Section 3. We model the CBDC as a direct liability of the central bank as shown in Figure 2.

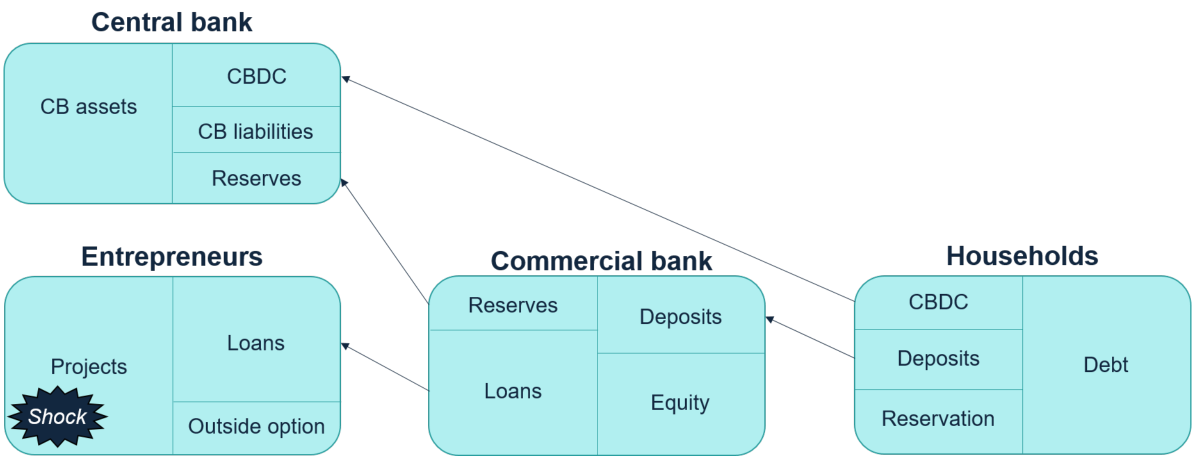

Figure 2. The figure illustrates the model of the banking sector with a CBDC. The bank intermediates between entrepreneurs and households. Entrepreneurs borrow one unit from the bank and invest it in a technology. The entrepreneurs' projects generate returns that depend on the economic shock. Households put their savings in bank deposits or CBDC. If the commercial bank does not have enough liquidity buffer to accomodate the demand for CBDC, it can borrow additional funds directly from the central bank. The central bank exogenously sets the interest rate on CBDC deposits. The bank decides the interest rates on loans and deposits.

In line with current working hypotheses,[5] we assume that a CBDC can pay an interest rate. For simplicity, we assume that the central bank exogenously sets a fixed interest rate r^C. Alternatively, the central bank could use the CBDC interest rate as a new monetary policy tool, observing the state of the economy and choosing the CBDC rate for the next period: $r_t^C(z_{t-1})$.

While we are agnostic concerning the exact characteristics of the technology underlying a CBDC, we assume that a certain share of the population will prefer such technology and extract utility from it. The reasons could be multiple. Firstly, a CBDC would introduce an element of technological innovation with features like money programmability, instantaneous settlement, smart contracts, and decentralized financial services. Secondly, as a CBDC is issued by the central bank, it could provide a safe and trustworthy instrument to citizens. Lastly, policymakers ensure the interoperability of the CBDC with other means of payments or saving instruments without the purpose of substituting them, so that households will be at worst indifferent. Therefore, in the

[5] See BIS (2020) or ECB (2020), for example.

model, households have a heterogeneous preference for CBDC. The preference of each household, γ_t, is drawn every period from the distribution function $\Gamma(\gamma_t)$, that we assume i.i.d. over time and uniform $\mathcal{U}\left(\underline{\gamma}, \overline{\gamma}\right)$. We also assume that nobody has a negative preference for technology, and thus we set $\underline{\gamma} = 0$ because a CBDC would add new possibilities without precluding current ones.

For the sake of simplicity, we assume that the preference for technology can be expressed as an extra yield, to be added on top of r^C, and compared against the interest rate on deposits and the reservation value θ_t. The reservation value is randomly drawn each period and independent of the preference. The bank deposit supply is the following:

$$D_t\left(r_t^D\right) = \int_{\underline{\gamma}}^{\overline{\gamma}} \int_{\underline{\theta}}^{\overline{\theta}} \mathbb{1}_{\{r^C+\gamma_t \leqslant r_t^D\}} \mathbb{1}_{\{\theta_t \leqslant r_t^D\}} \, d\Theta(\theta_t) d\Gamma(\gamma_t), \tag{26}$$

while the CBDC supply is:

$$C_t\left(r_t^D\right) = \int_{\underline{\gamma}}^{\overline{\gamma}} \int_{\underline{\theta}}^{\overline{\theta}} \mathbb{1}_{\{r^C+\gamma_t > r_t^D\}} \mathbb{1}_{\{r^C+\gamma_t > \theta_t\}} \, d\Theta(\theta_t) d\Gamma(\gamma_t). \tag{27}$$

The representative commercial bank chooses the optimal interest rate on deposits considering the CBDC interest rate exogenously set by the central bank. However, for any given r_t^D, the total amount of savings (in terms of bank deposits and CBDC) is higher when households have access to a CBDC. The reason is that there can be households for which r_t^D is lower than their reservation value, but their technological preference is so high that they choose the CBDC rather than the alternative saving option. Figure 3 shows this mechanism by representing the distribution of households' total assets. In the model, this increase in the amount of savings in the "regulated" banking sector (bank deposits and CBDC), and it could represents the financial inclusion that some central banks are seeking with the introduction of a CBDC.

Since financial institutions are the only ones that can create money by lending, we assume that all the savings pass through the commercial bank in the first place. After resources are allocated, the commercial bank accommodates the demand for CBDC by transferring households' savings to the central bank. The amount of money that households want to transfer from bank deposit to CBDC is:

$$\tau_t\left(r_t^D\right) = \int_{\underline{\theta}}^{\bar{\theta}} \mathbb{1}_{\theta_t \leqslant r_t^D} \, d\Theta(\theta_t) - D_t\left(r_t^D\right), \tag{28}$$

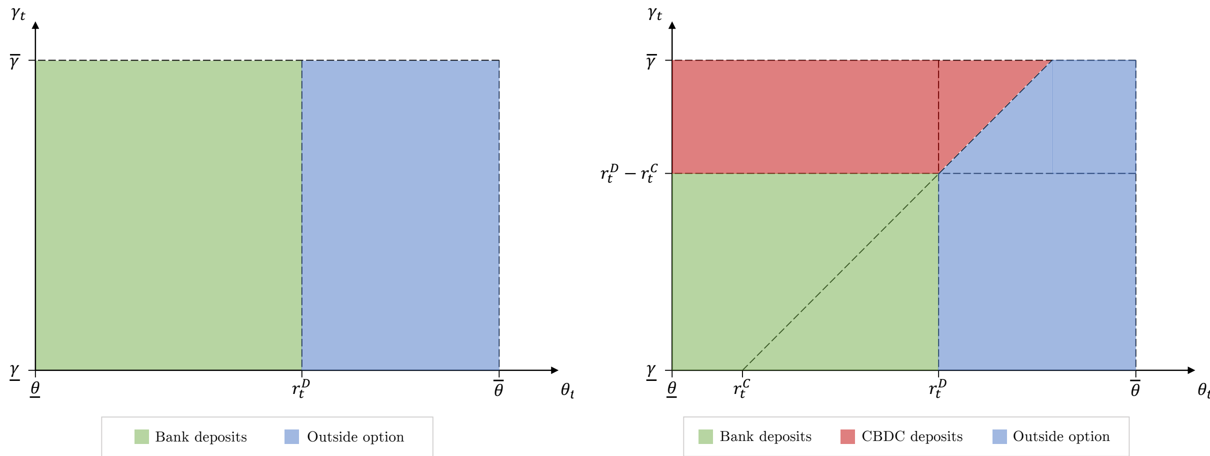

Figure 3. The figure shows the distribution of households' preferences along two dimensions: reservation value and preference for technology. Introducing a CBDC could increase the total amount of savings in terms of bank deposits and CBDC.

where the first term of the equation is the demand for deposit in the baseline model without CBDC, as in equation (9), and the second term is the demand for deposit that remains at the bank, considering the interest rate, the reservation value, and the preference for CBDC as in equation (26).

Following Fraschini et al. (2021), the commercial bank accommodate the demand for CBDC by optimally reducing its reserves, and the central bank swaps one type of liability (reserves) into another (CBDC). In other words, the commercial bank uses its liquidity buffer to compensate for the sudden loss of funds in terms of deposits. As the liquidity requirement remains valid, the transfer affects the bank reserves in the following way:

$$M_t\left(r_t^D, Q_t\right) = \delta D_t\left(r_t^D\right) + \max\left\{Q_t - \tau_t\left(r_t^D\right); \phi\right\}, \tag{29}$$

where ϕ is the threshold after which the commercial bank does not want to switch reserves into CBDC anymore. The commercial bank might want a bigger liquidity buffer than the liquidity requirement for different reasons, and ϕ represents this need. For simplicity, we set $\phi = 0$.[6]

If the demand for CBDC is higher than the amount of reserves that can be swapped, that means $Q_t - \tau_t < \phi$, then the commercial bank can ask for direct funding from the central bank. This

[6]We could set a $\phi > 0$ in the future to improve the tightening elasticity of the central bank's balance sheet.

scenario, where the central bank channels CBDC deposits back to banks, is often considered as the baseline in the literature (see e.g., Brunnermeier et al., 2019; Niepelt, 2020), even if its conditions are not extensively discussed in the central banks' reports. Here, we limit this possibility to the extreme case where the commercial bank does not have enough liquidity to deal with the loss of funds. Therefore, the amount of central bank's funding to the bank is:

$$F_t\left(r_t^D, Q_t\right) = \max\Big\{0; \tau_t\left(r_t^D\right) - Q_t + \phi\Big\}. \tag{30}$$

The central bank could ask an interest rate r_t^F on the direct finding. This interest rate could either match the interest rate on reserves, on CBDC or on deposits, or it could be used as an additional monetary policy tool.

The equation for the bank's profit changes to account for the possibility of this new type of liability on the commercial bank's balance sheet:

$$\begin{aligned}\pi_t\left(r_t^L, r_t^D, z_{t-1}, z_t, Q_t\right) = \mathcal{P}_t\left(r_t^L, z_{t-1}, z_t\right)\Big[L_t\left(r_t^L, z_{t-1}\right) - Q_t\Big] + \Big[1 + r_t^M(z_{t-1}, Q_t)\Big] M_t\left(r_t^D, Q_t\right) \\ - \Big[1 + r_t^D\Big] D_t\left(r_t^D\right) - \Big[1 + r_t^F\Big] F_t\left(r_t^D, Q_t\right).\end{aligned} \tag{31}$$

The same happens to the equity:

$$\begin{aligned}f_{t+1}\left(r_t^L, r_{t+1}^L, r_t^D, r_{t+1}^D, z_{t-1}, z_t, Q_t\right) = L_{t+1}\left(r_{t+1}^L, z_t\right) - Q_{t+1}\left(r_t^L, r_t^D, z_{t-1}, z_t, Q_t\right) \\ + M_{t+1}\left(r_t^L, r_t^D, r_{t+1}^D, z_{t-1}, z_t, Q_t\right) - D_{t+1}\left(r_{t+1}^D\right) - F_{t+1}\left(r_t^L, r_t^D, r_{t+1}^D, z_{t-1}, z_t, Q_t\right).\end{aligned} \tag{32}$$

Finally, the amount of loans on the central bank's balance sheet, Q_t, is not affected by the introduction of a CBDC. Also the equation for dividends and the maximization problem remain the same.

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A Model notation

Table 5
This table summarizes the notation for the variables used throughout the model.

Variable	Definition
r^L	Loan interest rate
r^D	Bank deposit interest rate
r^M	Reserve interest rate
r^C	CBDC deposit interest rate
L	Loans
D	Bank deposits
M	Reserves
Q	Central bank's loans after QE
C	CBDC deposits
π	Bank's profit
f	Bank's equity
d	Dividends
z	Aggregate shock
u	Aggregate shock innovation
$\mathcal{R}$	Project stochastic return
s	Project success
ε	Project success innovation
p	Project's probability of success
R	Entrepreneur's choice of technology
Π	Entrepreneur's payoff
$\mathcal{P}$	Project expected payoff to bank
v	Optimal project value
ω	Entrepreneur's outside option
θ	Household's reservation value
γ	Household's preference for CBDC

Table 6

This table summarizes the notation for the parameters used throughout the model.

Parameter	Definition
β	Discount factor
δ	Liquidity requirement
κ	Capital requirement
λ	Loss for project failure
ρ	Aggregate shock persistance
σ_u	Aggregate shock distribution
a	Success probability, weight shock
b	Success probability, weight risk
σ_ε	Project success distribution
$\underline{\omega}$	Min entrepreneur outside option
$\overline{\omega}$	Max entrepreneur outside option
$\underline{\theta}$	Min household reservation value
$\overline{\theta}$	Max household reservation value
$\underline{\gamma}$	Min household preference for CBDC
$\overline{\gamma}$	Max household preference for CBDC