FAI LAB 12
Knowledge representation and reasoning

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2024-25

Knowledge Representation and Reasoning (KRR)

KRR: use of **decidable** logics to reason on (among other things):

- *Subjective knowledge*: (epistemic) **modal** logics

(c) Italian Spiderman, Alrugo

- *Categories, objects, properties, their relations*: **description** logics

(c) Yingtong et al., Nature, 2021

Reasoning about knowledge with modal logics

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- Formulas can't be arguments of predicates

$$Knows(Clark, CanFly(SuperWoman)) \Rightarrow_{reify} Knows(Clark, CanFlySuperWoman)$$

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- **Referential transparency**:

  $(SuperWoman = Lois) \wedge$
  $Knows(Clark, CanFly(SuperWoman)) \models_{FOL} Knows(Clark, CanFly(Lois))$

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We use **modal operators** over formulas $\Box$ (necessity), $\Diamond$ (possibility)

$$\Box\alpha = \neg\Diamond\neg\alpha$$

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*"Agent A **knows** α"*: $\Box_A\alpha$ (or $K_A\alpha$)

$(SuperWoman = Lois) \wedge K_{Clark}(CanFly(SuperWoman)) \not\models K_{Clark}(CanFly(Lois))$

Reasoning about knowledge with modal logics

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Properties of **all** modal logics:

$$K_A(\alpha \wedge \beta) \Leftrightarrow K_A\alpha \wedge K_A\beta$$
$$K_A\alpha \vee K_A\beta \models K_A(\alpha \vee \beta)$$

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Axiom in **all** **normal** modal logics:

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Axioms in **some** **normal** modal logics:

T: $K_A\alpha \rightarrow \alpha$
4: $K_A\alpha \rightarrow K_AK_A\alpha$
5: $\neg K_A\alpha \rightarrow K_A\neg K_A\alpha$

Reasoning about knowledge with modal logics

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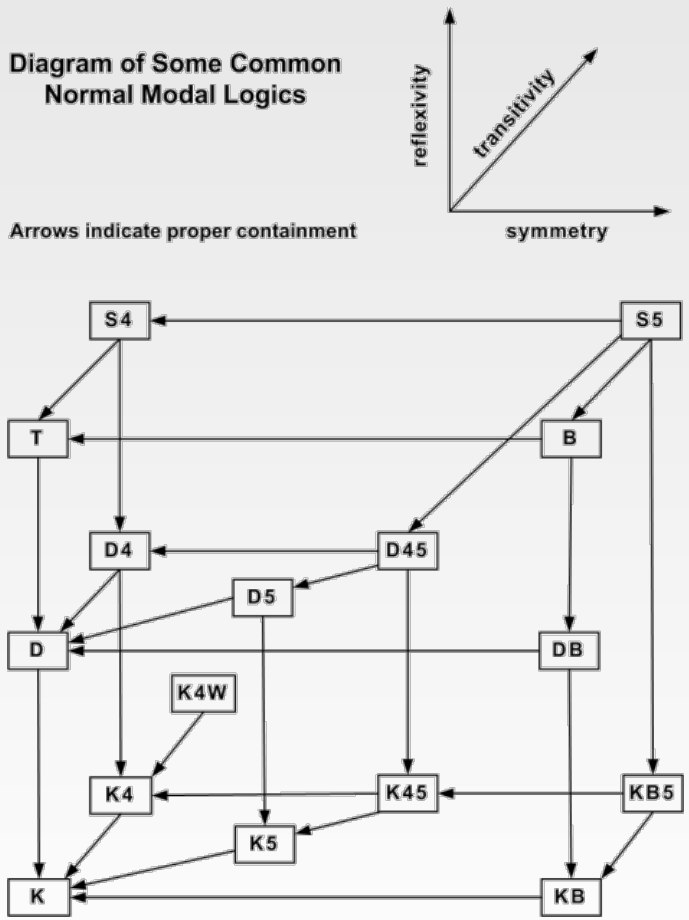
Warning: you can get lost in

https://en.wikipedia.org/wiki/Modal_logic



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Modal logic exercises: T/F

Let:

- EF = "The Earth is flat.", NWO = "New World Order is real!1!"
- P = "Paolo", R = "Roberto"
- $\models_{KT}$ = "entailed in modal logic KT"
- $\models_{K}$ = "entailed in modal logic K"
- $\models_{\_}$ = "entailed in (non-normal) modal logic"

Modal logic exercises: T/F

Answer **True** or **False**:

- $K_P[EF] \models_{KT} EF$?

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Modal logic exercises: T/F

Answer **True** or **False**:

- $K_P[EF] \wedge K_R[EF \rightarrow NWO] \models_{KT} NWO$? **True**
- $K_P[EF] \wedge K_R[EF \rightarrow NWO] \models_{K} NWO$? **False**
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  - Concept definitions, e.g.:
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- **A-Boxes** (Assertions):
  - Concept membership $(i : C)$, e.g.:
    $paolo : Scientist, ziltoid : AlienOverlord$
  - Role memberships $(\langle i1, i2 \rangle : R)$, e.g.:
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Description logics exercises

Given:

- the atomic concepts: $\{Person, Turtle, Cat, Female, Male, Pizza\}$
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Write a T-box in $\mathcal{ALCN}$ description logic defining the following concepts:

- WannabeTurtleLover: a person with at least 2 turtles but less than 5
- Turtlecats: creatures that are both turtles and cats
- Ninja turtles: turtles that only eat pizza
- FemaleTurleEatingCat: female cat that ate turtle
- PersonWithMaleTurtlecats: a person with male turtlecats
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Description logics exercises

Given:

- the atomic concepts: $\{Male, Female\}$
- the atomic roles: $\{hasChild\}$

Write a T-box in $\mathcal{ALCN}$ description logic defining the following concepts:

- Person
- Mother, Father
- Parent
- Childless
- Grandmother, Grandfather
- ParentOfSons: a parent with at least one son
- ParentOfOnlySons
- MotherWithManyChildren: a mother with more then three children
- GrandmotherOfOnlyGrandsons