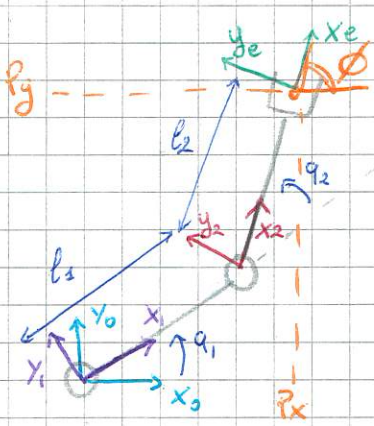
DIRECT DIFFERENTIAL KINEMATICS 2R Planar Arm



Compute DK: ${}_0T_1$ Pure rotation
${}_1T_2$ Roto-traslation
${}_2T_e$ Rigid-transform

$$ {}_0T_e = \begin{bmatrix} C_{12} & -S_{12} & 0 & l_2 C_{12} + l_1 C_1 \\ S_{12} & C_{12} & 0 & l_2 S_{12} + l_1 S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \rightarrow {}_0P_e $$

$$ \begin{bmatrix} V \\ w \end{bmatrix} = J(q)\dot{q} \qquad [J_1 \quad J_2] \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \qquad J_i = \begin{bmatrix} z_i \times ({}_wP_e - {}_wP_i) \\ z_i \end{bmatrix} $$ For revolute Joints

$$ J_1 = \begin{bmatrix} z_1 \times ({}_0P_e - {}_0P_1) \\ z_1 \end{bmatrix} \qquad J_2 = \begin{bmatrix} z_2 \times ({}_0P_e - {}_0P_2) \\ z_2 \end{bmatrix} $$

${}_0P_1$ e ${}_0P_2$ si ricavano da ${}_0T_1$ e ${}_0T_2$

$$ {}_0P_e = \begin{bmatrix} l_1 C_1 + l_1 C_1 \\ l_2 S_{12} + l_1 S_1 \\ 0 \end{bmatrix} \qquad {}_0P_2 = \begin{bmatrix} l_2 C_{12} \\ l_2 S_{12} \\ 0 \end{bmatrix} \qquad {}_0P_1 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$

$\cancel{J(q) = l_1 C_1 + l_1 C_1}$

Analytical Jacobian

$P_x = l_1 C_1 + l_2 C_{12}$ $\qquad \dot{P}_x = -l_1 S_1 \dot{q}_1 - l_2 S_{12} (\dot{q}_1 + \dot{q}_2)$

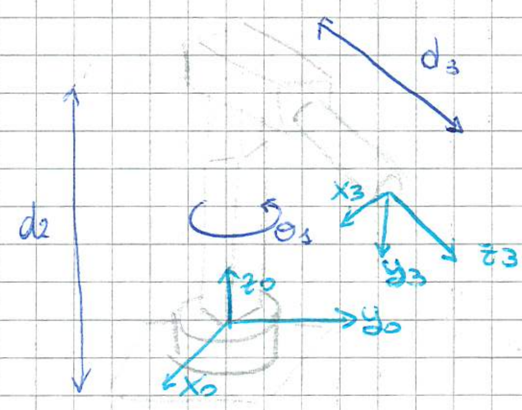
$P_y = l_1 S_1 + l_2 S_{12}$ $\qquad \dot{P}_y = l_1 C_1 \dot{q}_1 + l_2 C_{12} (\dot{q}_1 + \dot{q}_2)$

$\phi = q_1 + q_2$ $\qquad \dot{\phi} = \dot{q}_1 + \dot{q}_2$

$$ \begin{bmatrix} \dot{P}_x \\ \dot{P}_y \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -l_1 S_1 - l_2 S_{12} & -l_2 S_{12} \\ l_1 C_1 + l_2 C_{12} & l_2 C_{12} \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} $$

IK RPP

FWD: find ${}^0T_3 = \begin{bmatrix} C_1 & 0 & -S_1 & -d_3S_1 \\ S_1 & 0 & C_1 & d_3C_1 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$



INV. KIN: solve for (θ_1, d_2, d_3)

$$\begin{bmatrix} C_1 & 0 & -S_1 & -d_3S_1 \\ S_1 & 0 & C_1 & d_3C_1 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$d_2 = P_z \qquad P_x = -d_3S_1 \qquad P_y = d_3C_1$

$\theta_1 = atan_2(r_{21}, r_{11})$

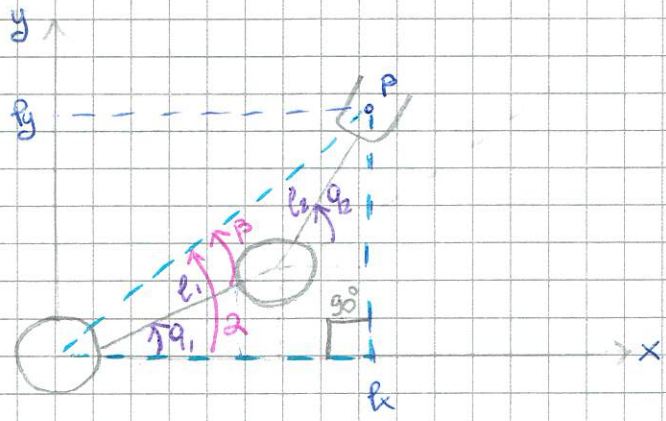
* $-P_xS_1 + P_yC_1 = S_1^2 d_3 + C_1^2 d_3 = d_3(S_1^2 + C_1^2) = d_3$

$\hookrightarrow d_3 = -P_xS_1 + P_yC_1$

*

$$\left.\begin{array}{l} P_x = -d_3S_1 \rightarrow -S_1P_x = -d_3S_1^2 \\ P_y = d_3C_1 \rightarrow C_1P_y = d_3C_1^2 \end{array}\right\}$$ metodo per poi arrivare ad avere $S_1^2 + C_2^2$ e isolare d_3

IK PLANAR RR closed-form method based on geometric intuition



(1) WRITE DK $\begin{cases} P_x = l_1 C_1 + l_2 C_{12} \\ P_y = l_1 S_1 + l_2 S_{12} \end{cases}$

Input data $P(P_x, P_y)$ in funzione di q_1 e q_2 (non noti)

(2) COMPUTE IK

INTUITION: distance between P and O only depends on q_2

$$P_x^2 + P_y^2 = (l_1 C_1 + l_2 C_{12})^2 + (l_1 S_1 + l_2 S_{12})^2 =$$
$$= l_1^2 C_1^2 + l_2^2 C_{12}^2 + 2 l_1 C_1 l_2 C_{12} + l_1^2 S_1^2 + l_2^2 S_{12}^2 + 2 l_1 S_1 l_2 S_{12} =$$
$$l_1^2 \underbrace{(C_1^2 + S_1^2)}_{=1} + l_2^2 \underbrace{(C_{12}^2 + S_{12}^2)}_{=1} + 2 l_1 l_2 \underbrace{(C_1 C_{12} + S_1 S_{12})}_{*}$$

$* \; C(A-B) = C_A C_B + S_A S_B \rightarrow C((\cancel{q_1} + q_2) - \cancel{q_1}) = C_2$

$$= l_1^2 + l_2^2 + 2 l_1 l_2 C_2$$ (Dimostra che la distanza di P dipende solo da q_2)

$$P_x^2 + P_y^2 = l_1^2 + l_2^2 + 2 l_1 l_2 C_2 \longrightarrow C_2 = \frac{P_x^2 + P_y^2 - l_1^2 - l_2^2}{2 l_1 l_2}$$

$S_2 = \pm\sqrt{1 - C_2^2} \rightarrow$ 2 solutions

- positive: elbow down
- Negative: elbow up

$q_2 = \arctan2(S_2, C_2)$

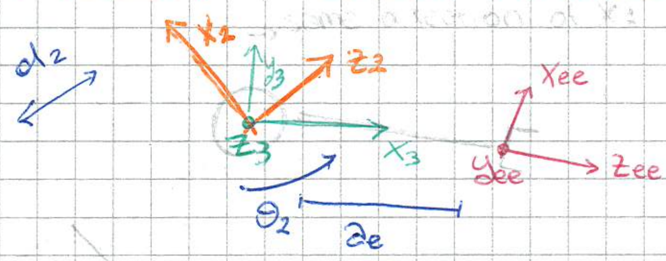
C_2 returns a value $\in [-1, 1] \Rightarrow -1 \leq \dfrac{P_x^2 + P_y^2 - l_1^2 - l_2^2}{2 l_1 l_2} \leq 1$

$\hookrightarrow P_x^2 + P_y^2 \leq (l_1 + l_2)^2$

$\quad P_x^2 + P_y^2 \geq (l_1 - l_2)^2$

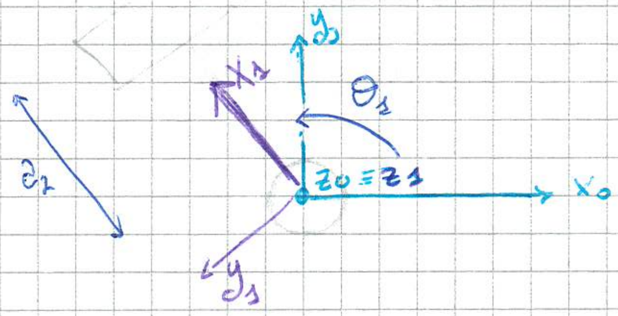
$q_1 = \alpha - \beta \qquad \alpha = \operatorname{atan2}(P_y, P_x) \qquad \beta = \operatorname{atan2}(\underbrace{l_2 S_2}_{\sin(\beta)}, \underbrace{l_1 + l_2 C_2}_{\cos(\beta)})$

RPR ROBOT - Direct Kinematics



θ: angle between x_{i-1} and x_i around z_{i-1}

d: distance between x_{i-1} and x_i along z_{i-1}



$${}_0T_1 = \text{Rotation of } x \text{ around } z = \begin{bmatrix} c(\theta) & -s(\theta) & 0 & 0 \\ s(\theta) & c(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}_1T_2 = \text{Traslation along } z_2 \text{ and rotation around } x_1 = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

${}_2T_3 =$ ① $-90°$ around x_2 (from z_2 To z_3) $R_x(-90) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

② θ_3 (generic) around z_3 (from x_2 To x_3) $R_z(\theta_3) = \begin{bmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$${}_2R_3 = \begin{bmatrix} c_3 & -s_3 & 0 \\ 0 & 0 & 1 \\ -s_3 & -c_3 & 0 \end{bmatrix}$$

$$\rightarrow {}_2T_3 = \begin{bmatrix} c_3 & -s_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_3 & -c_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}_3T_e = \begin{bmatrix} 0 & 0 & 1 & a_e \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$