FAI LAB ~~13~~ 1
Probability Theory & Bayesian Networks

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2024-25

Intro

- Schedule: Wednesday (or Friday) 11.30 - 13.30
- Material: `https://paolomorettin.github.io/`
- Contact: `paolo.morettin@unitn.it` (put `[FAI lab]` in subject)
- Interactive exercise sessions

Probability theory

Probability model:

- For every *possible world* $\omega \in \Omega: \ \mathbb{P}(\omega) \in [0, 1]$
- $\sum_{\omega \in \Omega} \mathbb{P}(\omega) = 1$
- Atomic representation ω: **impratical**

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- $\sum_{\omega \in \Omega} \mathbb{P}(\omega) = 1$
- Atomic representation ω: **impratical**

$\mathbb{P}(\textit{"Pollo and Paolo both at the conf"}) = 1/5$

$\mathbb{P}(\textit{"Pollo at the conf, but Paolo is not"}) = 1/20$

$\mathbb{P}(\textit{"Paolo at the conf, but Pollo is not"}) = 3/20$

$\mathbb{P}(\textit{"Neither Pollo nor Paolo at the conf"}) = 3/5$

Probability theory

Probability model:

- For every *possible world* $\omega \in \Omega$: $\mathbb{P}(\omega) \in [0, 1]$
- $\sum_{\omega \in \Omega} \mathbb{P}(\omega) = 1$
- Atomic representation ω: **imprатical**

Better use a **factored** representation of the world:

- $X_1, ..., X_N$ *random vars* (RVs), each having $dom(X_i) = \{v_1, ..., v_{si}\}$
- $\mathbf{P}(X_1, ..., X_N)$ *joint probability distribution* over the RVs

Probability theory

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- $\mathbf{P}(X_1, \ldots, X_N)$ *joint probability distribution* over the RVs

$\mathbf{P}(Pa, Po)$		Pa $\top$	Pa $\bot$
Po	$\top$	$1/5$	$1/20$
	$\bot$	$3/20$	$3/5$

Probability theory

$\mathbf{P}(Pa, Po)$		Pa $\top$	Pa $\bot$
Po	$\top$	$1/5$	$1/20$
	$\bot$	$3/20$	$3/5$

Basic **rules**:

Probability theory

$\mathbf{P}(Pa, Po)$		Pa $\top$	$\bot$
Po	$\top$	$1/5$	$1/20$
	$\bot$	$3/20$	$3/5$

Basic **rules**:

- $\mathbf{P}(A) = \sum_{b \in B} \mathbf{P}(A, B = b)$ (marginalization)

$\mathbf{P}(Pa, Po)$		Pa $\top$	$\bot$
Po	$\top$	$1/5$	$1/20$
	$\bot$	$3/20$	$3/5$

Basic **rules**:

- $\mathbf{P}(A) = \sum_{b \in B} \mathbf{P}(A, B = b)$ (marginalization)

$$\mathbf{P}(Po) = \mathbf{P}(Pa = \top, Po) + \mathbf{P}(Pa = \bot, Po) = \begin{cases} 5/20 & \text{if } Po = \top \\ 15/20 & \text{if } Po = \bot \end{cases}$$

Probability theory

$\mathbf{P}(Pa, Po)$		Pa $\top$	$\bot$
Po	$\top$	$1/5$	$1/20$
	$\bot$	$3/20$	$3/5$

Basic **rules**:

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- $\mathbf{P}(A \mid B = b) = \mathbf{P}(A, B = b)/\mathbf{P}(B = b)$ (conditional probability)

Probability theory

$\mathbf{P}(Pa, Po)$		Pa $\top$	$\bot$
Po	$\top$	$1/5$	$1/20$
	$\bot$	$3/20$	$3/5$

Basic **rules**:

- $\mathbf{P}(A) = \sum_{b \in B} \mathbf{P}(A, B = b)$ (marginalization)

$$\mathbf{P}(Po) = \mathbf{P}(Pa = \top, Po) + \mathbf{P}(Pa = \bot, Po) = \begin{cases} 5/20 & \text{if } Po = \top \\ 15/20 & \text{if } Po = \bot \end{cases}$$

- $\mathbf{P}(A \mid B = b) = \mathbf{P}(A, B = b)/\mathbf{P}(B = b)$ (conditional probability)

$$\mathbf{P}(Po \mid Pa = \top) = \mathbf{P}(Pa = \top, Po)/\mathbf{P}(Pa = \top)$$

Probability theory

$\mathbf{P}(Pa, Po)$		Pa $\top$	Pa $\bot$
Po	$\top$	$1/5$	$1/20$
	$\bot$	$3/20$	$3/5$

Basic **rules**:

- $\mathbf{P}(A) = \sum_{b \in B} \mathbf{P}(A, B = b)$ (marginalization)

$$\mathbf{P}(Po) = \mathbf{P}(Pa = \top, Po) + \mathbf{P}(Pa = \bot, Po) = \begin{cases} 5/20 & \text{if } Po = \top \\ 15/20 & \text{if } Po = \bot \end{cases}$$

- $\mathbf{P}(A \mid B = b) = \mathbf{P}(A, B = b) / \mathbf{P}(B = b)$ (conditional probability)

$$\begin{aligned} \mathbf{P}(Po \mid Pa = \top) &= \mathbf{P}(Pa = \top, Po) / \mathbf{P}(Pa = \top) \\ &= \frac{\mathbf{P}(Pa = \top, Po)}{7/20} = \begin{cases} 4/7 & \text{if } Po = \top \\ 3/7 & \text{if } Po = \bot \end{cases} \end{aligned}$$

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$, $B = \top$	$A = \top$, $B = \bot$	$A = \bot$, $B = \top$	$A = \bot$, $B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A = \top, B = \top, C = \bot)$

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A = \top, B = \top, C = \bot)$ **0**

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A = \top, B = \top, C = \bot)$ **0**
- $\mathbf{P}(A = \top, B = \top)$

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A = \top, B = \top, C = \bot)$ **0**
- $\mathbf{P}(A = \top, B = \top)$ **1/5**

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A = \top, B = \top, C = \bot)$ **0**
- $\mathbf{P}(A = \top, B = \top)$ **1/5**
- $\mathbf{P}(A = \top)$

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A = \top, B = \top, C = \bot)$ **0**
- $\mathbf{P}(A = \top, B = \top)$ **1/5**
- $\mathbf{P}(A = \top)$ **2/5**

Probability theory exercises

$\mathbf{P}(A,B,C)$	$A=\top$		$A=\bot$	
	$B=\top$	$B=\bot$	$B=\top$	$B=\bot$
$C=\top$	$2/10$	$0/10$	$3/10$	$1/10$
$C=\bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A=\top, B=\top, C=\bot)$ **0**
- $\mathbf{P}(A=\top, B=\top)$ **$1/5$**
- $\mathbf{P}(A=\top)$ **$2/5$**
- $\mathbf{P}(A=\top \mid B=\top)$

Probability theory exercises

$\mathbf{P}(A,B,C)$	$A=\top$		$A=\bot$	
	$B=\top$	$B=\bot$	$B=\top$	$B=\bot$
$C=\top$	$2/10$	$0/10$	$3/10$	$1/10$
$C=\bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A=\top, B=\top, C=\bot)$ **0**
- $\mathbf{P}(A=\top, B=\top)$ **1/5**
- $\mathbf{P}(A=\top)$ **2/5**
- $\mathbf{P}(A=\top \mid B=\top)$ **2/7**

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A = \top, B = \top, C = \bot)$ **0**
- $\mathbf{P}(A = \top, B = \top)$ **1/5**
- $\mathbf{P}(A = \top)$ **2/5**
- $\mathbf{P}(A = \top \mid B = \top)$ **2/7**
- $\mathbf{P}(B = \top \mid A = \top)$

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A = \top, B = \top, C = \bot)$ **0**
- $\mathbf{P}(A = \top, B = \top)$ **1/5**
- $\mathbf{P}(A = \top)$ **2/5**
- $\mathbf{P}(A = \top \mid B = \top)$ **2/7**
- $\mathbf{P}(B = \top \mid A = \top)$ **1/2**

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A = \top, B = \top, C = \bot)$ **0**
- $\mathbf{P}(A = \top, B = \top)$ **1/5**
- $\mathbf{P}(A = \top)$ **2/5**
- $\mathbf{P}(A = \top \mid B = \top)$ **2/7**
- $\mathbf{P}(B = \top \mid A = \top)$ **1/2**
- $\mathbf{P}(C = \top \mid A = \top, B = \top)$

Probability theory exercises

$\mathbf{P}(A,B,C)$	$A=\top$		$A=\bot$	
	$B=\top$	$B=\bot$	$B=\top$	$B=\bot$
$C=\top$	$2/10$	$0/10$	$3/10$	$1/10$
$C=\bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A=\top, B=\top, C=\bot)$ **0**
- $\mathbf{P}(A=\top, B=\top)$ **$1/5$**
- $\mathbf{P}(A=\top)$ **$2/5$**
- $\mathbf{P}(A=\top \mid B=\top)$ **$2/7$**
- $\mathbf{P}(B=\top \mid A=\top)$ **$1/2$**
- $\mathbf{P}(C=\top \mid A=\top, B=\top)$ **1**

Probability theory exercises

$$\mathbf{P}(B \mid C = c) = \frac{\sum_a \mathbf{P}(A = a, B, C = c)}{\mathbf{P}(C = c)}$$

About **normalization**:

Probability theory exercises

$$\mathbf{P}(B \mid C = c) = \frac{\sum_a \mathbf{P}(A = a, B, C = c)}{\mathbf{P}(C = c)}$$

About **normalization**:

- $\mathbf{P}(C = c) = \alpha$ is a constant wrt c
- it only serves as a **normalization** factor

Probability theory exercises

$$\mathbf{P}(B \mid C = c) = \frac{\sum_a \mathbf{P}(A = a, B, C = c)}{\mathbf{P}(C = c)}$$

About **normalization**:

- $\mathbf{P}(C = c) = \alpha$ is a constant wrt c
- it only serves as a **normalization** factor
- Can be **computed last**:
  1) We compute $\tilde{\mathbf{P}}(B \mid C = c) = \sum_a \mathbf{P}(A = a, B, C = c)$
  2) We normalize $\mathbf{P}(B \mid C = c) = \alpha \cdot \tilde{\mathbf{P}}(B \mid C = c)$

$$\text{with } \alpha = \frac{1}{\sum_b \tilde{\mathbf{P}}(B = b \mid C = c)}$$

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A \mid B = \top, C = \bot)$

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A \mid B = \top, C = \bot)$ $\qquad \alpha \cdot \begin{bmatrix} 0 \\ 2/10 \end{bmatrix} \; (\alpha = 5)$

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A \mid B = \top, C = \bot)$ $\qquad \alpha \cdot \begin{bmatrix} 0 \\ 2/10 \end{bmatrix} \; (\alpha = 5)$
- $\mathbf{P}(A \mid B = \top)$

Probability theory exercises

$\mathbf{P}(A,B,C)$	$A=\top$		$A=\bot$	
	$B=\top$	$B=\bot$	$B=\top$	$B=\bot$
$C=\top$	$2/10$	$0/10$	$3/10$	$1/10$
$C=\bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A \mid B=\top, C=\bot)$ $\qquad \alpha \cdot \begin{bmatrix} 0 \\ 2/10 \end{bmatrix} \ (\alpha = 5)$
- $\mathbf{P}(A \mid B=\top)$ $\qquad \alpha \cdot \left(\begin{bmatrix} 2/10 \\ 3/10 \end{bmatrix} + \begin{bmatrix} 0 \\ 2/10 \end{bmatrix} \right) = \alpha \cdot \begin{bmatrix} 2/10 \\ 5/10 \end{bmatrix} \ (\alpha = 10/7)$

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A \mid B = \top, C = \bot)$ $\qquad \alpha \cdot \begin{bmatrix} 0 \\ 2/10 \end{bmatrix} \; (\alpha = 5)$
- $\mathbf{P}(A \mid B = \top)$ $\qquad \alpha \cdot \left(\begin{bmatrix} 2/10 \\ 3/10 \end{bmatrix} + \begin{bmatrix} 0 \\ 2/10 \end{bmatrix} \right) = \alpha \cdot \begin{bmatrix} 2/10 \\ 5/10 \end{bmatrix} \; (\alpha = 10/7)$
- $\mathbf{P}(A)$

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A \mid B = \top, C = \bot)$ $\quad \alpha \cdot \begin{bmatrix} 0 \\ 2/10 \end{bmatrix} \; (\alpha = 5)$
- $\mathbf{P}(A \mid B = \top)$ $\quad \alpha \cdot (\begin{bmatrix} 2/10 \\ 3/10 \end{bmatrix} + \begin{bmatrix} 0 \\ 2/10 \end{bmatrix}) = \alpha \cdot \begin{bmatrix} 2/10 \\ 5/10 \end{bmatrix} \; (\alpha = 10/7)$
- $\mathbf{P}(A)$ $\quad \alpha \cdot (\begin{bmatrix} 2/10 \\ 3/10 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/10 \end{bmatrix} + \begin{bmatrix} 0 \\ 2/10 \end{bmatrix} + \begin{bmatrix} 2/10 \\ 0 \end{bmatrix}) = \begin{bmatrix} 4/10 \\ 6/10 \end{bmatrix} \; (\alpha = 1)$

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$		$A = \bot$	
	$B = \top$	$B = \bot$	$B = \top$	$B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A \mid B = \top, C = \bot)$ $\quad \alpha \cdot \begin{bmatrix} 0 \\ 2/10 \end{bmatrix} \; (\alpha = 5)$
- $\mathbf{P}(A \mid B = \top)$ $\quad \alpha \cdot (\begin{bmatrix} 2/10 \\ 3/10 \end{bmatrix} + \begin{bmatrix} 0 \\ 2/10 \end{bmatrix}) = \alpha \cdot \begin{bmatrix} 2/10 \\ 5/10 \end{bmatrix} \; (\alpha = 10/7)$
- $\mathbf{P}(A)$ $\quad \alpha \cdot (\begin{bmatrix} 2/10 \\ 3/10 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/10 \end{bmatrix} + \begin{bmatrix} 0 \\ 2/10 \end{bmatrix} + \begin{bmatrix} 2/10 \\ 0 \end{bmatrix}) = \begin{bmatrix} 4/10 \\ 6/10 \end{bmatrix} \; (\alpha = 1)$
- $\mathbf{P}(B \mid A = \top)$

Probability theory exercises

$\mathbf{P}(A, B, C)$	$A = \top$, $B = \top$	$A = \top$, $B = \bot$	$A = \bot$, $B = \top$	$A = \bot$, $B = \bot$
$C = \top$	$2/10$	$0/10$	$3/10$	$1/10$
$C = \bot$	$0/10$	$2/10$	$2/10$	$0/10$

Compute:

- $\mathbf{P}(A \mid B = \top, C = \bot)$ $\quad \alpha \cdot \begin{bmatrix} 0 \\ 2/10 \end{bmatrix} \; (\alpha = 5)$
- $\mathbf{P}(A \mid B = \top)$ $\quad \alpha \cdot (\begin{bmatrix} 2/10 \\ 3/10 \end{bmatrix} + \begin{bmatrix} 0 \\ 2/10 \end{bmatrix}) = \alpha \cdot \begin{bmatrix} 2/10 \\ 5/10 \end{bmatrix} \; (\alpha = 10/7)$
- $\mathbf{P}(A)$ $\quad \alpha \cdot (\begin{bmatrix} 2/10 \\ 3/10 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/10 \end{bmatrix} + \begin{bmatrix} 0 \\ 2/10 \end{bmatrix} + \begin{bmatrix} 2/10 \\ 0 \end{bmatrix}) = \begin{bmatrix} 4/10 \\ 6/10 \end{bmatrix} \; (\alpha = 1)$
- $\mathbf{P}(B \mid A = \top)$ $\quad \alpha \cdot (\begin{bmatrix} 2/10 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 2/10 \end{bmatrix}) = \begin{bmatrix} 2/10 \\ 2/10 \end{bmatrix} \; (\alpha = 10/4)$

Probability theory exercises

$$\mathbf{P}(\mathit{Cause} \mid \mathit{Effect}) = \frac{\mathbf{P}(\mathit{Effect} \mid \mathit{Cause}) \cdot \mathbf{P}(\mathit{Cause})}{\mathbf{P}(\mathit{Effect})} \text{ (Bayes' rule)}$$

Ex.:

- 25% of the population has a flu
- 9 out of 10 positives to flu have cough
- 2 out of 10 negatives to flu have cough

What is $\mathbf{P}(\mathit{Flu} = \top \mid \mathit{Cough} = \top)$?

What is $\mathbf{P}(\mathit{Flu} = \top \mid \mathit{Cough} = \bot)$?

What is $\mathbf{P}(\mathit{Flu} = \bot \mid \mathit{Cough} = \top)$?

Probability theory exercises

$$\mathbf{P}(Cause \mid Effect) = \frac{\mathbf{P}(Effect \mid Cause) \cdot \mathbf{P}(Cause)}{\mathbf{P}(Effect)} \text{ (Bayes' rule)}$$

Ex.:

- 25% of the population has a flu
- 9 out of 10 positives to flu have cough
- 2 out of 10 negatives to flu have cough

What is $\mathbf{P}(Flu = \top \mid Cough = \top)$? $3/5$

What is $\mathbf{P}(Flu = \top \mid Cough = \bot)$?

What is $\mathbf{P}(Flu = \bot \mid Cough = \top)$?

Probability theory exercises

$$\mathbf{P}(\mathit{Cause} \mid \mathit{Effect}) = \frac{\mathbf{P}(\mathit{Effect} \mid \mathit{Cause}) \cdot \mathbf{P}(\mathit{Cause})}{\mathbf{P}(\mathit{Effect})} \text{ (Bayes' rule)}$$

Ex.:

- 25% of the population has a flu
- 9 out of 10 positives to flu have cough
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What is $\mathbf{P}(\mathit{Flu} = \top \mid \mathit{Cough} = \top)$? $3/5$

What is $\mathbf{P}(\mathit{Flu} = \top \mid \mathit{Cough} = \bot)$? $1/25$

What is $\mathbf{P}(\mathit{Flu} = \bot \mid \mathit{Cough} = \top)$?

Probability theory exercises

$$\mathbf{P}(\mathit{Cause} \mid \mathit{Effect}) = \frac{\mathbf{P}(\mathit{Effect} \mid \mathit{Cause}) \cdot \mathbf{P}(\mathit{Cause})}{\mathbf{P}(\mathit{Effect})} \text{ (Bayes' rule)}$$

Ex.:

- 25% of the population has a flu
- 9 out of 10 positives to flu have cough
- 2 out of 10 negatives to flu have cough

What is $\mathbf{P}(\mathit{Flu} = \top \mid \mathit{Cough} = \top)$? $3/5$

What is $\mathbf{P}(\mathit{Flu} = \top \mid \mathit{Cough} = \bot)$? $1/25$

What is $\mathbf{P}(\mathit{Flu} = \bot \mid \mathit{Cough} = \top)$? $2/5$

Probability theory

$$\mathbf{P}(A, B) = \mathbf{P}(A \mid B) \cdot \mathbf{P}(B) \text{ (chain rule)}$$

Probability theory

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- We can **factorize** the joint

$$\begin{aligned} \mathbf{P}(An, Pa, Po) &= \mathbf{P}(Po, Pa \mid An) \cdot \mathbf{P}(An) \\ &= \mathbf{P}(Po \mid Pa, An) \cdot \mathbf{P}(Pa \mid An) \cdot \mathbf{P}(An) \end{aligned}$$

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- Exploit **independencies** among RVs:
  - A and B are **independent** ($A \perp\!\!\!\perp B \mid \emptyset$)
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- More compact models!

$$Po \perp\!\!\!\perp An \mid Pa \Rightarrow \mathbf{P}(An, Pa, Po) = \mathbf{P}(Po \mid Pa) \cdot \mathbf{P}(Pa \mid An) \cdot \mathbf{P}(An)$$

$$\mathbf{P}(X_1, \ldots., X_N) = \prod_i \mathbf{P}(X_i \mid \mathrm{Pa}(X_i))$$

Bayesian Networks

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Bayesian Networks

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Bayesian Networks

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$$An \longrightarrow Pa \longrightarrow Po$$

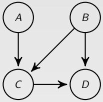
$$\mathbf{P}(An, Pa, Po) = \mathbf{P}(Po \mid Pa) \cdot \mathbf{P}(Pa \mid An) \cdot \mathbf{P}(An)$$

$\mathbf{P}(An = \top)$
$2/5$

	$\mathbf{P}(Pa = \top \mid An)$
$An = \top$	$2/5$
$An = \bot$	$1/4$

	$\mathbf{P}(Po = \top \mid Pa)$
$Pa = \top$	$4/5$
$Pa = \bot$	$1/10$

Write $\mathbf{P}(A, B, C, D)$ and min. # of params (with binary RVs):



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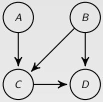

$$\mathbf{P}(A) \cdot \mathbf{P}(B) \cdot \mathbf{P}(C \mid A, B) \cdot \mathbf{P}(D \mid B, C)$$

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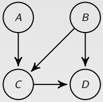

$$\overbrace{\mathbf{P}(A)}^{1} \cdot \overbrace{\mathbf{P}(B)}^{1} \cdot \overbrace{\mathbf{P}(C \mid A, B)}^{1 \cdot 2^2} \cdot \overbrace{\mathbf{P}(D \mid B, C)}^{1 \cdot 2^2}$$

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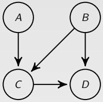

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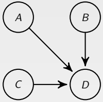
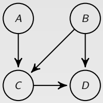
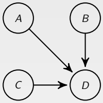


Write $\mathbf{P}(A, B, C, D)$ and min. # of params (with binary RVs):



$$\overbrace{\mathbf{P}(A)}^{1} \cdot \overbrace{\mathbf{P}(B)}^{1} \cdot \overbrace{\mathbf{P}(C \mid A, B)}^{1 \cdot 2^2} \cdot \overbrace{\mathbf{P}(D \mid B, C)}^{1 \cdot 2^2}$$



$$\mathbf{P}(A) \cdot \mathbf{P}(B) \cdot \mathbf{P}(C) \cdot \mathbf{P}(D \mid A, B, C)$$

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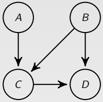

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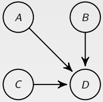

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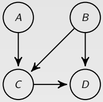
Bayesian Networks exercises

Write $\mathbf{P}(A, B, C, D)$ and min. # of params (with binary RVs):



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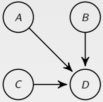

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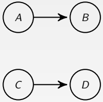
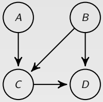


Bayesian Networks exercises

Write $\mathbf{P}(A, B, C, D)$ and min. # of params (with binary RVs):



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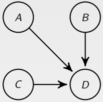

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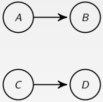

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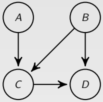

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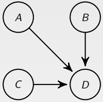

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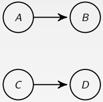

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Bayesian Networks exercises

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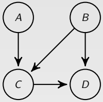

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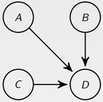

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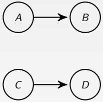

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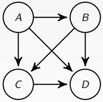

Bayesian Networks exercises

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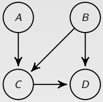

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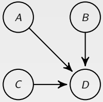

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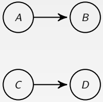

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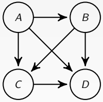

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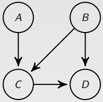
Bayesian Networks exercises

Write $\mathbf{P}(A, B, C, D)$ and min. # of params (with binary RVs):



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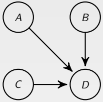

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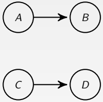

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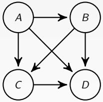

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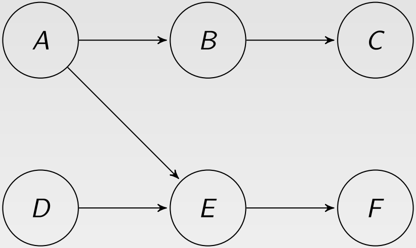
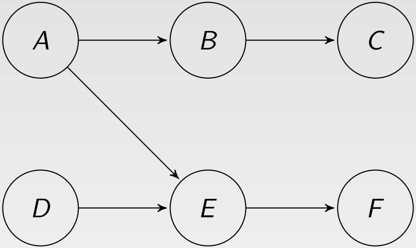
Bayesian Networks exercises

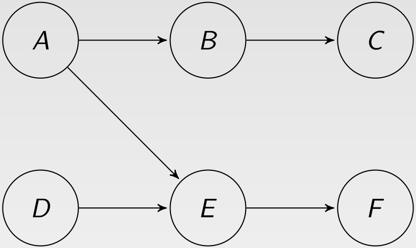


Bayesian Networks exercises



$$\mathbf{P}(A) \cdot \mathbf{P}(D) \cdot \mathbf{P}(B \mid A) \cdot \mathbf{P}(C \mid B) \cdot \mathbf{P}(E \mid A, D) \cdot \mathbf{P}(F \mid E)$$

Bayesian Networks exercises



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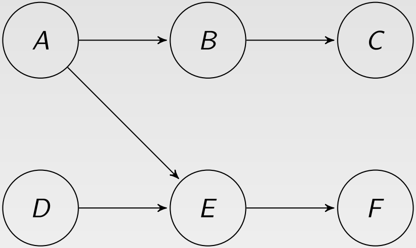
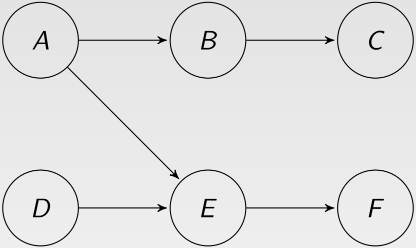
- $A \perp\!\!\!\perp C \mid B$?
- $A \perp\!\!\!\perp C$?
- $A \perp\!\!\!\perp D$?
- $B \perp\!\!\!\perp E \mid A$?

Bayesian Networks exercises



$$\mathbf{P}(A) \cdot \mathbf{P}(D) \cdot \mathbf{P}(B \mid A) \cdot \mathbf{P}(C \mid B) \cdot \mathbf{P}(E \mid A, D) \cdot \mathbf{P}(F \mid E)$$

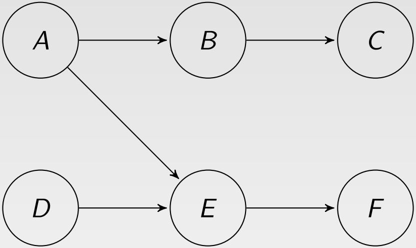
- $A \perp\!\!\!\perp C \mid B$? $\mathbf{P}(A, C \mid B) = \mathbf{P}(A \mid B) \cdot \mathbf{P}(C \mid B)$?
- $A \perp\!\!\!\perp C$?
- $A \perp\!\!\!\perp D$?
- $B \perp\!\!\!\perp E \mid A$?



$$\mathbf{P}(A) \cdot \mathbf{P}(D) \cdot \mathbf{P}(B \mid A) \cdot \mathbf{P}(C \mid B) \cdot \mathbf{P}(E \mid A, D) \cdot \mathbf{P}(F \mid E)$$

- $A \perp\!\!\!\perp C \mid B$? $\mathbf{P}(A, C \mid B) = \mathbf{P}(A \mid B) \cdot \mathbf{P}(C \mid B)$? **Yes**
- $A \perp\!\!\!\perp C$?
- $A \perp\!\!\!\perp D$?
- $B \perp\!\!\!\perp E \mid A$?

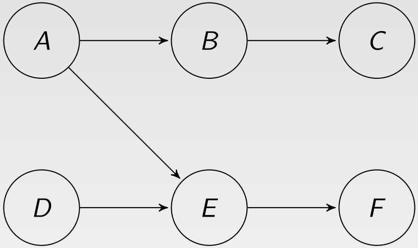
Bayesian Networks exercises



$$\mathbf{P}(A) \cdot \mathbf{P}(D) \cdot \mathbf{P}(B \mid A) \cdot \mathbf{P}(C \mid B) \cdot \mathbf{P}(E \mid A, D) \cdot \mathbf{P}(F \mid E)$$

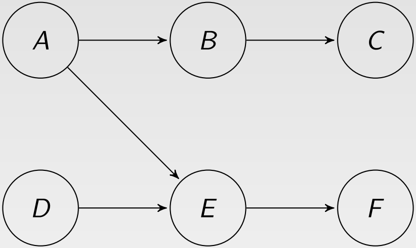
- $A \perp\!\!\!\perp C \mid B$? $\mathbf{P}(A, C \mid B) = \mathbf{P}(A \mid B) \cdot \mathbf{P}(C \mid B)$? **Yes**
- $A \perp\!\!\!\perp C$? $\mathbf{P}(A, C) = \mathbf{P}(A) \cdot \mathbf{P}(C)$?
- $A \perp\!\!\!\perp D$?
- $B \perp\!\!\!\perp E \mid A$?

Bayesian Networks exercises



$$\mathbf{P}(A) \cdot \mathbf{P}(D) \cdot \mathbf{P}(B \mid A) \cdot \mathbf{P}(C \mid B) \cdot \mathbf{P}(E \mid A, D) \cdot \mathbf{P}(F \mid E)$$

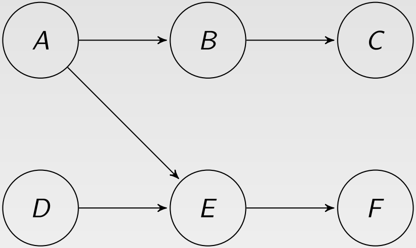
- $A \perp\!\!\!\perp C \mid B$? $\mathbf{P}(A, C \mid B) = \mathbf{P}(A \mid B) \cdot \mathbf{P}(C \mid B)$? **Yes**
- $A \perp\!\!\!\perp C$? $\mathbf{P}(A, C) = \mathbf{P}(A) \cdot \mathbf{P}(C)$? **No**
- $A \perp\!\!\!\perp D$?
- $B \perp\!\!\!\perp E \mid A$?



$$\mathbf{P}(A) \cdot \mathbf{P}(D) \cdot \mathbf{P}(B \mid A) \cdot \mathbf{P}(C \mid B) \cdot \mathbf{P}(E \mid A, D) \cdot \mathbf{P}(F \mid E)$$

- $A \perp\!\!\!\perp C \mid B$? $\mathbf{P}(A, C \mid B) = \mathbf{P}(A \mid B) \cdot \mathbf{P}(C \mid B)$? **Yes**
- $A \perp\!\!\!\perp C$? $\mathbf{P}(A, C) = \mathbf{P}(A) \cdot \mathbf{P}(C)$? **No**
- $A \perp\!\!\!\perp D$? $\mathbf{P}(A, D) = \mathbf{P}(A) \cdot \mathbf{P}(D)$?
- $B \perp\!\!\!\perp E \mid A$?

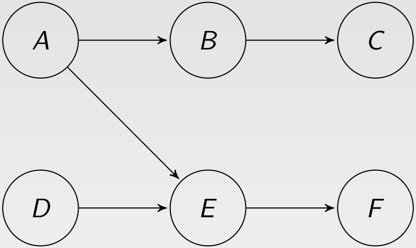
Bayesian Networks exercises



$$\mathbf{P}(A) \cdot \mathbf{P}(D) \cdot \mathbf{P}(B \mid A) \cdot \mathbf{P}(C \mid B) \cdot \mathbf{P}(E \mid A, D) \cdot \mathbf{P}(F \mid E)$$

- $A \perp\!\!\!\perp C \mid B$? $\mathbf{P}(A, C \mid B) = \mathbf{P}(A \mid B) \cdot \mathbf{P}(C \mid B)$? **Yes**
- $A \perp\!\!\!\perp C$? $\mathbf{P}(A, C) = \mathbf{P}(A) \cdot \mathbf{P}(C)$? **No**
- $A \perp\!\!\!\perp D$? $\mathbf{P}(A, D) = \mathbf{P}(A) \cdot \mathbf{P}(D)$? **Yes**
- $B \perp\!\!\!\perp E \mid A$?

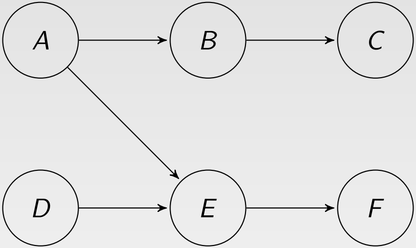
Bayesian Networks exercises



$$\mathbf{P}(A) \cdot \mathbf{P}(D) \cdot \mathbf{P}(B \mid A) \cdot \mathbf{P}(C \mid B) \cdot \mathbf{P}(E \mid A, D) \cdot \mathbf{P}(F \mid E)$$

- $A \perp\!\!\!\perp C \mid B$? $\mathbf{P}(A, C \mid B) = \mathbf{P}(A \mid B) \cdot \mathbf{P}(C \mid B)$? **Yes**
- $A \perp\!\!\!\perp C$? $\mathbf{P}(A, C) = \mathbf{P}(A) \cdot \mathbf{P}(C)$? **No**
- $A \perp\!\!\!\perp D$? $\mathbf{P}(A, D) = \mathbf{P}(A) \cdot \mathbf{P}(D)$? **Yes**
- $B \perp\!\!\!\perp E \mid A$? $\mathbf{P}(B, E \mid A) = \mathbf{P}(B \mid A) \cdot \mathbf{P}(E \mid A)$?

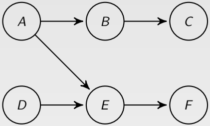
Bayesian Networks exercises



$$\mathbf{P}(A) \cdot \mathbf{P}(D) \cdot \mathbf{P}(B \mid A) \cdot \mathbf{P}(C \mid B) \cdot \mathbf{P}(E \mid A, D) \cdot \mathbf{P}(F \mid E)$$

- $A \perp\!\!\!\perp C \mid B$? $\mathbf{P}(A, C \mid B) = \mathbf{P}(A \mid B) \cdot \mathbf{P}(C \mid B)$? **Yes**
- $A \perp\!\!\!\perp C$? $\mathbf{P}(A, C) = \mathbf{P}(A) \cdot \mathbf{P}(C)$? **No**
- $A \perp\!\!\!\perp D$? $\mathbf{P}(A, D) = \mathbf{P}(A) \cdot \mathbf{P}(D)$? **Yes**
- $B \perp\!\!\!\perp E \mid A$? $\mathbf{P}(B, E \mid A) = \mathbf{P}(B \mid A) \cdot \mathbf{P}(E \mid A)$? **Yes**

Bayesian Networks exercises



$$\mathbf{P}(a) = 4/10 \qquad \mathbf{P}(d) = 3/10$$
$$\mathbf{P}(b \mid a) = 6/10 \qquad \mathbf{P}(b \mid \neg a) = 1/10$$
$$\mathbf{P}(c \mid b) = 5/10 \qquad \mathbf{P}(c \mid \neg b) = 0/10$$
$$\mathbf{P}(e \mid a, d) = 2/10 \qquad \mathbf{P}(e \mid a, \neg d) = 1/10$$
$$\mathbf{P}(e \mid \neg a, d) = 2/10 \qquad \mathbf{P}(e \mid \neg a, \neg d) = 4/10$$
$$\mathbf{P}(f \mid e) = 3/10 \qquad \mathbf{P}(f \mid \neg e) = 4/10$$

- Compute $\mathbf{P}(A \mid B, E)$
- Using the factors above, compute $\mathbf{P}(A \mid b, \neg e)$
- Using the factors above, compute $\mathbf{P}(A \mid b, e)$

Bayesian Networks exercises

$$\mathbf{P}(a) = 4/10 \qquad \mathbf{P}(d) = 3/10 \qquad \mathbf{P}(b \mid a) = 6/10 \qquad \mathbf{P}(b \mid \neg a) = 1/10$$

$$\mathbf{P}(e \mid a, d) = 2/10 \qquad \mathbf{P}(e \mid a, \neg d) = 1/10$$

$$\mathbf{P}(e \mid \neg a, d) = 2/10 \qquad \mathbf{P}(e \mid \neg a, \neg d) = 4/10$$

$$\begin{aligned}
\mathbf{P}(A \mid B, E) &= \alpha \cdot \sum_C \sum_D \sum_F \mathbf{P}(A, B, C, D, E, F) \\
&= \alpha \cdot \mathbf{P}(A) \cdot \mathbf{P}(B \mid A) \cdot \sum_d \mathbf{P}(D) \cdot \mathbf{P}(E \mid A, D) \\
&= \alpha \cdot \begin{bmatrix} 4/10 \\ 6/10 \end{bmatrix} \cdot \begin{bmatrix} \overbrace{\begin{matrix} 6/10 \\ 1/10 \end{matrix}}^{b} & \overbrace{\begin{matrix} 4/10 \\ 9/10 \end{matrix}}^{\neg b} \end{bmatrix} \cdot \left(\frac{3}{10} \cdot \begin{bmatrix} \overbrace{\begin{matrix} 2/10 \\ 2/10 \end{matrix}}^{e} & \overbrace{\begin{matrix} 8/10 \\ 8/10 \end{matrix}}^{\neg e} \end{bmatrix} + \frac{7}{10} \cdot \begin{bmatrix} \overbrace{\begin{matrix} 1/10 \\ 4/10 \end{matrix}}^{e} & \overbrace{\begin{matrix} 9/10 \\ 6/10 \end{matrix}}^{\neg e} \end{bmatrix} \right) \\
&= \alpha \cdot \begin{bmatrix} 4/10 \\ 6/10 \end{bmatrix} \cdot \begin{bmatrix} \overbrace{\begin{matrix} 6/10 \\ 1/10 \end{matrix}}^{b} & \overbrace{\begin{matrix} 4/10 \\ 9/10 \end{matrix}}^{\neg b} \end{bmatrix} \cdot \begin{bmatrix} \overbrace{\begin{matrix} 13/100 \\ 34/100 \end{matrix}}^{e} & \overbrace{\begin{matrix} 87/100 \\ 66/100 \end{matrix}}^{\neg e} \end{bmatrix}
\end{aligned}$$

Bayesian Networks exercises

$$\mathbf{P}(E \mid A, D) = \alpha \cdot \begin{bmatrix} 4/10 \\ 6/10 \end{bmatrix} \cdot \begin{bmatrix} \overbrace{6/10}^{b} & \overbrace{4/10}^{\neg b} \\ 1/10 & 9/10 \end{bmatrix} \cdot \begin{bmatrix} \overbrace{13/100}^{e} & \overbrace{87/100}^{\neg e} \\ 34/100 & 66/100 \end{bmatrix}$$

$$\mathbf{P}(A \mid b, \neg e) = \alpha_{b,\neg e} \cdot \begin{bmatrix} 4/10 \\ 6/10 \end{bmatrix} \cdot \begin{bmatrix} 6/10 \\ 1/10 \end{bmatrix} \cdot \begin{bmatrix} 87/100 \\ 66/100 \end{bmatrix} = \begin{bmatrix} 2088/10000 \\ 396/10000 \end{bmatrix}$$

$$\alpha_{b,\neg e} = \frac{10000}{2484}$$

$$\mathbf{P}(A \mid b, e) = \alpha_{b,e} \cdot \begin{bmatrix} 4/10 \\ 6/10 \end{bmatrix} \cdot \begin{bmatrix} 6/10 \\ 1/10 \end{bmatrix} \cdot \begin{bmatrix} 13/100 \\ 34/100 \end{bmatrix} = \begin{bmatrix} 312/10000 \\ 204/10000 \end{bmatrix}$$

$$\alpha_{b,e} = \frac{10000}{516}$$