C0 - Actuators

Actuators Types

Outline

- Actuator types
- Electrical motors
- Transmissions
- Non idealities in actuation systems
- Simulation of actuators

Actuator

- A device that produce motion:
  - Linear
  - rotary

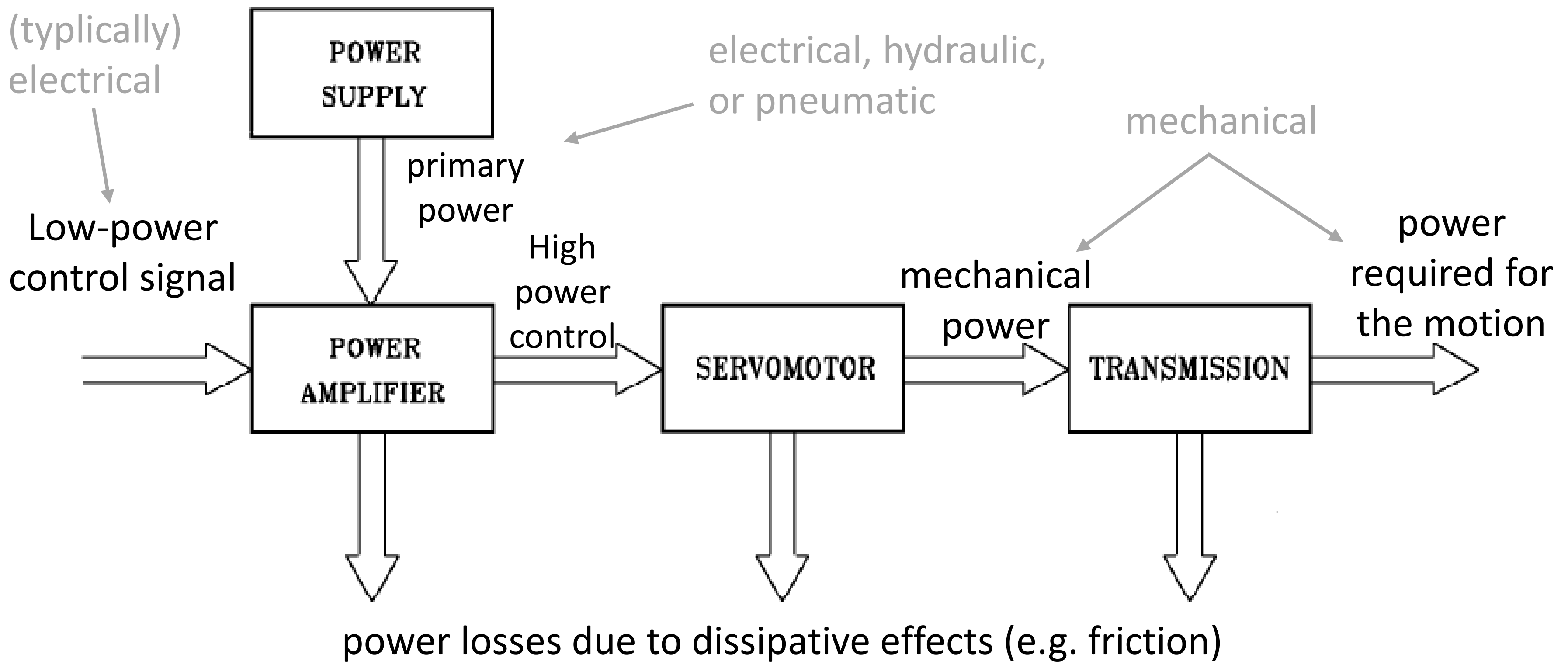
- The output of an actuator can be considered either:
  - a linear/angular velocity
  - a force/torque

We will consider actuators in this course mainly as force /torque sources

- Types of Actuators:
  - Pneumatic
  - Hydraulic
  - Electrical
  - More exotic: EHA, SEA, Piezo, …

Overview on Actuation System

- Actuators are the devices through which the controller acts on the system



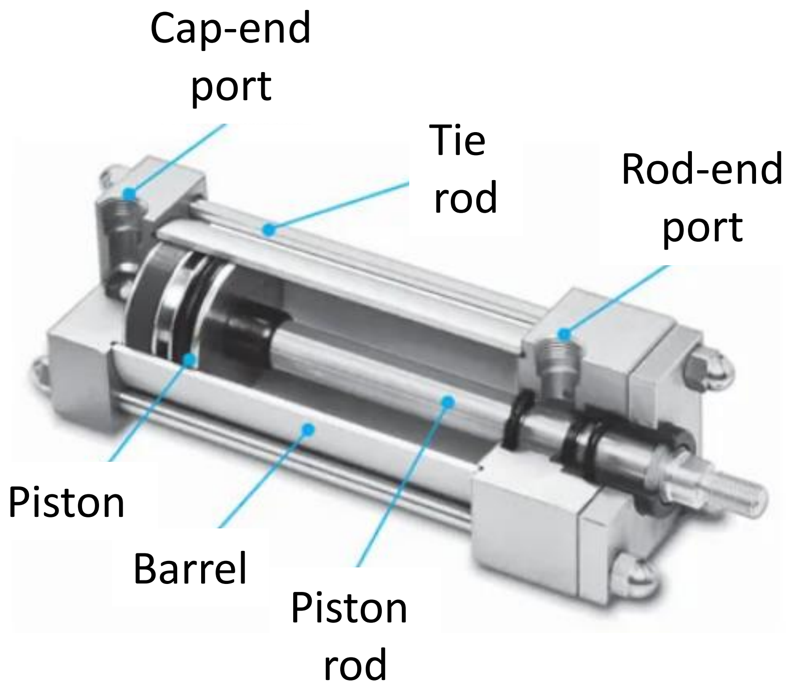
power = voltage · current = pressure · flow rate = force · speed = torque · angular speed [W]
efficiency = power out/power in [%]
energy ~ work = power · time [Wh, Nm, J]

Common requirements in robotic actuators

- high power-to-weight ratio and low inertia
- Possibility of overload and delivery impulse torques
- High acceleration capabilities
- Large range of operational velocities
  - from 1 to 1000 rpm
- High accuracy in positioning
  - at least 1/1000 of turn
- Low torque ripple
  - to guarantee continuous rotation at low speed
- Power
  - from 80 W to 10 kW

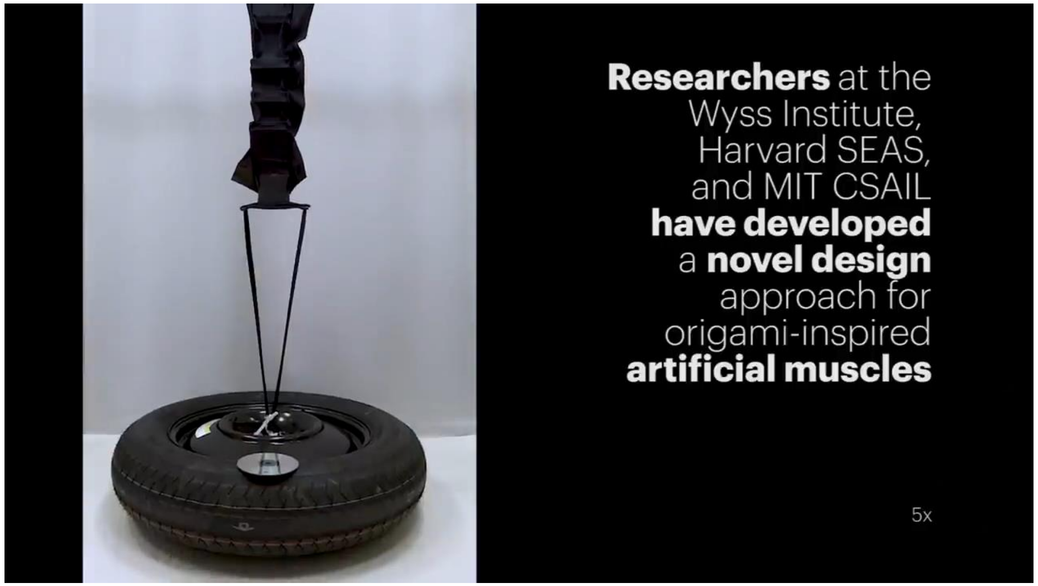
Pneumatic actuators

- Utilize the pneumatic energy provided by a compressor and transform it into mechanical energy by means of pistons and valves
  - (-) Difficult to control accurately: change of fluid compressibility makes trajectory control harder (slow transients)
  - Suitable for **low force** applications that requires intrinsic compliance (i.e. moving objects on a conveyor belt, open/close grippers)



Pneumatic Motors

Festo BionicSoftArm
https://youtu.be/JbGhtpSfPmU

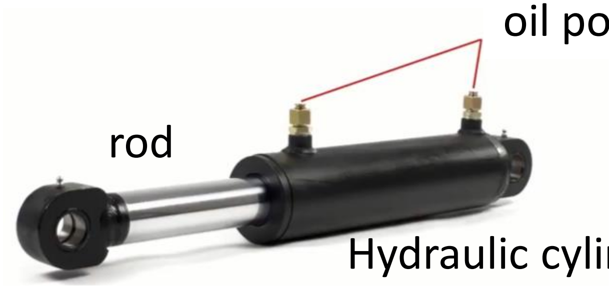


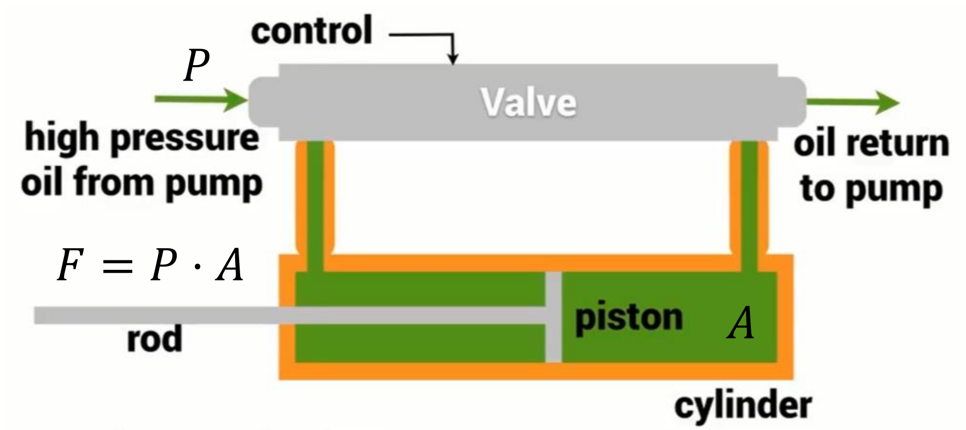
Origami-inspired Artificial Muscles
https://youtu.be/Ir69MXyOvFs

Hydraulic actuators

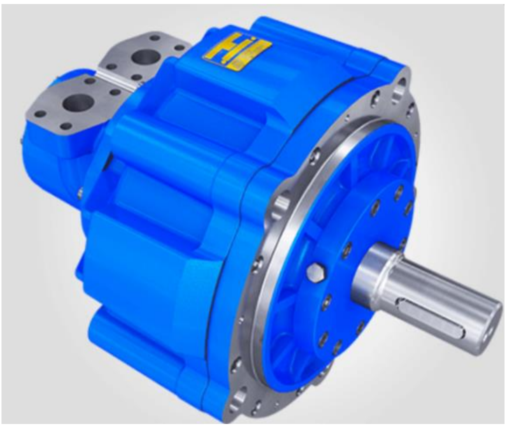
- Transform the **hydraulic** energy generated by a pump (that pressurizes the oil coming from an accumulation tank) into **mechanical** energy by means of valves

Linear

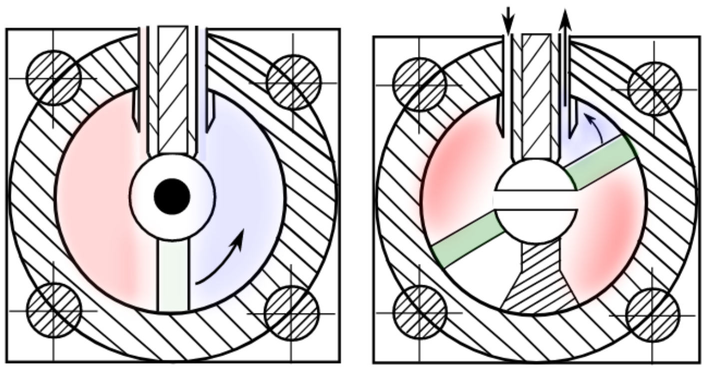




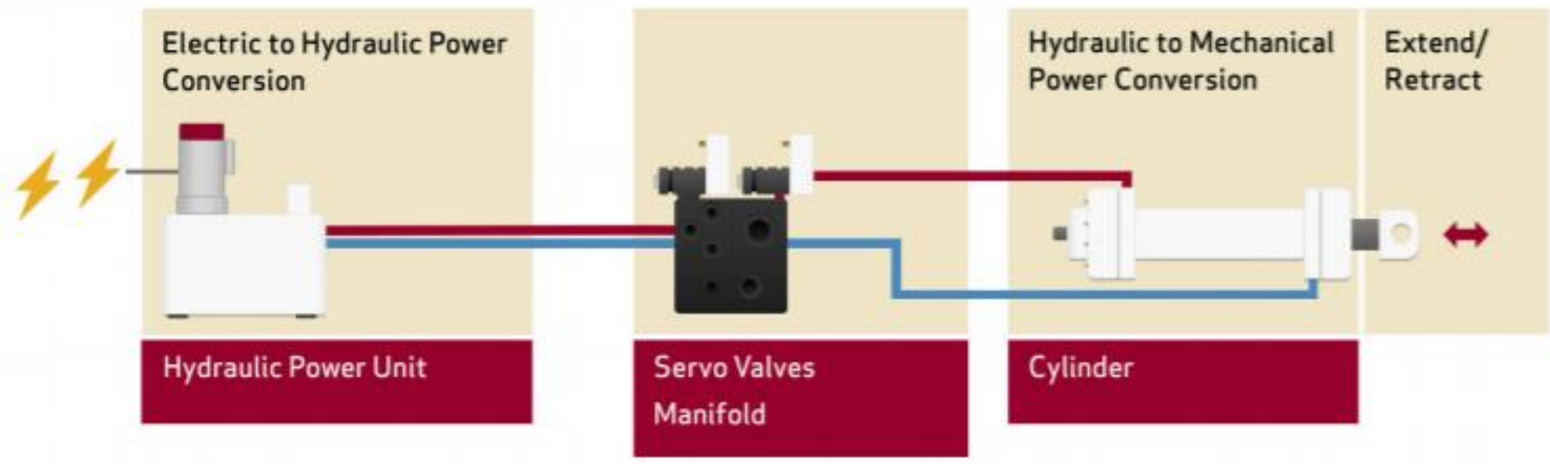
Rotational



Hydraulic motor/ rotary actuator



Hydraulic actuators



Advantages

- **excellent power-to-weight ratio**
- no static overheating
- self-lubricated
- inherently safe (no sparks)
- large torques/large velocity (w/o reduction)
- Passive compliance (due to oil compressibility)

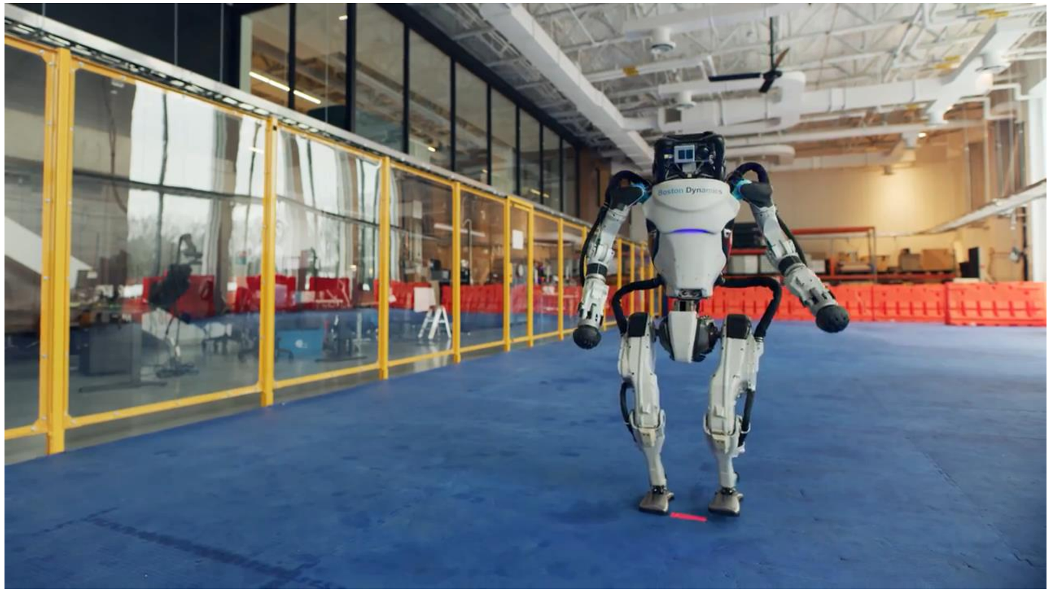
Disadvantages

- **needs hydraulic supply**
- Harder to control (non-linearity)
- large size
- low efficiency (friction losses due to throttling valves)
- increased maintenance (oil leaking)

Suitable for heavy load applications

Hydraulically powered robots

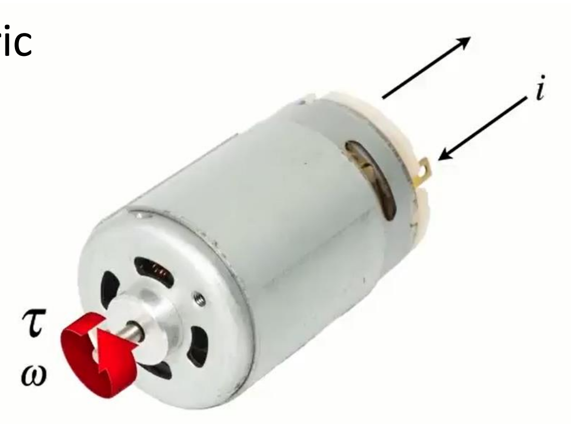
HyQ Real
https://www.youtube.com/watch?v=pLsNs1ZS_TI



Boston Dynamics
https://www.youtube.com/watch?v=fn3KWM1kuAw

Electric Motors

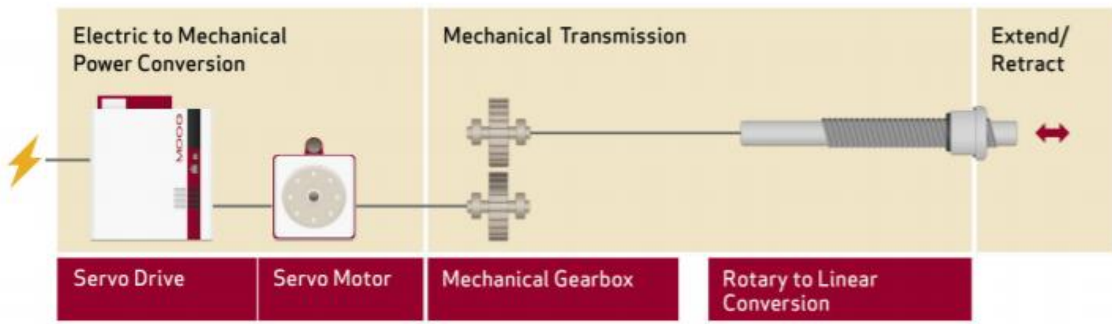
Converts electrical energy into mechanical energy thanks to the interaction between the motor’s magnetic field and the electric current in a wire winding/coil (in the stator)



Advantages

complex co

- power supply available everywhere
- low cost
- **Easy to control**
- Reliable
- high power conversion efficiency
- easy maintenance
- no pollution in working environment



Disadvantages

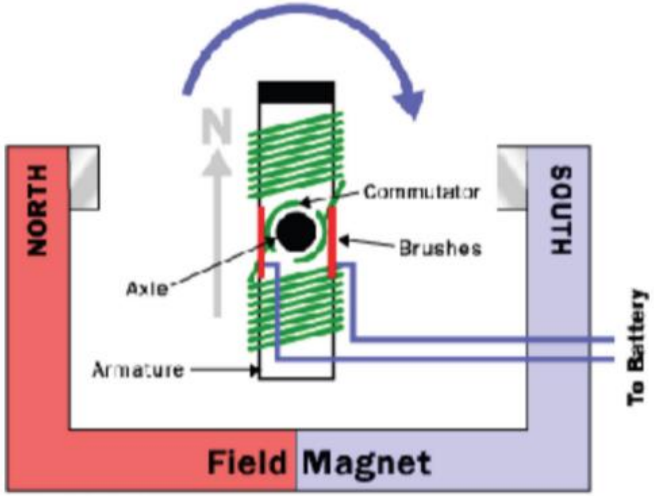
- Coil overheating in static condition
- **Low power to weight ratio**
- some advanced models require more

Electric Motors - Categories

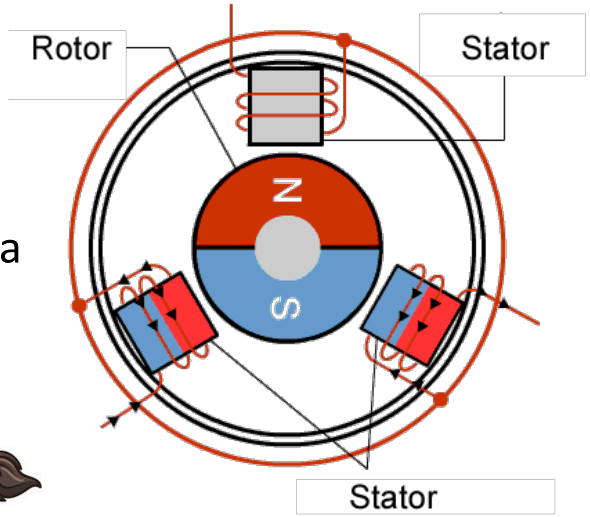
- **DC motors**

  Have large torque and speed ranges

  - **Brushed**: the stator is a permanent magnet, the rotor is a coil (electromagnet). Uses a mechanical commutator to switch polarity of the rotor electromagnet (sparks, brush wearing)



  - **Brushless:** the rotor is a ring of permanent magnets, the stator are stationary coils. Use control electronics to switch polarity of the stator coils. Very flexible. Variable speed and torque. Mostly used in robotics. Coupled with a transmission and an encoder it becomes a **servo-motor.**



C1 - Actuators

Electrical Actuators

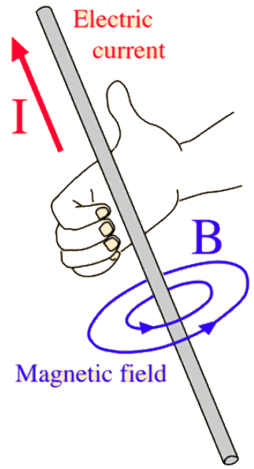
Outline

- Useful notions of physics
- DC motors
  - Principles of operation
  - Brushed motors
  - Brushless motors
  - Mathematical model
  - Steady state response
  - Characteristic curves
  - Voltage and current controls

Physics Laws review

- **Biot–Savart law:**

A wire with **constant** current flow it generates a **constant** magnetic field B =f(i)

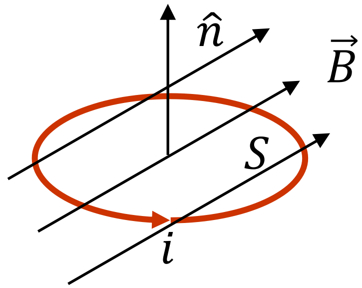


- **Lenz's Law (action and reaction)**

When a closed conductor (i.e. coil) is placed in the path of a **changing magnetic field** B, an induced current is produced in the conductor by magnetic **induction**. This current produces a magnetic field that opposes the direction of change of the magnetic field producing it.

- **Faraday law (induction)**

This current is generated by a voltage called Electro Motive Force (EMF) that can be computed as:



$$\mathrm{EMF} = -\frac{d\Phi_S(\vec{B})}{d\,t}$$

Where S is the surface of the coil conductor, $\vec{B}$ the induced magnetic field whose flux $\Phi_S(\vec{B})$ is time-varying and it is defined as:

$$\Phi_S(\vec{B}) = \int_S \vec{B} \cdot \hat{n}\, dS$$

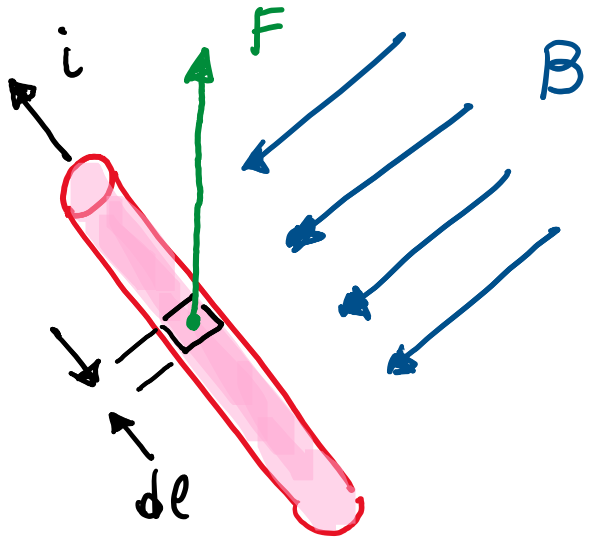
Because EMF is a "reaction" we call it **back EMF**

Physics Laws

- **Lorentz's force law**

A straight conductor of length $d\vec{l}$ travelled by a current i and placed in a magnetic field $\vec{B}$ is subject to a force:

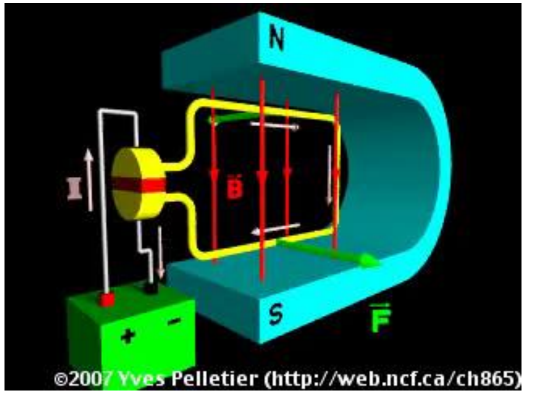
$$d\vec{F} = dl\,\vec{i} \times \vec{B}$$



Non infinitesimal conductor:

$$\vec{F} = l\,\vec{i} \times \vec{B}$$

Principles of Operation of a DC Motor



- The stator generates a **constant** (DC) magnetic field

- The coil, located in the rotor, is travelled by a **constant** current (creating a **constant** magnetic field)

- Each side of the rotor coil is subject to a force:

$$\vec{F} = L\left(\vec{\imath} \times \vec{B}\right)$$

  - On the sides orthogonal to the axis of rotation, the forces annihilates because they have same magnitude but opposite direction
  - On the sides parallel to the axis of rotation, the forces generates a torque which rotates the coil in the position **orthogonal** to the magnetic field

- When the coil becomes perpendicular to the field the forces are no longer producing torque

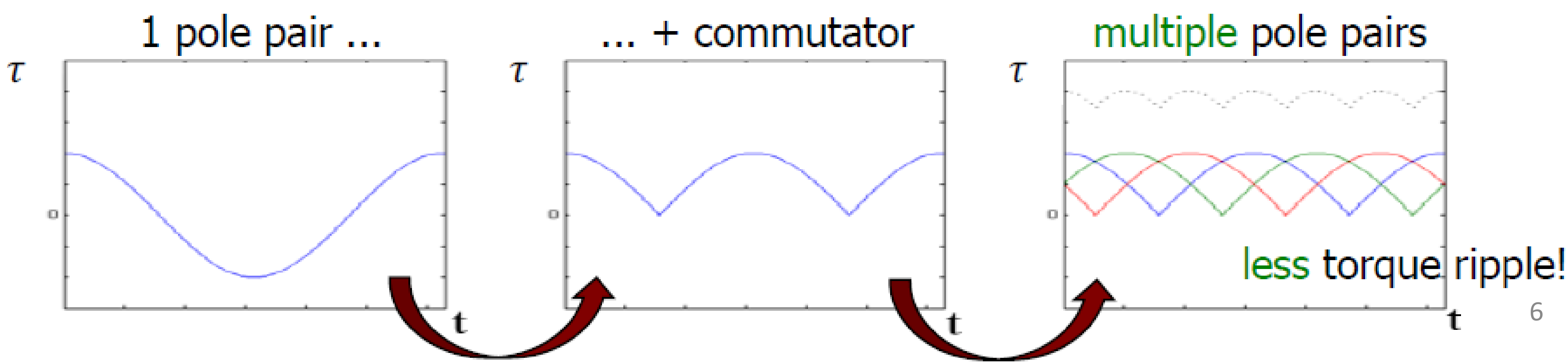
Commutator and Poles

- The **commutator** allows to have continuous rotation: at each half a turn of the coil, the commutator inverts the current direction, changing the orientation of rotoric the magnetic field

- The magnitude of this Lorentz force, and thus the output torque, is a function of the rotor angle (going to zero when coil is perpendicular to B), leading to a phenomenon known as **torque ripple**, i.e. the output torque increases and decreases as the motor shaft rotates

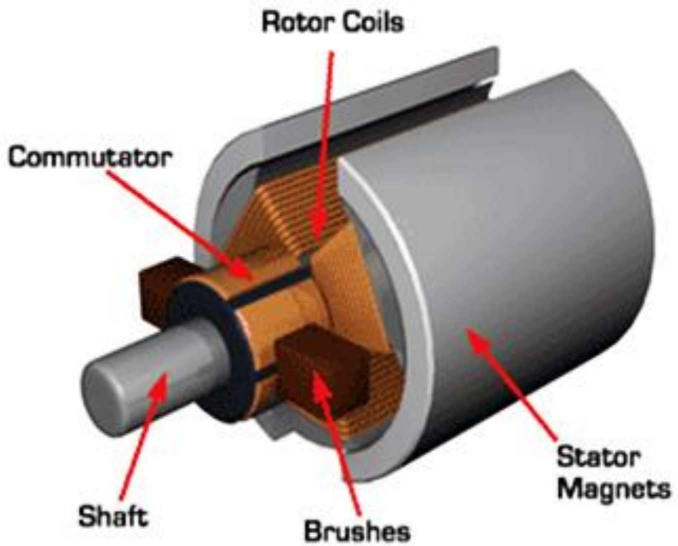
- Each coil introduces a pole pair: the total torque is the sum of the torques generated by a single coil.

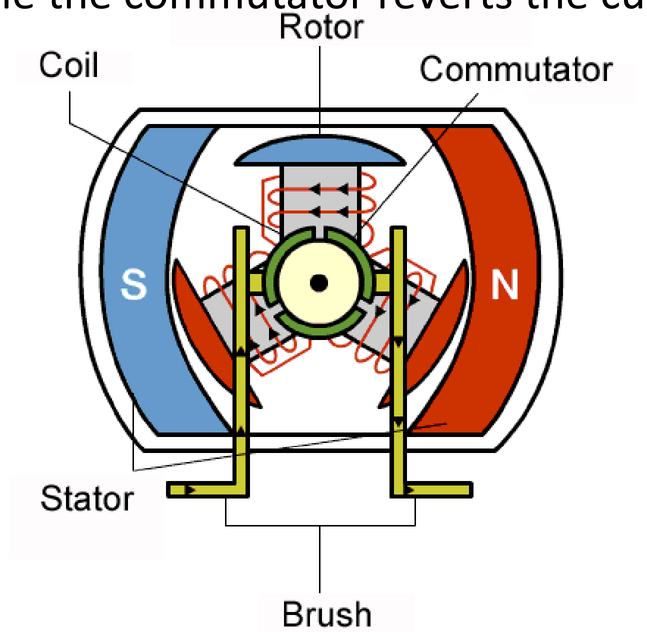
- Having multiple pole pairs give a total torque with **less** torque ripple



Brushed DC Motor

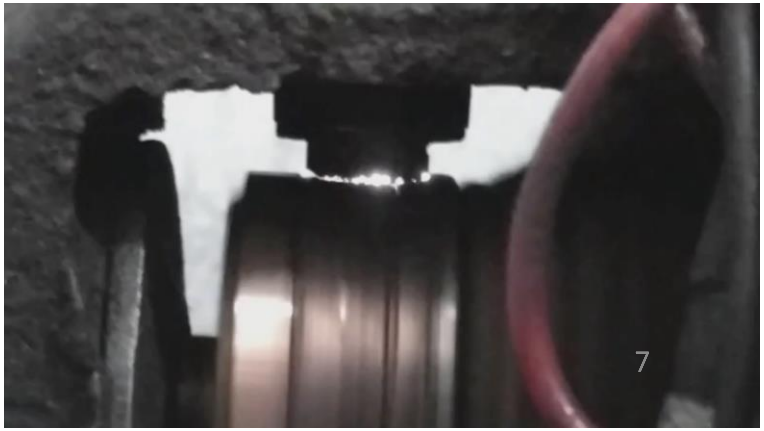
- The power is delivered to the rotor using the commutator and two **brushes**. The brushes provide the exciting voltages to the rotor coil, while the commutator reverts the current in the rotor





Brushed DC motors use mechanical design to switch polarity of the electromagnet

- The brushes are subject to **wear** and can create friction and **sparks** at high speed (EMI)



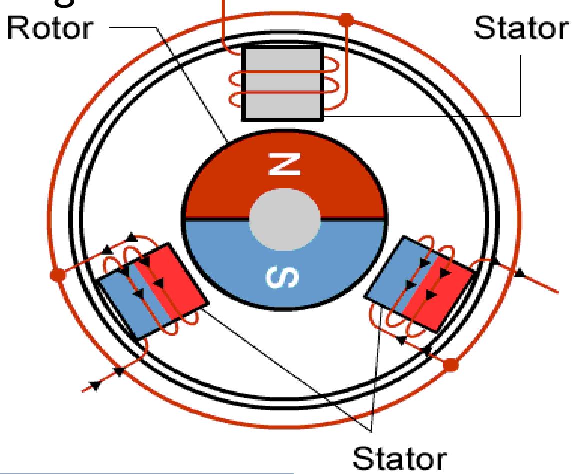
Brushless DC Motor (BLDC)

- Uses a permanent magnet on the **rotor** and use windings (coil) on the **stator**. There is no need to use brushes and commutator to switch the polarity of the voltage in the coil.

- A BLDC motor accomplishes commutation electronically using rotor position feedback to determine when to switch the current (Hall sensor or a rotary encoder).

- Each coil is separately controlled: the coils are switched on to attract or repel the permanent magnet rotor. The coils are driven by a constant DC voltage (hence the name brushless DC), which simply switches from one stator coil to the next to generate a magnetic field waveform with a trapezoidal (not sinusoidal) shape.

- To continuously rotate, the current in the stator coils must alternate continuously. A feedback loop is needed to properly switch the voltage on the stator coils



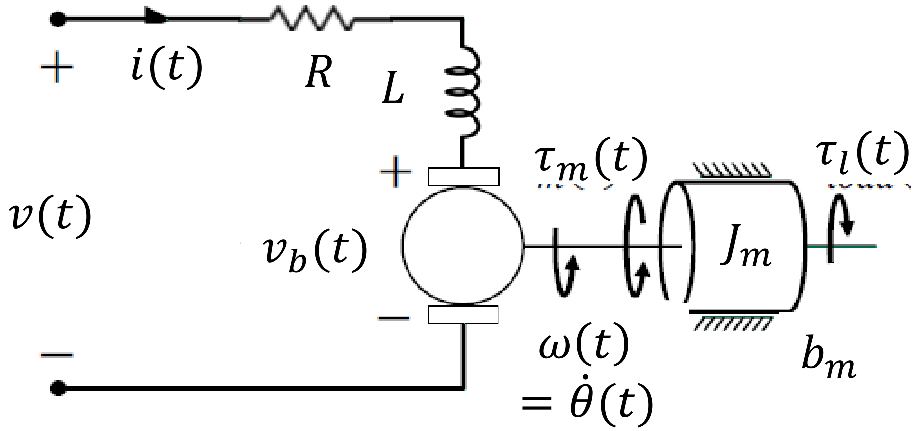
Brushless DC motors use control electronics to vary the polarity of the coils

Brushless DC Motor (BLDC)

www.youtube.com/watch?v=bCEiOnuODac

Mathematical Model of a (monophase) DC motor – in time domain

- We model in the linear domain (assuming we do not hit max current/voltage limits)
- We have some torque generated due to current but also a back emf because the magnetic flux is changing in the coil due to rotation



Electrical balance

$$v(t) = Ri(t) + L\frac{d\,i(t)}{d\,t} + v_b(t)$$

Kirchoff 's voltage rule on the equivalent armature circuit

$$v_b(t) = k_v\omega(t)$$

Faraday Law

Mechanical balance

$$\tau_m(t) = J_m\frac{d\,\omega(t)}{d\,t} + b_m\omega(t) + \tau_l(t)$$

Newton law on torques

$$\tau_m(t) = k_\tau i(t)$$

Lorenz Law

Parameters		Variables	
R	Resistance of the winding (coil)	i	Current of the winding (coil)
L	Induttance of the winding (coil)	v	Supply Voltage of the windind (coil)
		v_b	Back emf
k_v, k_τ	back emf and torque constants	$\dot{\omega}, \theta$	Motor velocity and position
J_m	Motor inertia	τ_m	Motor torque
b_m	Motor damping	τ_l	Load torque

Mathematical Model – in Laplace domain

Recall: Laplace Transform

$$X(s) = \mathcal{L}[x(t)] = \int_{0}^{+\infty} x(t)e^{-st}dt$$

In Laplace domain, differential equations become algebraic equations:

d/dt => s

i(t) ⇒ I(s), v(t) ⇒ V(s), τ(t) ⇒ T(s)

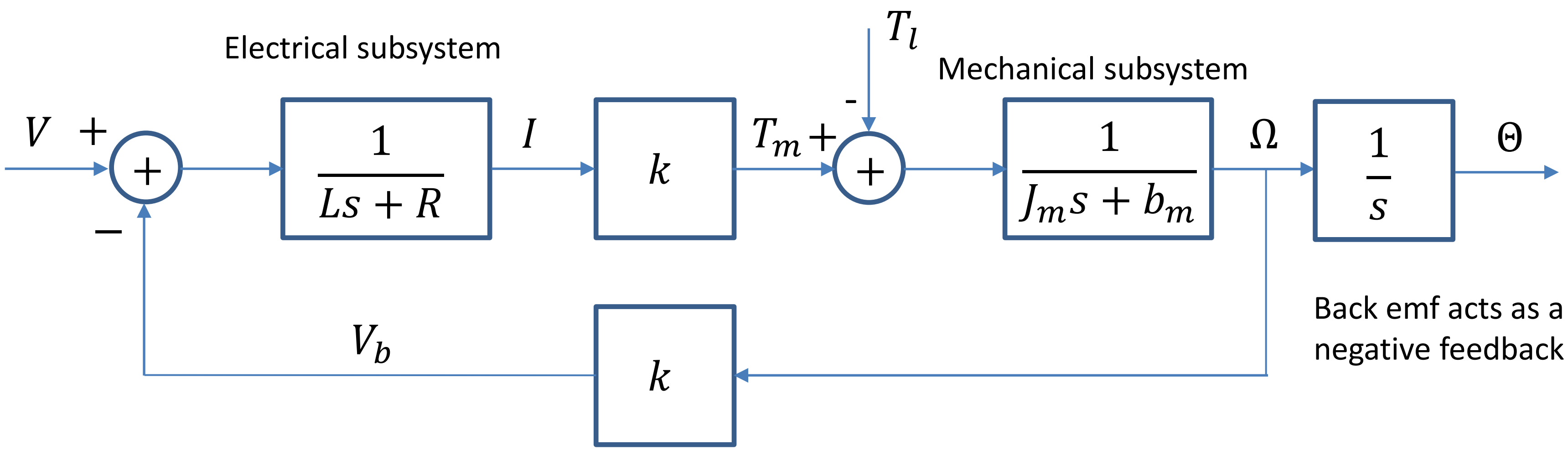
$\omega(t) \Rightarrow \Omega(\mathrm{s}), \theta(t) \Rightarrow \Theta(\mathrm{s})$

Electrical balance

$$V = (Ls + R)I + V_b$$
$$V_b = k\Omega$$

Mechanical balance

$$\mathrm{T}_m = (J_m s + b_m)\Omega + \mathrm{T}_l$$
$$\mathrm{T}_m = kI$$



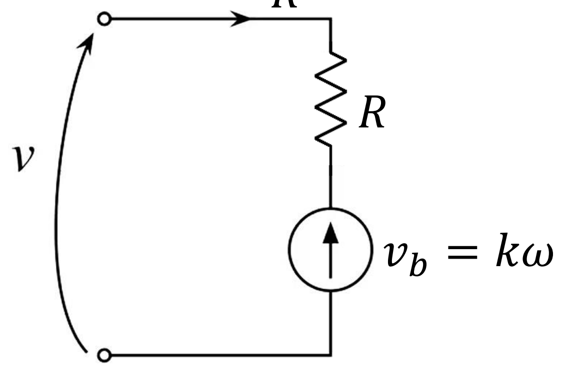
Transfer function from Voltage V(s) of the coil to angular velocity Ω(s) of the shaft:

$$\frac{\Omega(\mathrm{s})}{\mathrm{V}(\mathrm{s})} = \frac{k}{(Ls + R)(J_m s + b_m) + k^2}$$

Voltage Control

- There are two ways to control an electric motor: adjusting the voltage /adjusting the current
- Let's consider the electrical model (ignoring dynamical component)

$$i = \frac{v - k\omega}{R}$$



- Imagine that the motor starts at rest and apply a voltage v, current flows, a torque is generated, and the motor speed will start to increase.
- The back EMF will increase too and eventually the back EMF will be equal to the applied voltage and then no current will flow into the motor. The motor will then stop accelerating and, when this occurs, we have

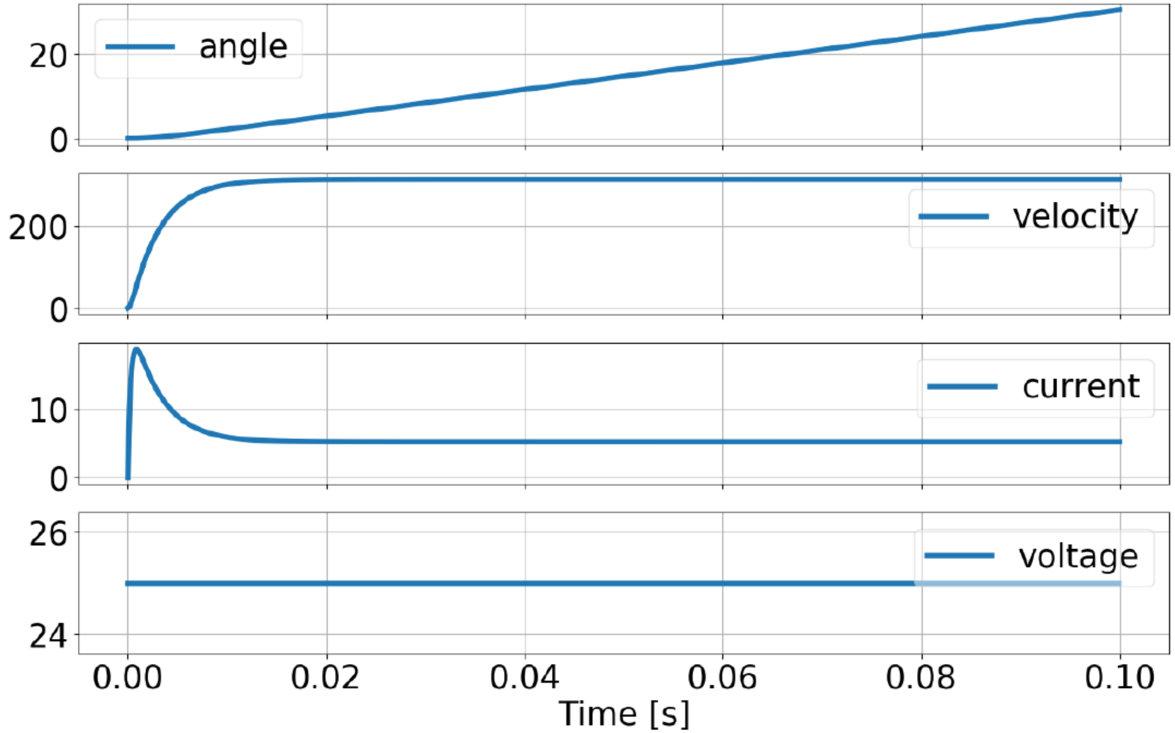
$$\omega = \frac{v}{k}$$

- In practice the motor has (viscous) friction that makes the motor speed a bit lower $\omega < \frac{v}{k}$

Steady State Response to Constant Input

- Constant voltage $v(t) = \overline{v}$ as input.
- After a **transient** a steady situation is reached where the motor will stop accelerating because the motor torque is all balancing friction (or a load torque if there is one)

Maxon148877



- At **steady state**, current and motor velocity are **constant**

Steady state response to constant input

- Studying the **steady state response,** it is useful to study the **static** characteristics of the motor (e.g., steady velocity ω or capability to balance a load torque τ_l)

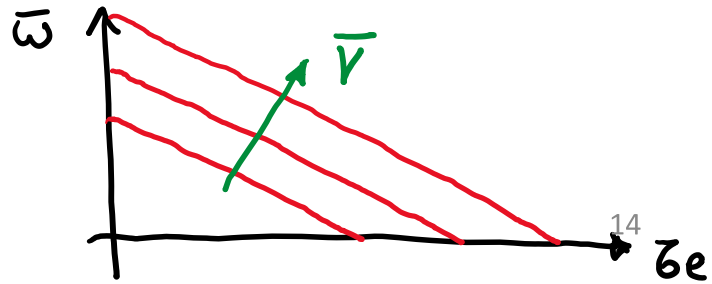
- Constant voltage $v(t) = \bar{v}$ and load torque $\tau_l(t) = \bar{\tau}_l$ as input. At steady state, current and motor velocity are constant, so their derivatives are zero:

$$\begin{cases} \dfrac{di}{dt} = 0 \Rightarrow \bar{V} - R\bar{i} - K\bar{\omega} = 0 \\ \dfrac{d\omega}{dt} = 0 \Rightarrow K\bar{i} - b_m\bar{\omega} - \tau_l = 0 \end{cases}$$

solving this algebraic system:

$$\begin{cases} \boxed{\bar{i} = \dfrac{\bar{V} - K\bar{\omega}}{R}} \\ K\left(\dfrac{\bar{V} - K\bar{\omega}}{R}\right) = b_m\bar{\omega} + \tau_l \Rightarrow \left(\dfrac{K^2}{R} + b_m\right)\bar{\omega} = \dfrac{K}{R}\bar{V} - \tau_l \end{cases}$$

$$\boxed{\bar{\omega}} = \frac{\dfrac{K\bar{V}}{R} - \tau_l}{\dfrac{K^2}{R} + b_m} = \boxed{\frac{K\bar{V} - \tau_l R}{K^2 + b_m R}}$$



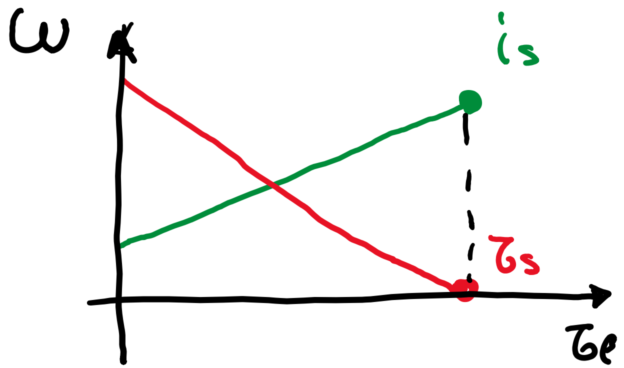
$$\bar{\omega} = \frac{k\bar{V} - \tau_e R}{k^2 + b_m R} \qquad \bar{i} = \frac{1}{R}(\bar{V} - k\bar{\omega})$$

- Let's compute some relevant points of the these steady-state relationships
- Stall torque represents the point on the graph at which the torque (and current) is maximum, but the shaft is not rotating

Ⓐ Stall Torque and current (max load)

$$\bar{\omega} = 0 \Rightarrow k\bar{V} - \tau_e R = 0 \Rightarrow \tau_s = \frac{k\bar{V}}{R}$$

$$\Rightarrow i_s = \frac{1}{R}(\bar{V} - \emptyset) \Rightarrow i_s = \frac{\bar{V}}{R}$$



- No load velocity is the maximum output velocity of the motor (i.e. when no load torque is applied to the output shaft).

(B) velocity and current at (null load)

$$\tau_e = 0 \Rightarrow \boxed{\omega_m} = \frac{K\bar{V}}{K^2 + b_m R} = \frac{K\bar{V}}{R}\,\frac{1}{\frac{K^2}{R} + b_m} = \boxed{\tau_s \frac{1}{\frac{K^2}{R} + b_m}}$$

if we neglect friction:

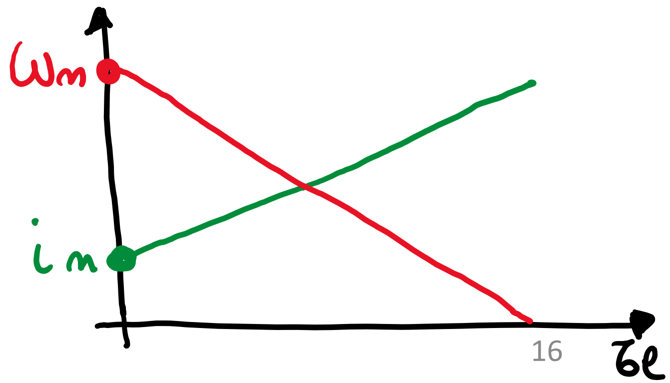
$$\boxed{\omega_m = \frac{\bar{V}}{K}}$$

for current we use mechanical equation:

$$K i_m - b_m \omega_m = \tau_e = 0$$

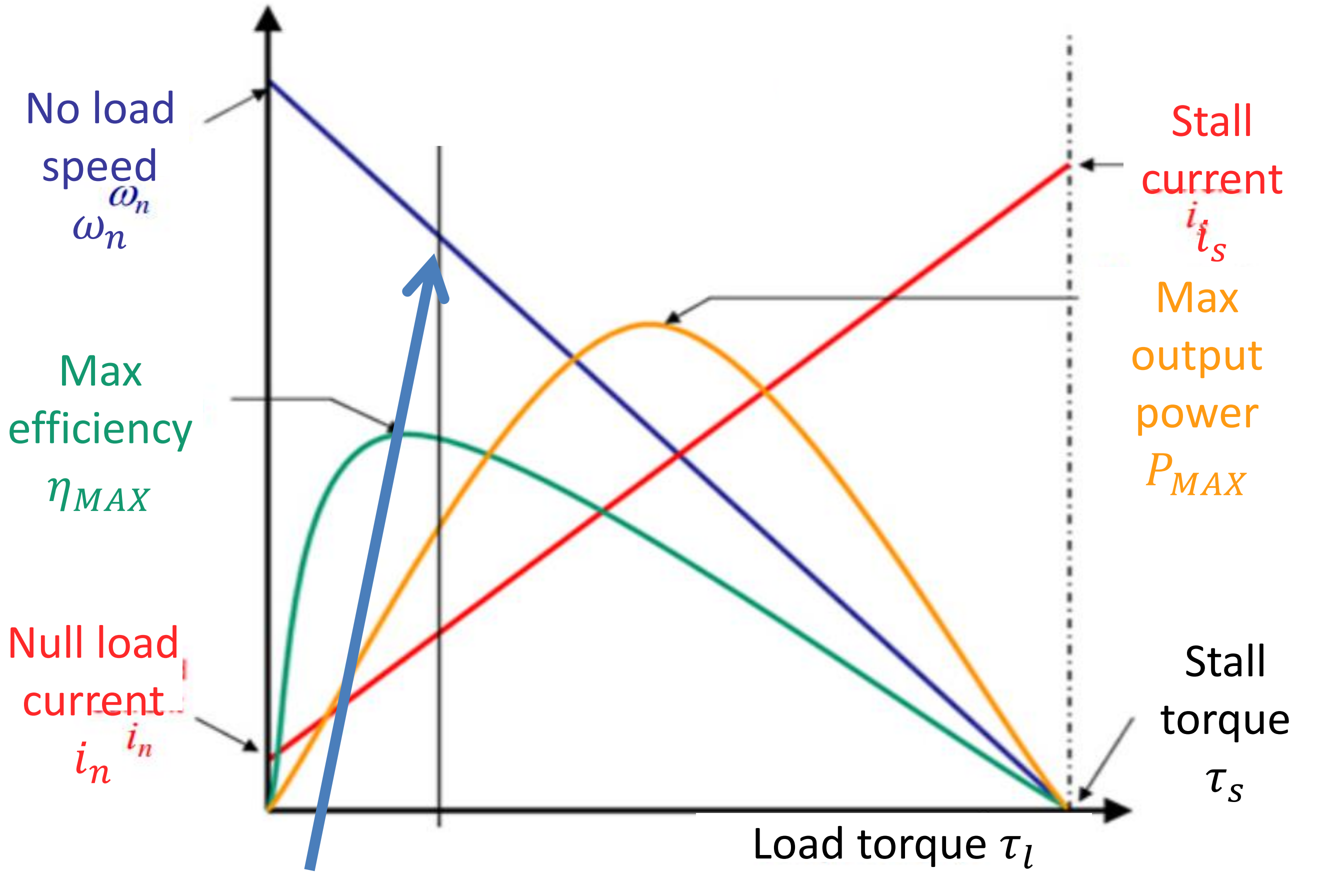
$$\Rightarrow \boxed{i_m = \frac{b_m \omega_m}{K}}$$

↳ balance friction



Typical Characteristics Curves of DC Motors (steady-state)

- Maximum **efficiency** and maximum **output power** do not occur at the same torque.

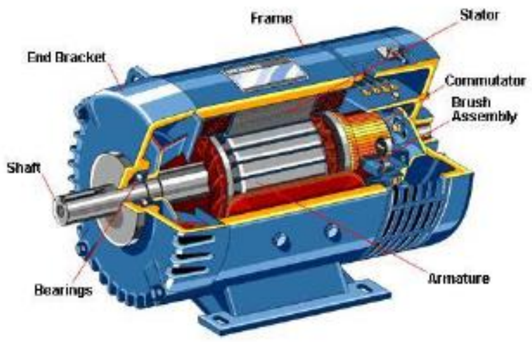


Rated operating point is the speed and torque point at which the motor can operate continuously (generating the max torque it can) without **overheating.** The rated current derives from the motor's thermally maximum permissible continuous current; the rated speed is the speed correspondent to the rated current

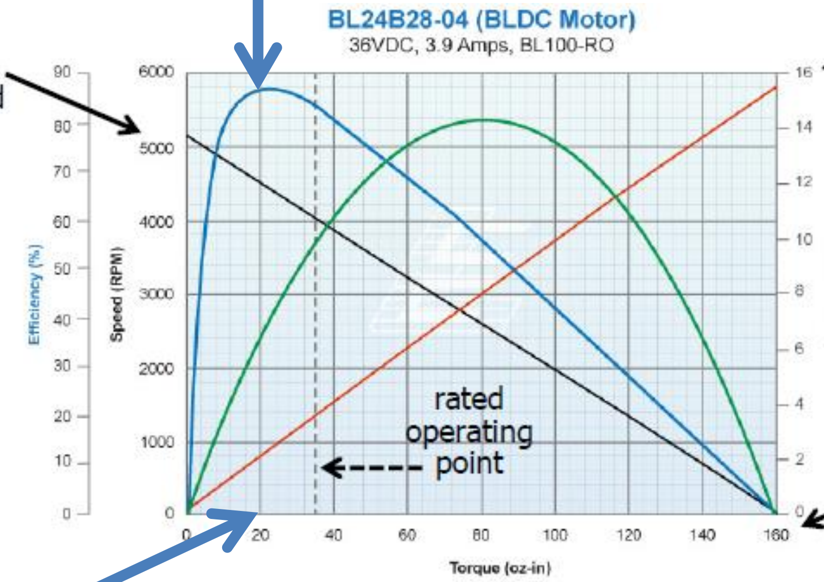
Example of characteristics curves of DC Motors

efficiency

at steady-state, for constant applied tension $\overline{v}$



no-load max speed



stall current

medium size motor 160 W

stall load torque

Optimal operating point: maximum efficiency

- The choice of the supply voltage follows from considerations of where the maximum no load speed should be
- The choice of the rated current depends on how much is the output torque we need to move the load (on average)

Data sheet of electrical DC motors: relevant parameters

- Motor manufacturers make the motor characteristics below available to you to help you decide which motor to purchase.
- We will use all SI units (which is not the case on most motor data sheets).

Motor Property	Symbol	Units	Comments
Nominal voltage	V_{nom}	V	Chosen so the no-load speed is safe for brushes, commutator, and bearings.
Rated power	P_{rated}	W	The max continuous mechanical power output at the nominal voltage without overheating
Max mechanical power	P_{max}	W	The max mechanical power output at the nominal voltage (with overheating).
No-load speed	ω_n	rad/s	Speed when no load and powered by V_{nom}. Usually given in rpm
No-load current	i_n	A	Non-zero because of (viscous) friction torque. You can determine i_n by using a current-sensing resistor and an oscilloscope/multimeter
Rated/Nominal/Max continuous current	I_{rated}/i_c	A	Max continuous current without overheating
Starting current	i_s	A	Same as stall current: V_{nom}/R

Data sheet of electrical DC motors

Motor Property	Symbol	Units	Comments
Rated/Nominal/ Max continuous torque	T_{rated} / T_c	Nm	Directly proportional to max continuous current. This is determined by thermal considerations.
Stall torque	T_s	Nm	Same as starting torque.
Terminal resistance	R	Ω	Coil resistance. Increases with heat. You can measure with a multimeter
Terminal inductance	L	H	Due to the coils.
Electrical time constant	τ_{el}	s	The time for the motor current to reach 63% of its final value. Equal to L/R.
Mechanical time constant	τ_m	s	The time for the motor to go from rest to 63% of its final speed under constant voltage. (defined as RJ_m/k^2 in case you consider the voltage as input, or as J_m/b_m in case you consider the torque)
Max efficiency	η_{max}	%	occurring at approximately 1/7 of stall torque. The wasted power is due to coil heating and friction losses.
Torque constant	k_t	Nm/A	Also called the motor constant.
Speed constant	k_v	Vs/rad	Same numerical value as the torque constant (in SI units). Also called voltage or back-emf constant.

Example of data sheet electrical DC motors

Model 8DCG(W)□-25-30: Gear Type Shaft 8DCD□-25-30: D-Cut Type Shaft	Output	Voltage	Starting Current	Starting Torque		No Load		Rated Load			
						Current	Speed	Current	Speed	Torque	
	W	V	A	kgfcm	N.m	A	r/min	A	r/min	kgfcm	N.m
8DCG(W)12-25-30	25	12	48.00	15.50	1.500	1.80	3300	3.30	3100	0.811	0.081
8DCG(W)24-25-30	25	24	29.00	18.00	1.800	0.80	3050	1.90	2900	0.811	0.081
8DCG(W)90-25-30	25	90	10.00	21.50	2.150	0.04	3200	0.35	3000	0.811	0.081

Model 8DCG(W)□-40-30: Gear Type Shaft 8DCD□-40-30: D-Cut Type Shaft	Output	Voltage	Starting Current	Starting Torque		No Load		Rated Load			
						Current	Speed	Current	Speed	Torque	
	W	V	A	kgfcm	N.m	A	r/min	A	r/min	kgfcm	N.m
8DCG(W)12-40-30	40	12	47.00	15.00	1.500	1.50	3300	4.80	3000	1.30	0.130
8DCG(W)24-40-30	40	24	37.00	23.00	2.300	0.60	3250	1.90	3000	1.30	0.130
8DCG(W)90-40-30	40	90	1.50	24.00	2.400	0.03	3400	0.60	3000	1.30	0.130

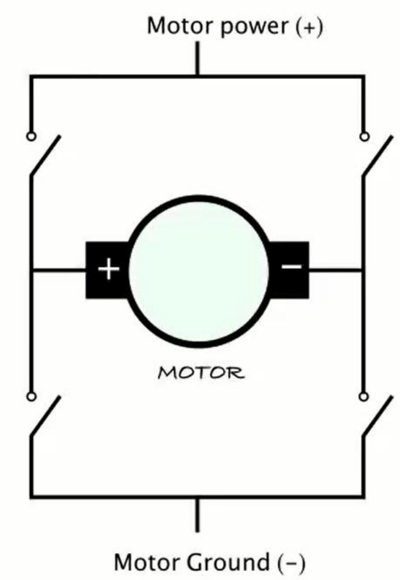
The motor driver

- The purpose of a motor driver is to take a signal representing the desired speed, and to drive a motor at that speed.
- If we keep the voltage **constant**, the speed (steady state) will be **dictated** by the load torque according to the torque-speed characteristics
- However, the speed can also change with the supply voltage, so to control the speed we need to be able to control the voltage
- The controller works by varying the **average** voltage sent to the motor. This can be done:
    - adjusting the voltage sent to the motor (e.g. by a voltage divider), quite inefficient to do (produces a lot of heat)
    - H bridge circuit

Voltage Control for a DC motor: H bridge circuit

- Most common way to drive an electric motor today: H bridge circuit
- In the 1-phase case, it is composed by 4 switches (power MOSFETS):
  - **small dimension**
  - small resistance
  - small power loss in the ON/OFF states, only during transient (~ 100 ns/1kHz) there are both voltage and current

Here, no current can flow through the motor. But if two switches are closed…



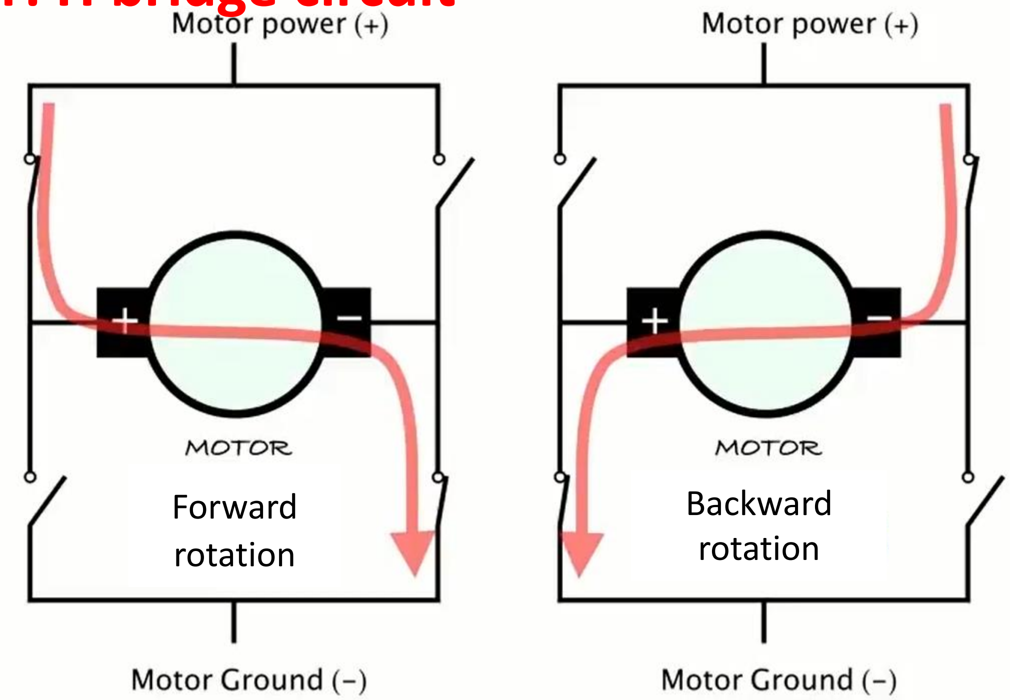
Voltage Control for a DC motor: H bridge circuit

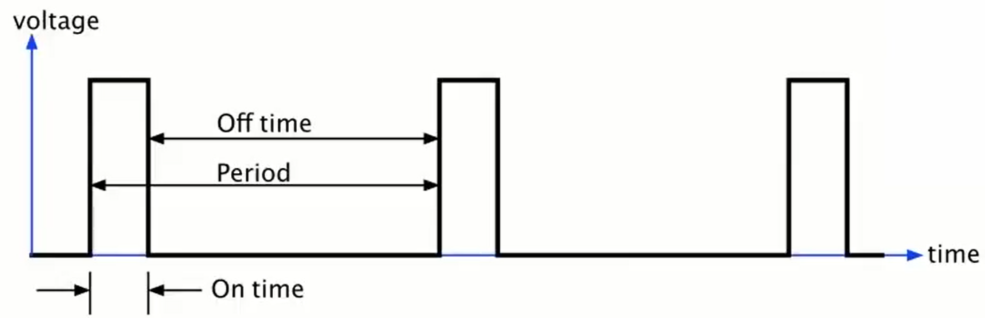
- each side of the motor can be connected either to battery positive, or to battery negative. Note that only one MOSFET on each side of the motor must be **turned on** at any one time.

- (+) we always apply the **maximum** voltage. The electronics which is driving the H bridge circuit, is able to control the duty cycle:

$$\text{Duty cycle} = \frac{\text{On time}}{\text{Period}}$$

- The **On time** is the pulse width, hence we call this **pulse-width modulation** (PWM) at high frequency





- Most **PWM drives** operate with a fixed PWM **frequency** that is several times **higher** than the highest output **frequency** that is to be used (range 2 kHz up to about 10 kHz)

C2 - Actuators

Transmission

Why Transmissions?

- Optimize the transfer of mechanical torque from actuating motors to driven loads (i.e links)

- **Quantitative** transformation (from low torque/high velocity to high torque/low velocity). They:
  - work as torque amplifiers
  - can use smaller motors (lighter robot)

- **Qualitative** transformation (e.g. from rotational motion of an electrical motor to a linear motion in the case of a prismatic joint)

- locating the motors closer to the robot base, allows improvement of static and dynamic performance by reducing the inertia of the actual robot structure in motion

Quantitative Transformation

- Typical robot applications need actuators able to provide:
  - LARGE TORQUES $1 \div 10^3\ Nm$
  - LOW SPEED $1 \div 10^2\ rad/s$

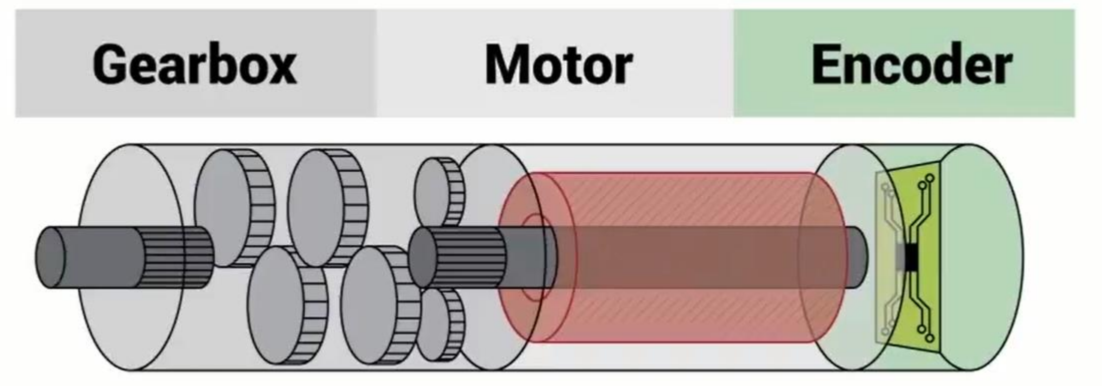
$$\boxed{1\ \frac{rad}{s} = \frac{60}{2\pi} rpm \approx 9.55\ rpm}$$

- Unfortunately, typical DC motors provide:
  - LOW TORQUES
  - HIGH SPEED

- The transmission trades off speed for torque: from low torque/high velocity to high torque/low velocity but there is some inefficiency and loss (as friction)

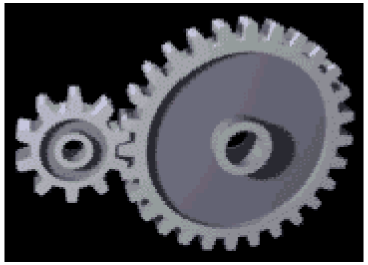
- In this case, the transmission acts as a **speed reducer/torque amplifier**

Load



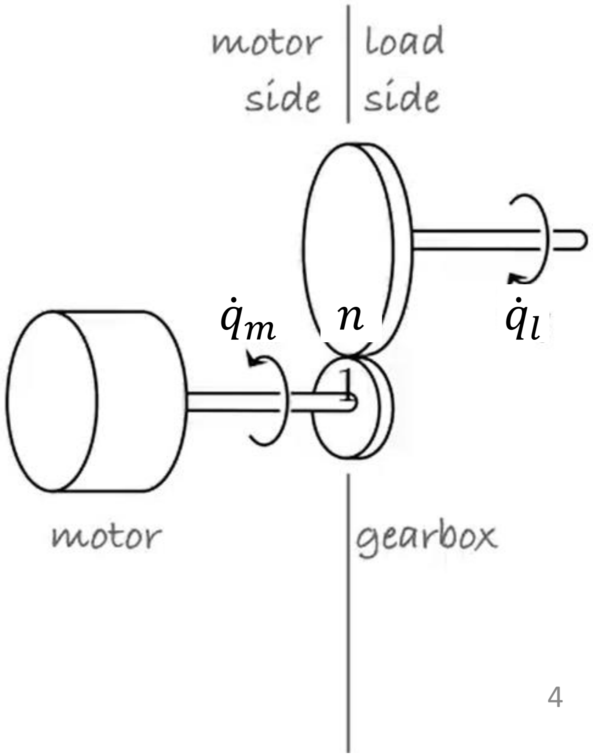
Quantitative transformation

- Here we have a motor with a single stage reduction gearbox. We call it a reduction gearbox because for every rotation of the motor, the output shaft (on the load side) rotates less than once



$$n = \frac{\#\, teeth\ of\ the\ load\ side\ wheel}{\#\, teeth\ of\ the\ motor\ side\ wheel}$$

- For a reduction gearbox $\mathrm{n} > 1$, where n is the **gear reduction ratio**



- Output speed:
$$\dot{q}_l = \dot{q}_m / n$$

- Power equivalence (neglecting losses):
$$P_{IN} = P_{OUT} \Leftrightarrow \tau_m \dot{q}_m = \tau_l \dot{q}_l$$
torque × angular velocity

- Output torque:
$$\tau_l = n\tau_m$$

- The output shaft speed is lower than the motor speed and the output torque is greater than the motor torque

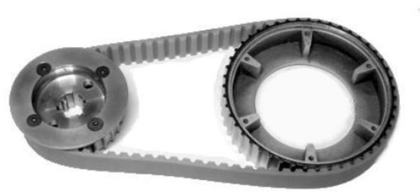
Transmissions in Robotics

Spur gears

Bevel gears

- **Spur gears, Bevel gears**: modify direction and/or translate axis of (rotational or translational) motor displacement
    - **Problems**: deformations, backlash

- **Worm gear**: convert rotational into translational motion (prismatic joints). Composed of a lead screw and a gear. High reduction ratio (up to 1000)
    - Problems: friction, elasticity, backlash

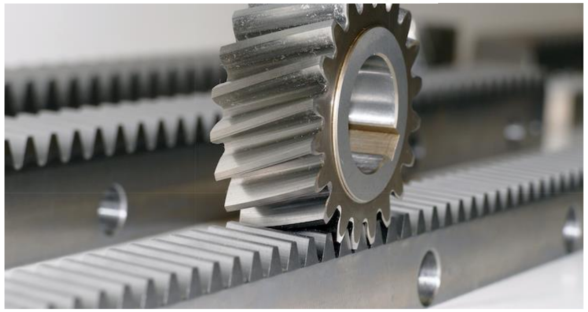


- **Toothed belts and chains**: dislocate the motor axis w.r.t. the joint axis. Reduction ratio (up to 10).
    - Problems: compliance (belts) or vibrations induced by larger mass at high speed (chains)

Transmissions in Robotics

- **Lead screw**: High reduction, high friction, high vibration, high force capability

- **Rack and pionion**: provide extremely long stroke lengths. Used for applications that require long travel (where properly tensioning a toothed belt drive becomes difficult). High dynamics, high thrust forces, and high positioning accuracy.



- **Harmonic drives**: compact, in-line, power efficient, with high reduction ratio (up to 150-200:1)
    - Problems: elasticity

C3 - Actuators

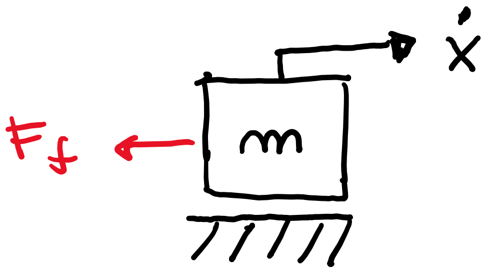
Non idealities in Motion Control

Outline

- Joint friction
- Back-lash
- Dead-band

Joint Friction

- All mechanical systems exhibit friction: a force always **opposing** the motion



bearings / sliding parts

- Different components: Friction = Coulomb Friction + Viscous Friction + Stiction

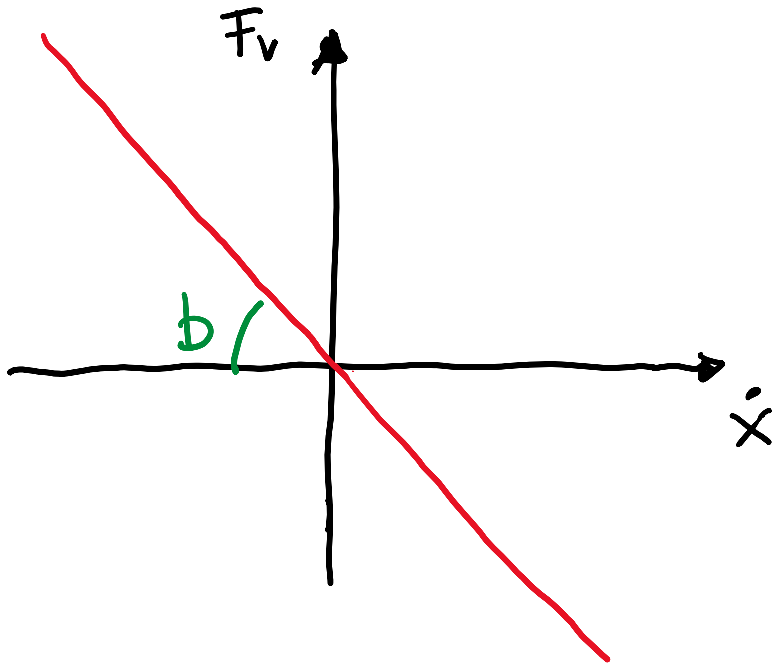
$$F_f = \underbrace{F_c}_{\text{coulomb friction}} + \underbrace{F_v}_{\text{viscous friction}} + \underbrace{F_s}_{\text{stiction (static friction)}}$$

viscous friction $\downarrow$ proportional to velocity

Viscous friction (aka damping)

- Is proportional to velocity through the damping coefficient **b**:

$$F_v = -b\dot{x}$$



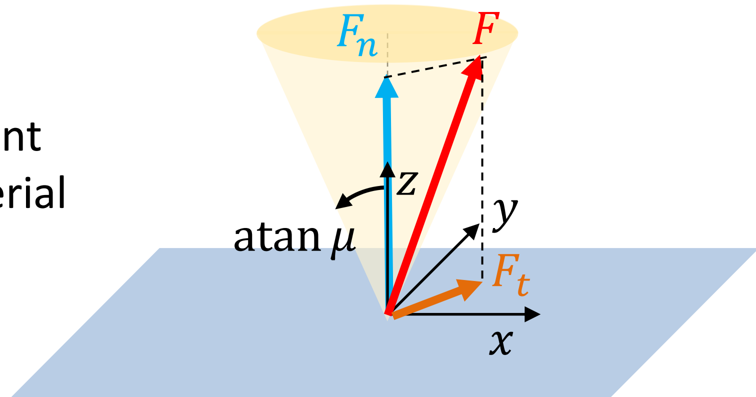
Coulomb friction

- The Coulomb law is the most common and practical model of friction available. It is, however, an experimental law. Was derived from a great deal of experimental work in 1785.

- Coulomb's law says that the (tangential) friction force F_t is bounded in magnitude by the normal contact force F_n times the coefficient of friction (μ):

$$f_c = F_t = \mu F_n$$

μ: Coulomb friction coefficient (depending on the two material in contact)

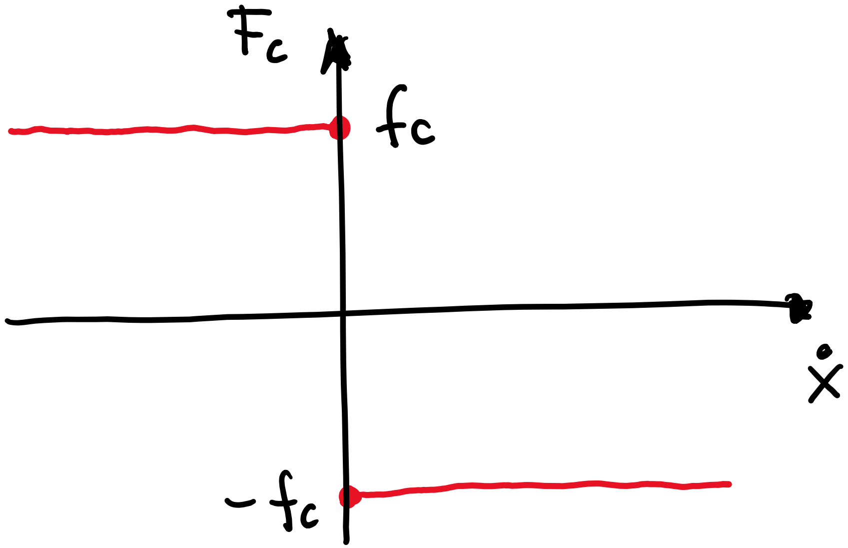


<u>Geometric interpretation</u>

To avoid slippage F must lie inside on at the boundary the friction cone, i.e. the cone whose axis is on the normal to the contact surface at the contact point, has vertex at the contact point, and a angle size of $\mathrm{atan}(\mu)$

- If the contact is **sliding**, then the magnitude of the friction force is exactly μF_n (i.e. lies on the cone boundary) in the opposite direction to the relative velocity at the contact

$$F_c = \begin{cases} -f_c & \text{if } \dot{x} > 0 \\ f_c & \text{if } \dot{x} < 0 \end{cases} = -fc\ \mathrm{sign}(\dot{x})$$



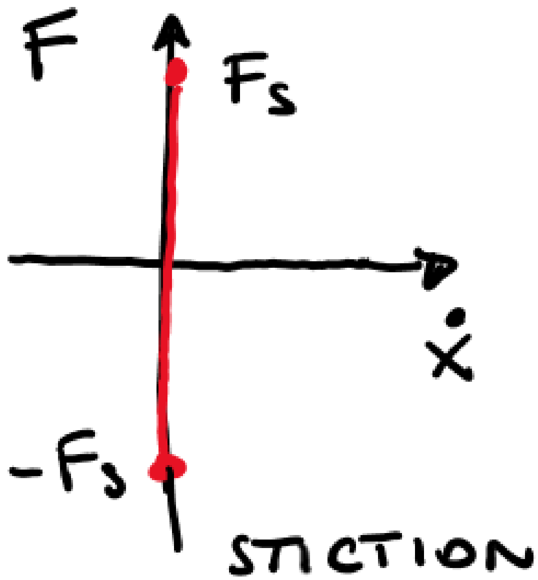
- If the contact is not sliding: stiction $f_s > f_c$

Stiction (static friction)

- Force to be overcome to enable relative motion of **stationary** objects in contact

- Similar to Coulomb friction, but for **zero** velocity

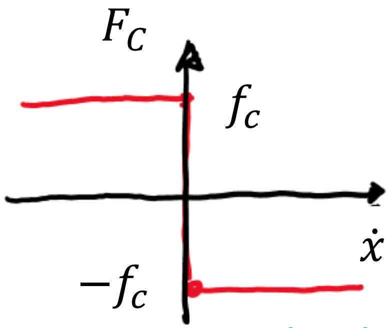
$$F_s = \begin{cases} -f_s & \text{if } \dot{x} = 0^+ \\ f_s & \text{if } \dot{x} = 0^- \end{cases}$$



- In general is slightly higher than Coulomb friction

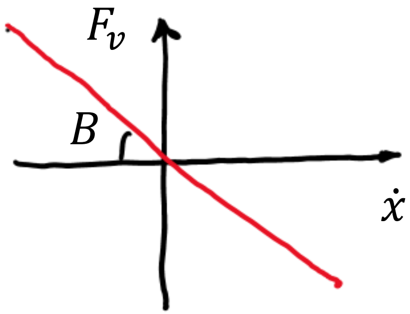
Friction model

- Total friction: $F_f = -\mathrm{f_c}\mathrm{sign}(\dot{x}) - b\dot{x} + F_s$



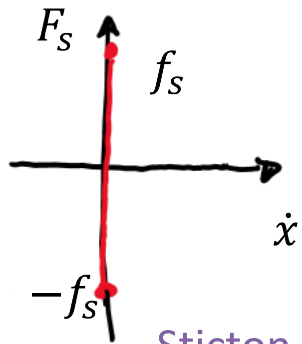
Coulomb

$+$



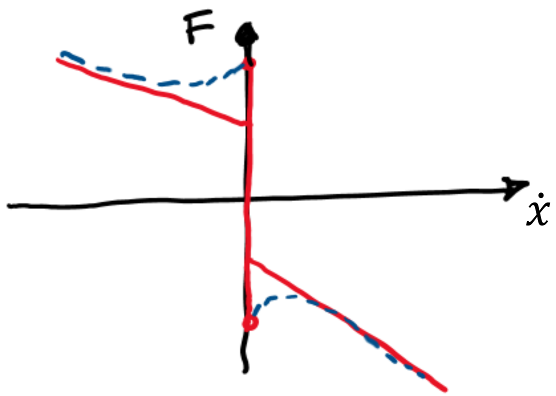
Viscous

$+$



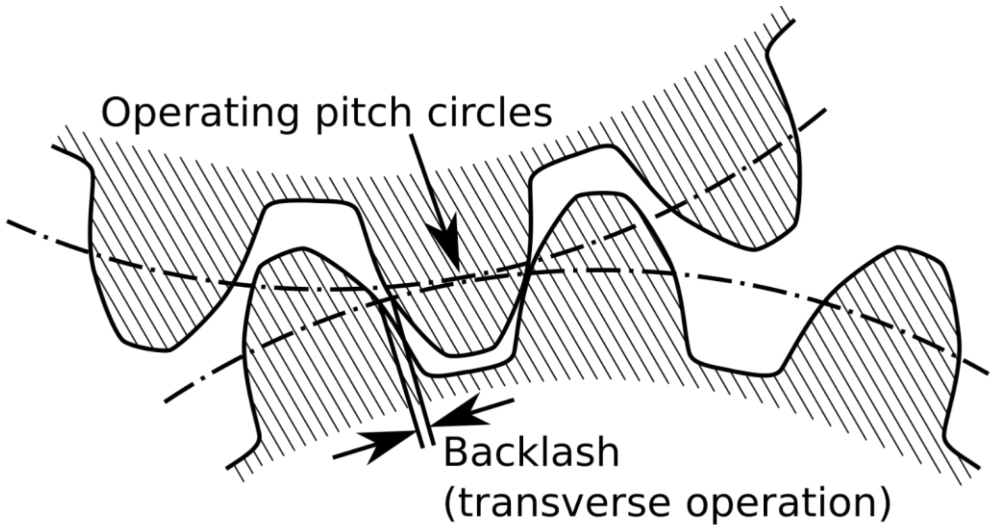
Sticton

$=$



Stribeck effect
Positive slope, destabilizing effect/stick-slip

Backlash



- Maximum clearance in a mechanism due to the gaps between parts (e.g. teeth of a gear wheel)
- The system may be moved in one direction without applying appreciable force or motion to the next part in mechanical sequence

- There is no drive from the input to the output shaft in either direction while the teeth are not meshed

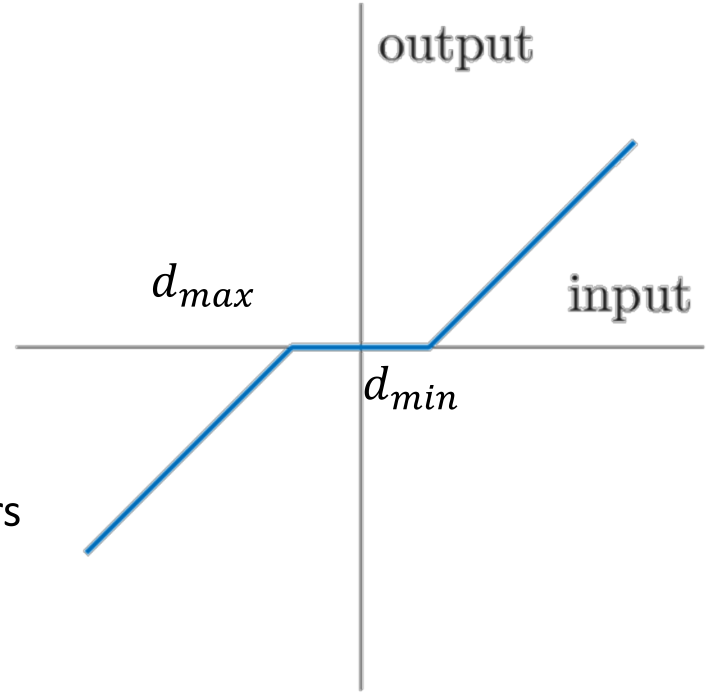
- Effect on control: limit cycle, instabilities, vibrations

- Solution: improve mechanical design (e.g. conic fits)

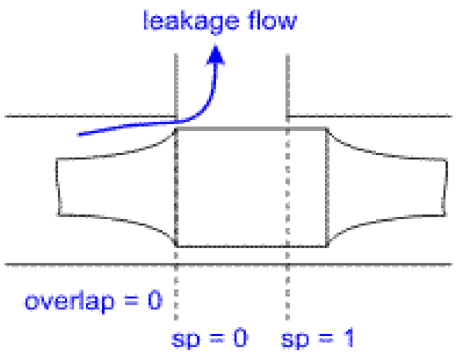
Dead-band

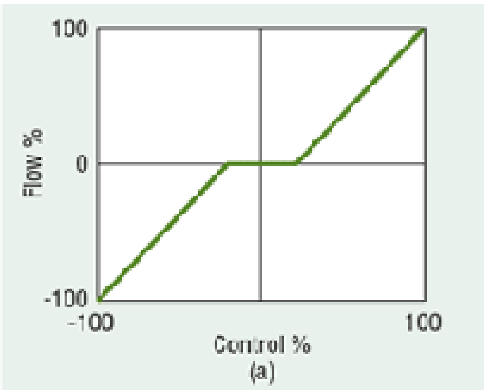
- A dead-band is a range of input values for which the system output is **zero**
- gain is zero for very low outputs

$$\text{output} = \begin{cases} \text{input} - d_{min} & \text{if input} < d_{min} \\ 0 & \text{if } d_{min} \leq \text{input} \leq d_{max} \\ \text{input} - d_{max} & \text{if input} > d_{max} \end{cases}$$



- present in (overlapped) hydraulic valves/motor drivers





- backlash is an example of dead-band for gears train

- To compensate it, we “invert” the dead-band