# Fundamentals of Artificial Intelligence
## Laboratory

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# Exercise 8.1 - Which of the following are correct?

a. False |= True. This is true because False has no models and hence entails every sentence and because True is true in all models and hence is entailed by every sentence.

b. True |= False. This is false.

c. (A ∧ B) |= (A ⇔ B). This is true because the left-hand side has exactly one model that is one of the two models of the right-hand side.

d. A ⇔ B |= A ∨ B. This is false because one of the models of A ⇔ B has both A and B false, which does not satisfy A ∨ B.

e. A ⇔ B |= ¬A ∨ B. This is true because the RHS is A ⇒ B, one of the conjuncts in the definition of A ⇔ B.

f. (A ∧ B) ⇒ C |= (A ⇒ C) ∨ (B ⇒ C). This is true because the RHS is false only when both disjuncts are false, i.e., when A and B are true and C is false, in which case the LHS is also false. This may seem counterintuitive, and would not hold if ⇒ is interpreted as "causes."

g. (A ∨ B) ∧ (¬C ∨ ¬D ∨ E) |= (A ∨ B). This is true; removing a conjunct only allows more models.

h. (A ∨ B) ∧ (¬C ∨ ¬D ∨ E) |= (A ∨ B) ∧ (¬D ∨ E). This is false; removing a disjunct allows fewer models.

i. (A ∨ B) ∧ ¬(A ⇒ B) is satisfiable. This is satisfiable; model has A and ¬B.

j. (A ⇔ B) ∧ (¬A ∨ B) is satisfiable. This is satisfiable; RHS is entailed by LHS so models are those of A ⇔ B.

# Exercise 8.2 - Homework

Using truth tables, verify the following equivalences:

a.  $P \wedge Q \Leftrightarrow Q \wedge P$

b.  $P \vee Q \Leftrightarrow Q \vee P$

c.  $P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$

d.  $P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R$

# Exercise 8.3

Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?

**a.** B ∨ C.
Sentence is false only if B and C are false, which occurs in 4 cases for A and D, leaving 12.

**b.** ¬A ∨ ¬B ∨ ¬C ∨ ¬D.
Sentence is false only if A, B, C, and D are true, which occurs in 1 case, leaving 15.

**c.** (A ⇒ B) ∧ A ∧ ¬B ∧ C ∧ D.
The last four conjuncts specify a model in which the first conjunct is false, so 0.

# Exercise 8.4

According to some political pundits, a person who is radical (R) is electable (E) if he/she is conservative (C), but otherwise is not electable.

a.   Which of the following are correct representations of this assertion?

(i) $(R \wedge E) \Leftrightarrow C$

(ii) $R \Rightarrow (E \Leftrightarrow C)$

(iii) $R \Rightarrow ((C \Rightarrow E) \vee \neg E)$

# Exercise 8.4

According to some political pundits, a person who is radical (R) is electable (E) if he/she is conservative (C), but otherwise is not electable.

a. Which of the following are correct representations of this assertion?

(i) $(R \wedge E) \Leftrightarrow C$

No.
This sentence asserts, among other things, that all conservatives are radical, which is not what was stated.

# Exercise 8.4

According to some political pundits, a person who is radical (R) is electable (E) if he/she is conservative (C), but otherwise is not electable.

a.  Which of the following are correct representations of this assertion?

(ii) $R \Rightarrow (E \Leftrightarrow C)$

Yes.
This says that if a person is a radical then they are electable if and only if they are conservative.

# Exercise 8.4

According to some political pundits, a person who is radical (R) is electable (E) if he/she is conservative (C), but otherwise is not electable.

a.  Which of the following are correct representations of this assertion?

(iii) R ⇒ ((C ⇒ E) ∨ ¬E)

No.
This is equivalent to ¬R ∨ ¬C ∨ E ∨ ¬E which is a tautology, true under any assignment.

# Exercise 8.5

Show using the propositional resolution calculus that:

    (1)  P ∨ ¬Q

    (2)  ¬P ∨ R ∨ S

    (3)  Q

    (4)  ¬P ∨ ¬R

    (5)  Q ∨ P ∨ R

    (6)  S ∨ ¬Q

    (7)  R

is not satisfiable;

# Exercise 8.5 - Solution

Show using the propositional resolution calculus that:

**(1)** **P ∨ ¬Q**

(2) ¬P ∨ R ∨ S

**(3)** **Q**

(4) ¬P ∨ ¬R

(5) Q ∨ P ∨ R

(6) S ∨ ¬Q

(7) R

**(8)** **P**

# Exercise 8.5 - Solution

Show using the propositional resolution calculus that:

**(1) P** ∨ **¬Q**

(2) ¬P ∨ R ∨ S

**(3) Q**

**(4) ¬P ∨ ¬R**

(5) Q ∨ P ∨ R

(6) S ∨ ¬Q

(7) R

**(8) P**

**(9) ¬R**

# Exercise 8.5 - Solution

Show using the propositional resolution calculus that:

**(1)  P ∨ ¬Q**

(2)  ¬P ∨ R ∨ S

**(3)  Q**

**(4)  ¬P ∨ ¬R**

(5)  Q ∨ P ∨ R

(6)  S ∨ ¬Q

**(7)  R**

**(8)  P**

**(9) ¬R**

**(10) {}**

# Exercise 8.6

Show using the propositional resolution calculus that:

(1)  P ∨ Q

(2)  ¬Q ∨ P ∨ ¬R

(3)  ¬P ∨ ¬R

(4)  R

is not satisfiable.

# Exercise 8.6

Show using the propositional resolution calculus that:

**(1)** **P ∨ Q**

**(2)** **¬Q ∨ P ∨ ¬R**

(3) ¬P ∨ ¬R

(4) R

**(5) P ∨ P ∨ ¬R**

## Exercise 8.6

Show using the propositional resolution calculus that:

**(1)  P ∨ Q**

**(2)  ¬Q ∨ P ∨ ¬R**

(3)  ¬P ∨ ¬R

(4)  R

**(5)  P ∨ P ∨ ¬R**

**(6)  P ∨ ¬R**

# Exercise 8.6

Show using the propositional resolution calculus that:

(1) **P ∨ Q**

(2) **¬Q ∨ P ∨ ¬R**

(3) **¬P ∨ ¬R**

(4) R

(5) **P ∨ P ∨ ¬R**

(6) **P ∨ ¬R**

(7) **¬R**

# Exercise 8.6

Show using the propositional resolution calculus that:

(1)  **P ∨ Q**

(2)  **¬Q ∨ P ∨ ¬R**

(3)  **¬P ∨ ¬R**

(4)  **R**

(5)  **P ∨ P ∨ ¬R**

(6)  **P ∨ ¬R**

(7)  **¬R**

(8)  **{}**

# Exercise 8.7

Show using the propositional resolution calculus that:

    (1)   G ∨ D ∨ ¬B

    (2)  ¬E ∨ C

    (3)   G

    (4)   F ∨ C ∨ ¬A

    (5)  ¬A

    (6)  ¬C ∨ B

    (7)  ¬G ∨ ¬F ∨ E

    (8)   F

    (9)  ¬F ∨ ¬E ∨ D

    (10) ¬F ∨ ¬B ∨ A

is not satisfiable;

# Exercise 8.7

(1)  G ∨ D ∨ ¬B

(2)  ¬E ∨ C
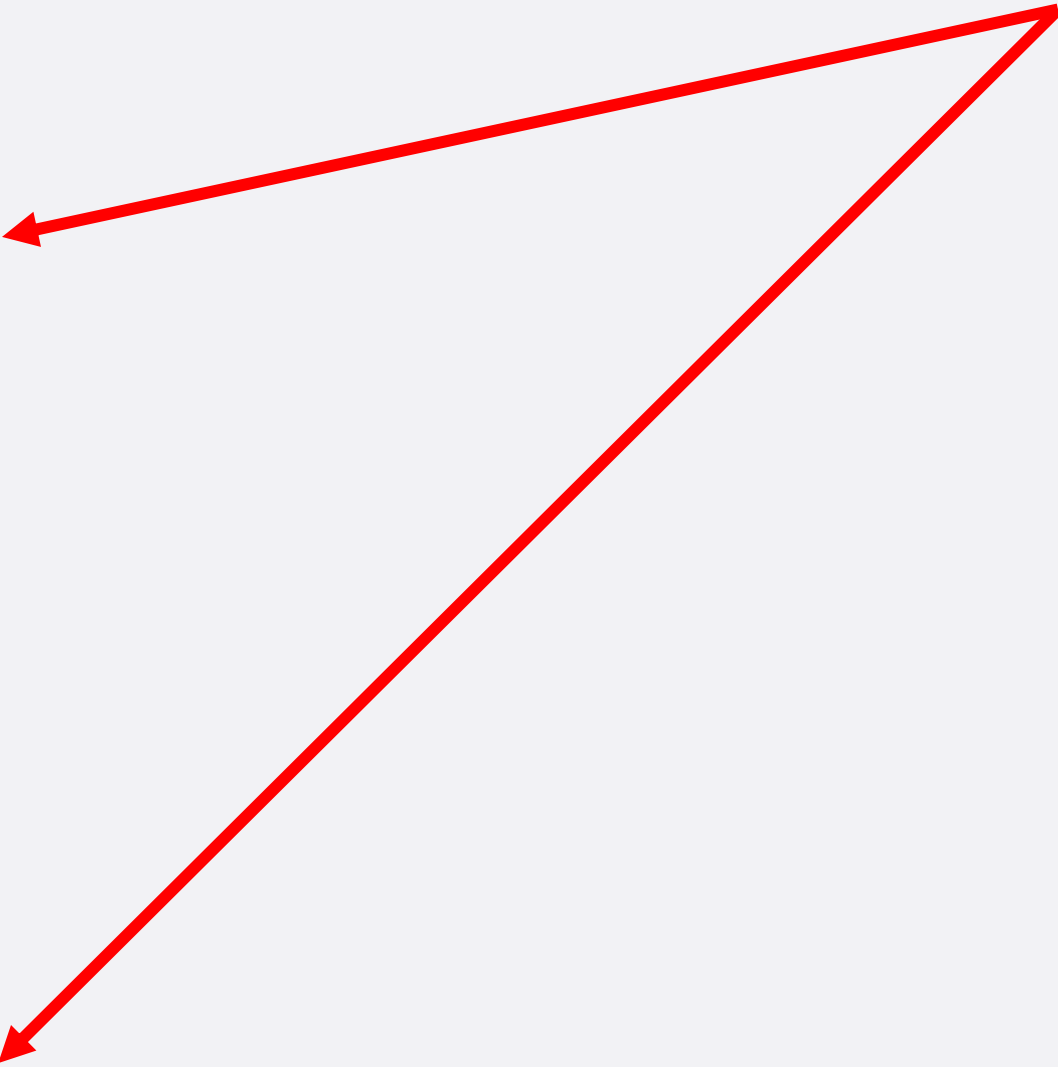
(3)  G

(4)  F ∨ C ∨ ¬A

(5)  ¬A

(6)  ¬C ∨ B

(7)  ¬G ∨ ¬F ∨ E

(8)  F

(9)  ¬F ∨ ¬E ∨ D

(10) ¬F ∨ ¬B ∨ A

Resolution: deduction of a new clause from a pair of clauses with EXACTLY ONE incompatible variable (resolvent)

(11) [(7), (3)] ⇒ (¬F ∨ E)

(12) [(8), (11)] ⇒ (E)

(13) [(12), (2)] ⇒ (C)

(14) [(13), (6)] ⇒ (B)

(15) [(8), (10)] ⇒ (¬B ∨ A)

(16) [(14), (15)] ⇒ (A)

(17) [(5), (16)] ⇒ {}

# Exercise 8.7

(1)  G ∨ D ∨ ¬B

(2)  ¬E ∨ C
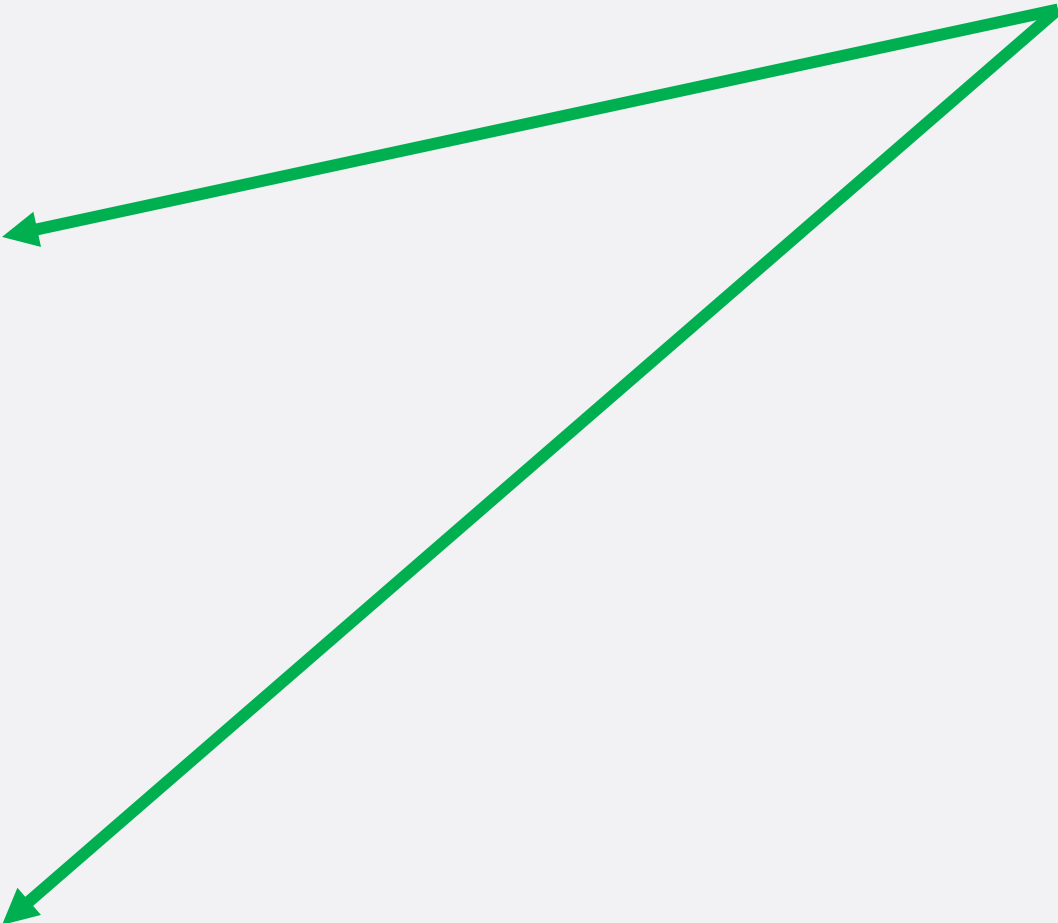
(3)  G

(4)  F ∨ C ∨ ¬A

(5)  ¬A

(6)  ¬C ∨ B

(7)  ¬G ∨ ¬F ∨ E

(8)  F

(9)  ¬F ∨ ¬E ∨ D

(10) ¬F ∨ ¬B ∨ A

Resolution: deduction of a new clause from a pair of clauses with EXACTLY ONE incompatible variable (resolvent)

(11) [(7), (3)] ⇒ (¬F ∨ E)

(12) [(8), (11)] ⇒ (E)

(13) [(12), (2)] ⇒ (C)

(14) [(13), (6)] ⇒ (B)

(15) [(8), (10)] ⇒ (¬B ∨ A)

(16) [(14), (15)] ⇒ (A)

(17) [(5), (16)] ⇒ {}

# Exercise 8.12

Show using the propositional resolution calculus that:
- (1) ¬C ∨ ¬D ∨ E
- (2) F
- (3) ¬D ∨ B
- (4) ¬B ∨ A
- (5) F ∨ E ∨ ¬A
- (6) C ∨ B ∨ ¬G
- (7) ¬C ∨ ¬A ∨ G
- (8) ¬F ∨ ¬C ∨ D
- (9) C
- (10) ¬G

is not satisfiable;

# Exercise 8.12

(1)   ¬C ∨ ¬D ∨ E

(2)    F

(3)   ¬D ∨ B

(4)   ¬B ∨ A

(5)    F ∨ E ∨ ¬A

(6)    C ∨ B ∨ ¬G

(7)  ¬C ∨ ¬A ∨ G

(8)   ¬F ∨ ¬C ∨ D

(9)    C

(10) ¬G


(11) [(8), (2)] ⇒ (¬C ∨ D)

(12) [(9), (11)] ⇒ (D)

(13) [(12), (3)] ⇒ (B)

(14) [(13), (4)] ⇒ (A)

(15) [(9), (7)] ⇒ (¬A ∨ G)

(16) [(14), (15)] ⇒ (G)

(17) [(10), (16)] ⇒ {}

# Exercise 8.8

Using DPLL algorithm to find a model for the following proposition:

- $(A \lor B) \land (C \lor D) \land \neg B$

# Exercise 8.8

Using DPLL algorithm to find a model for the following proposition:

- $(A \lor B) \land (C \lor D) \land \neg B$

- $(A \lor \textbf{B}) \land (C \lor D) \land \neg\textbf{B}$

Propagate: B = False

# Exercise 8.8

Using DPLL algorithm to find a model for the following proposition:

- (A ∨ B) ∧ (C ∨ D) ∧ ¬B

- (A ∨ **B**) ∧ (C ∨ D) ∧ **¬B**

- (**A** ∨ **B**) ∧ (C ∨ D) ∧ **¬B**


Propagate: B = False
Propagate: A = True

# Exercise 8.8

Using DPLL algorithm to find a model for the following proposition:

- (A ∨ B) ∧ (C ∨ D) ∧ ¬B

- (A ∨ **B**) ∧ (C ∨ D) ∧ **¬B**

- (**A** ∨ **B**) ∧ (C ∨ D) ∧ **¬B**

- (**A** ∨ **B**) ∧ (**C** ∨ D) ∧ **¬B**

Propagate: B = False
Propagate: A = True
Decide: C = True

Interpretation: A=True; B=False; C=True

# Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- (¬A ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (A ∨ D)

# Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- (¬A ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (A ∨ D)

- (**¬A** ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (**A** ∨ D)

Decide: A = True

# Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- $(\neg A \lor B) \land (\neg C \lor \neg B) \land (C \lor \neg B) \land (A \lor D)$

- $(\neg A \lor B) \land (\neg C \lor \neg B) \land (C \lor \neg B) \land (A \lor D)$

Decide: A = True
Propagate: B = True

# Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- $(¬A ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (A ∨ D)$

- $(¬A ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (A ∨ D)$

- $(¬A ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (A ∨ D)$

Decide: A = True
Propagate: B = True
Propagate: C = False

# Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- (¬A ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (A ∨ D)

- (¬A ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (A ∨ D)

- (¬A ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (A ∨ D)

Decide: A = True
Propagate: B = True
Propagate: C = False

# Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- (¬A ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (A ∨ D)

- (**¬A** ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (**A** ∨ D)

Decide: A = False

# Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- (¬A ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (A ∨ D)

- (¬**A** ∨ B) ∧ (¬C ∨ ¬B) ∧ (C ∨ ¬B) ∧ (**A** ∨ **D**)
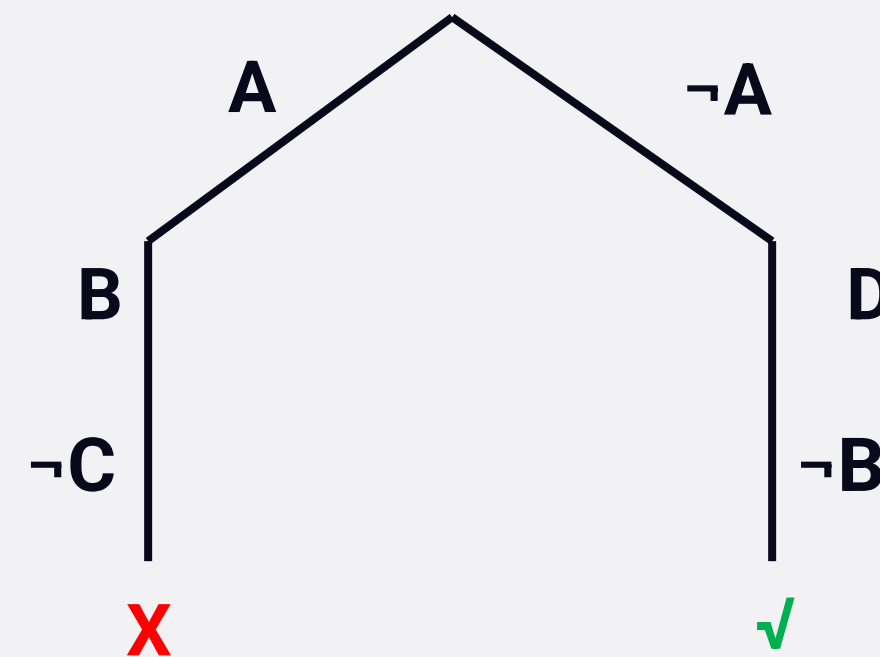
Decide: A = False
Propagate: D = True

# Exercise 8.9

Using DPLL algorithm to find a model for the following proposition:

- $(\neg A \lor B) \land (\neg C \lor \neg B) \land (C \lor \neg B) \land (A \lor D)$

- $(\textcolor{green}{\neg A} \lor \textcolor{red}{B}) \land (\neg C \lor \textcolor{green}{\neg B}) \land (C \lor \textcolor{green}{\neg B}) \land (\textcolor{red}{A} \lor \textcolor{green}{D})$

Decide: A = False
Propagate: D = True
Decide: B = False

Interpretation: A=False; B=False; D=True

# Exercise 8.10

Consider the following CNF formula in PL:

$$
\begin{array}{llll}
(\neg L & ) & & \wedge \\
( M & \vee N & \vee L) & \wedge \\
(\neg N & \vee I & ) & \wedge \\
(\neg H & \vee \neg N & \vee G) & \wedge \\
(\neg H & \vee \neg G & \vee \neg N) & \wedge \\
( F & \vee \neg N & \vee H) & \wedge \\
( H & \vee \neg F & \vee \neg N) & \wedge \\
( N & \vee E & \vee L) & \wedge \\
(\neg E & \vee \neg D & \vee N) & \wedge \\
( D & \vee C & \vee B) & \wedge \\
(\neg C & \vee L & \vee \neg B) & \wedge \\
( N & \vee \neg C & \vee B) & \wedge \\
( C & \vee A & \vee \neg E) & \wedge \\
( D & \vee C & \vee \neg A)
\end{array}
$$

Draw the search tree obtained by applying to the above formula the DPLL algorithm without the pure-symbol rule. Variables should be chosen according to **reverse alphabetic order**, and assigned **true** first.

# Exercise 8.10

¬L

M V  N V  L

¬N V  I

¬H V ¬N V  G

¬H V ¬G V ¬N

F V ¬N V  H

H V ¬F V ¬N

N V  E V  L

¬E V ¬D V  N

D V  C V  B

¬C V  L V ¬B

N V ¬C V  B

C V  A V ¬E

D V  C V ¬A

$$
\begin{array}{llll}
(\neg L & ) & & \wedge \\
( M & \vee N & \vee L) \wedge \\
(\neg N & \vee I & ) \wedge \\
(\neg H & \vee \neg N & \vee G) \wedge \\
(\neg H & \vee \neg G & \vee \neg N) \wedge \\
( F & \vee \neg N & \vee H) \wedge \\
( H & \vee \neg F & \vee \neg N) \wedge \\
( N & \vee E & \vee L) \wedge \\
(\neg E & \vee \neg D & \vee N) \wedge \\
( D & \vee C & \vee B) \wedge \\
(\neg C & \vee L & \vee \neg B) \wedge \\
( N & \vee \neg C & \vee B) \wedge \\
( C & \vee A & \vee \neg E) \wedge \\
( D & \vee C & \vee \neg A)
\end{array}
$$

# Exercise 8.13

Consider the following CNF formula in PL:

$$
\begin{aligned}
&(\neg C\ ) \ \wedge \\
&(\ B \ \vee \ A \ \vee \ C) \wedge \\
&(\neg A \ \vee \ D \ ) \wedge \\
&(\neg E \ \vee \neg A \ \vee \ F) \wedge \\
&(\neg E \ \vee \neg F \ \vee \neg A) \wedge \\
&(\ G \ \vee \neg A \ \vee \ E) \wedge \\
&(\ E \ \vee \neg G \ \vee \neg A) \wedge \\
&(\ A \ \vee \ H \ \vee \ C) \wedge \\
&(\neg H \ \vee \neg I \ \vee \ A) \wedge \\
&(\ I \ \vee \ L \ \vee \ M) \wedge \\
&(\neg L \ \vee \ C \ \vee \neg M) \wedge \\
&(\ A \ \vee \neg L \ \vee \ M) \wedge \\
&(\ L \ \vee \ N \ \vee \neg H) \wedge \\
&(\ I \ \vee \ L \ \vee \neg N)
\end{aligned}
$$

Draw the search tree obtained by applying to the above formula the DPLL algorithm without the pure-symbol rule. Variables should be chosen according to **alphabetic order**, and assigned **true** first.

# Exercise 8.13

¬C

B V A V C

¬A V D

¬E V ¬A V F

¬E V ¬F V ¬A

G V ¬A V E

E V ¬G V ¬A

A V H V C

¬H V ¬I V A

I V L V M

¬L V C V ¬M

A V ¬L V M

L V N V ¬H

I V L V ¬N

# Exercise 8.13

¬C

B ∨ A ∨ C

¬A ∨ D

¬E ∨ ¬A ∨ F

¬E ∨ ¬F ∨ ¬A

G ∨ ¬A ∨ E

E ∨ ¬G ∨ ¬A

A ∨ H ∨ C

¬H ∨ ¬I ∨ A

I ∨ L ∨ M

¬L ∨ C ∨ ¬M

A ∨ ¬L ∨ M

L ∨ N ∨ ¬H

I ∨ L ∨ ¬N
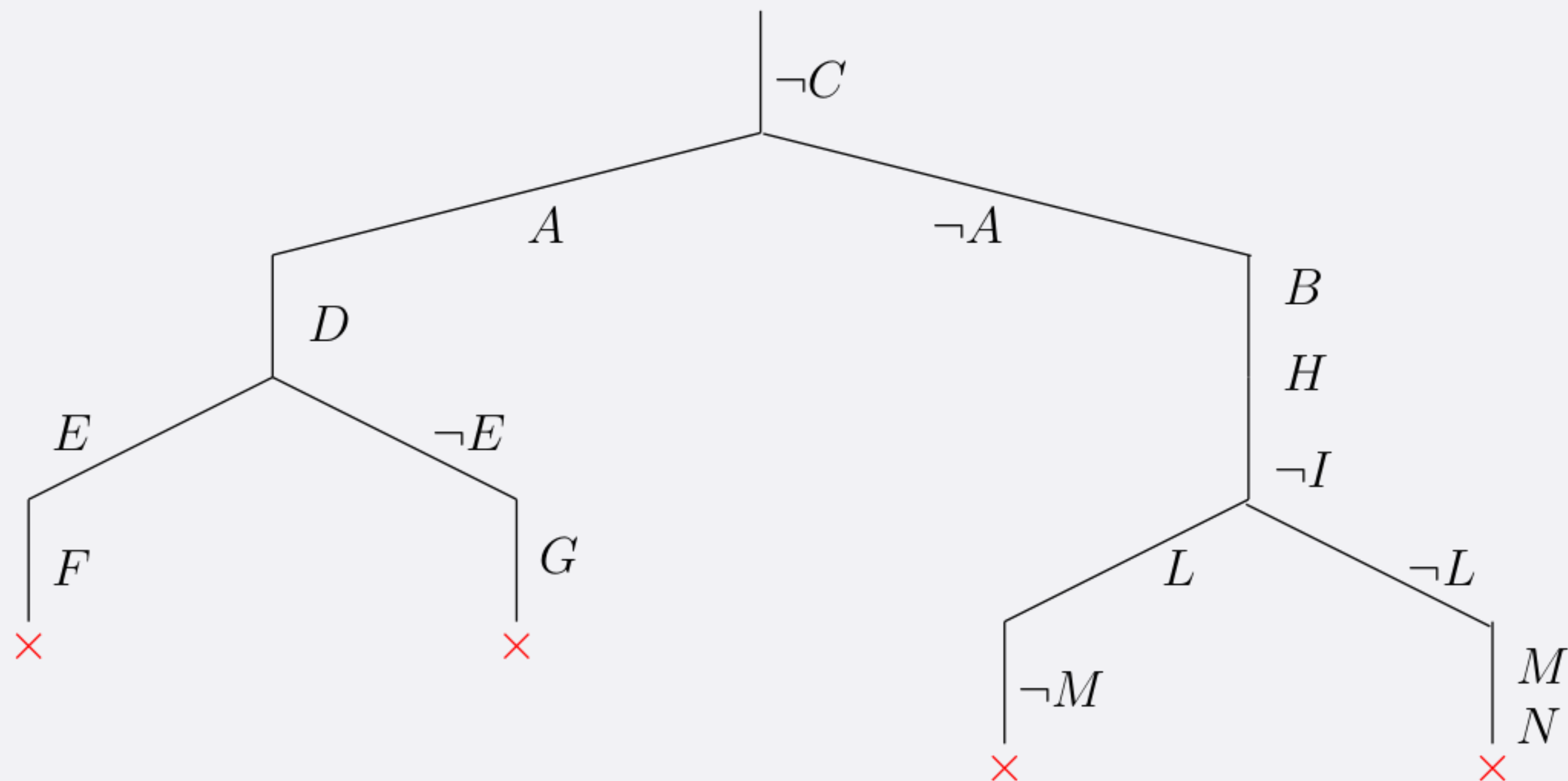
# Exercise 8.11 - Homework

Prove each of the following assertions:

a.  α is valid if and only if True |= α. Forward: If alpha is valid it is true in all models, hence it is true in all models of True. Backward: if True |= α then α must be true in all models of True, i.e., in all models, hence α must be valid.

b.  For any α, False |= α. False doesn't hold in any model, so α trivially holds in every model of False.

c.  α |= β if and only if the sentence (α ⇒ β) is valid. Both sides are equivalent to the assertion that there is no model in which α is true and β is false, i.e., no model in which α ⇒ β is false.

d.  α ≡ β if and only if the sentence (α ⇔ β) is valid. Both sides are equivalent to the assertion that α and β have the same truth value in every model.

a.  α |= β if and only if the sentence (α ∧ ¬β) is unsatisfiable. As in c, both sides are equivalent to the assertion that there is no model in which α is true and β is false.