# Course "Automated Planning: Theory and Practice"
# Chapter 08: Heuristics: An Overview

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# Heuristic Search (repetition)

```
function SEARCH(problem)
    initial-node ← MAKE-INITIAL-NODE(problem)
    open ← {initial-node}
    while (open ≠ ∅) do
        node ← SEARCH-STRATEGY-REMOVE-FROM(open)
        if IS-SOLUTION(node) then
            return EXTRACT-PLAN-FROM(node)
        end if
        for each newnode ∈ SUCCESSORS(node) do
            open ← open ∪ {newnode}
        end for
    end while
    return Failure
end function
```

An *heuristic strategy* bases decisions on:
$\Longrightarrow$ Heuristic value $h(n)$
$\Longrightarrow$ Often other factors e.g. $g(n)$ i.e. the cost of reaching $n$

Best first search: Greedy, A*, ...
Modifications: IDA*, D*, ...
Simulated annealing, hill climbing, ...

Requires an heuristic function!

How do we *calculate* $h(n)$?

Landmarks,
Pattern databases,
Relaxed plan graph,
...

# Example

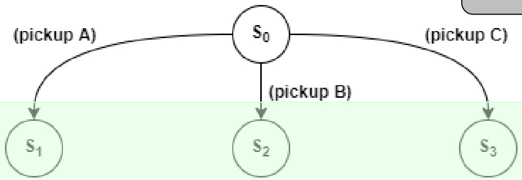
## 3 blocks, all on the table in $S_0$


We now have
1 open node,
which is *unexpanded*

# Example (cont.)

## We visit $S_0$ and we expand it!

Forward search: node ≈ state
so we may write $h(n)$ or $h(s)$

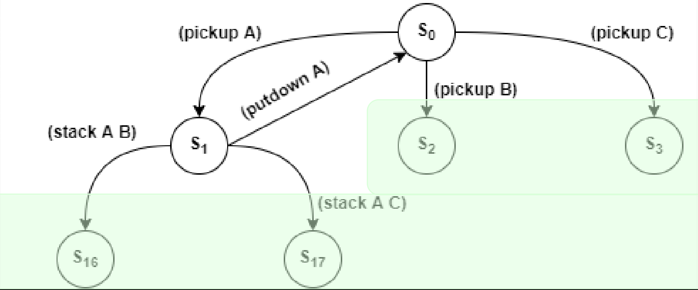



We now have
3 open nodes,
which are *unexpanded*

An heuristic function estimates the distance from each open node to the goal:
We calculate $h(S_1)$, $h(S_2)$, $h(S_3)$
An heuristic strategy uses this value (and other info) to prioritize the search

# Example (cont.)

## Suppose the strategy chooses to visit $S_1$!




We now have
4 open nodes,
which are *unexpanded*

2 new heuristic values are calculated $h(S_{16})$, $h(S_{17})$
The search strategy now has 4 nodes to prioritize

# What to measure?

## Question 1A: What should an heuristic function measure?

- An heuristic strategy bases its decisions on:
  - Heuristic value $h(s)$
  - Other factors: e.g. $g(s)$ i.e. cost of reaching $s$

- A very general definition!
  - $\Longrightarrow$ could measure anything that some strategy might find useful!

## Question 1B: What is "cost"?

- Often: $h(s)$ *tries* to approximate the cost of achieving the goal from $s$!
  - Useful for finding cheap plans, and often as a side effect, for finding plans cheaply!
  - But... What is "cost"?

# Plan Quality and Action Costs

- Maybe: long plan = expensive plan
  - $c(\pi) = |\pi|$, i.e. number of actions in plan $\pi$
    - Reasonable in some domains: e.g. Tower of Hanoi
    - But: How to make sure your car is clean?

go to car wash | get supplies | wash car

go to car dealer | buy new car | shortest plan is best?

Heuristic $h(s)$ estimates:
"How many actions are needed
to reach the goal from $s$"

- Would prefer to support different action costs
  - Supported by most current planners
    - Each action $a \in A$ is associated with a cost $c(a)$
  - Total cost: $c(\pi) = \sum_{a \in \pi} c(a)$

Heuristic $h(s)$ estimates:
"How *expensive* actions are needed
to reach the goal from $s$"

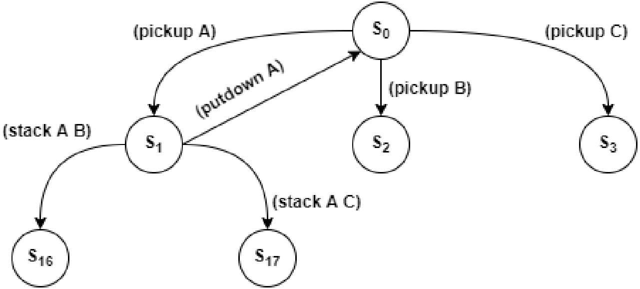
# Action Costs in PDDL

- PDDL: Specify requirements:
  - `(:requirements :action-costs)`
- Numeric state variables for the total cost, called `(total-cost)`
  - And possibly numeric variables to *calculate* action costs
  - ```
    (:functions (total-cost)
                (travel-slow-cost ?f1 - count ?f2 - count)
                (travel-fast-cost ?f1 - count ?f2 - count))
    ```
- Initial state
  - ```
    (:init (= (total-cost) 0)
           (= (travel-slow-cost n0 n1) 6) (= (travel-slow-cost n0 n2) 7) ...
           (= (travel-fast-cost n0 n1) 8) (= (travel-fast-cost n0 n2) 9) ...
           ...)
    ```
- Special increase effects to increase total cost
  - ```
    (:action move-up-slow
      :parameters (?l - slow-elevator ?f1 - count ?f2 - count)
      :precondition (and (lift-at ?l ?f1) (above ?f1 ?f2) (reachable-floor ?l ?f2))
      :effect (and (lift-at ?l ?f2) (not (lift-at ?l ?f1))
                   (increase (total-cost) (travel-slow-cost ?f1 ?f2))) )
    ```
    ```

# Remaining Costs

- The remaining cost in any search state $s$
  - The cost of a cheapest (optimal) solution starting in $s$
  - Denoted by $h^*(s)$
  - Star $* \Longrightarrow$ the best, optimal, estimate: *exact* cost

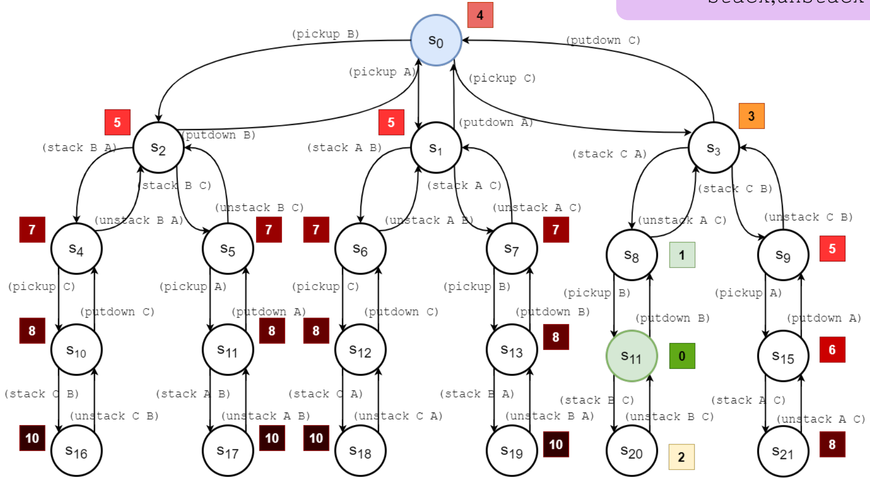
- The cost of an optimal solution to $(\Sigma, S_0, S_g)$
  - $h^*(S_0)$



# True Remaining Costs

- True cost of reaching a goal node from $n$: $h^*(n)$

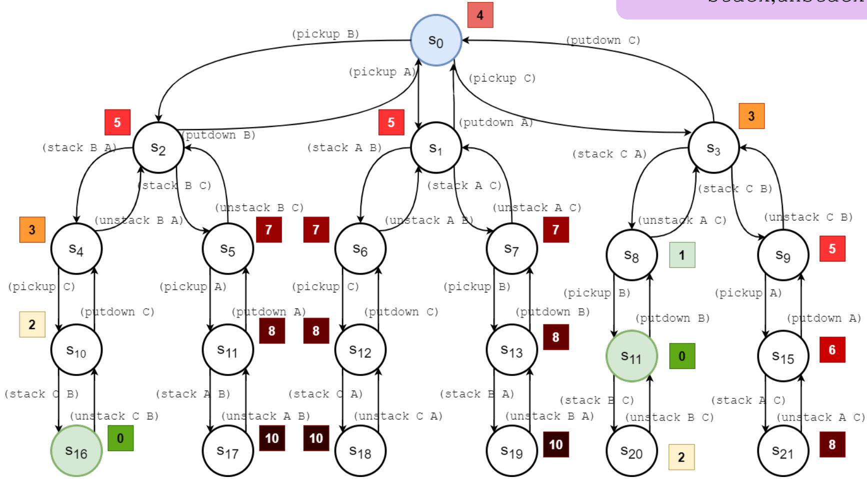
Initially A,B,C on the table
`pickup`,`putdown` cost 1
`stack`,`unstack` cost 2



# True Remaining Costs (cont.)

- True cost of reaching a goal node from $n$: $h^*(n)$

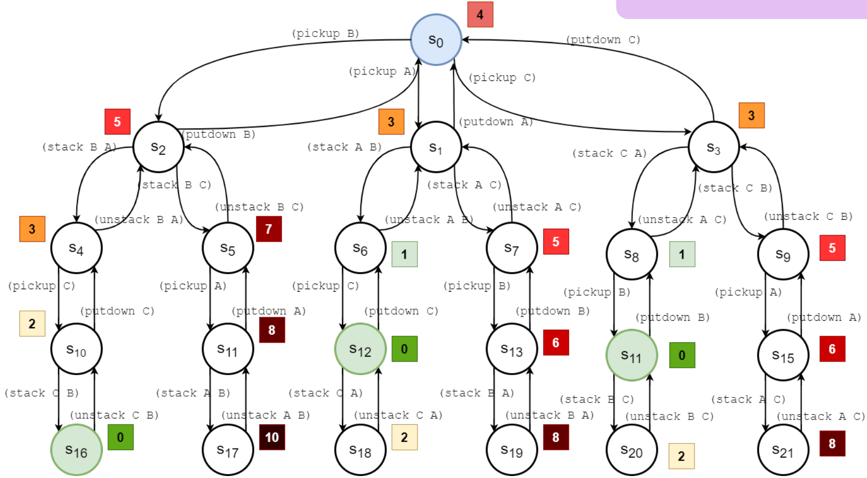
Two reachable goal nodes
pickup,putdown cost 1
stack,unstack cost 2



# True Remaining Costs (cont.)

- True cost of reaching a goal node from $n$: $h^*(n)$

Three reachable goal nodes
(there can be many)

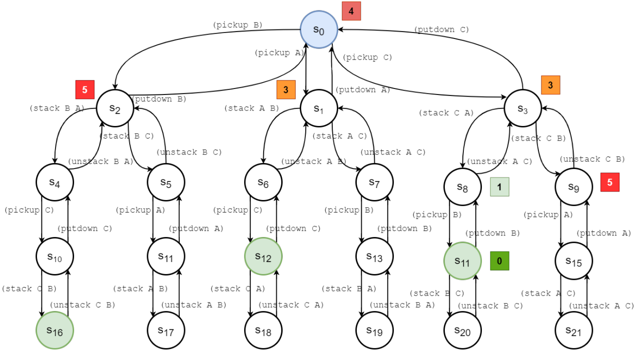


# True Remaining Costs (cont.)

- If we *knew* the true remaining cost $h^*(n)$ for every node:

**function** ALGORITHM SIMPLEPLAN(problem)
    initial-node ← MAKE-INITIAL-NODE(problem)
    **while** (not reached goal) **do**
        node ← A-SUCCESSOR-NODE-MINIMAL-H*(node)
    **end while**
**end function**

Trivial straight-line path minimizing $h^*$ values gives an *optimal* solution!



# Reflections

- What does this mean?
  - Calculating $h^*(n)$ is a good idea, because then we can easily find optimal plans?

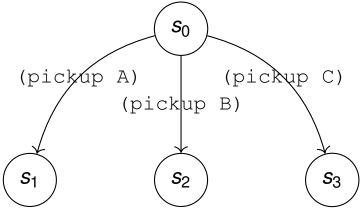
- No!!! - because we can prove that finding optimal plans is hard!
  - So the hard part must be calculating $h^*(n)$ ...

1. We can always quickly compute $h^*(n)$
2. Given $h^*(n)$ we can quickly find optimal solutions
3. We can quickly find optimal solutions for any classical planning problem
4. Known to be false! (PSPACE-complete)!
5. ... so one of these premises must be false!

- Must settle for an estimate that helps us search less than otherwise!

# Minimization: Introduction

- Example strategy: *depth-first search*: select a child with minimal $h(s)$

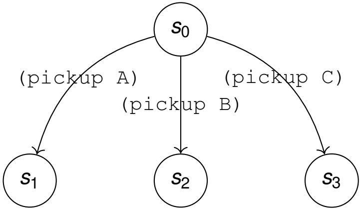



$h^*(s_1) = 55$ $h^*(s_2) = 57$ $h^*(s_3) = 62$

- If I start with `(pickup A)`, then make optimal choices:
  - Plan cost = 55
- If I start with `(pickup C)`, then make optimal choices:
  - Plan cost = 62

# Minimization: Case 1

- Example strategy: *depth-first search*: select a child with minimal $h(s)$

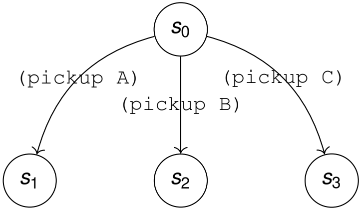



| | | | |
|---|---|---|---|
| $h^*(s_1) = 55$ | $h^*(s_2) = 57$ | $h^*(s_3) = 62$ | |
| $h^A(s_1) = 50$ | $h^A(s_2) = 53$ | $h^A(s_3) = 55$ | Close! |
| $h^B(s_1) = 4$ | $h^B(s_2) = 20$ | $h^B(s_3) = 21$ | Far from truth... |

- Which is best?
  - The strategy only cares about relative values!
    - $h^*$, $h^A$, and $h^B$ all results in identical choices: $s_1$ first!

# Minimization: Case 2

- Example strategy: *depth-first search*: select a child with minimal $h(s)$

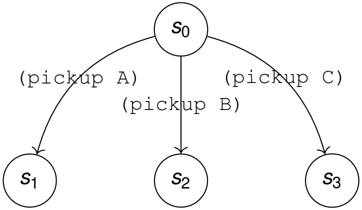



| | | | |
|---|---|---|---|
| $h^*(s_1) = 55$ | $h^*(s_2) = 57$ | $h^*(s_3) = 62$ | |
| $h^A(s_1) = 50$ | $h^A(s_2) = 53$ | $h^A(s_3) = 55$ | Close! |
| $h^B(s_1) = 107$ | $h^B(s_2) = 258$ | $h^B(s_3) = 522$ | Large overestimate! |

- Which is best?
  - The strategy only cares about relative values!
    - $h^*$, $h^A$, and $h^B$ all results in identical choices: $s_1$ first!

# Minimization: Case 3

- Example strategy: *depth-first search*: select a child with minimal $h(s)$

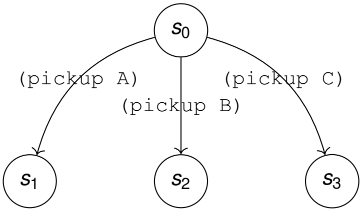



| | | | |
|---|---|---|---|
| $h^*(s_1) = 55$ | $h^*(s_2) = 57$ | $h^*(s_3) = 62$ | |
| $h^A(s_1) = 54$ | $h^A(s_2) = 53$ | $h^A(s_3) = 47$ | Close! |
| $h^B(s_1) = 4$ | $h^B(s_2) = 20$ | $h^B(s_3) = 21$ | Far from truth... |

- Which is best?
  - The strategy only cares about relative values!
    - $h^A$ is worse for this strategy, despite being closer to $h^*$: goes to $s_3$ first!
    - Even if we continue optimally, cost $\geq 62$!

# A*: Case 1

- Example strategy: $A*$

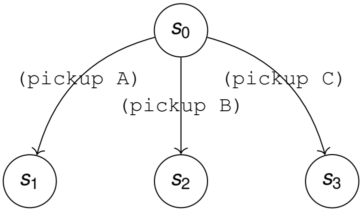



| | | |
|---|---|---|
| $h^*(s_1) = 55$ | $h^*(s_2) = 57$ | $h^*(s_3) = 62$ |
| $h^A(s_1) = 50$ | $h^A(s_2) = 53$ | $h^A(s_3) = 55$ |
| $h^B(s_1) = 4$ | $h^B(s_2) = 20$ | $h^B(s_3) = 21$ |

- Which is best?
  - A* expands all nodes where $h(s) + g(s) \leq h^*(s)$
    - As long as $h$ is admissible [$\forall s.h(s) \leq h^*(s)$], increasing it is always better

# A*: CASE 2

- Example strategy: *A**

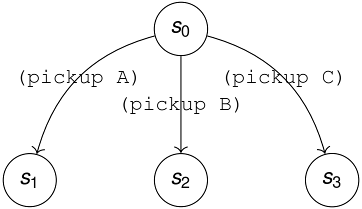



| | | |
|---|---|---|
| $h^*(s_1) = 55$ | $h^*(s_2) = 57$ | $h^*(s_3) = 62$ |
| $h^A(s_1) = 50$ | $h^A(s_2) = 53$ | $h^A(s_3) = 55$ |
| $h^B(s_1) = 107$ | $h^B(s_2) = 258$ | $h^B(s_3) = 522$ |

- Which is best?
  - A* expands all nodes where $h(s) + g(s) \leq h^*(s)$
    - Because $h^B$ is not admissible, optimal solutions may be missed!

# A*: CASE 3

- Example strategy: $A*$




| | | |
|---|---|---|
| $h^*(s_1) = 55$ | $h^*(s_2) = 57$ | $h^*(s_3) = 62$ |
| $h^A(s_1) = 54$ | $h^A(s_2) = 53$ | $h^A(s_3) = 47$ |
| $h^B(s_1) = 4$ | $h^B(s_2) = 20$ | $h^B(s_3) = 21$ |

- Which is best?
  - A* expands all nodes where $h(s) + g(s) \leq h^*(s)$
    - As long as $h(s)$ is admissible $[\forall s. h(s) \leq h^*(s)]$, increasing is always better: $\Longrightarrow h^A$ better then $h^B$

# Two requirements for Heuristic Guidance

## Define search strategy able to take guidance into account

- Example:
  - A* uses a heuristic function
  - Hill-climbing uses a heuristic … differently!
  - …

## Find a heuristic function suitable for the selected strategy

- Example:
  - Find an heuristic function suitable specifically for A*
  - Find an heuristic function suitable specifically for hill-climbing
  - …

- Can be domain specific, given as input to the planning problem!
- Can be domain independent, generated automatically by the planner given the problem domain!

We will consider both – heuristics more than strategies!

# Some Desired Properties

- What properties do good heuristic functions have?
  - Shall be Informative: provide good guidance to the specific search strategy we use
    - Admissible?
    - Close to $h^*(n)$?
    - Correct "ordering"?
    - ...

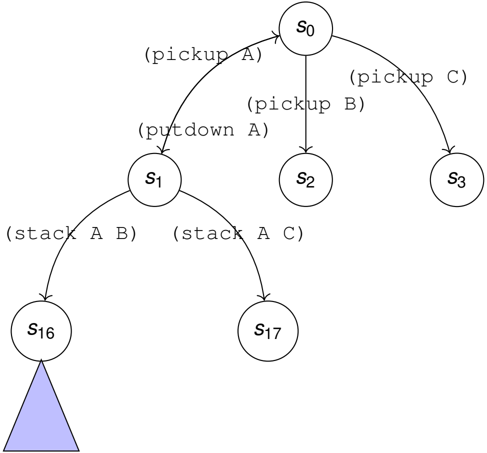
# Some Desired Properties (cont.)

- What properties do good heuristic functions have?
  - Shall be efficiently computable
    - Spend as little time as possible deciding which nodes to expand
  - Shall be balanced...
    - Many planners spend almost all their time calculating heuristics
    - But: Don't spend more time computing $h$ than the time you gain by expanding few nodes!
    - Illustrative (made-up) example:

| Heuristic quality | Nodes expanded | Exp. 1 node | Calc. $h$ one node | Total time |
|---|---|---|---|---|
| Worst | 100000 | 100$\mu s$ | 1 $\mu s$ | 10100ms |
| Better | 20000 | 100$\mu s$ | 10 $\mu s$ | 2200ms |
| ... | 5000 | 100$\mu s$ | 100 $\mu s$ | 1000ms |
| ... | 2000 | 100$\mu s$ | 1000 $\mu s$ | 2200ms |
| ... | 500 | 100$\mu s$ | 10000 $\mu s$ | 5050ms |
| Best | 200 | 100$\mu s$ | 100000$\mu s$ | 20020ms |

# Cheap Plans, Cheap Planning

- Cost can be indirectly related to plan generation time!




- If we can find a cheap plan "under" $s_{16}$
  - $\Longrightarrow$ might find a plan in few steps
  - $\Longrightarrow$ might not need to search so many nodes
  - $\Longrightarrow$ might find a plan cheaply
- Maybe!
  - Or maybe $s_{16}$ opens up a vast number of alternatives, so finding a solution may take more time...

# Prioritizing Speed or Plan Cost

- Can design strategies to prioritize speed or plan cost

### Find a solution quickly

Expand nodes where you think you can easily find a way to a goal node

### Find a good solution

Expand nodes where you think you can find a way to a good (high-quality) solution, even if finding it will be difficult!

Should prefer

Should prefer

Open nodes

Accumulated plan cost $g(n) = 50$, estimated "cost distance" $h(n) = 10$

Accumulated plan cost $g(n) = 5$, estimated "cost distance" $h(n) = 30$

Often one strategy+heuristic can achieve *both* reasonably well, but for optimum performance, the distinction can be important!

# REFERENCES I


[1] Hector Geffner and Blai Bonet. *A Concise Introduction to Models and Methods for Automated Planning*. Synthesis Lectures on Artificial Intelligence and Machine Learning. Morgan & Claypool Publishers, 2013. ISBN 9781608459698. doi: 10.2200/S00513ED1V01Y201306AIM022. URL `https://doi.org/10.2200/S00513ED1V01Y201306AIM022`.

[2] Malik Ghallab, Dana S. Nau, and Paolo Traverso. *Automated planning - theory and practice*. Elsevier, 2004. ISBN 978-1-55860-856-6.

[3] Malik Ghallab, Dana S. Nau, and Paolo Traverso. *Automated Planning and Acting*. Cambridge University Press, 2016. ISBN 978-1-107-03727-4. URL `http://www.cambridge.org/de/academic/subjects/computer-science/artificial-intelligence-and-natural-language-processing/automated-planning-and-acting?format=HB`.

[4] Patrik Haslum, Nir Lipovetzky, Daniele Magazzeni, and Christian Muise. *An Introduction to the Planning Domain Definition Language*. Synthesis Lectures on Artificial Intelligence and Machine Learning. Morgan & Claypool Publishers, 2019. doi: 10.2200/S00900ED2V01Y201902AIM042. URL `https://doi.org/10.2200/S00900ED2V01Y201902AIM042`.