Course "Automated Planning: Theory and Practice"
Chapter 12: Delete Relaxation

Teacher: Marco Roveri - marco.roveri@unitn.it
M.S. Course: Artificial Intelligence Systems (LM)
A.A.: 2025-2026
Where: DISI, University of Trento
URL: https://shorturl.at/A81hf

Last updated: Wednesday 29th October, 2025

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Re-Achieving Conditions

- To make actions applicable and achieve goals:
  - We often have to re-achieve what was already achieved
  - Example: Driving
    - Initial state: { `(at A) (have-fuel)` }
    - Goal: { `(at D) (have-fuel)` }
    - Actions: `(drive ?x ?y)` - must be in ?x, must follow road from ?x to ?y, must `(have-fuel)`, consume fuel, is no longer in ?x, it is in ?y!
      `(refuel)` - must have no fuel, it make `(have-fuel)` true!
    - Solution:
      ```
      (drive A B)
      (refuel)
      (drive B C)
      (refuel)
      (drive C D)
      (refuel)
      ```

Re-Achieving Conditions (cont.)

- *Suppose* conditions always remained achieved
  - If `(have-fuel)` is true, it always remains true
  - New Solution:
    ```
    (drive A B)
    (drive B C)
    (drive C D)
    ```

Can we exploit this observation to construct a relaxation?

Positive and Negative Effects

- Let's consider the classical representation used in Ghallab et al. [2]:
  - Precondition = set of literals that must be true
  - Goal = set of literals that must be true
  - Effects = set of literals (making atoms true or false)

- Suppose we have a solution `<A1,A2>`:
  - Initially `(have-fuel)`
  - Action `drive` $\Longrightarrow$ requires `(have-fuel)`, makes `(have-fuel)` false
  - Action `refuel` $\Longrightarrow$ requires `(not (have-fuel))`, makes `(have-fuel)` true

- Symmetry
  - Positive effects can *achieve* positive conditions, *un-achieve* negative conditions
  - Negative effects can *achieve* negative conditions, *un-achieve* positive conditions

Positive and Negative Effects (cont.)

- Let's consider the PDDL's plain `:strips` level
  - Forbids negative preconditions/goals
    - Precondition = set of atoms (no negations!)
    - Goal = set of atoms (no negations!)
    - Effects = set of literals (making atoms true or false)
  - In this setting:
    - Positive effects are never "problematic":
      Adding more facts to the state can only make *more* preconds/goals satisfied
    - Only negative effects can "un-achieve" goals or preconditions
    - And negative effects can only "un-achieve" goals or preconditions:
      We never *need* them

Delete Relaxation

- Assuming positive conditions, let's remove all negative effects
  - Example: `(unstack ?x ?y)`
    - Before transformation:

```
:precondition (and (handempy) (clear ?x) (on ?x ?y))
:effect       (and (not (handempy)) (holding ?x) (not (clear ?x))
                   (clear ?y) (not (on ?x ?y)))
```

    - After transformation:

```
:precondition (and (handempy) (clear ?x) (on ?x ?y))
:effect       (and (holding ?x)
                   (clear ?y))
```

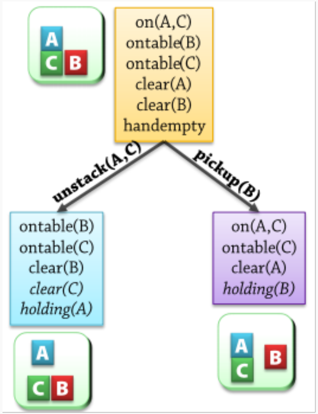
  - A fact that is true stays true

Is this a relaxation?

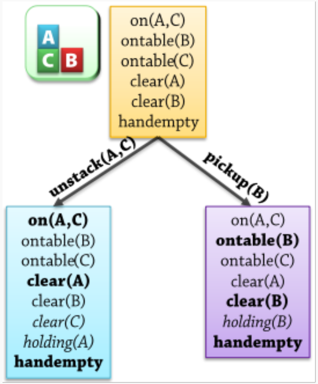
  - Positive conditions $\Longrightarrow$
    - No solution can *depend on a fact being false* in a visited state
    - No solution can *disappear* because we avoid making facts false

Delete Relaxation: Example

STS for the original problem



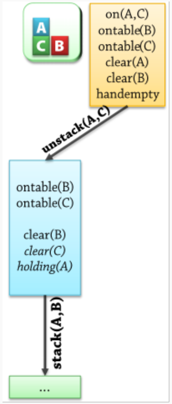
Delete-relaxed STRIPS problem



No physical "meaning"!

Delete Relaxation: Example (cont.)

STS for the original problem



Initial state does not change

$=$

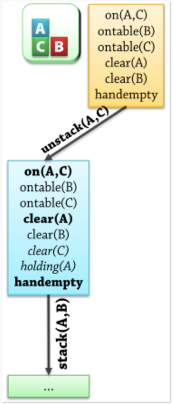
Same "origin", fewer facts removed

$\subseteq$

Different "origin" but same *action sequence*, fewer facts removed

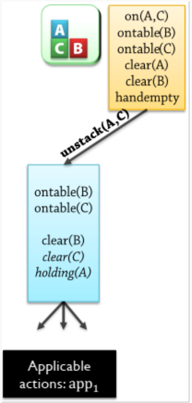
$\subseteq$

Delete-relaxed STRIPS problem



Delete Relaxation: Example (cont.)

STS for the original problem

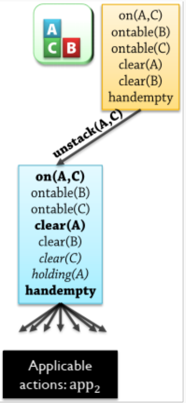


$\subseteq$

No action requires the *absence* of a fact

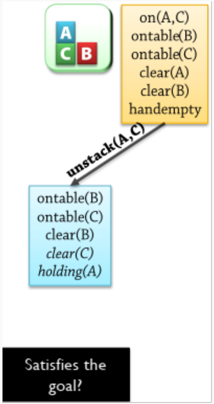
$\subseteq$

Delete-relaxed STRIPS problem



DELETE RELAXATION: EXAMPLE (CONT.)

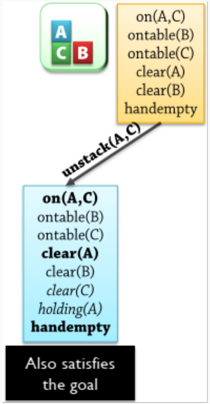
STS FOR THE ORIGINAL PROBLEM



No goal requires the *absence* of a fact

$\Longrightarrow$

DELETE-RELAXED STRIPS PROBLEM



Delete Relaxation

- Negative effects are also called "delete effects"
  - They delete facts from the state
- So this is called delete relaxation
  - "*Relaxing* the problem by getting rid of the *delete effects*"
- "Relaxed plan for P" = plan for the delete-relaxed version of P

Delete relaxation does not mean that we "delete the relaxation" (anti-relax)!

Delete relaxation is only a relaxation if preconditions and goals are positive!

Delete Relaxation (cont.)

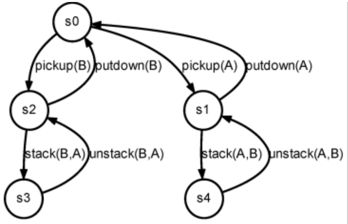
- Since solutions are preserved when action are added:

A state where additional facts are true can never be "worse"!
(Given positive preconds/goals)

$$h^*\left(\begin{array}{l}\text{ontable(B)}\\ \text{ontable(C)}\\ \text{clear(B)}\\ \text{clear(C)}\\ \text{holding(A)}\\ \textbf{handempty}\end{array}\right) \leq h^*\left(\begin{array}{l}\text{ontable(B)}\\ \text{ontable(C)}\\ \text{clear(B)}\\ \text{clear(C)}\\ \text{holding(A)}\end{array}\right)$$

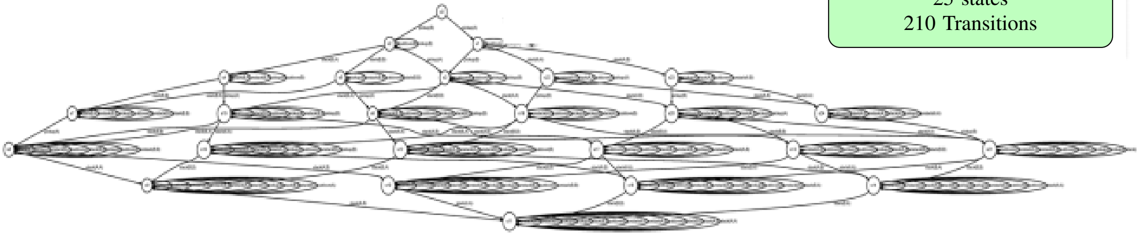
Given two states (sets of true atoms) s_1, s_2:
$s_2 \subset s_1 \rightarrow h^*(s_2) \geq h^*(s_1)$

Reachable State Space: BW size 2



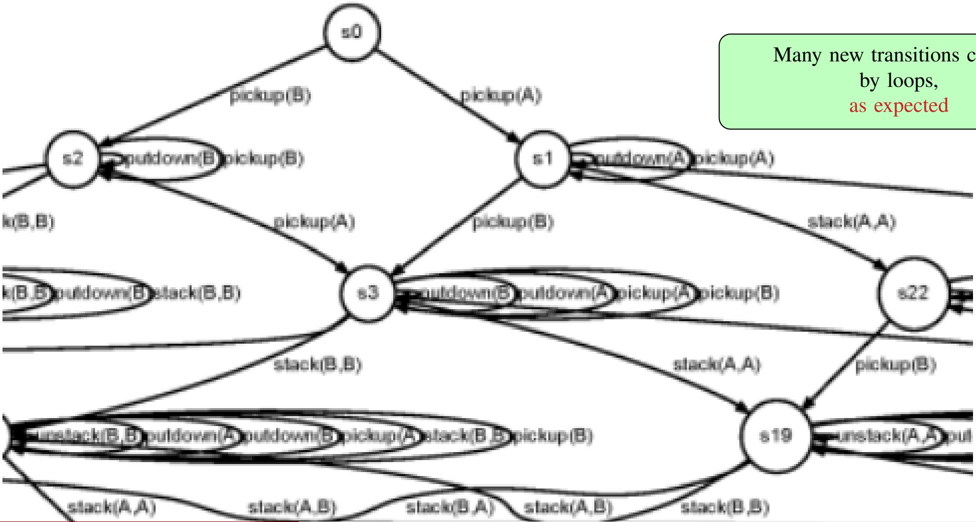
5 states
8 Transitions

$\Downarrow$



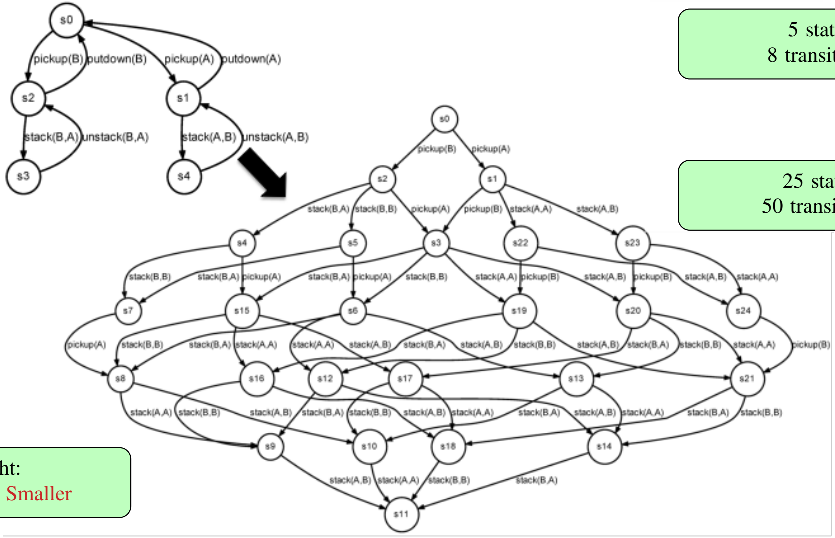
25 states
210 Transitions

Reachable State Space: BW size 2 - Detailed view



Many new transitions caused
by loops,
as expected

Delete Relaxed: "Loops" Removed



5 states
8 transitions

25 states
50 transitions

Insight:
Relaxed $\neq$ Smaller

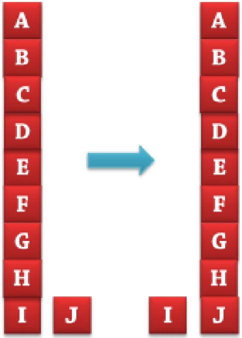
Optimal Delete Relaxation Heuristic

- If only delete relaxation is applied:
  - We can calculate the optimal delete relaxation heuristic, $h^{+}(n)$
  - $h^{+}(n) =$ the cost of an optimal solution
    to a delete-relaxed problem
    starting in node n

Accuracy of h^+ in Selected Domains

- How close is $h^+(n)$ to the true goal distance $h^*(n)$?
  - Worst case asymptotic accuracy as problem size approaches infinity:
    - Blocks world: $\frac{1}{4} \Longrightarrow h^+(n) \geq \frac{1}{4} h^*(n)$

Optimal plans in delete-relaxed Blocks World can be down to 25% of the length of optimal plans in "real" Blocks World and goals are positive!



Standard:

unstack(A,B)	pickup(G)
putdown(A)	stack(G,H)
unstack(B,C)	pickup(F)
putdown(B)	stack(F,G)
unstack(C,D)	pickup(E)
putdown(C)	stack(E,F)
...	pickup(D)
unstack(H,I)	stack(D,E)
stack(H,J)	...

Relaxed:

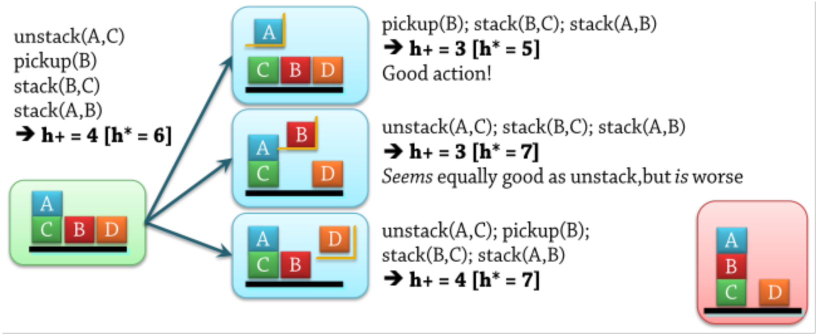
unstack(A,B)
unstack(B,C)
unstack(C,D)
unstack(D,E)
unstack(E,F)
unstack(F,G)
unstack(G,H)
unstack(H,I)
stack(H,J)
DONE!

ACCURACY OF h^+ IN SELECTED DOMAINS

- How close is $h^+(n)$ to the true goal distance $h^*(n)$?
  - Worst case asymptotic accuracy as problem size approaches infinity:
    - Blocks world: $\frac{1}{4}$ $\implies h^+(n) \geq \frac{1}{4} h^*(n)$
    - Gripper domain: $\frac{2}{3}$ (single robot moving balls)
    - Logistics domain: $\frac{3}{4}$ (move packages using trucks, airplanes)
    - Miconic STRIPS: $\frac{6}{7}$ (elevators)
    - Miconic-Simple-ADL: $\frac{3}{4}$ (elevators)
    - Schedule: $\frac{1}{4}$ (job shop scheduling)
    - Satellite: $\frac{1}{2}$ (satellite observations)
  - Details:
    - Malte Helmert and Robert Mattmüller
      Accuracy of Admissible Heuristic Functions in Selected Planning Domains [5]

Example of Accuracy

- How close is $h^+(n)$ to the true goal distance $h^*(n)$?
  - In practice: Also depends on the problem instance!



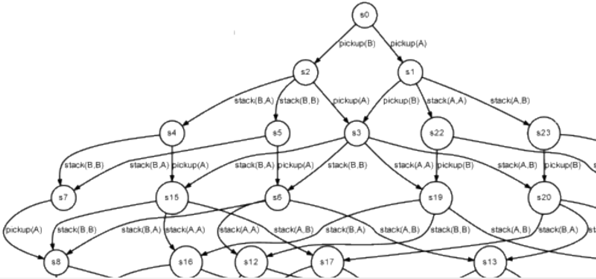
  - Performance also depends on the search strategy
    - How sensitive it is to specific types of inaccuracy

Computing h^+

- Is h^+ easier to compute than h^*?
  - h^* = length of optimal plan for arbitrary planning problem
    - Supports negative effects
    - If we can execute either `a1;a2` or `a2;a1`:
      `a1` removes p, `a2` adds $p \Longrightarrow$ net result: add p
      `a2` adds p, `a1` removes $p \Longrightarrow$ net result: remove p
      Both orders must be considered
  - h^+ = length of optimal plan after removing negative effects
    - If we can execute either `a1;a2` or `a2;a1`:
      Must lead to the same state (add `a1` before `a2`, or `a2` before `a1`)
      Sufficient to consider one order - simpler?
  - Incomplete analysis
    - But the worst case for h^+ is easier than the worst case for h^*

COMPUTING h^+ (CONT.)

- Still difficult to calculate in general!
  - NP-equivalent (reduced from PSPACE-equivalent)
    - Since you must find optimal solutions to the relaxed problem
  - Even a constant-factor approximation is NP-equivalent to compute!
    - Finding $h(n)$ so that $\forall n . h(n) \geq c \cdot h^+(n)$
- Therefore, rarely used "as is"
  - But forms the basis of many other heuristics



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[1] Hector Geffner and Blai Bonet. *A Concise Introduction to Models and Methods for Automated Planning*. Synthesis Lectures on Artificial Intelligence and Machine Learning. Morgan & Claypool Publishers, 2013. ISBN 9781608459698. doi: 10.2200/S00513ED1V01Y201306AIM022. URL `https://doi.org/10.2200/S00513ED1V01Y201306AIM022`.

[2] Malik Ghallab, Dana S. Nau, and Paolo Traverso. *Automated planning - theory and practice*. Elsevier, 2004. ISBN 978-1-55860-856-6. 5

[3] Malik Ghallab, Dana S. Nau, and Paolo Traverso. *Automated Planning and Acting*. Cambridge University Press, 2016. ISBN 978-1-107-03727-4. URL `http://www.cambridge.org/de/academic/subjects/computer-science/artificial-intelligence-and-natural-language-processing/automated-planning-and-acting?format=HB`.

[4] Patrik Haslum, Nir Lipovetzky, Daniele Magazzeni, and Christian Muise. *An Introduction to the Planning Domain Definition Language*. Synthesis Lectures on Artificial Intelligence and Machine Learning. Morgan & Claypool Publishers, 2019. doi: 10.2200/S00900ED2V01Y201902AIM042. URL `https://doi.org/10.2200/S00900ED2V01Y201902AIM042`.

[5] Malte Helmert and Robert Mattmüller. Accuracy of admissible heuristic functions in selected planning domains. In Dieter Fox and Carla P. Gomes, editors, *Proceedings of the Twenty-Third AAAI Conference on Artificial Intelligence, AAAI 2008, Chicago, Illinois, USA, July 13-17, 2008*, pages 938–943. AAAI Press, 2008. URL `http://www.aaai.org/Library/AAAI/2008/aaai08-149.php`. 19