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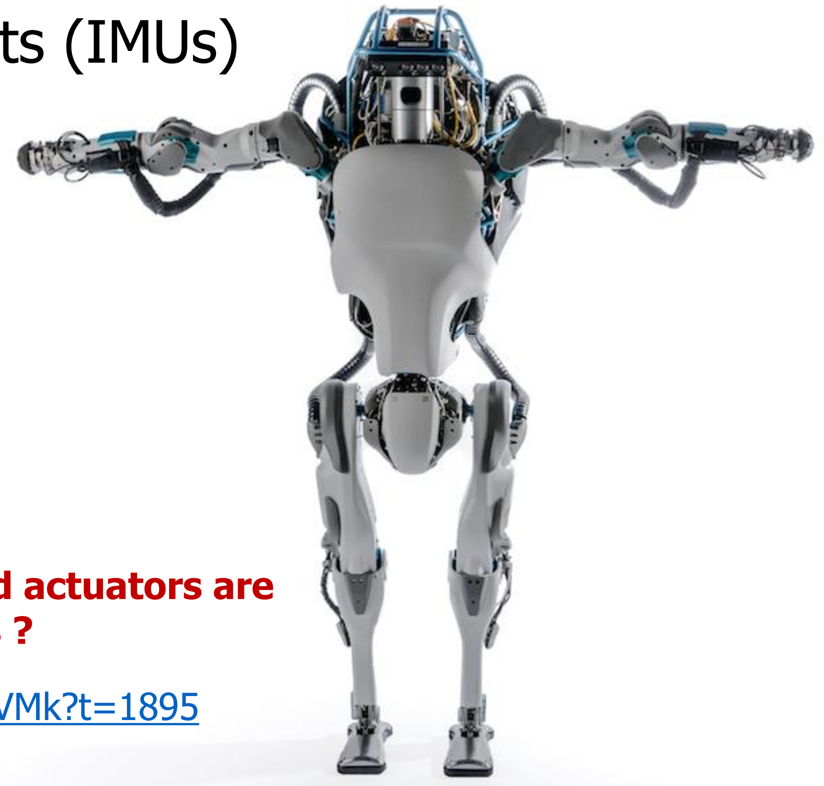
Mobile Robotics – Lecture 3

Anatomy of a Mobile Robot

Marco Camurri

Anatomy of a Mobile Robot

- **Proprioceptive Sensors:**
  - Inertial Measurement Units (IMUs)
  - Force/Torque sensors
  - Encoders
- **Exteroceptive Sensors:**
  - Lidar
  - Cameras
  - GNSS
- **Actuators:**
  - Hydraulic
  - Electric



What sensors and actuators are installed on Atlas ?

youtu.be/yLtdzJ6mVMk?t=1895

Anatomy of a Mobile Robot

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  - Lidar
  - Cameras
  - GNSS
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  - Hydraulic
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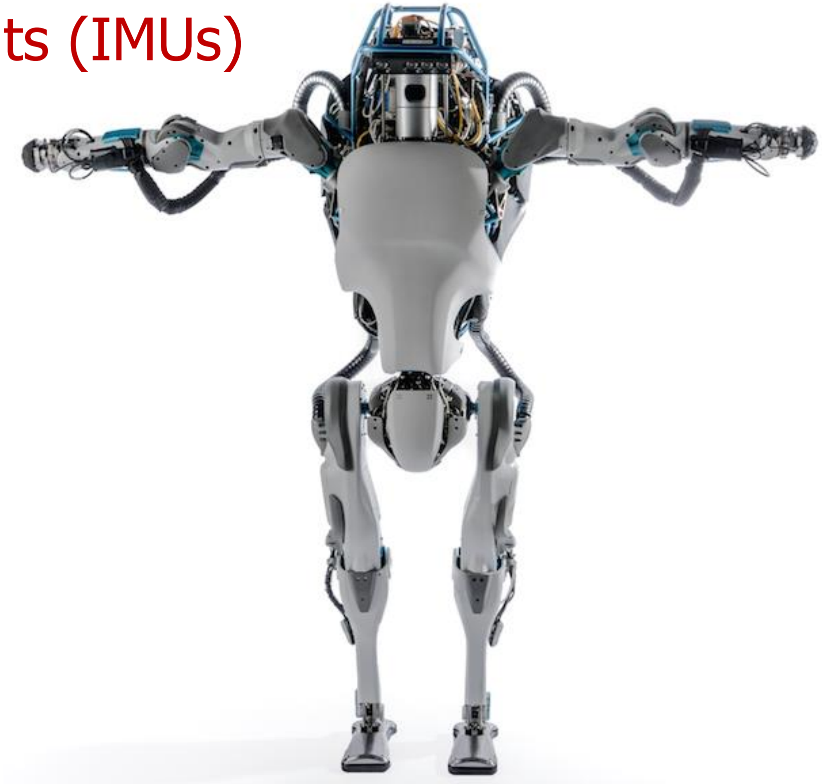

Proprio- vs. Exteroceptive sensors

Proprioceptive sensors → measure properties of the robot




Exteroceptive sensor → measure properties of the environment


NOTE: images not in scale

Normally, both are used with either in a loosely or tightly coupled methods to estimate the state of the robot

From raw data to sensor model

- Sensors transduce physical quantities into signals
- Those signals need to be interpreted
- The interpreted signal will become a measurement which will give us evidence about the state we want to estimate

Example:

- lidar gives time of flight of a beam reflected by a surface
- Interpreted as a range between sensor and surface
- Range can be used to estimate our incremental motion (e.g. by using a matching algorithm)

Remember Bayes Rule?

It’s time to use it!

Let’s see the Bayes Filter …

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What we get at the end of this lesson?

- Understand the physics behind the sensor
- How to get from pyhsical phaenomena to a quantifiable signal
- Understand the sources of noise that affect that measurement and how to model them
- Get a relationship between the the signal and the quantity we want to estimate

Inertial Measurement Unit (IMU)

Inertial Measurement Units

Embedded Sensors

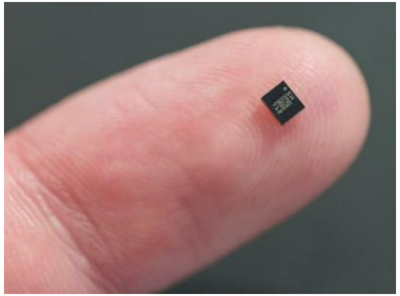
- 3-axis accelerometers
- 3-axis gyros
- Optional: 3-axis magnetometer
- Optional: integrated GNSS

Technologies

- Mechanical
- MEMS (Micro-Electro-Mechanical)
- Fiber Optics (FOG)

Quantities measured:

- Proper Acceleration
- Angular Velocity
- Absolute Orientation (like a compass)



$1

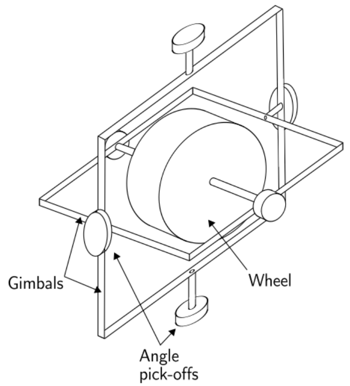
$1000

$20000

Principles of Operation: Gyroscopes

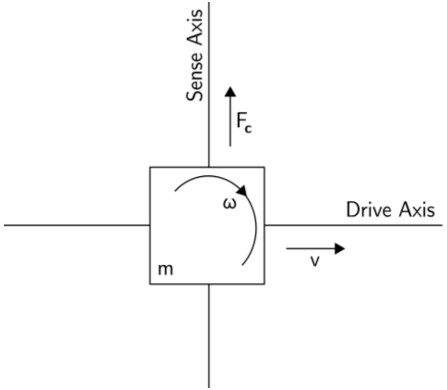
Gyroscopes measure angular orientation or angular velocity, depending on their principle.

Mechanical gyroscope



Conservation angular momentum

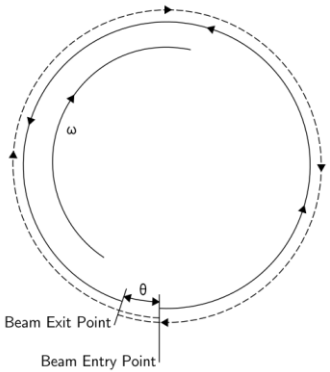
Microelectromechanical System (MEMS) Gyroscope



Coriolis Effect

$$\boldsymbol{F}_c = -2m(\boldsymbol{\omega} \times \boldsymbol{v})$$

Fiber Optic Gyroscope (FOG)

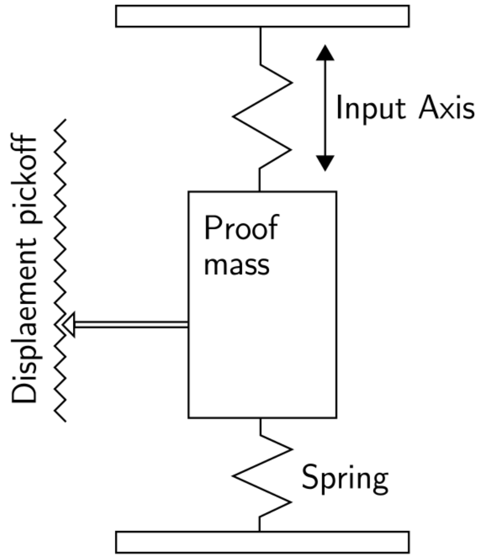


Sagnac Effect

$$\Delta\phi \approx \frac{8\pi}{\lambda c}\boldsymbol{\omega} \cdot \mathbf{A}$$

Principles of Operation: Accelerometers

Accelerometers measure proper acceleration, which is <u>different</u> than coordinate acceleration. An accelerometer at rest measures ~9.81 m/s. It measures 0.0 m/s in free fall.



$$F_s = kx$$

Spring Mass
Hooke's law

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Types of errors

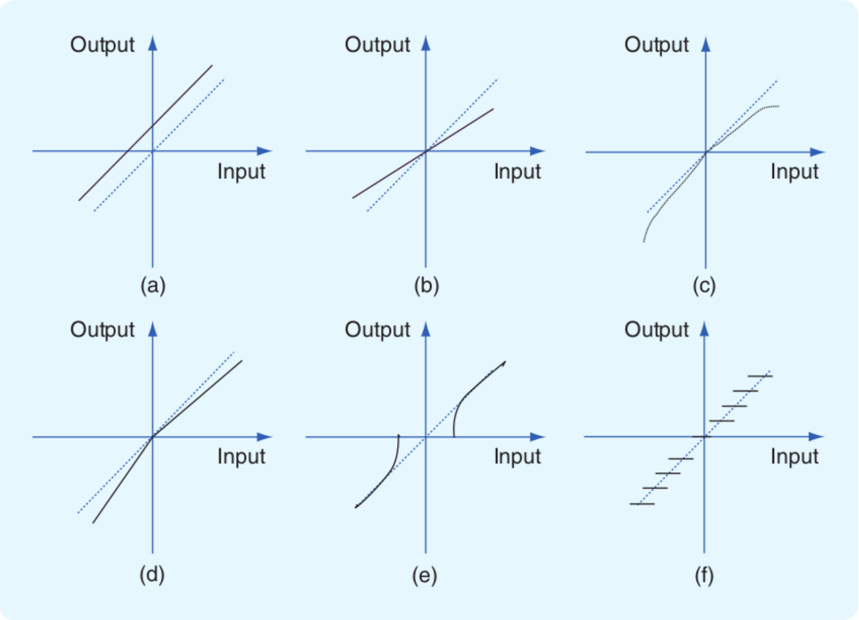

FIGURE 1 Common input/output gyro error types. The figure illustrates the errors in typical gyros as a function of the input/output relationship. These errors are modeled in the measured angular rate of the gyros and can be estimated by using other sensors, such as the global positioning system. Bias (a) is any nonzero sensor output when the input is zero. Scale-factor error (b) often results from aging or manufacturing tolerances. Nonlinearity (c) is present in most sensors to some degree. Scale-factor sign asymmetry (d) is often from mismatched push-pull amplifiers. A deadzone (e) is usually due to mechanical stiction or lock-in (for example, in a ring laser gyro). Quantization error (f) is inherent in all digitized systems. Quantization error may not be zero mean when the input is held constant.

TABLE 2 Typical performance grades. The typical magnitudes and units of the inertial, intermediate, and moderate gyros are shown for the various performance parameters.

Typical Performance Grades

Performance Parameter	Performance Units	Performance Grades: Inertial	Intermediate	Moderate
Maximum input	deg/h	10^{2}–10^{6}	10^{2}–10^{6}	10^{2}–10^{6}
	deg/s	10^{-2}–10^{2}	10^{-2}–10^{2}	10^{-2}–10^{2}
Scale factor	part/part	10^{-6}–10^{-4}	10^{-4}–10^{-3}	10^{-3}–10^{-2}
Bias stability	deg/h	10^{-4}–10^{-2}	10^{-2}–10	10–10^{2}
	deg/s	10^{-8}–10^{-6}	10^{-6}–10^{-3}	10^{-3}–10^{-2}
Bias drift	deg/$\sqrt{h}$	10^{-4}–10^{-3}	10^{-2}–10^{-1}	1–10
	deg/$\sqrt{s}$	10^{-6}–10^{-5}	10^{-5}–10^{-4}	10^{-4}–10^{-3}

Source:
How Good Is Your Gyro? [Ask the Experts]
https://ieeexplore.ieee.org/abstract/document/5386159

MEMS Gyro Noise Characteristics

Error Type	Description	Result of Integration
Bias	A constant bias ϵ	A steadily growing angular error $$\theta(t) = \epsilon \cdot t$$
White Noise	White noise with some standard deviation σ	An angle random walk, whose standard deviation $$\sigma_\theta(t) = \sigma \cdot \sqrt{\delta t \cdot t}$$ grows with the square root of time
Temperature Effects	Temperature dependent residual bias	Any residual bias is integrated into the orientation, causing an orientation error which grows linearly with time
Calibration	Deterministic errors in scale factors, alignments and gyro linearities	Orientation drift proportional to the rate and duration of motion
Bias Instability	Bias fluctuations, usually modelled as a bias random walk	A second-order random walk

MEMs Gyro Noise Characteristics

Constant

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Constant	Calibration	Deterministic errors in scale factors, alignments and gyro linearities	Orientation drift proportional to the rate and duration of motion
Changes Slowly	Bias Instability	Bias fluctuations, usually modelled as a bias random walk	A second-order random walk

Source:
An introduction to inertial navigation
https://www.cl.cam.ac.uk/techreports/UCAM-CL-TR-696.pdf

Governing Equations

What is the relationship between measured values and pysical properties of interest?

$$\boxed{{}_{\mathrm{B}}\tilde{\boldsymbol{\omega}}_{\mathrm{WB}}(t)} = \boxed{{}_{\mathrm{B}}\boldsymbol{\omega}_{\mathrm{WB}}(t)} + \boxed{\mathbf{b}^g(t)} + \boldsymbol{\eta}^g(t)$$

$$\boxed{{}_{\mathrm{B}}\tilde{\mathbf{a}}(t)} = \boxed{\mathrm{R}_{\mathrm{WB}}^{\mathsf{T}}(t)}\left(\boxed{{}_{\mathrm{W}}\mathbf{a}(t)} - {}_{\mathrm{W}}\mathbf{g}\right) + \boxed{\mathbf{b}^a(t)} + \boldsymbol{\eta}^a(t)$$

What we want:

- angular velocity of B from W expressed in B
- coordinate acceleration of B expressed in W

What we also want:

- angular velocity of B from W expressed in B
- coordinate acceleration of B expressed in W

What we get:

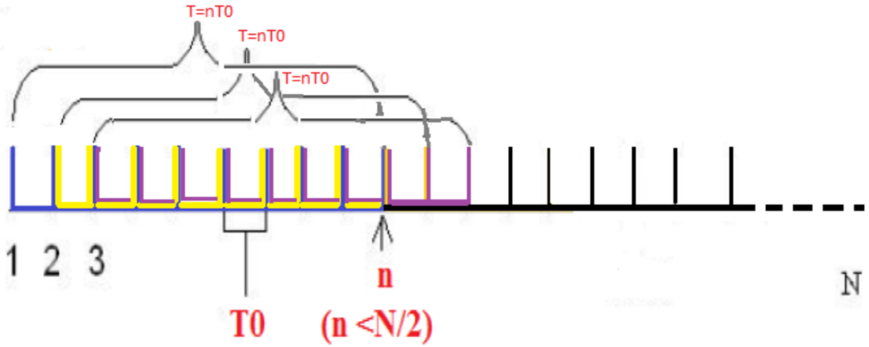
- angular velocity, affected by noise and bias
- Proper acceleration, affected by noise and bias

What we need:

- angular velocity of B from W expressed in B
- coordinate acceleration of B expressed in W

Allan Variance (AVAR)

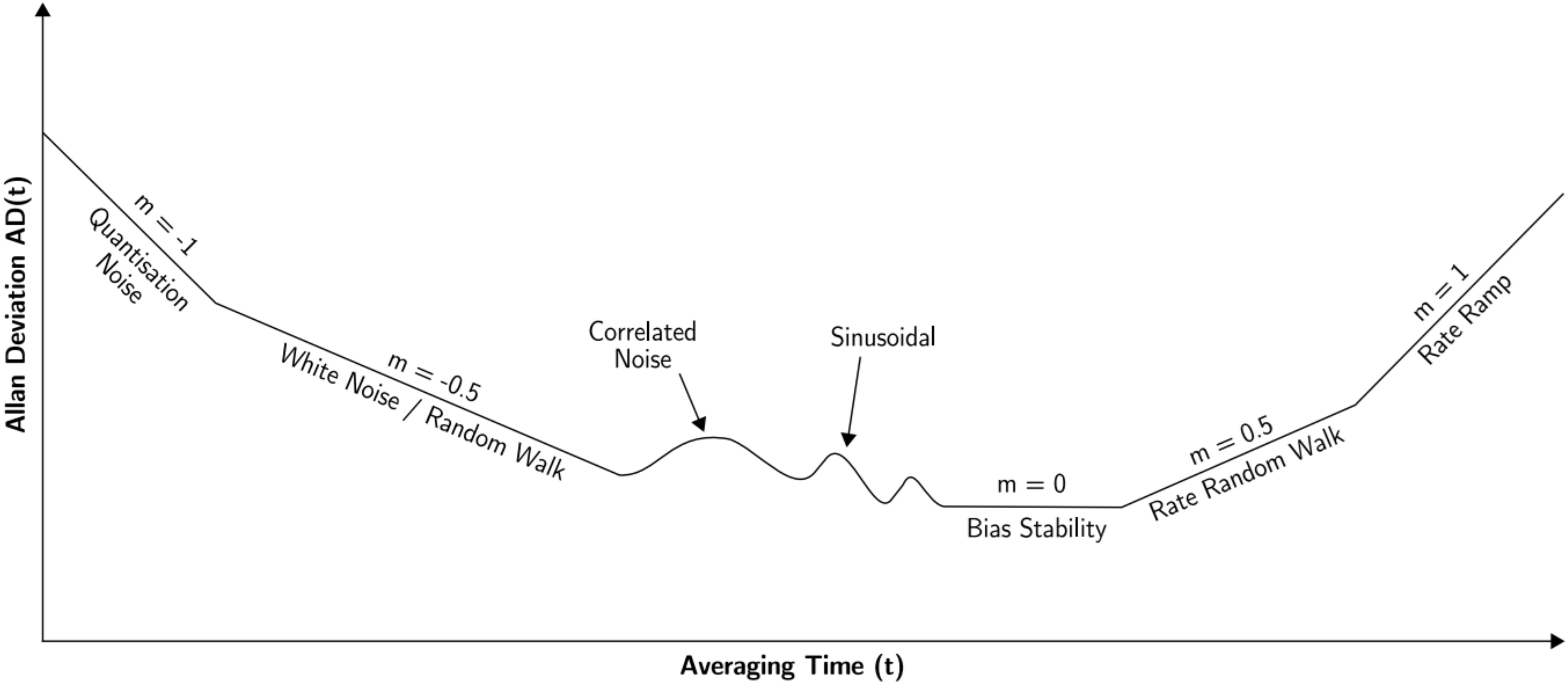
1. Collect long sequence of data
2. Divide into bins of size T (which can be overlapping)
3. Compute average of each bin
4. AVAR is computed between consecutive bins



$$\overline{\Omega}_k(T) = \frac{1}{T}\int_{t_k}^{t_k+T} \Omega(t)\,dt$$

$$\sigma^2(T) = \frac{1}{2}\langle[\overline{\Omega}_{next}(T) - \overline{\Omega}_k(T)]^2\rangle \equiv \frac{1}{2(K-1)}\sum_{k=1}^{K-1}[\overline{\Omega}_{next}(T) - \overline{\Omega}_k(T)]^2,$$

Sensor Noise Analysis - Allan Deviation

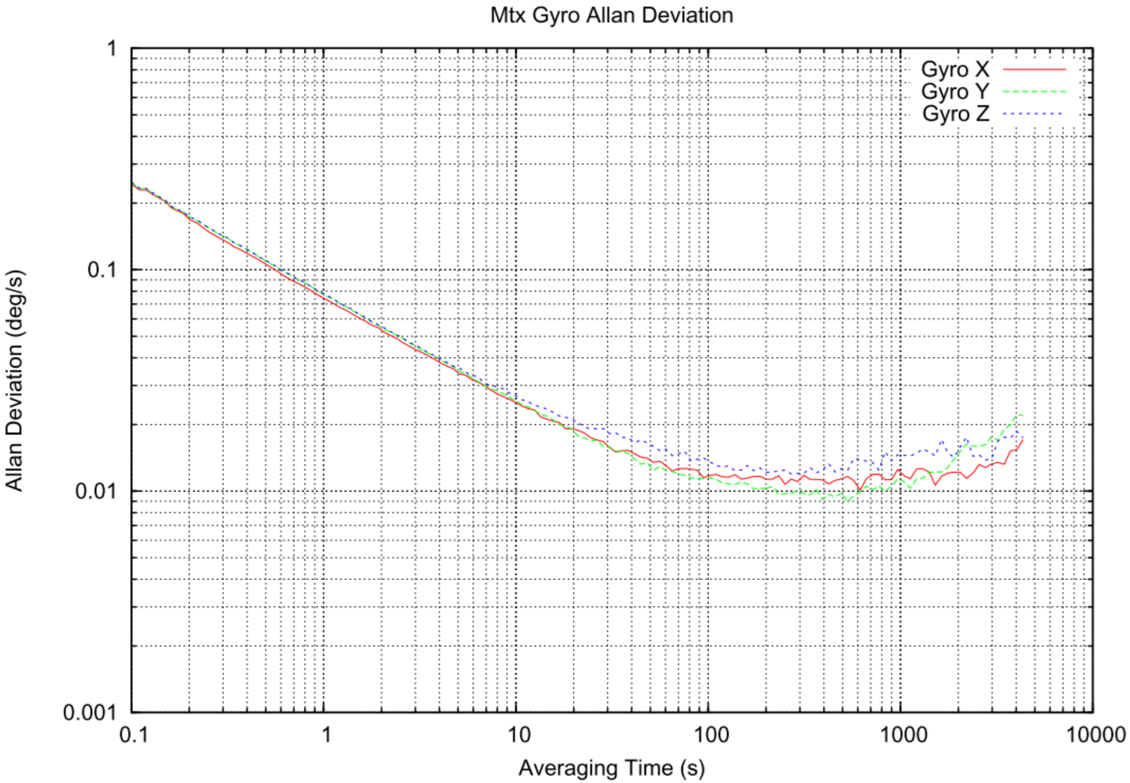


Another possible log-log plot of Allan Deviation analysis results

Allan Deviation

Allan Deviation is the square root of AVAR

- Can be computed for varied lengths of T
- Log-Log plot shows dominant noise at different frequencies



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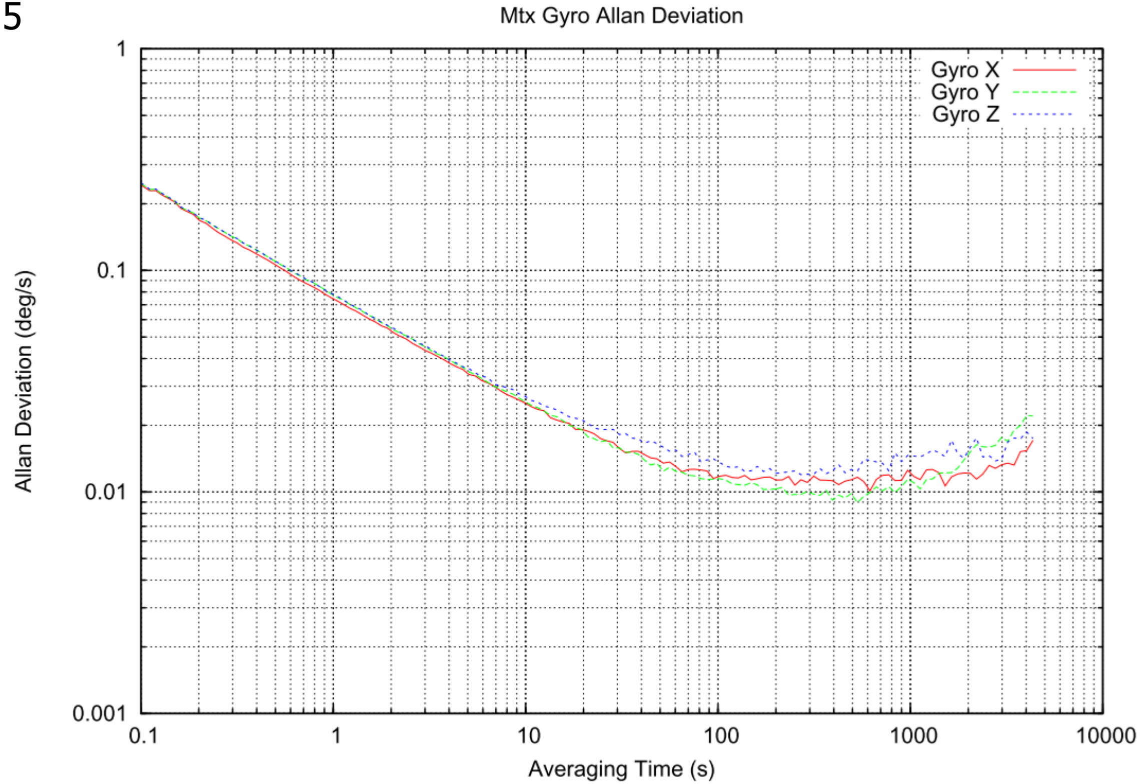
Allan Deviation Plotting

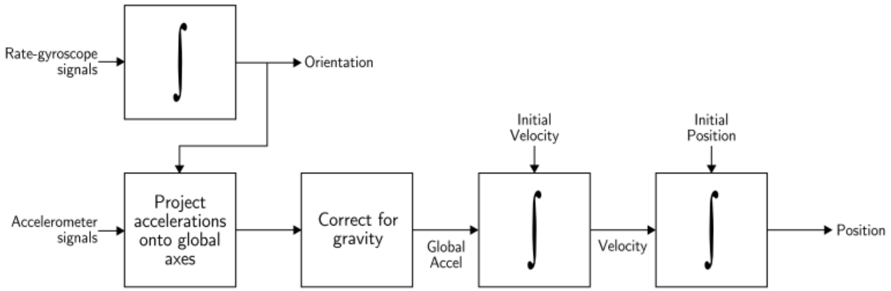
- White noise: slope with gradient −0.5 between 0.1 and 10s.
- Bias Instability: flat region

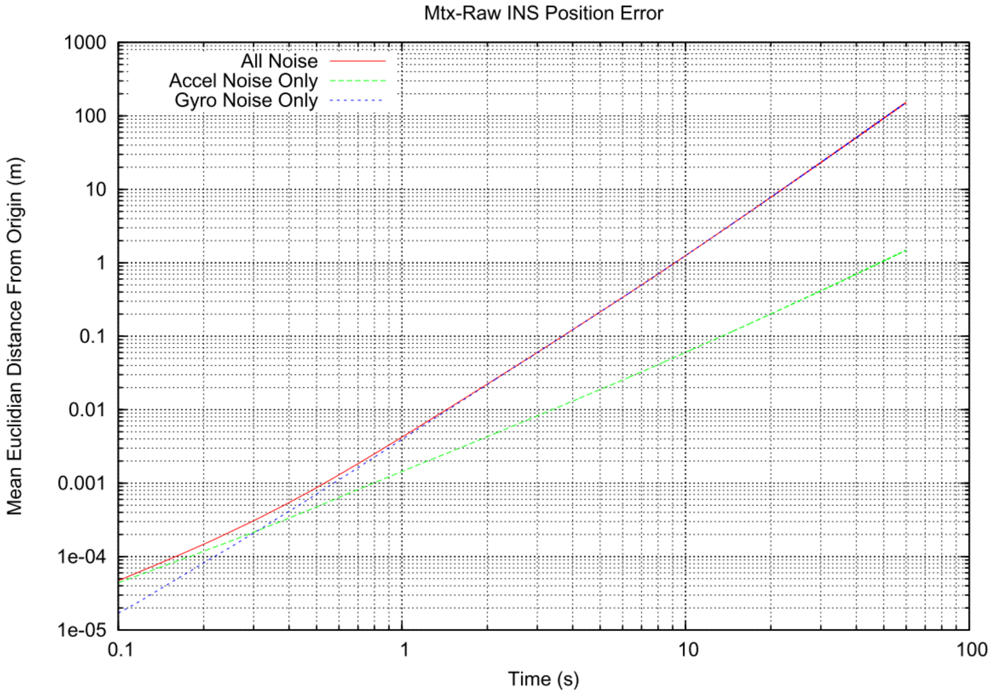
	Bias Instability	Angle Random Walk
X Axis	$0.010°/\mathrm{s} = 36°/\mathrm{h}$ (at 620 s)	$0.075°/\sqrt{\mathrm{s}} = 4.6°/\sqrt{\mathrm{h}}$
Y Axis	$0.009°/\mathrm{s} = 32°/\mathrm{h}$ (at 530 s)	$0.078°/\sqrt{\mathrm{s}} = 4.8°/\sqrt{\mathrm{h}}$
Z Axis	$0.012°/\mathrm{s} = 43°/\mathrm{h}$ (at 270 s)	$0.079°/\sqrt{\mathrm{s}} = 4.8°/\sqrt{\mathrm{h}}$



Some more insights

- Over longer periods of time, INS position error is primarily due to Gyroscope noise.
  - From Oliver Woodman





Sources

- “Characterization of Errors and Noises in MEMS Inertial Sensors Using Allan Variance Method”. Leslie Barreda Pupo, MSc Thesis
- “An introduction to inertial navigation”, Oliver J. Woodman, Technical Report
- “**Strapdown inertial navigation technology**”, David H. Titterton and John L. Weston. Textbook

Project Idea

An interesting idea for a project could be characterising the noise of an IMU of our lab and see if there is a way to predict its evolution over the course of these months…

Encoders

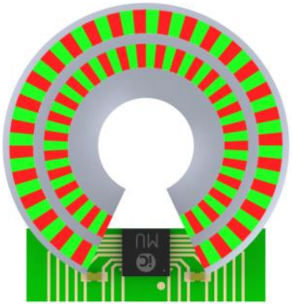
Encoders

Working principle:
- Optical
- Magnetic

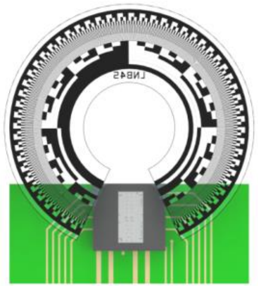
Type of measurement:
- Incremental
- Absolute

Type of signal:
- Digital
- Analog

What do they measure:
- Angular (or linear) position of a joint (e.g. wheel)
- Angular (or linear) velocity (mostly via differentiation)



Magnetic pole wheel and single-chip encoder iC-MU



Optical code wheel and single-chip encoder iC-LNB

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F/T Sensors

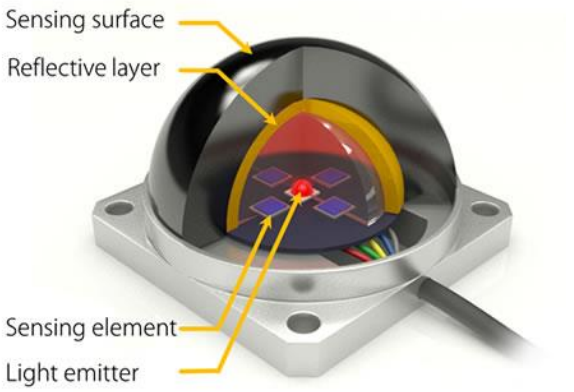
F/T Sensors

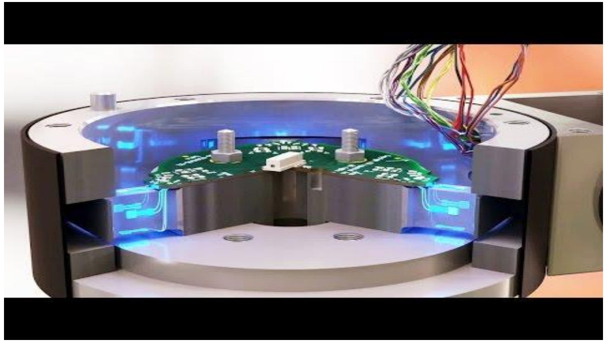
Working principle:
- Strain gauge
- Optical

Type of signal:
- Digital
- Analog

What do they measure:
- 6-axis Force and or Torque
- 1- Axis Force

What is used for
- torque control
- contact estimation





Cameras

What do Cameras Measure?

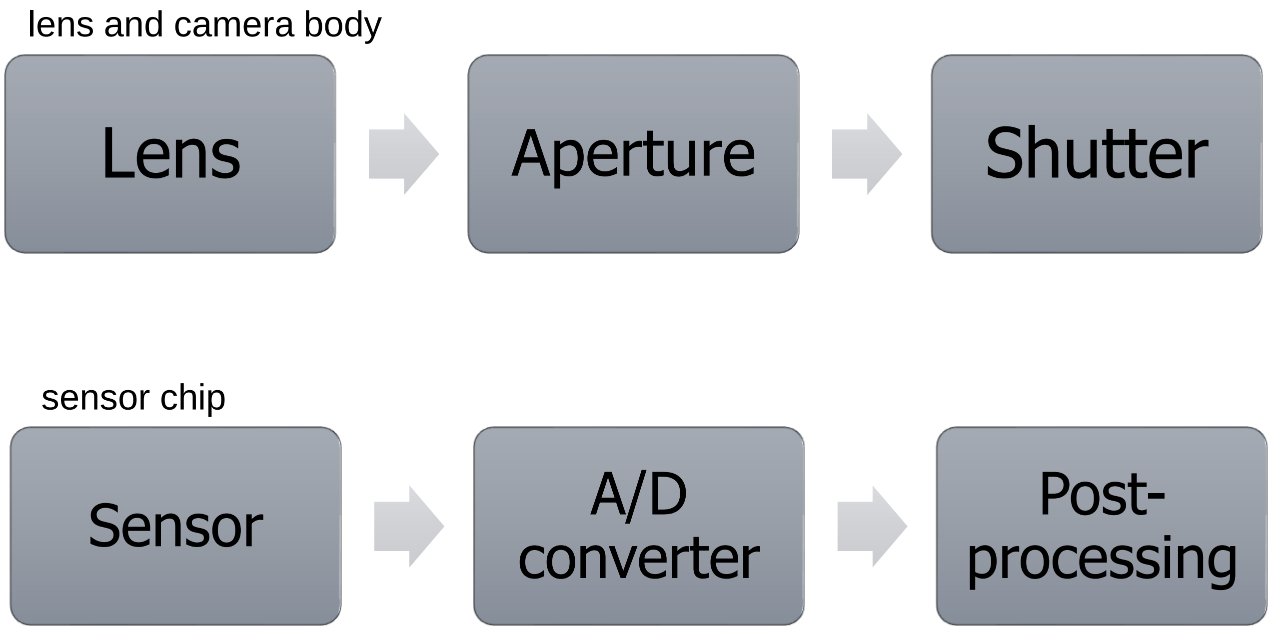
Cameras provide 2D images consisting of pixels ("picture elements")

Cameras measure the **light intensity** for each pixel

Each position in an image (=**pixel**) corresponds to a **specific direction** in the 3D world

Each pixel measures the amount of light coming from a certain direction.

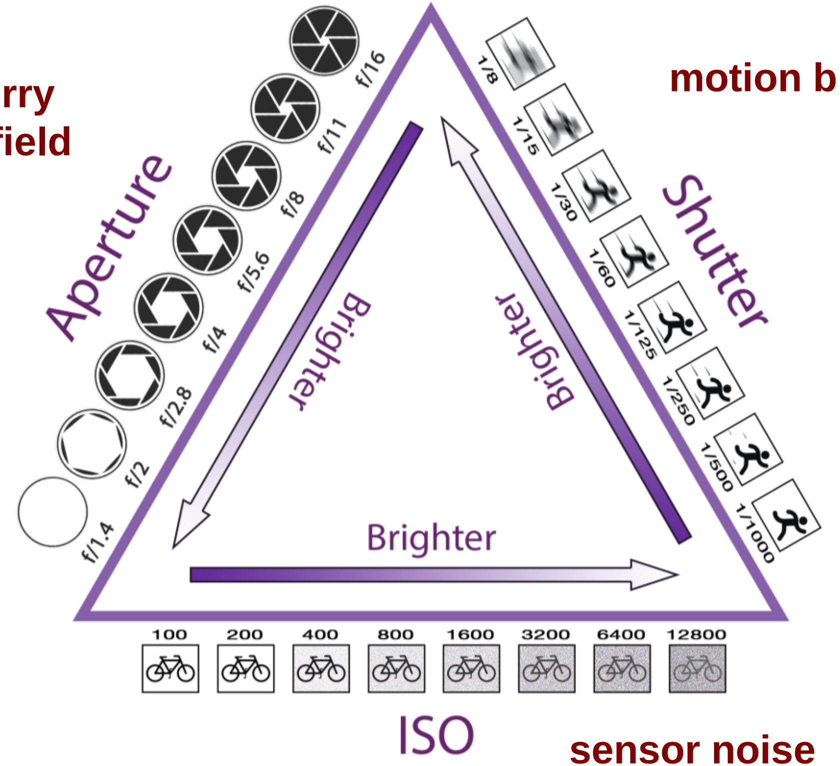
Elements of a Digital Camera



Exposure Triangle

sharp/blurry
depth of field

motion blur



sensor noise

See: https://actioncamera.blog/2017/02/22/the-exposure-triangle/

Image courtesy: M. Walsh

Shutter Speed / Exposure Time

Controls the amount of light reaching the sensor

Longer exposure time = more light = brighter images

Long exposure time leads to motion blur

Rolling Shutter

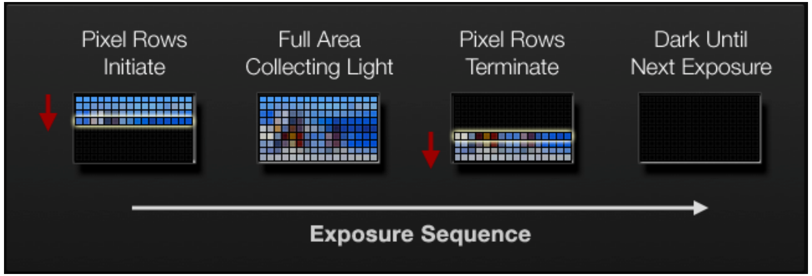
The shutter rolls (moves) across the exposable image area

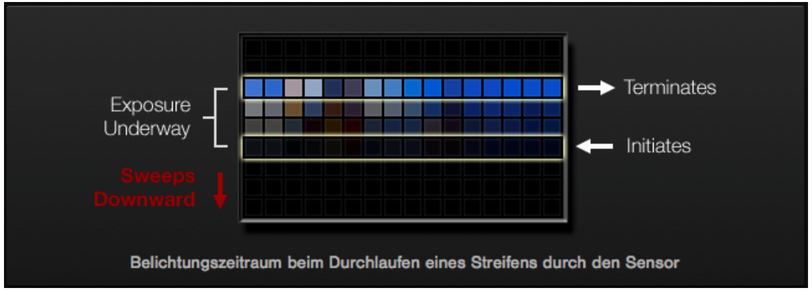
The pixels at the same line of the image are recorded at the same time

Produces distortions in case of fast-moving objects or cameras

Often found in CMOS cameras

Rolling Shutter





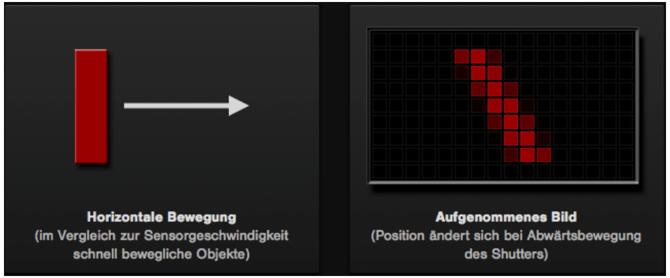


Image Courtesy:
Red.com, Inc.

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Rolling Shutter Effects

Image Courtesy:
Axel1963
(wikipedia)

Image Courtesy:
Richmilliron
(wikipedia)

Rolling vs. Global Shutter

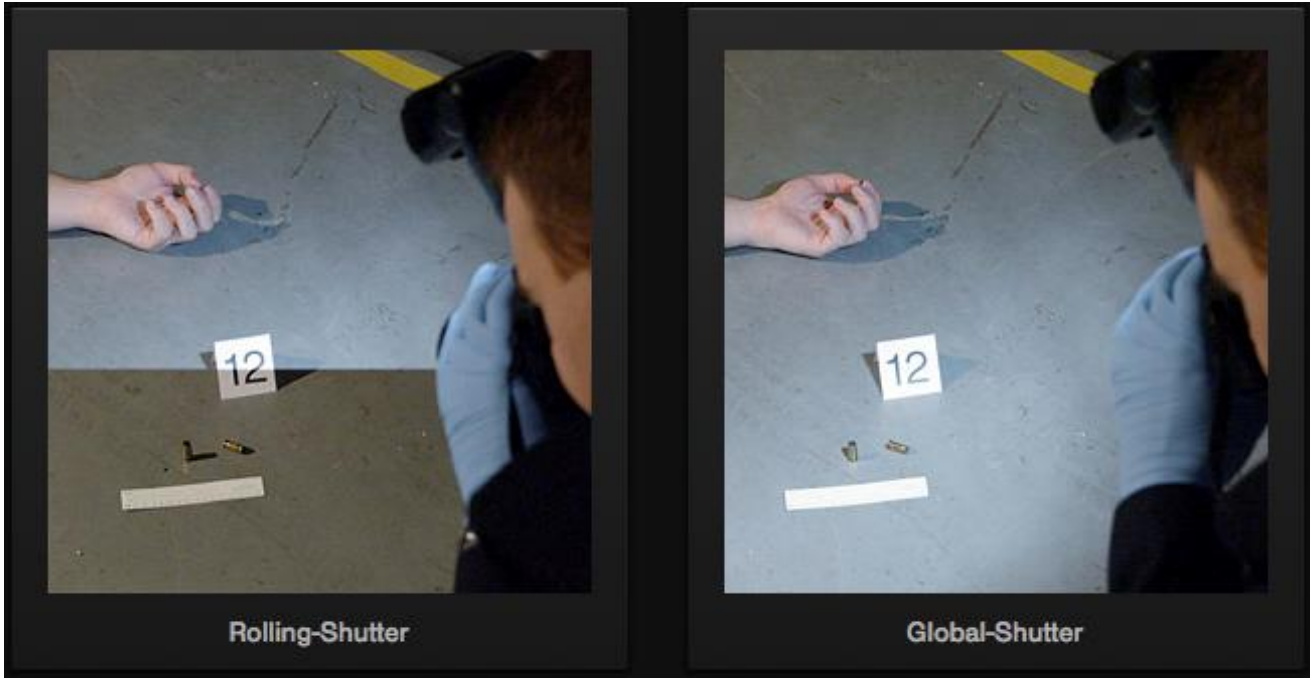
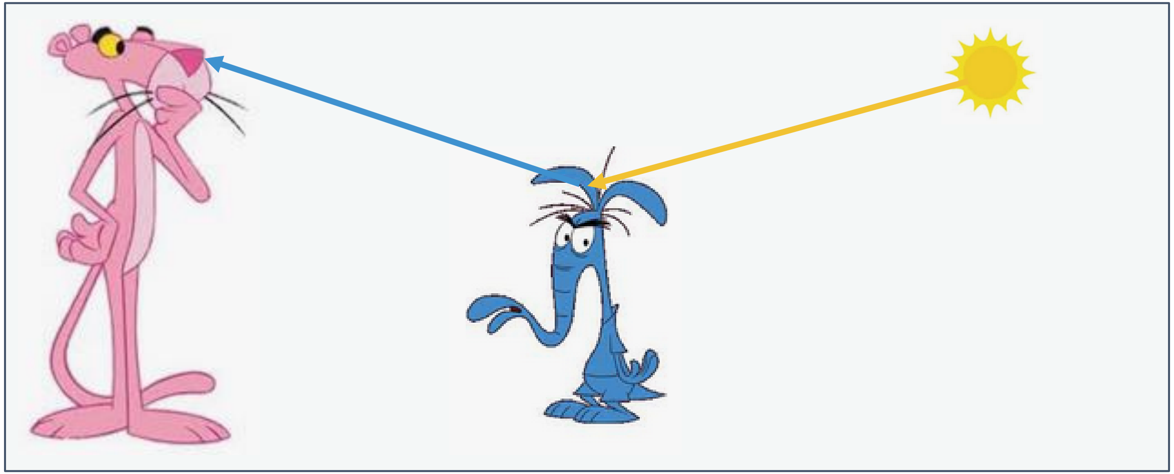


Image Courtesy: Red.com, Inc.

Fundamentals

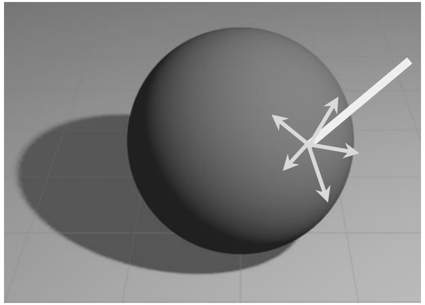
How do we see?

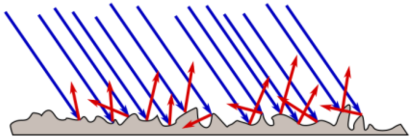
- Light emitted from a source, interacts with objects in the environment
- After interaction, this light reaches our eyes



Interaction of light with objects

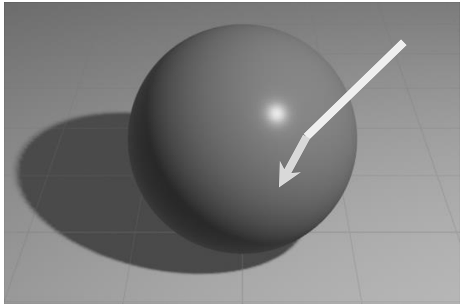
Diffuse

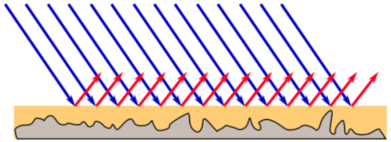




Parallel rays reflect
in
all directions

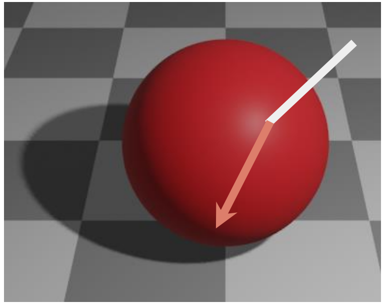
Specular





Parallel rays reflect
in
a single direction

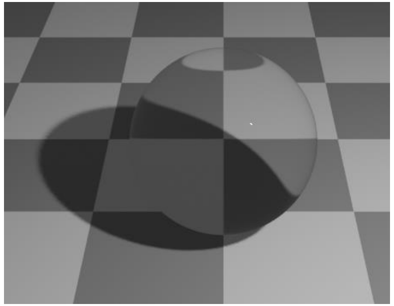
Colour

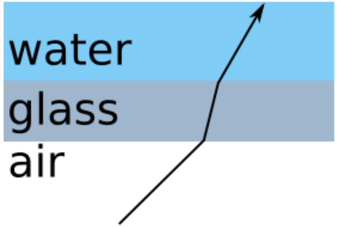


Colour is determined by
the
wavelength of the light
reflected back

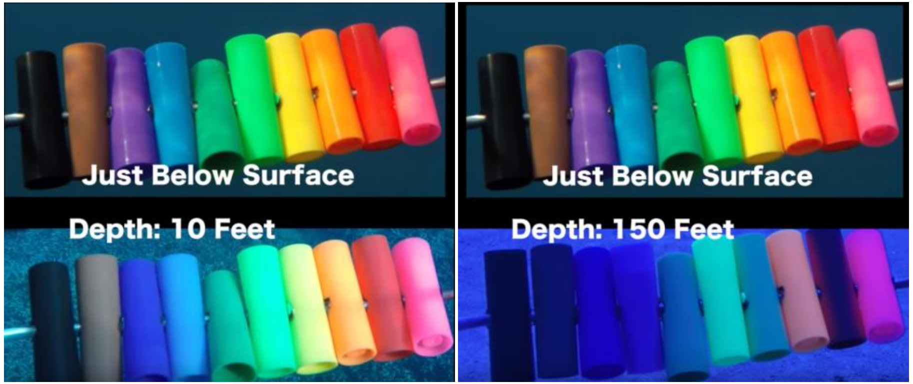
Interaction of light with participating media

Refraction





Attenuation



Courtesy of Kendall Roberg

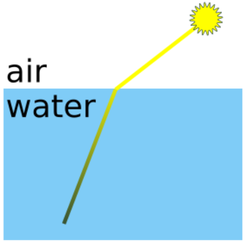


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Image Formation

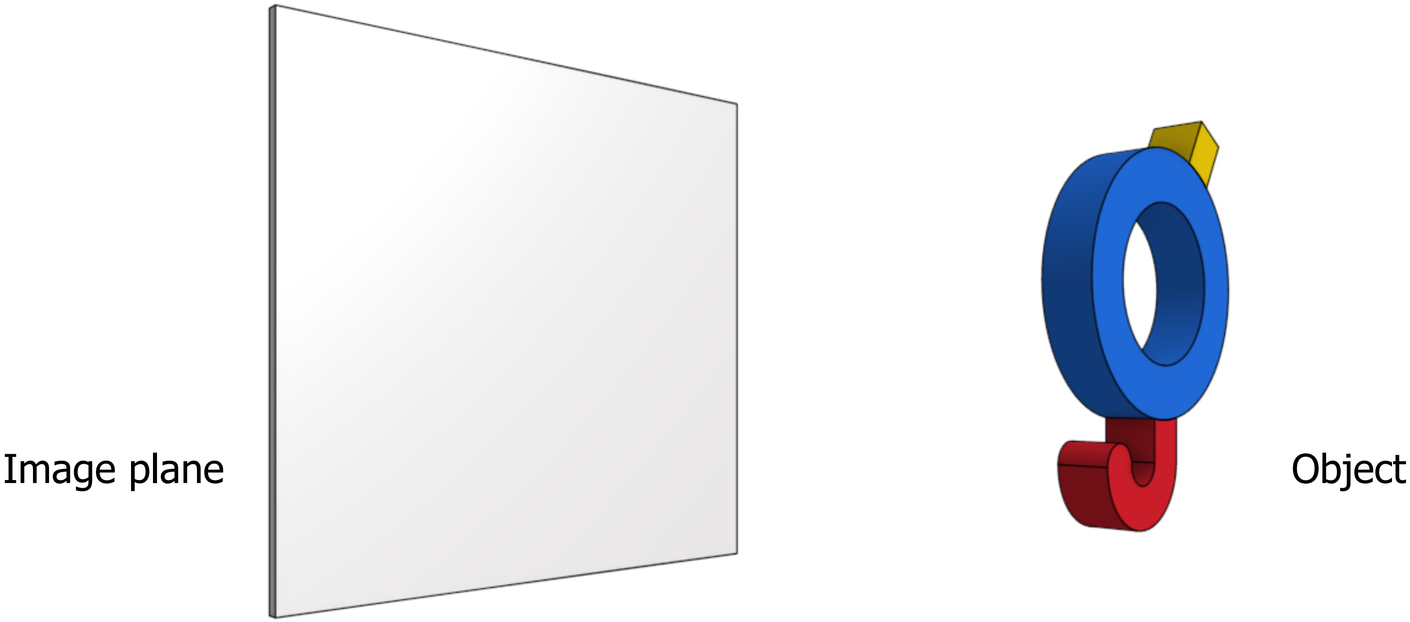

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Image Formation

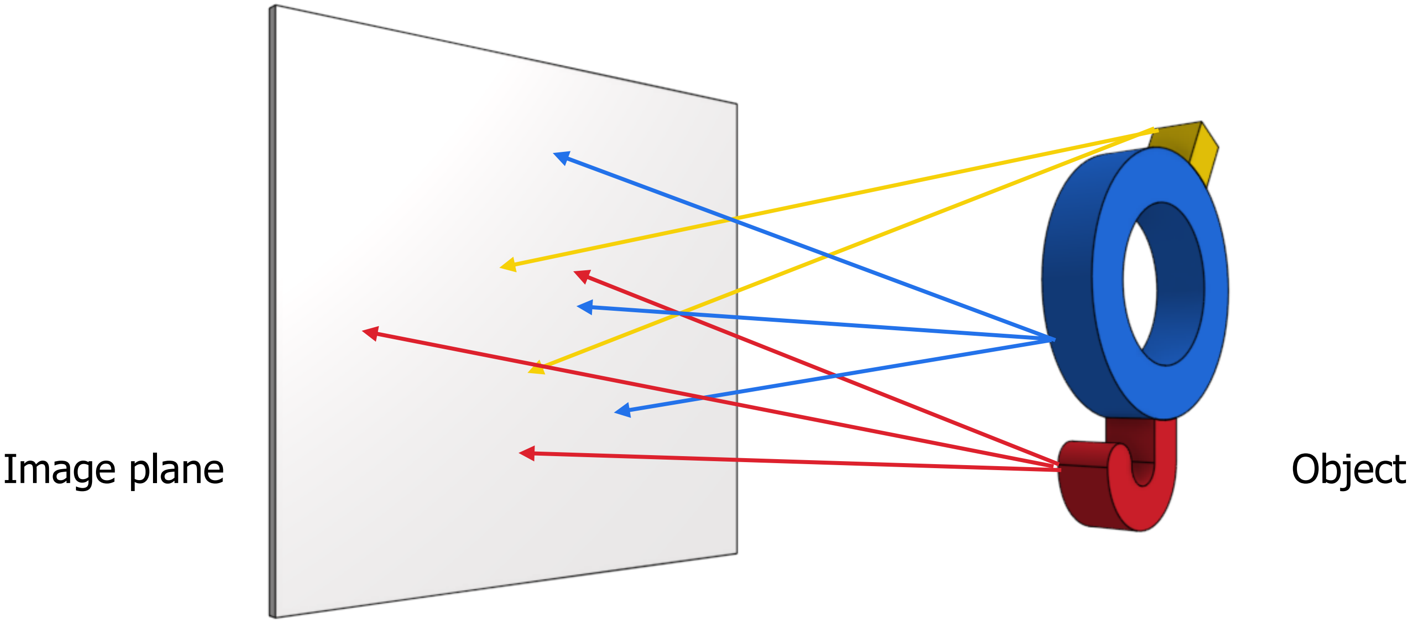
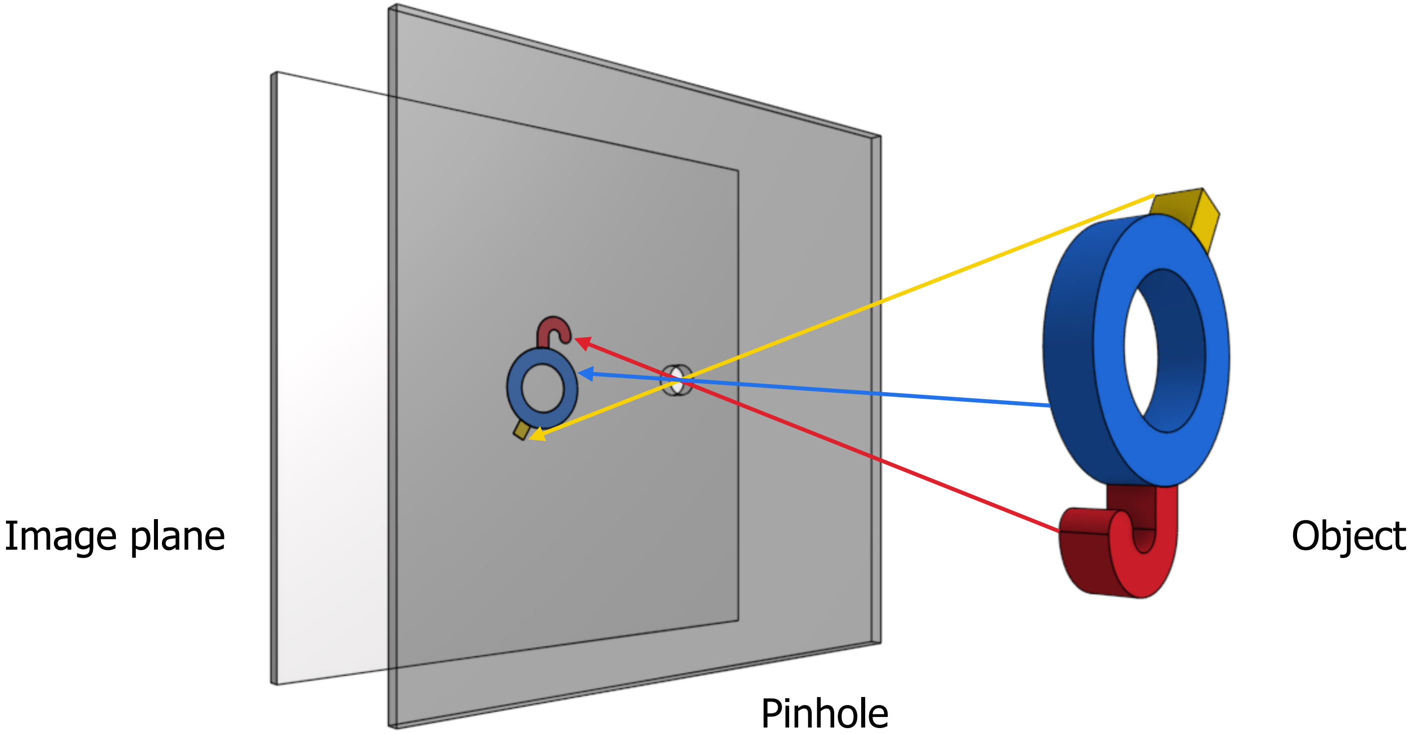


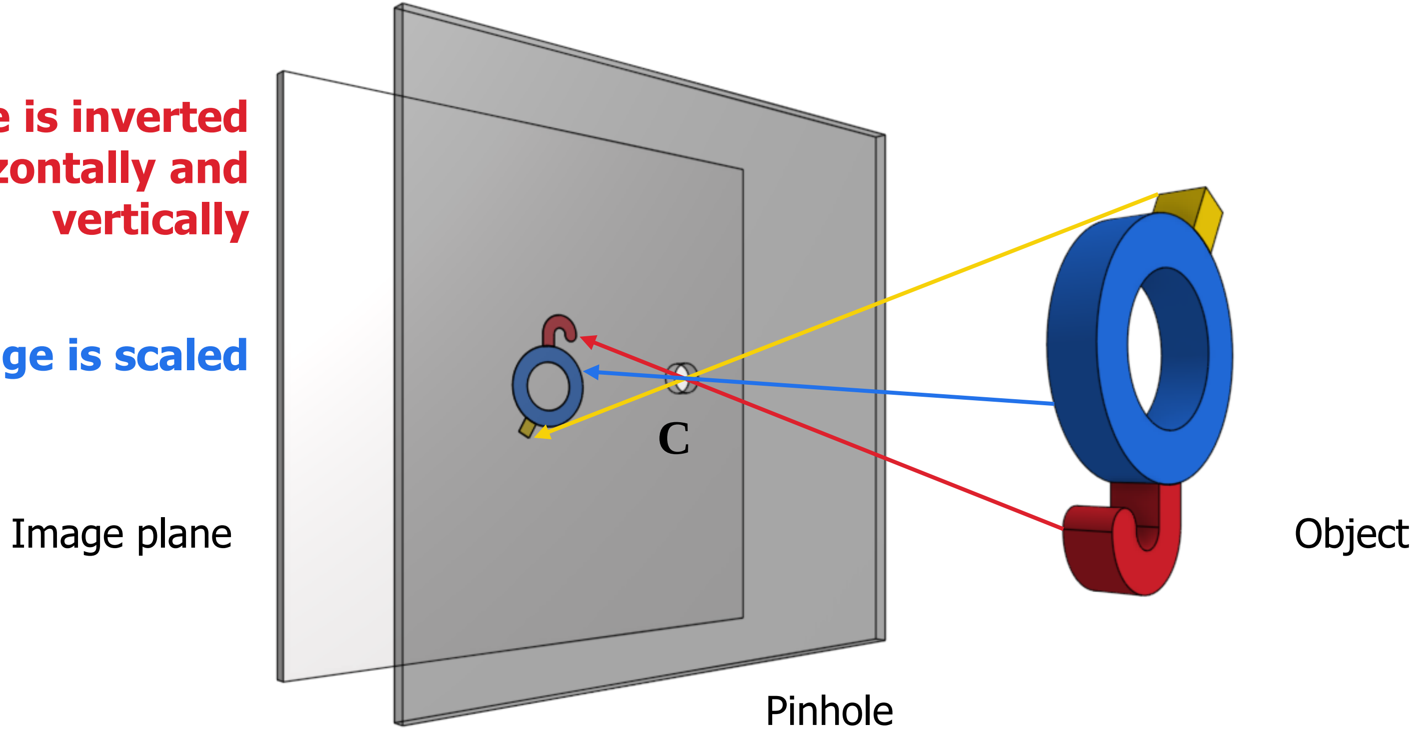
Image Formation



Pinhole imaging model

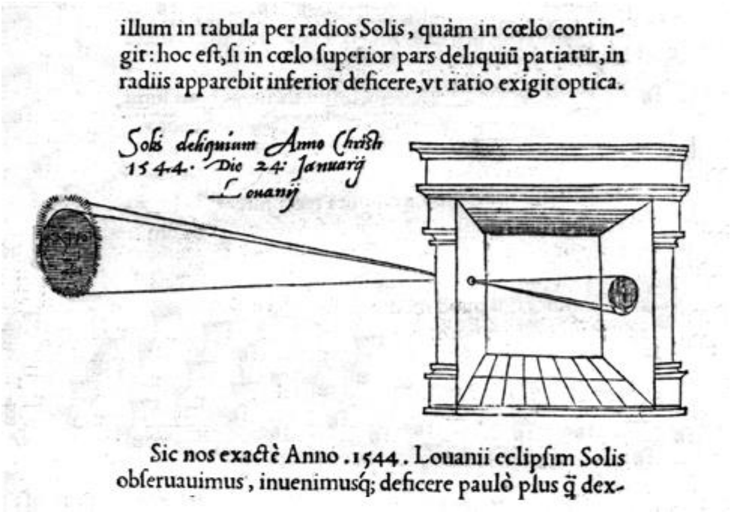
Image is inverted horizontally and vertically

Image is scaled



Captures a **pencil of rays** passing through a common point.
This point is called **centre of projection (focal point) - C**

Camera Obscura (1544)



illum in tabula per radios Solis, quàm in cœlo contingit: hoc est, si in cœlo superior pars deliquiũ patiatur, in radiis apparebit inferior deficere, vt ratio exigit optica.

Solis deliquium Anno Christi
1544. Die 24. Januarij
Louanij

Sic nos exactè Anno .1544. Louanii eclipsim Solis obseruauimus, inuenimusq; deficere paulò plus q̃ dex-

In Latin, means “dark room”

"**Reinerus Gemma-Frisius**, observed an eclipse of the sun at Louvain on January 24, 1544, and later he used this illustration of the event in his book De Radio Astronomica et Geometrica, 1545. It is thought to be the first published illustration of a camera obscura..."
Hammond, John H., The Camera Obscura, A Chronicle

Image Courtesy: http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html

Camera Obscura at Home

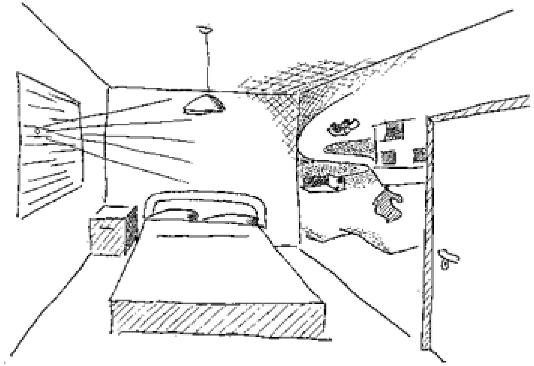


Figure 1 - A lens on the window creates the image of the external world on the opposite wall and you can see it every morning, when you wake up.

Sketch from:
http://www.funsci.com/fun3_en/sky/sky.htm

Image Courtesy:
http://blog.makezine.com/archive/2006/02/how_to_room_sized_camera_obscu.html

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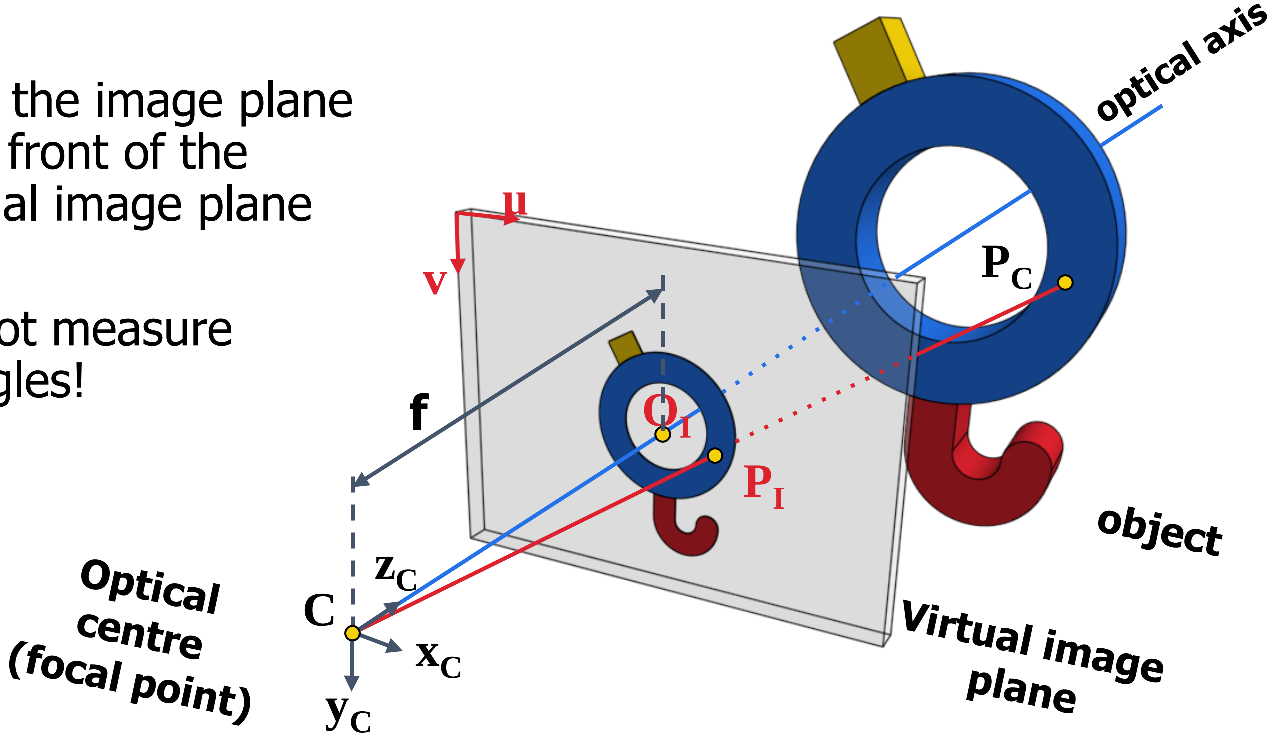
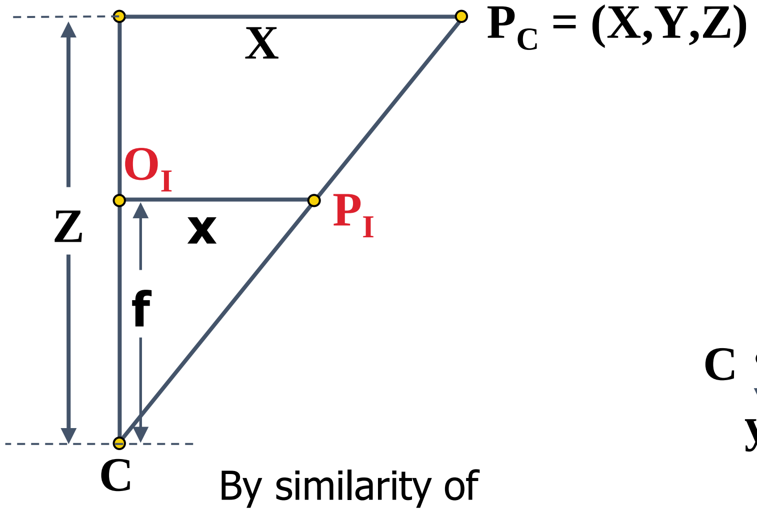
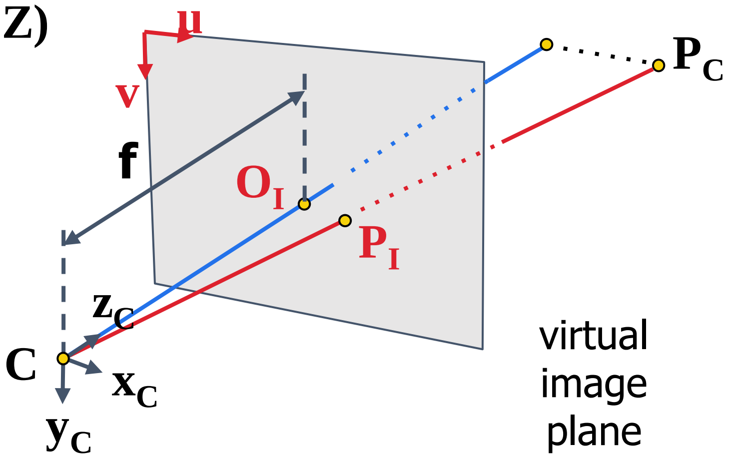
Pinhole camera model

- For convenience, the image plane is represented in front of the focal point - virtual image plane
- A camera does not measure distances but angles!



Pinhole camera model: perspective projection





By similarity of triangles

$$\frac{x}{f} = \frac{X}{Z} \rightarrow$$

similarly

$$\boxed{x = X\frac{f}{Z} \qquad y = Y\frac{f}{Z}}$$

Maps 3D points onto a 2D plane

Pinhole Perspective Projection

Maps points in 3D to points in 2D.

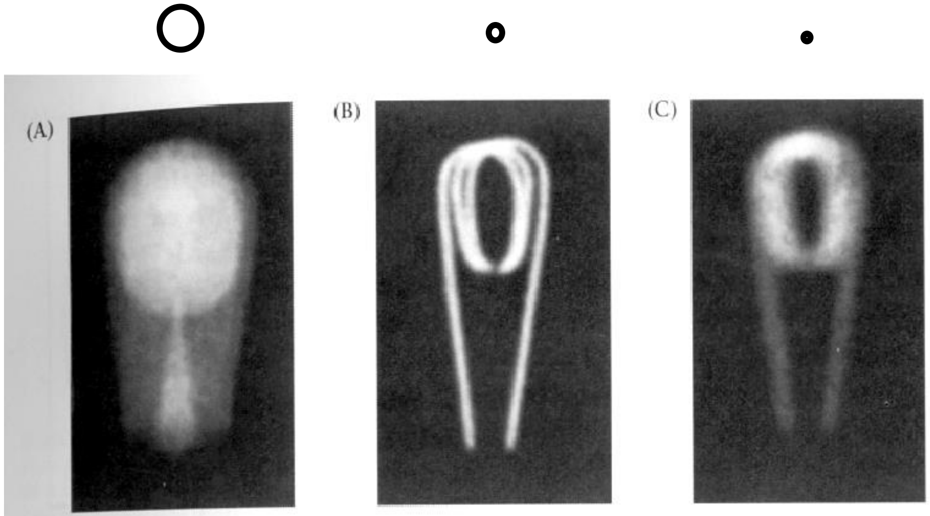
Distant objects appear smaller than nearer objects.

Parallel lines converge at the '**vanishing point**'.

The human eye works in the same way, therefore perspective projection looks most realistic

Courtesy of Dustin Resch (http://dustinresch.blogspot.com.au/2012/03/grad-school-perspective-assignments.html)

Pinhole model: Limitations



2.18 DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruechardt, 1958.

Courtesy Prof. Stefan B. Williams

The pinholes size (aperture) impacts image formation

- **Wide aperture** - more light, better exposure, blurry image, reduced depth of field
- **Narrow aperture** - less light, poorer exposure, but larger depth of field
  - Can cause diffraction effects

Solution: Lenses

Three Assumptions Made in the Pinhole Camera/Thin Lens

1. All rays from the object point intersect in a single point
2. All image points lie on a plane
3. The ray from the object point to the image point is a straight line

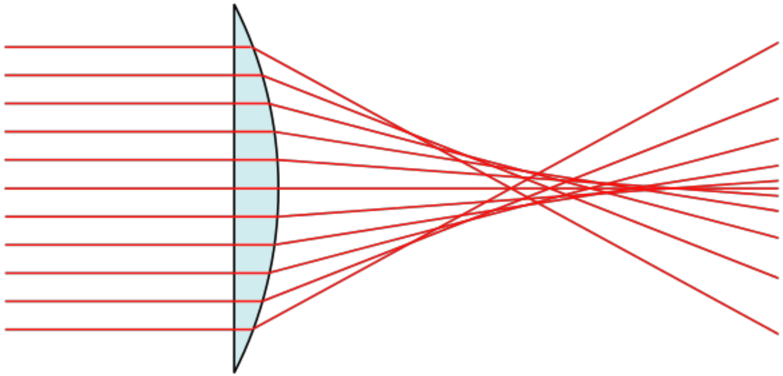
Often these assumption do not hold and leads to imperfect images

Aberrations

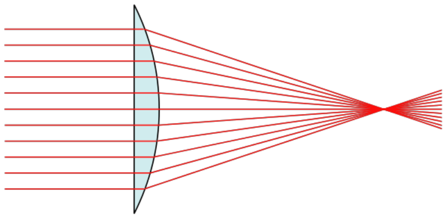
- A deviation from the ideal mapping with a thin lens is called aberration
- Main types of aberrations:
  - Distortion
  - Spherical aberrations
  - Chromatic aberrations
  - Astigmatism
  - Comatic aberrations
  - Vignetting
  - ...

Spherical Aberration

Effect in a lens due to the increased refraction of light rays when they strike a lens



spherical aberration
(DE: Sphärische Aberration)

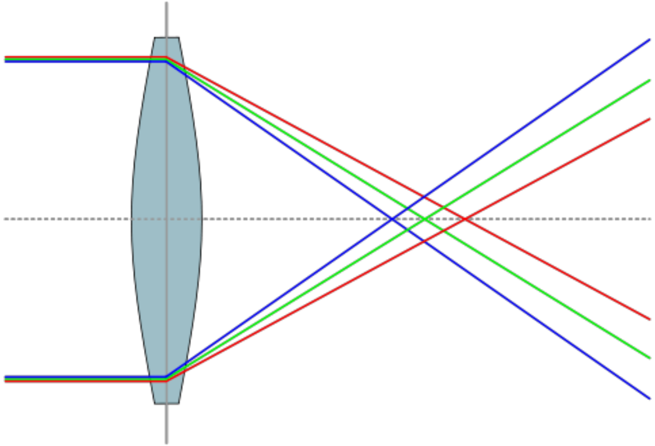


ideal

Image courtesy: Wikipedia

Chromatic Aberration

- Index of refraction for glass varies slightly as a function of wavelength
- Light at different wave length are not projected to the same point (are focused with a different focal length)

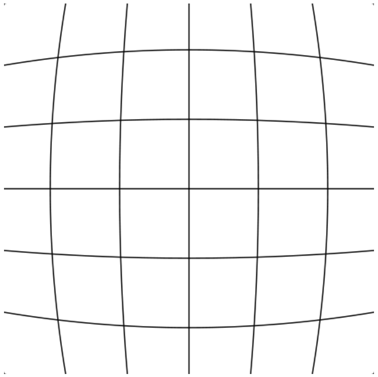


chromatic aberration
(DE: Chromatische Aberration)

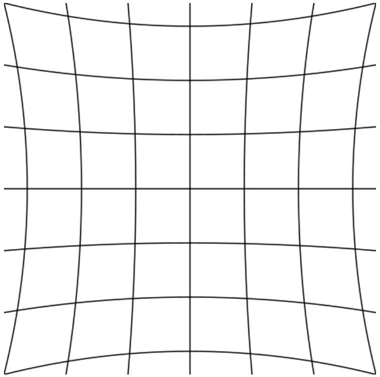
Image courtesy: Wikipedia

Distortion

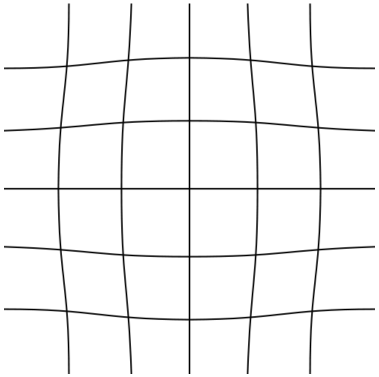
Deviation from rectilinear projection, a projection in which straight lines in a scene remain straight in an image



barrel distortion



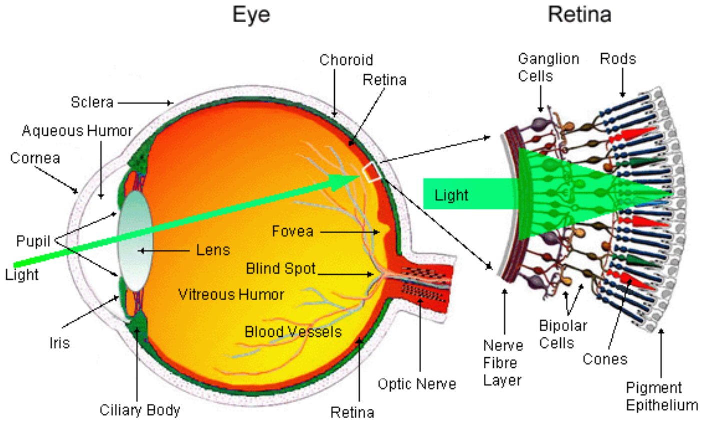
pincushion distortion



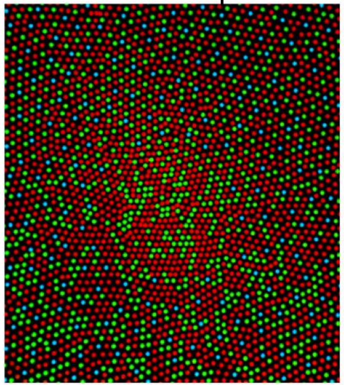
mustache distortion

Image courtesy: Wikipedia

Human eye

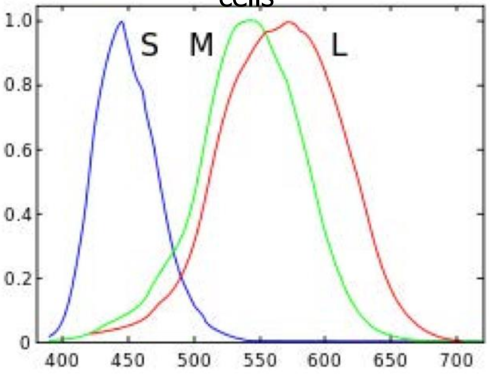


Cone cells under microscope



Courtesy: Mark Fairchild

Normalized spectral response of cone cells



The human eye has two types of photosensitive cells

- Rods: high light sensitivity – mono-chromatic
- Cones: colour sensitivity – tri-chromatic – sensitive to three different wavelengths of the visible spectrum

Astigmatism

A different focus point in vertical and horizontal direction

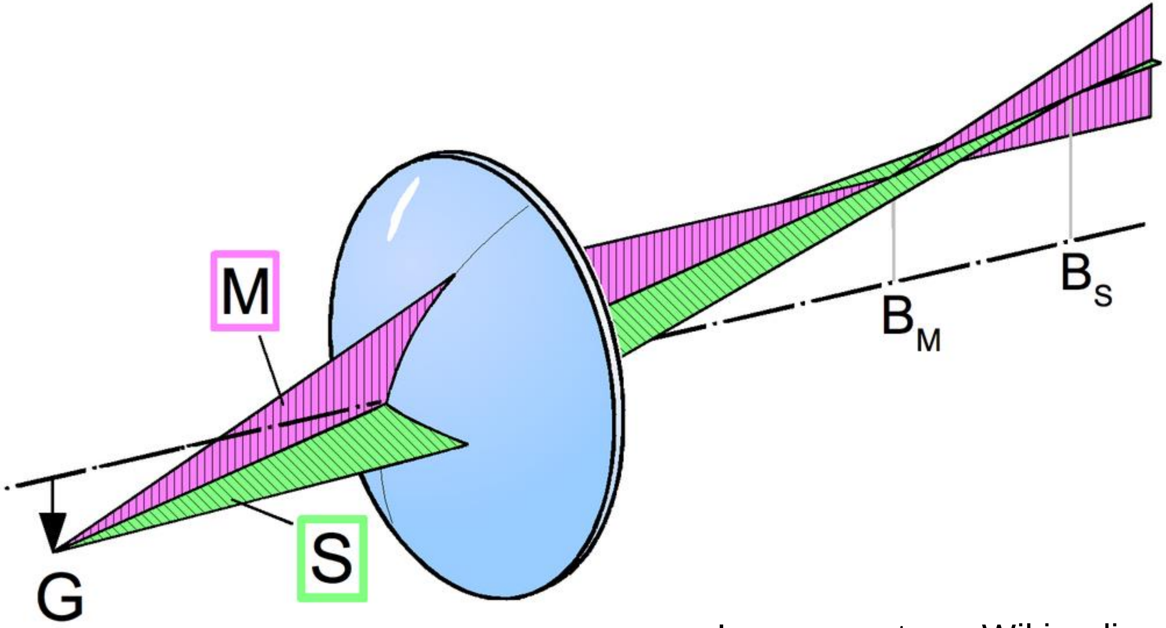
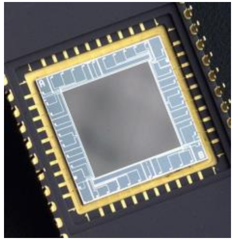


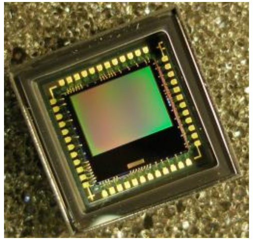
Image courtesy: Wikipedia

Digital Camera

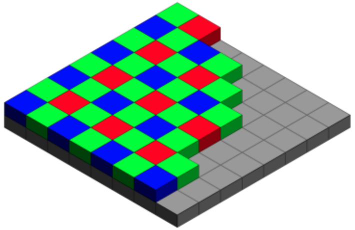
CCD Sensor



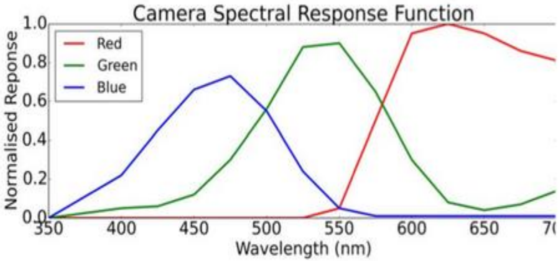
CMOS Sensor



Bayer Mosaic



Camera spectral response



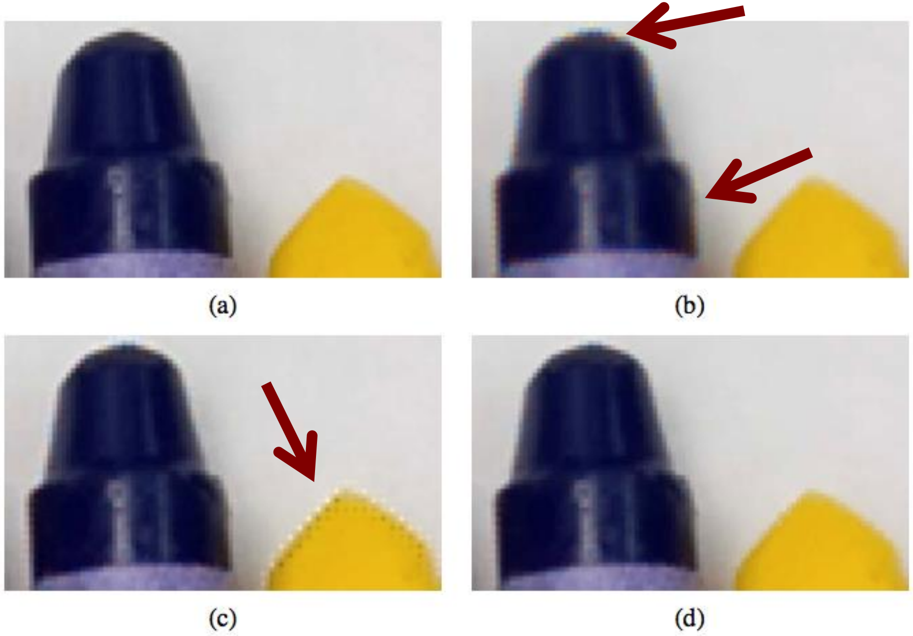
Typical bayer camera spectral response functions (Sony/Prosilica 12B CCD)

The sensor of a camera triggers an electrical response when light is incident on it – Photoelectric Effect

Colour is detected through band pass filters – using separate sensors for each colour or through a Bayer Mosaic

Demosaicing

Interpolating the missing color values to obtain RGB values for all the pixels is called **demosaicing**



(a) original full-resolution image

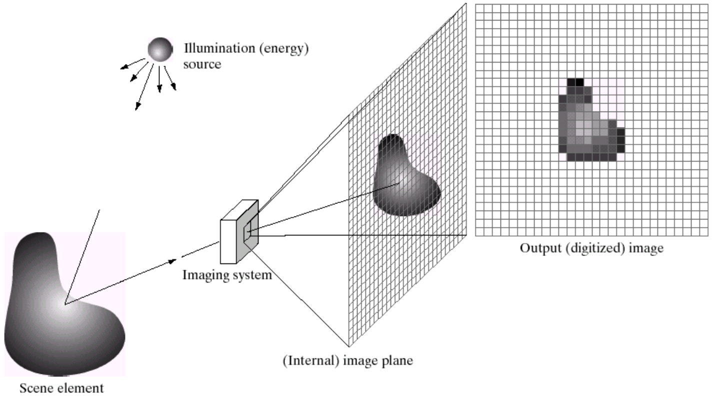
(b) bilinear interpolation

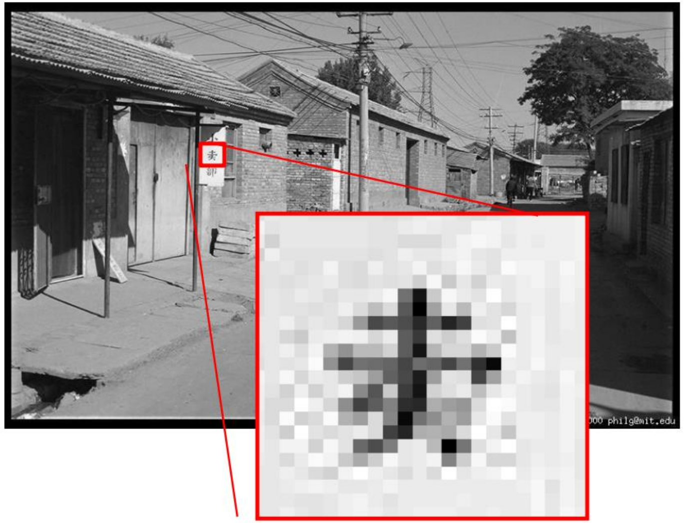
(c) the high-quality linear interpolation

(d) using the local two-color prior

Image Courtesy: Szeliski

Digital Image: Pixel Matrix



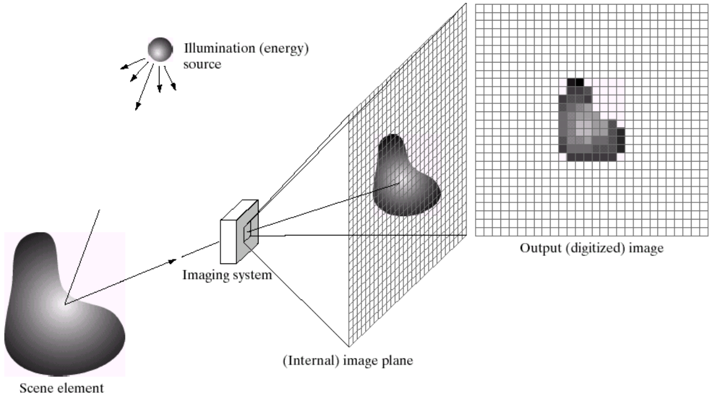


Digitize
r

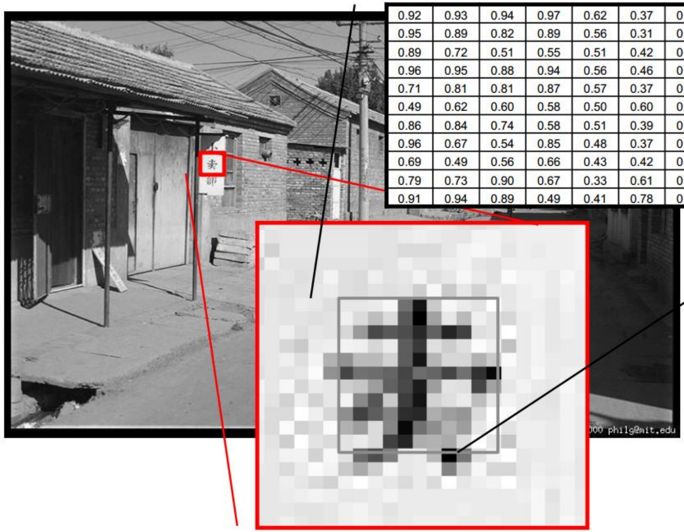
An Analog to digital convertor digitizes (discretizes) the image into pixels

Courtesy Prof. Stefan B. Williams

Digital Image: Pixel Matrix



Digitize
r

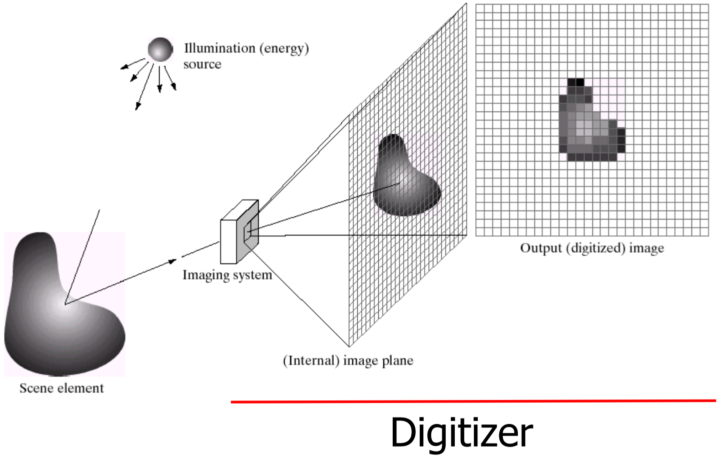


0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

Each pixel is mathematically represented as a number – corresponding to the intensity of the incident light

Courtesy Prof. Stefan B. Williams

Digital Image – Pixel Matrix



For a colour image – 3 separate matrices of pixels, one for each colour

Courtesy Prof. Stefan B. Williams

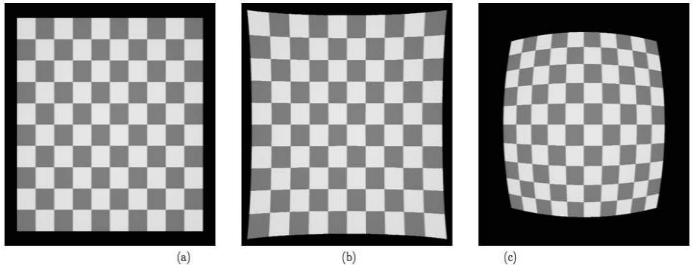
Calibration

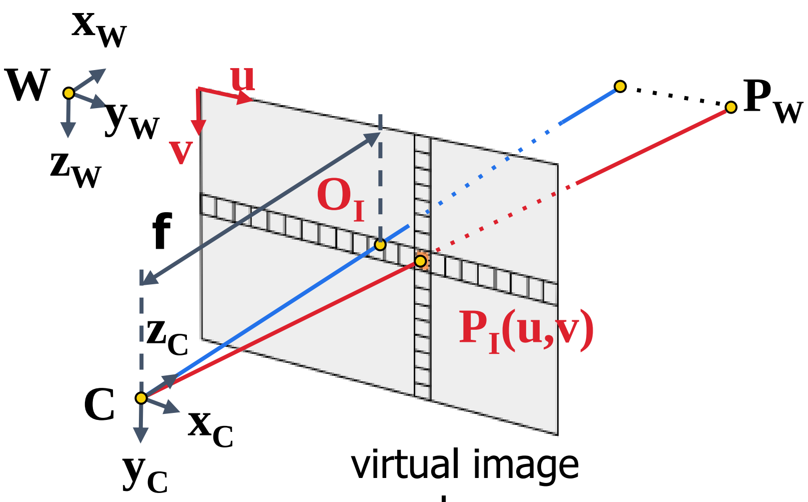
Intrinsic Calibration: Determine the internal parameters of the camera (with lens)

Extrinsic Calibration: Determine the pose of the camera with respect to some reference frame

Intrinsic Parameters:

- Focal length: **f**
- Pixel size: $\mathbf{s_x, s_y}$
- Image centre: $\mathbf{u_0, v_0}$
- Distortion parameters: $\mathbf{k_1, k_2, k_3, r_1, r_2}$





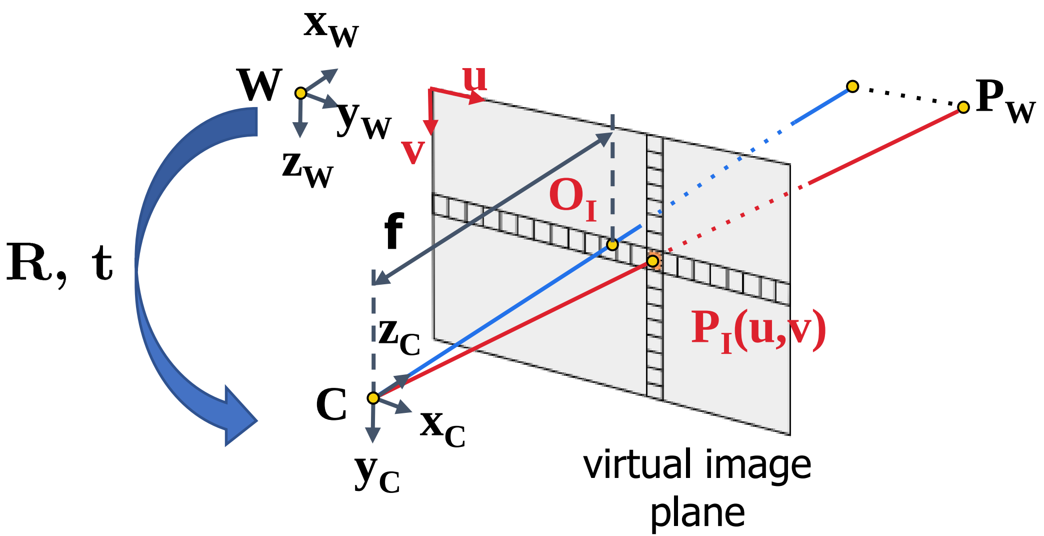
Calibration

Intrinsic Calibration: Determine the internal parameters of the camera (with lens)

Extrinsic Calibration: Determine the pose of the camera with respect to some reference frame

Extrinsic Parameters:

- Position of the camera: Translation, T = [t_x, t_y, t_z]
- Orientation of the camera: Rotation Matrix, R (3x3)



Pinhole projection Model

Intrinsics matrix

Camera model

Coord. of point in the image

$$\begin{pmatrix} su \\ sv \\ s \end{pmatrix} = \begin{pmatrix} \alpha_u & 0 & u_o & 0 \\ 0 & \alpha_v & v_o & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} R & T \\ 0 & 1 \end{pmatrix} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}$$

Coord. of point in "world" frame

s = depth of 3D point in scene (scale factor)

Model parameters

$$\alpha_u = k_u f$$
$$\alpha_v = k_v f$$

Transf. between camera frame and "world" frame

Extrinsics

[Devy 2003]

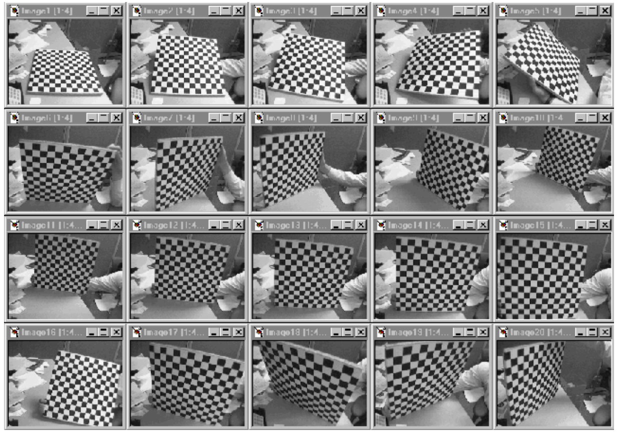
How to determine these parameters?

Calibration

How to determine the model parameters?

Points in the world → Camera model and parameters → Points in the image

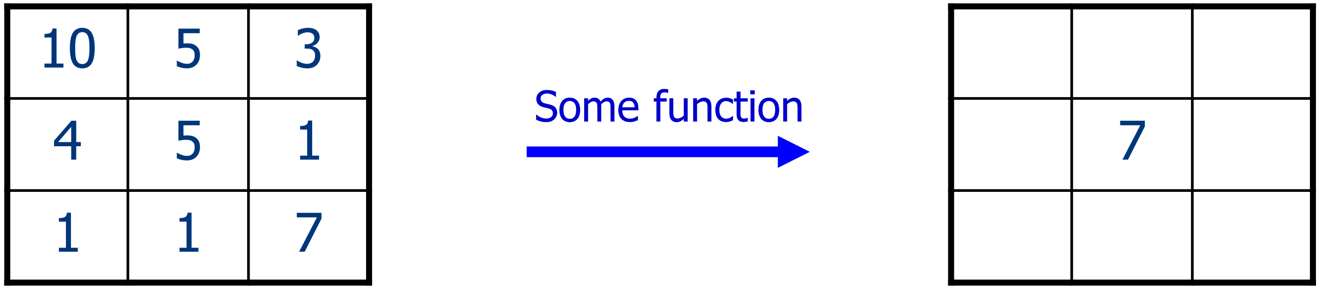
Designed object



Ok I got my picture. What now?

Filtering

Modify pixels in an image based on some function over a local neighbourhood



Filtering

Linear Filtering

This function is a linear function

- Replace each pixel by a linear combination of its neighbours: weighted averaging
- The prescription/function for the linear combination is called the mask or convolutional kernel

10	5	3
4	5	1
1	1	7

$\otimes$

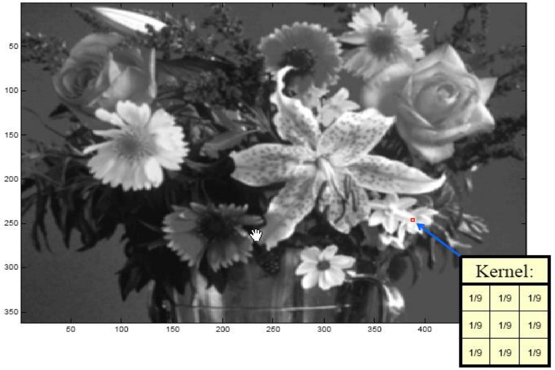
0	0	0
0	0.5	0
0	1.0	0.5

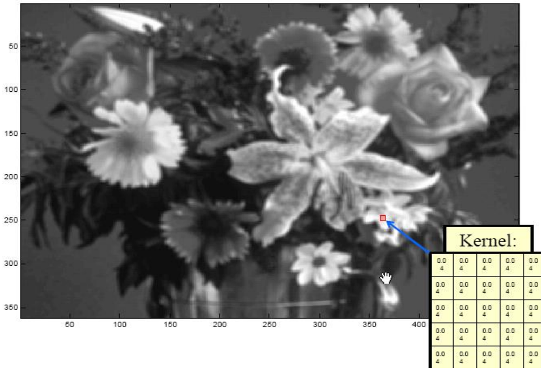
kernel

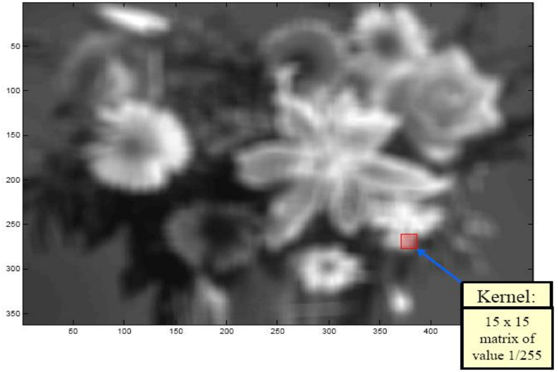
$=$

	7	

Blurring - With a uniform kernel

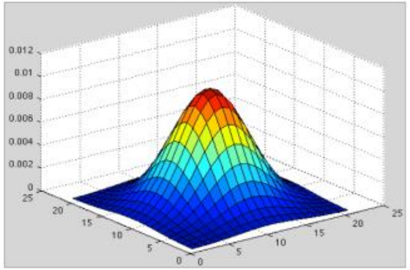




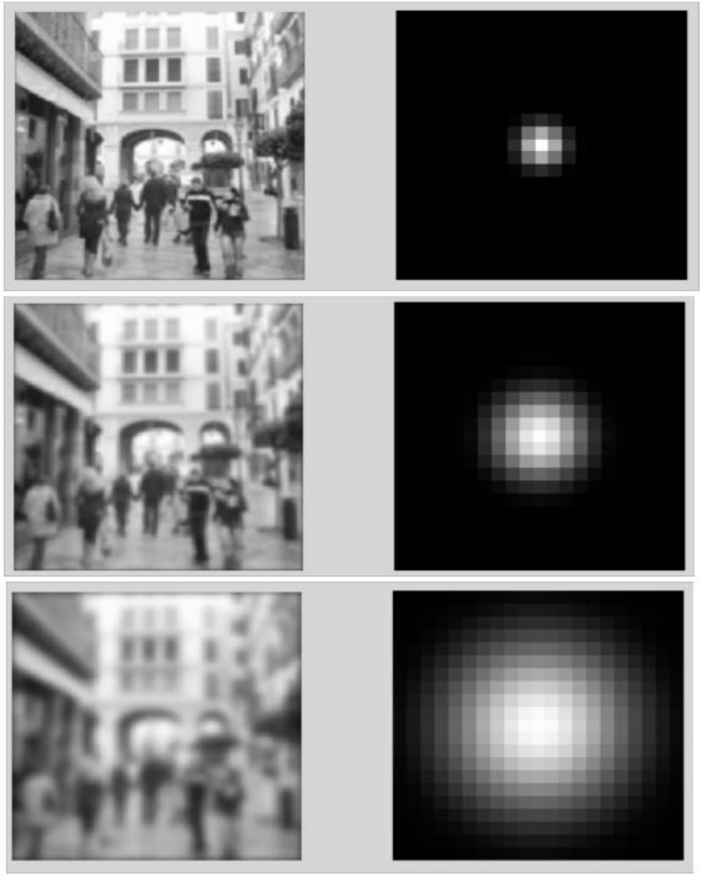


Blurring - With a Gaussian kernel

$$G(x, y; \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



Remove the high frequency information.



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Edge detection - With a Gaussian kernel

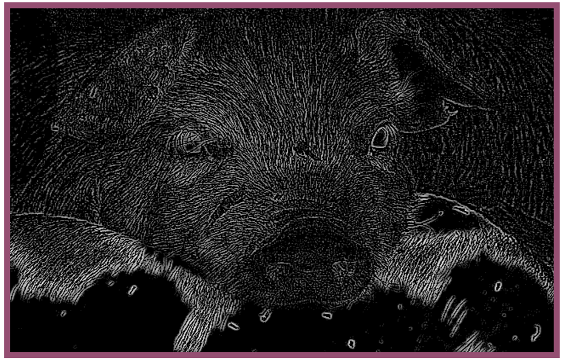
Original Image

−

Gaussian Blurred Image

=

Edges



Features

A feature is a region (patch, point, line) that is salient – representing a characteristic part of the image

Image from the KITTI dataset

Edge Features – Sobel filter

1	0	-1
2	0	-2
1	0	-1

Vertical
Sobel Kernel

Vertical Edges

Edge Features – Sobel filter

1	2	1
0	0	0
-1	-2	-1

Horizontal Sobel Kernel

Horizontal Edges

Edge Features – Sobel filter

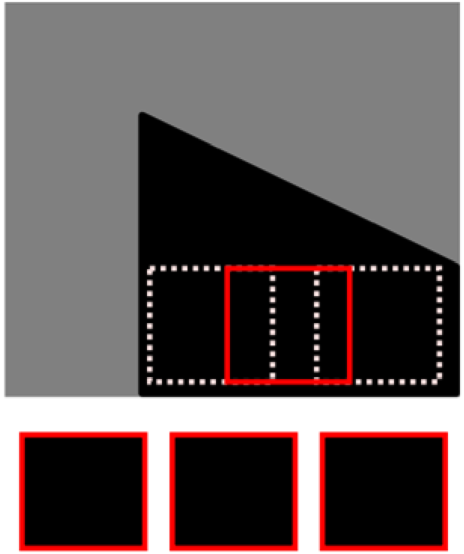
$$\mathbf{G} = \sqrt{{\mathbf{G}_x}^2 + {\mathbf{G}_y}^2}$$

All Edges

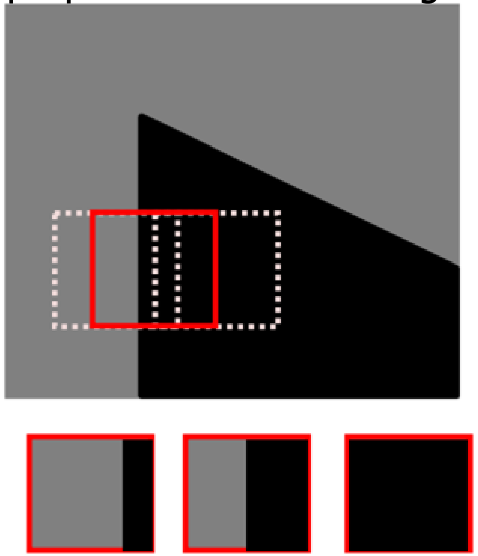
Corner Features – Harris Corner Detector

Intuition: In a region around the corner the intensity gradient changes in two directions

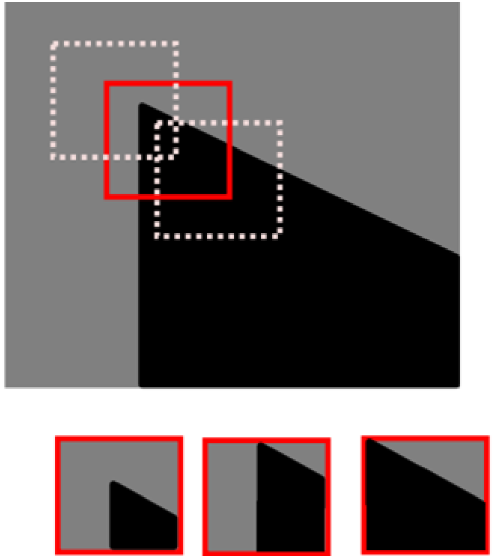
Flat: all patches look similar



Edge: patches change perpendicular to the edge



Corner: All patches look different



Corner Features – Harris Corner Detector

Corner Features – Harris Corner Detector

Change in intensity for a shift of (u, v)

$$E(u,v) = \sum_{x,y \in W} [I(x+u, y+v) - I(x,y)]^2$$

Window

Intensity at shifted location

Intensity at current location

We want to find regions where the $E(u, v)$ is large.

$$I(x+u, y+v) = I(x,y) + u\,I_x(x,y) + v\,I_y(x,y)$$ First Order Taylor's expansion

Corner Features – Harris Corner Detector

Corner Features – Harris Corner Detector

Change in intensity for a shift of (u, v)

$$E(u,v) = \sum_{x,y \in W} [\, I(x+u, y+v) - I(x,y) \,]^2$$

$$= \sum [\, I(x,y) + u\, I_x(x,y) + v\, I_y(x,y) - I(x,y) \,]^2$$

$$= \sum [\, u^2 I_x{}^2 + 2uvI_xI_y + v^2 I_y{}^2 \,]$$

$$= \sum [u\ v] \begin{bmatrix} I_x{}^2 & I_{xy} \\ I_{xy} & I_y{}^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = [u\ v] \begin{bmatrix} \sum I_x{}^2 & \sum I_{xy} \\ \sum I_{xy} & \sum I_y{}^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

Features

Corner Features – Harris Corner Detector

Corners from the Structure Tensor

$$C = \begin{bmatrix} \sum {I_x}^2 & \sum I_{xy} \\ \sum I_{xy} & \sum {I_y}^2 \end{bmatrix}$$

Structure Tensor

Compute Eigenvalues of this matrix

Flat: λ_1, λ_2 are small

Edge: $\lambda_1 \gg \lambda_2$ or $\lambda_2 \gg \lambda_1$

Corner: λ_1, λ_2 are large, $\lambda_1 \sim \lambda_2$

Light detection and ranging (lidar)

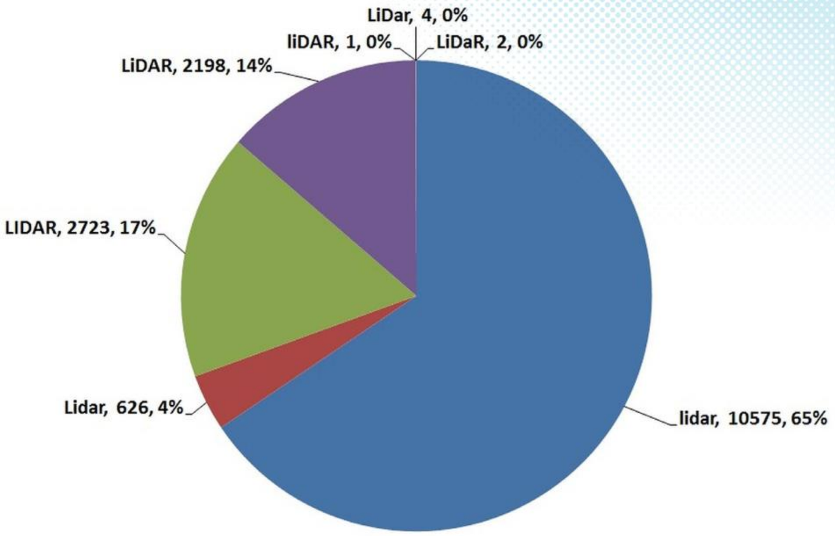
Lidar sensors

Light **d**etection **a**nd **r**anging

Method for determining ranges by targeting an object or a surface with a laser and measuring the time for the reflected light to return to the receiver

Typical wavelength is 905 nm Class 1 (safe)

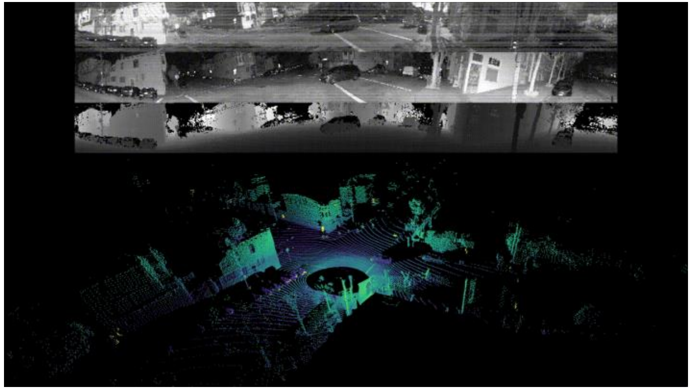
Lidar, LiDAR, LIDAR, ???



There is no consensus. I'll just call it "lidar" in this course because that's what's written on the vocabulary and that is how it should be called.

"Let's Agree on the Casing of Lidar" (**link**)

Principle Of Operation



- Pulsed light is emitted and received by the sensor
- The time of flight for each pulse indicates the distance from the object for that pulse
- Multi-beam lidars are the most common
- Solid state lidars are down the market

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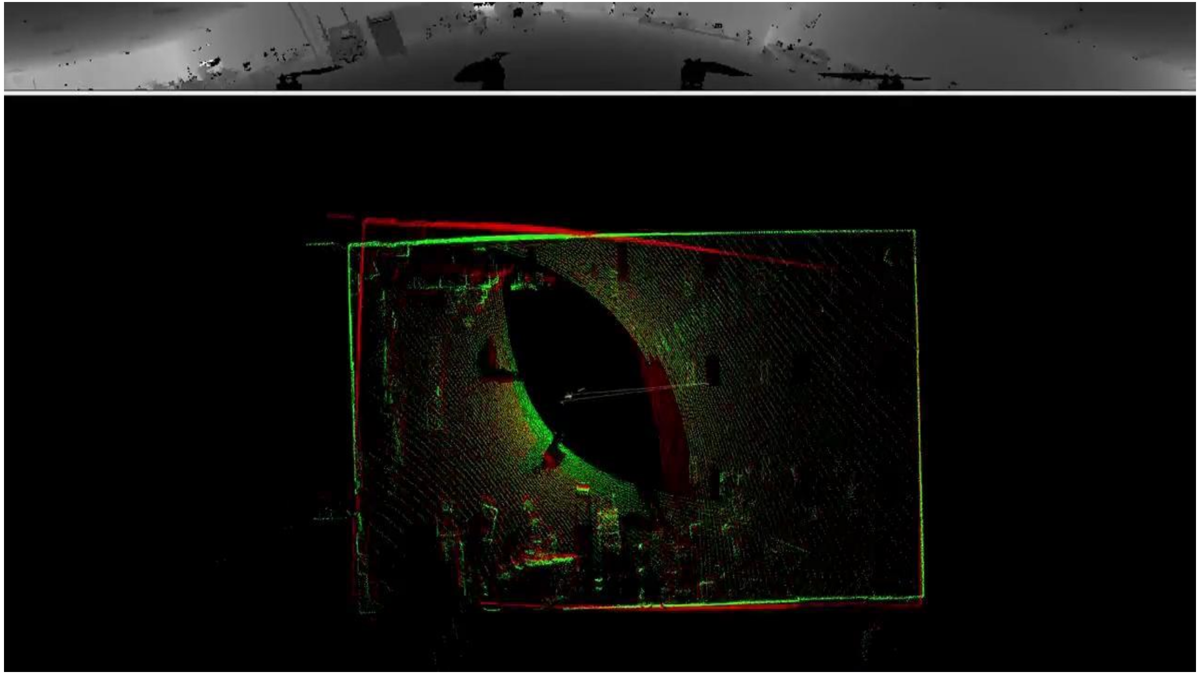
The problem of motion compensation

- Typical lidars emit one point cloud with 300k per each revolution
- This is at 10 Hz
- The position of the sensors might NOT be the same at start and end of the scan
- Luckily, each point is timestamped, so if we have a motion prior we can "correct" the point cloud



Global Navigation Satellite Systems (GNSS)

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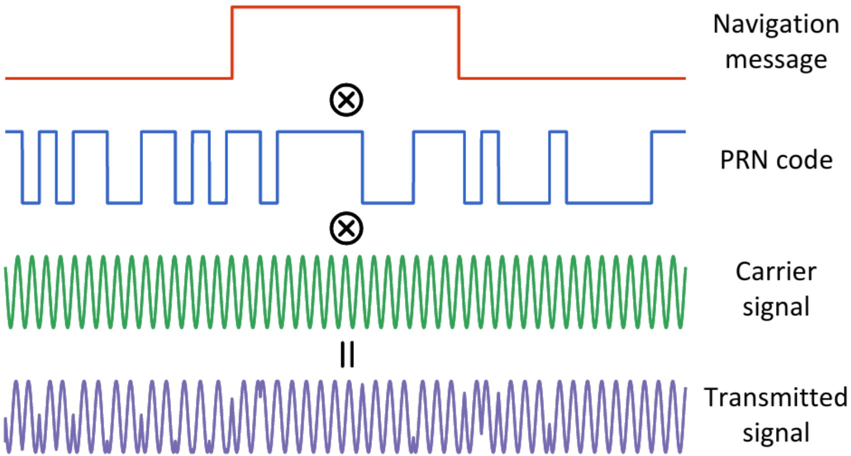
Global Navigation Satellyte System (GNSS)

GPS is GNSS
GNSS is NOT GPS

Examples of Global Navigation Satellite Systems: GPS, Galileo, GLONASS, Baidu etc.

Satellite signals

- **Navigation data** ~ 50 Hz
- **Code** ~ 1 MHz (~ 300 m)
- **Carrier** ~ 1-2 GHz (~ 20 cm)
- **Transmitted signal** - multiplication



Sources of error

- Electromagnetic noise
- Clock drift
- Imprecise satellite orbits
- Ionospheric delay
- Multipath effect