Fundamentals of Artificial Intelligence

Laboratory

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Academic Year 2022/2023

Exercise 9.1

- Consider the following axioms:
  1. If something is intelligent, it has common sense
  2. Deep Blue does not have common sense
  3. (Conclusion) Deep Blue is not intelligent

Exercise 9.1

- Resolution

1. If something is intelligent, it has common sense
   $\forall x\ (I(x) \Rightarrow H(x))$

2. Deep Blue does not have common sense
   $\neg H(D)$

3. (Conclusion) Deep Blue is not intelligent
   $\neg I(D)$

Exercise 9.1

- Resolution

1. If something is intelligent, it has common sense
   $\forall x\ (I(x) \Rightarrow H(x))$
   $\boldsymbol{\neg I(x) \vee H(x)}$

2. Deep Blue does not have common sense
   $\boldsymbol{\neg H(D)}$

3. (Conclusion) Deep Blue is not intelligent
   $\neg I(D)$
   $\boldsymbol{I(D)}$ (negated conclusion)

Exercise 9.1

- Resolution

1. If something is intelligent, it has common sense
   $\forall x\ (I(x) \Rightarrow H(x))$
   $\mathbf{\neg I(x) \lor H(x)}$

2. Deep Blue does not have common sense
   $\mathbf{\neg H(D)}$

3. (Conclusion) Deep Blue is not intelligent
   $\neg I(D)$
   $\mathbf{I(D)}$ (negated conclusion)

4. [1, 2] $\neg I(D)$ $\{x/D\}$

Exercise 9.1

- Resolution

1. If something is intelligent, it has common sense
   $\forall x\ I(x) \Rightarrow H(x)$
   $\mathbf{\neg I(x) \lor H(x)}$

2. Deep Blue does not have common sense
   $\mathbf{\neg H(D)}$

3. (Conclusion) Deep Blue is not intelligent
   $\neg I(D)$
   $\mathbf{I(D)}$ (negated conclusion)

4. [1, 2] $\neg I(D)$ $\{x/D\}$

5. [3, 4] $\square$

Exercise 9.2

- Consider the following axioms:

  1. Mother(Lulu, Fifi)
  2. Alive(Lulu)
  3. $\forall x \, \forall y \, (Mother(x,y) \Rightarrow Parent(x,y))$
  4. $\forall x \, \forall y \, ((Parent(x,y) \land Alive(x)) \Rightarrow Older(x,y))$
  5. (Conclusion) Older(Lulu, Fifi)

Exercise 9.2

- Consider the following axioms:

1. **Mother(Lulu, Fifi)**

2. **Alive(Lulu)**

3. $\forall x\ \forall y\ (\text{Mother}(x,y) \Rightarrow \text{Parent}(x,y))$
   $\neg$Mother(x,y) $\lor$ Parent(x,y)

4. $\forall x\ \forall y\ ((\text{Parent}(x,y) \land \text{Alive}(x)) \Rightarrow \text{Older}(x,y))$
   $\neg$Parent(x,y) $\lor$ $\neg$Alive(x) $\lor$ Older(x,y)

5. (Conclusion) Older(Lulu, Fifi)
   $\neg$Older(Lulu, Fifi)

Exercise 9.2

- Consider the following axioms:

1. **Mother(Lulu, Fifi)**
2. **Alive(Lulu)**
3. $\forall x \forall y$ (Mother(x,y) $\Rightarrow$ Parent(x,y))
   ¬Mother(x,y) ∨ Parent(x,y)
4. $\forall x \forall y$ ((Parent(x,y) $\wedge$ Alive(x)) $\Rightarrow$ Older(x,y))
   ¬Parent(x,y) ∨ ¬Alive(x) ∨ Older(x,y)
5. (Conclusion) Older(Lulu, Fifi)
   ¬Older(Lulu, Fifi)
6. [1, 3] Parent(Lulu,Fifi) {x/Lulu, y/Fifi}

Exercise 9.2

- Consider the following axioms:

1. **Mother(Lulu, Fifi)**
2. **Alive(Lulu)**
3. ∀x ∀y (Mother(x,y) ⇒ Parent(x,y))
   ¬Mother(x,y) ∨ Parent(x,y)
4. ∀x ∀y ((Parent(x,y) ∧ Alive(x)) ⇒ Older(x,y))
   ¬Parent(x,y) ∨ ¬Alive(x) ∨ Older(x,y)
5. (Conclusion) Older(Lulu, Fifi)
   ¬Older(Lulu, Fifi)
6. [1, 3] Parent(Lulu,Fifi) {x/Lulu, y/Fifi}
7. [4, 6] ¬Alive(Lulu) ∨ Older(Lulu, Fifi) {x/Lulu, y/Fifi}

Exercise 9.2

- Consider the following axioms:

1. **Mother(Lulu, Fifi)**
2. **Alive(Lulu)**
3. $\forall x\ \forall y\ (\text{Mother}(x,y) \Rightarrow \text{Parent}(x,y))$
   ¬Mother(x,y) ∨ Parent(x,y)
4. $\forall x\ \forall y\ ((\text{Parent}(x,y) \wedge \text{Alive}(x)) \Rightarrow \text{Older}(x,y))$
   ¬Parent(x,y) ∨ ¬Alive(x) ∨ Older(x,y)
5. (Conclusion) Older(Lulu, Fifi)
   ¬Older(Lulu, Fifi)
6. [1, 3] Parent(Lulu,Fifi) {x/Lulu, y/Fifi}
7. [4, 6] ¬Alive(Lulu) ∨ Older(Lulu, Fifi) {x/Lulu, y/Fifi}
8. [2, 7] Older(Lulu, Fifi)

Exercise 9.2

- Consider the following axioms:

1. **Mother(Lulu, Fifi)**
2. **Alive(Lulu)**
3. ∀x ∀y (Mother(x,y) ⇒ Parent(x,y))
   ¬Mother(x,y) ∨ **Parent(x,y)**
4. ∀x ∀y ((Parent(x,y) ∧ Alive(x)) ⇒ Older(x,y))
   ¬Parent(x,y) ∨ **¬Alive(x)** ∨ **Older(x,y)**
5. (Conclusion) Older(Lulu, Fifi)
   ¬Older(Lulu, Fifi)
6. [1, 3] Parent(Lulu,Fifi) {x/Lulu, y/Fifi}
7. [4, 6] ¬Alive(Lulu) ∨ Older(Lulu, Fifi) {x/Lulu, y/Fifi}
8. [2, 7] Older(Lulu, Fifi)
9. [5, 8] □

Exercise 9.3

- Axioms:

1. $Allergies(x) \Rightarrow Sneeze(x)$
2. $Cat(y) \wedge AllergicToCats(x) \Rightarrow Allergies(x)$
3. $Cat(Felix)$
4. $AllergicToCats(Mary)$
5. (Conclusion) $Sneeze(Mary)$

Exercise 9.3

- Axioms:

1. Allergies(x) ⇒ Sneeze(x)
   ¬Allergies(w) v Sneeze(w)

2. Cat(y) ∧ AllergicToCats(x) ⇒ Allergies(x)
   ¬Cat(y) v ¬AllergicToCats(z) ∨ Allergies(z)

3. **Cat(Felix)**

4. **AllergicToCats(Mary)**

5. (Conclusion) Sneeze(Mary)
   ¬Sneeze(Mary)

Exercise 9.3

- Axioms:

1. ¬Allergies(w) v Sneeze(w)
2. ¬Cat(y) v ¬AllergicToCats(z) v Allergies(z)
3. Cat(Felix)
4. AllergicToCats(Mary)
5. (Conclusion) ¬Sneeze(Mary)
6. [1, 2] ¬Cat(y) v Sneeze(z) v ¬AllergicToCats(z) {w/z}

Exercise 9.3

- Axioms:

1. ¬Allergies(w) v Sneeze(w)
2. ¬Cat(y) v ¬AllergicToCats(z) v Allergies(z)
3. Cat(Felix)
4. AllergicToCats(Mary)
5. (Conclusion) ¬Sneeze(Mary)
6. [1, 2] ¬Cat(y) v Sneeze(z) v ¬AllergicToCats(z) {w/z}
7. [3, 6] Sneeze(z) v ¬AllergicToCats(z) {y/Felix}

Exercise 9.3

- Axioms:
  1. ¬Allergies(w) ∨ Sneeze(w)
  2. ¬Cat(y) ∨ ¬AllergicToCats(z) ∨ Allergies(z)
  3. Cat(Felix)
  4. AllergicToCats(Mary)
  5. (Conclusion) ¬Sneeze(Mary)
  6. [1, 2] ¬Cat(y) ∨ Sneeze(z) ∨ ¬AllergicToCats(z) {w/z}
  7. [3, 6] Sneeze(z) ∨ ¬AllergicToCats(z) {y/Felix}
  8. [4, 7] Sneeze(Mary) {z/Mary}

Exercise 9.3

- Axioms:

1. ¬Allergies(w) v Sneeze(w)
2. ¬Cat(y) v ¬AllergicToCats(z) v Allergies(z)
3. Cat(Felix)
4. AllergicToCats(Mary)
5. (Conclusion) ¬Sneeze(Mary)
6. [1, 2] ¬Cat(y) v Sneeze(z) v ¬AllergicToCats(z) {w/z}
7. [3, 6] Sneeze(z) v ¬AllergicToCats(z) {y/Felix}
8. [4, 7] Sneeze(Mary) {z/Mary}
9. [5, 8] □

Exercise 9.6

Consider the following set of clauses, build a refutation using the Hyper-Resolution strategy:

(1) $Q(c) \vee R(b) \vee T(e)$

(2) $\neg Q(c) \vee T(e) \vee \neg S(d)$

(3) $Q(c) \vee S(d)$

(4) $S(d) \vee T(e)$

(5) $\neg T(e) \vee R(b)$

(6) $\neg R(b) \vee \neg P(a)$

(7) $R(b) \vee P(a)$

(8) $P(a)$

Exercise 9.6

Consider the following set of clauses, build a refutation using the Hyper-Resolution strategy:

(1) Q(c) ∨ R(b) ∨ T(e)

(2) ¬Q(c) ∨ T(e) ∨ ¬S(d)

(3) Q(c) ∨ S(d)

(4) S(d) ∨ T(e)

(5) ¬T(e) ∨ R(b)

(6) ¬R(b) ∨ ¬P(a)

(7) R(b) ∨ P(a)

(8) P(a)

(9) [1,2] R(b) ∨ T(e) ∨ ¬S(d)

(10) [9,4] R(b) ∨ T(e)

(11) [10,5] R(b)

(12) [6,8] ¬R(b)

(13) [11,12] {}

Exercise 9.7

Consider the following set of clauses, build a refutation using the Hyper-Resolution strategy:

(1) $P(a) \lor Q(c) \lor R(b)$

(2) $\neg U(a) \lor R(b)$

(3) $\neg P(a) \lor R(b) \lor T(e)$

(4) $\neg Q(c) \lor T(e) \lor \neg S(d)$

(5) $\neg R(b) \lor U(a)$

(6) $S(d) \lor T(e)$

(7) $\neg T(e) \lor R(b)$

(8) $\neg R(b)$

Exercise 9.7

Consider the following set of clauses, build a refutation using the Hyper-Resolution strategy:

(1) P(a) v Q(c) v R(b)

(2) ¬U(a) v R(b)

(3) ¬P(a) v R(b) v T(e)

(4) ¬Q(c) v T(e) v ¬S(d)

(5) ¬R(b) v U(a)

(6) S(d) v T(e)

(7) ¬T(e) v R(b)

(8) ¬R(b)

(9) [1,3] Q(c) v R(b) v T(e)

(10) [9,4] R(b) v T(e) v ¬S(d)

(11) [10,6] R(b) v T(e)

(12) [11,7] R(b)

(13) [8,12] {}

Exercise 9.8

Consider the following set of clauses, build a refutation using the Hyper-Resolution strategy:

(1) Q(c) ∨ S(b) ∨ T(e)

(2) ¬Q(c) ∨ T(e) ∨ ¬U(d)

(3) Q(c) ∨ U(d)

(4) U(d) ∨ T(e)

(5) ¬T(e) ∨ S(b)

(6) ¬S(b) ∨ ¬R(a)

(7) S(b) ∨ R(a)

(8) R(a)

Exercise 9.8

Consider the following set of clauses, build a refutation using the Hyper-Resolution strategy:

(1) Q(c) ∨ S(b) ∨ T(e)

(2) ¬Q(c) ∨ T(e) ∨ ¬U(d)

(3) Q(c) ∨ U(d)

(4) U(d) ∨ T(e)

(5) ¬T(e) ∨ S(b)

(6) ¬S(b) ∨ ¬R(a)

(7) S(b) ∨ R(a)

(8) R(a)

(9) [1,2] S(b) ∨ T(e) ∨ ¬U(d)

(10) [9,4] S(b) ∨ T(e)

(11) [10,5] S(b)

(12) [6,8] ¬S(b)

(13) [11,12] {}

Exercise 9.4

- Consider the following axioms:
  (predicates to use: HOUND, HOWL, HAVE, CAT, MOUSE, LS)

  1. All hounds howl at night.
  2. Anyone who has any cats will not have any mice.
  3. Light sleepers do not have anything which howls at night.
  4. John has either a cat or a hound.
  5. (Conclusion) If John is a light sleeper, then John does not have any mice.

Exercise 9.4

- The conclusion can be proved using Resolution as shown in the next slides.
- The first step is to write each axiom as a well-formed formula in first-order predicate calculus. The clauses written for the above axioms are shown below, using LS(x) for "light sleeper".

1. All hounds howl at night.
   $\forall x\ (\mathbf{HOUND}(x) \Rightarrow \mathbf{HOWL}(x))$
2. Anyone who has any cats will not have any mice.
   $\forall x\ \forall y\ (\mathbf{HAVE}(x,y) \wedge \mathbf{CAT}(y) \Rightarrow \neg\exists z\ (\mathbf{HAVE}(x,z) \wedge \mathbf{MOUSE}(z)))$
3. Light sleepers do not have anything which howls at night.
   $\forall x\ (\mathbf{LS}(x) \Rightarrow \neg\exists y\ (\mathbf{HAVE}(x,y) \wedge \mathbf{HOWL}(y)))$
4. John has either a cat or a hound.
   $\exists x\ (\mathbf{HAVE}(\mathbf{John},x) \wedge (\mathbf{CAT}(x) \vee \mathbf{HOUND}(x)))$
5. (Conclusion) If John is a light sleeper, then John does not have any mice.
   $\mathbf{LS}(\mathbf{John}) \Rightarrow \neg\exists z\ (\mathbf{HAVE}(\mathbf{John},z) \wedge \mathbf{MOUSE}(z))$

Exercise 9.4

- The next step is to transform each well-formed formula into Prenex Normal Form, skolemize, and rewrite as clauses in conjunctive normal form.

1. All hounds howl at night.

   $\forall x \ (HOUND(x) \Rightarrow HOWL(x))$

   $\boldsymbol{\neg HOUND(x) \lor HOWL(x)}$

Exercise 9.4

- The next step is to transform each wff into Prenex Normal Form, skolemize, and rewrite as clauses in conjunctive normal form; these transformations are shown below.

2. Anyone who has any cats will not have any mice.

$\forall x\ \forall y\ (HAVE(x,y) \wedge CAT(y) \Rightarrow \neg\exists z\ (HAVE(x,z) \wedge MOUSE(z)))$

$\forall x\ \forall y\ (HAVE\ (x,y) \wedge CAT\ (y) \Rightarrow \forall z\ \neg(HAVE(x,z) \wedge MOUSE\ (z)))$

$\forall x\ \forall y\ \forall z\ (\neg(HAVE\ (x,y) \wedge CAT\ (y)) \vee \neg(HAVE(x,z) \wedge MOUSE\ (z)))$

$\mathbf{\neg HAVE(x,y) \vee \neg CAT(y) \vee \neg HAVE(x,z) \vee \neg MOUSE(z)}$

Exercise 9.4

- The next step is to transform each wff into Prenex Normal Form, skolemize, and rewrite as clauses in conjunctive normal form; these transformations are shown below.

3. Light sleepers do not have anything which howls at night.

$\forall x\ (LS(x) \Rightarrow \neg\exists y\ (HAVE(x,y) \wedge HOWL(y)))$

$\forall x\ (LS(x) \Rightarrow \forall y\ \neg(HAVE\ (x,y) \wedge HOWL(y)))$

$\forall x\ \forall y\ (LS(x) \Rightarrow \neg HAVE(x,y) \vee \neg HOWL(y))$

$\forall x\ \forall y\ (\neg LS(x) \vee \neg HAVE(x,y) \vee \neg HOWL(y))$

$\boldsymbol{\neg LS(x) \vee \neg HAVE(x,y) \vee \neg HOWL(y)}$

Exercise 9.4

- The next step is to transform each wff into Prenex Normal Form, skolemize, and rewrite as clauses in conjunctive normal form; these transformations are shown below.

4. John has either a cat or a hound.

$\exists x\ (HAVE(John,x) \wedge (CAT(x) \vee HOUND(x)))$

$\mathbf{HAVE(John,a) \wedge (CAT(a) \vee HOUND(a))}$

Exercise 9.4

- The next step is to transform each wff into Prenex Normal Form, skolemize, and rewrite as clauses in conjunctive normal form; these transformations are shown below.

5. (Conclusion) If John is a light sleeper, then John does not have any mice.

   $LS(John) \Rightarrow \neg\exists z\ (HAVE(John,z) \land MOUSE(z))$

   $\neg[LS(John) \Rightarrow \neg\exists z\ (HAVE(John,z) \land MOUSE(z))]$ (negated conclusion)

   $\neg[\neg LS(John) \lor \neg\exists z\ (HAVE(John, z) \land MOUSE(z))]$

   $LS(John) \land \exists z\ (HAVE(John, z) \land MOUSE(z)))$

   $\mathbf{LS(John) \land HAVE(John,b) \land MOUSE(b)}$

Exercise 9.4

- The set of CNF clauses for this problem is thus as follows:
    1. ¬HOUND(x) ∨ HOWL(x)
    2. ¬HAVE(x,y) ∨ ¬CAT(y) ∨ ¬HAVE(x,z) ∨ ¬MOUSE(z)
    3. ¬LS(x) ∨ ¬HAVE(x,y) ∨ ¬HOWL(y)
    4.
        a. HAVE(John,a)
        b. CAT(a) ∨ HOUND(a)
    5.
        a. LS(John)
        b. HAVE(John,b)
        c. MOUSE(b)