FAI LAB 9
FOL inference

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Reasoning on FOL: Substitutions

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- $\alpha\theta$ is the formula obtained by applying θ to α

e.g. $\alpha = (SurvivedPhD(x) \wedge Coauthor(x, y)) \qquad \theta = \{x/Paolo\}$

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- **Equal-term / equivalent-formula** substitution rules:

$$\frac{\Gamma \wedge (t_1 = t_2) \wedge \alpha}{\Gamma \wedge (t_1 = t_2) \wedge \alpha\{t_1/t_2\}} \qquad \frac{\Gamma \wedge (\beta_1 \leftrightarrow \beta_2) \wedge \alpha}{\Gamma \wedge (\beta_1 \leftrightarrow \beta_2) \wedge \alpha\{\beta_1/\beta_2\}}$$

Reasoning on FOL: Instantiations

Universal (UI)

$$\frac{\Gamma \wedge \forall x \,.\, \alpha}{\Gamma \wedge \forall x \,.\, \alpha \wedge \alpha\{x/t\}}$$

- the result is **equivalent**
- t is an arbitrary term
- can be applied multiple times

Existential (EI)

$$\frac{\Gamma \wedge \exists x \,.\, \alpha}{\Gamma \wedge \alpha\{x/C\}}$$

- the result is **not equivalent** (but SAT-preserving)
- C is a fresh (Skolem) constant
- can be applied only once

Before applying both, convert α in NNF!

Reasoning on FOL: Generalized Modus Ponens (GMP)

"Socrates is a man. All men are mortals. Therefore, Socrates is mortal."

$$Man(Socrates) \land \forall x \,.\, (Man(x) \to Mortal(x)) \models Mortal(Socrates)$$

GMP If there exists θ s.t. $\alpha_i\theta = \alpha_i'\theta$ for all i, then:

$$\frac{\alpha_1', \ldots, \alpha_n', (\alpha_1 \land \ldots \land \alpha_n) \to \beta}{\beta\theta}$$

- Variables are universally quantified (implicitly)
- Different formulas must use different var names (standardize)
- θ substitutes variables with terms (**unifier**)
- **Most-general unifier (MGU)**, minimal, unique modulo renaming

Reasoning on FOL: unification exercises

$$\text{MGU}(Knows(Alice, PyTorch), Knows(x, PyTorch)) =$$

$$\text{MGU}(Knows(Alice, y), Knows(x, z)) =$$

$$\text{MGU}(Knows(Alice, y), Knows(Bob, z)) =$$

$$\text{MGU}(Knows(Alice, y), Knows(x, F(x))) =$$

Reasoning on FOL: unification exercises

$$\mathrm{MGU}(Knows(Alice, PyTorch), Knows(x, PyTorch)) = \{x/Alice\}$$

$$\mathrm{MGU}(Knows(Alice, y), Knows(x, z)) =$$

$$\mathrm{MGU}(Knows(Alice, y), Knows(Bob, z)) =$$

$$\mathrm{MGU}(Knows(Alice, y), Knows(x, F(x))) =$$

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$$\mathrm{MGU}(Knows(Alice, y), Knows(Bob, z)) = \textit{fail}$$

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$$\mathrm{MGU}(Knows(Alice, PyTorch), Knows(x, PyTorch)) = \{x/Alice\}$$

$$\mathrm{MGU}(Knows(Alice, y), Knows(x, z)) = \{x/Alice, y/z\}$$

$$\mathrm{MGU}(Knows(Alice, y), Knows(Bob, z)) = fail$$

$$\mathrm{MGU}(Knows(Alice, y), Knows(x, F(x))) = \{x/Alice, y/F(Alice)\}$$

Reasoning on FOL: definite clauses

Definite clauses:

- Disjunctions with **exactly one** positive literal

$$\neg T_1 \vee \dots \vee \neg T_n \vee H \quad \equiv \quad (T_1 \wedge \dots \wedge T_n) \rightarrow H$$

- (implicitly) universally quant., no $\exists$ in scope of $\forall$, $\exists x \,.\, \alpha \Rightarrow \alpha\{x/C\}$
- **Datalog KBs**: DC with no function symbols

Reasoning on FOL-DC: forward chain reasoning

function FOL-FC-ASK(KB, α) **returns** a substitution or *false*
  inputs: KB, the knowledge base, a set of first-order definite clauses
          α, the query, an atomic sentence
  local variables: new, the new sentences inferred on each iteration

  repeat until new is empty
    $new \leftarrow \{\}$
    for each $rule$ **in** KB **do**
      $(p_1 \wedge \ldots \wedge p_n \Rightarrow q) \leftarrow$ STANDARDIZE-VARIABLES($rule$)
      for each θ such that SUBST($\theta, p_1 \wedge \ldots \wedge p_n$) = SUBST($\theta, p'_1 \wedge \ldots \wedge p'_n$)
              for some $p'_1, \ldots, p'_n$ in KB
        $q' \leftarrow$ SUBST(θ, q)
        if q' does not unify with some sentence already in KB or new **then**
          add q' to new
          $\phi \leftarrow$ UNIFY(q', α)
          if ϕ is not *fail* **then return** ϕ
    add new to KB
  return *false*

- If $KB \models \alpha$, correct and complete (for DC)
- If $KB \not\models \alpha$, might not terminate
- (**backward**-chain reasoning applies GMP backwards from goals)

Reasoning on FOL-DC: exercises

Try answering the **queries** using forward and backward chain reasoning:

$1. Enrolled(Alice)$

$2. Enrolled(Bob)$

$3. Mandatory(FAI)$

$4. Covers(FAI, Logics)$

$5. Covers(ML, NeuralNets)$

$6. Knows(Bob, NeuralNets)$

$7. Takes(s, c) \leftarrow (Enrolled(s) \wedge Mandatory(c))$

$8. Knows(s, t) \leftarrow (Takes(s, c) \wedge Covers(c, t))$

Q1: $Knows(Alice, Logics)$?

Q2: $Mandatory(ML)$?

Reasoning on FOL: CNFization

- Convert into **NNF** (convert $\rightarrow, \leftrightarrow$, push $\neg$ to the atoms)

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- **Standardize variables** (for each $\forall, \exists$ introduce fresh names)
- **Skolemize** (i.e. remove $\exists$):

$$\exists y \,.\, \alpha \Rightarrow \alpha\{y/C\}$$

$$\forall x_1, ..., x_n \,.\, \exists y \,.\, \alpha \Rightarrow \forall x_1, ..., x_n \,.\, \alpha\{y/F(x_1, ..., x_n)\}$$

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- **Drop universal quantifiers** ($\forall$ now implicit)

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- **CNFize** propositionally (DeMorgan/renaming subformulas)

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- **Standardize variables** (for each $\forall, \exists$ introduce fresh names)
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$$\exists y \,.\, \alpha \Rightarrow \alpha\{y/C\}$$

$$\forall x_1, ..., x_n \,.\, \exists y \,.\, \alpha \Rightarrow \forall x_1, ..., x_n \,.\, \alpha\{y/F(x_1, ..., x_n)\}$$

- **Drop universal quantifiers** ($\forall$ now implicit)
- **CNFize** propositionally (DeMorgan/renaming subformulas)
- **Standardize variables** again (Prof. Sebastiani's suggestion)

Reasoning on FOL: CNFization exercises

- $\exists x \,.\, [\forall y \,.\, Gt(x, y)] \rightarrow \exists x \,.\, [\forall y \,.\, S(x, y, x)]$
- $\neg \exists x \,.\, \exists y \,.\, [\neg(x = y) \wedge Gt(x, y) \wedge Gt(y, x)]$

Reasoning on FOL: CNFization exercises

$$\exists x \,.\, [\forall y \,.\, Gt(x,y)] \rightarrow \exists x \,.\, [\forall y \,.\, S(x,y,x)]$$

$$\exists x \,.\, [\forall y \,.\, Gt(x,y)] \rightarrow \exists x \,.\, [\forall y \,.\, S(x,y,x)]$$

$$\neg\exists x \,.\, [\forall y \,.\, Gt(x,y)] \vee \exists x \,.\, [\forall y \,.\, S(x,y,x)]$$

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$$\neg\exists x \,.\, [\forall y \,.\, Gt(x,y)] \vee \exists x \,.\, [\forall y \,.\, S(x,y,x)]$$

$$\forall x \,.\, [\exists y \,.\, \neg Gt(x,y)] \vee \exists x \,.\, [\forall y \,.\, S(x,y,x)]$$

Reasoning on FOL: CNFization exercises

$$\exists x \,.\, [\forall y \,.\, Gt(x,y)] \rightarrow \exists x \,.\, [\forall y \,.\, S(x,y,x)]$$

$$\neg \exists x \,.\, [\forall y \,.\, Gt(x,y)] \vee \exists x \,.\, [\forall y \,.\, S(x,y,x)]$$

$$\forall x \,.\, [\exists y \,.\, \neg Gt(x,y)] \vee \exists x \,.\, [\forall y \,.\, S(x,y,x)]$$

$$\forall x_1 \,.\, [\exists y_1 \,.\, \neg Gt(x_1,y_1)] \vee \exists x_2 \,.\, [\forall y_2 \,.\, S(x_2,y_2,x_2)]$$

Reasoning on FOL: CNFization exercises

$$\exists x\,.\,[\forall y\,.\,Gt(x,y)] \rightarrow \exists x\,.\,[\forall y\,.\,S(x,y,x)]$$

$$\neg\exists x\,.\,[\forall y\,.\,Gt(x,y)] \vee \exists x\,.\,[\forall y\,.\,S(x,y,x)]$$

$$\forall x\,.\,[\exists y\,.\,\neg Gt(x,y)] \vee \exists x\,.\,[\forall y\,.\,S(x,y,x)]$$

$$\forall x_1\,.\,[\exists y_1\,.\,\neg Gt(x_1,y_1)] \vee \exists x_2\,.\,[\forall y_2\,.\,S(x_2,y_2,x_2)]$$

$$\forall x_1\,.\,[\neg Gt(x_1,F_1(x_1))] \vee \forall y_2\,.\,[S(F_2,y_2,F_2)]$$

Reasoning on FOL: CNFization exercises

$$\exists x \,.\, [\forall y \,.\, Gt(x,y)] \rightarrow \exists x \,.\, [\forall y \,.\, S(x,y,x)]$$

$$\neg\exists x \,.\, [\forall y \,.\, Gt(x,y)] \vee \exists x \,.\, [\forall y \,.\, S(x,y,x)]$$

$$\forall x \,.\, [\exists y \,.\, \neg Gt(x,y)] \vee \exists x \,.\, [\forall y \,.\, S(x,y,x)]$$

$$\forall x_1 \,.\, [\exists y_1 \,.\, \neg Gt(x_1,y_1)] \vee \exists x_2 \,.\, [\forall y_2 \,.\, S(x_2,y_2,x_2)]$$

$$\forall x_1 \,.\, [\neg Gt(x_1,F_1(x_1))] \vee \forall y_2 \,.\, [S(F_2,y_2,F_2)]$$

$$[\neg Gt(x_1,F_1(x_1))] \vee [S(F_2,y_2,F_2)]$$

Resolution rule Let $\theta = MGU(l_i, \neg m_j)$, then:

$$\frac{(l_1 \vee ... \vee l_i \vee ... \vee l_k) \quad (m_1 \vee ... \vee m_j \vee ... \vee m_k)}{(l_1 \vee ... \vee l_{i-1} \vee l_{i+1} \vee ... \vee l_k \vee m_1 \vee ... \vee m_{j-1} \vee m_{j+1} \vee ... \vee m_k)\theta}$$

Reasoning on FOL: resolution

Resolution rule Let $\theta = MGU(l_i, \neg m_j)$, then:

$$\frac{(l_1 \vee \ldots \vee l_i \vee \ldots \vee l_k) \quad (m_1 \vee \ldots \vee m_j \vee \ldots \vee m_k)}{(l_1 \vee \ldots \vee l_{i-1} \vee l_{i+1} \vee \ldots \vee l_k \vee m_1 \vee \ldots \vee m_{j-1} \vee m_{j+1} \vee \ldots \vee m_k)\theta}$$

FOL resolution To prove $\Gamma \models \alpha$:

- $CNFize(\Gamma \wedge \neg\alpha)$
- Repeatedly apply RR to the formula above until:
  - The empty clause is generated ($\Gamma \models \alpha$)
  - RR can't be applied anymore ($\Gamma \not\models \alpha$)
  - We run out of resources
- Hint: apply resolution to unit clauses first!

Reasoning on FOL: exercises

Solve $KB \models Knows(Alice, Logics)$ using general resolution:

$1. Enrolled(Alice)$ $2. Enrolled(Bob)$ $3. Mandatory(FAI)$ $4. Covers(FAI, Logics)$
$5. Covers(ML, NeuralNets)$ $6. Knows(Bob, NeuralNets)$

$7. Takes(s_1, c_1) \vee \neg Enrolled(s_1) \vee \neg Mandatory(c_1)$
$8. Knows(s_2, t_2) \vee \neg Takes(s_2, c_2) \vee \neg Covers(c_2, t_2)$

$9. \neg Knows(Alice, Logics)$

Reasoning on FOL: exercises

Solve $KB \models Knows(Alice, Logics)$ using general resolution:

$1. Enrolled(Alice) \quad 2. Enrolled(Bob) \quad 3. Mandatory(FAI) \quad 4. Covers(FAI, Logics)$
$5. Covers(ML, NeuralNets) \quad 6. Knows(Bob, NeuralNets)$

$7. Takes(s_1, c_1) \vee \neg Enrolled(s_1) \vee \neg Mandatory(c_1)$
$8. Knows(s_2, t_2) \vee \neg Takes(s_2, c_2) \vee \neg Covers(c_2, t_2)$

$9. \neg Knows(Alice, Logics)$

$10. \neg Takes(Alice, c_2) \vee \neg Covers(c_2, Logics)$ $(8 + 9)$

Reasoning on FOL: exercises

Solve $\text{KB} \models \text{Knows(Alice, Logics)}$ using general resolution:

1.$Enrolled(Alice)$ 2.$Enrolled(Bob)$ 3.$Mandatory(FAI)$ 4.$Covers(FAI, Logics)$
5.$Covers(ML, NeuralNets)$ 6.$Knows(Bob, NeuralNets)$

7.$Takes(s_1, c_1) \lor \neg Enrolled(s_1) \lor \neg Mandatory(c_1)$
8.$Knows(s_2, t_2) \lor \neg Takes(s_2, c_2) \lor \neg Covers(c_2, t_2)$

9.$\neg Knows(Alice, Logics)$

10.$\neg Takes(Alice, c_2) \lor \neg Covers(c_2, Logics)$ $(8 + 9)$
11.$\neg Takes(Alice, FAI)$ $(4 + 10)$

Reasoning on FOL: exercises

Solve KB $\models$ Knows(Alice, Logics) using general resolution:

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$7. Takes(s_1, c_1) \vee \neg Enrolled(s_1) \vee \neg Mandatory(c_1)$
$8. Knows(s_2, t_2) \vee \neg Takes(s_2, c_2) \vee \neg Covers(c_2, t_2)$

$9. \neg Knows(Alice, Logics)$

$10. \neg Takes(Alice, c_2) \vee \neg Covers(c_2, Logics)$ (8 + 9)
$11. \neg Takes(Alice, FAI)$ (4 + 10)
$12. Takes(s_1, FAI) \vee \neg Enrolled(s_1)$ (3 + 7)

Reasoning on FOL: exercises

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$5. Covers(ML, NeuralNets)$ $6. Knows(Bob, NeuralNets)$

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$8. Knows(s_2, t_2) \vee \neg Takes(s_2, c_2) \vee \neg Covers(c_2, t_2)$

$9. \neg Knows(Alice, Logics)$

$10. \neg Takes(Alice, c_2) \vee \neg Covers(c_2, Logics)$ (8 + 9)
$11. \neg Takes(Alice, FAI)$ (4 + 10)
$12. Takes(s_1, FAI) \vee \neg Enrolled(s_1)$ (3 + 7)
$13. Takes(Alice, FAI)$ (1 + 12)

Reasoning on FOL: exercises

Solve $\text{KB} \models \text{Knows(Alice, Logics)}$ using general resolution:

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$7. Takes(s_1, c_1) \lor \neg Enrolled(s_1) \lor \neg Mandatory(c_1)$
$8. Knows(s_2, t_2) \lor \neg Takes(s_2, c_2) \lor \neg Covers(c_2, t_2)$

$9. \neg Knows(Alice, Logics)$

$10. \neg Takes(Alice, c_2) \lor \neg Covers(c_2, Logics)$ $(8 + 9)$
$11. \neg Takes(Alice, FAI)$ $(4 + 10)$
$12. Takes(s_1, FAI) \lor \neg Enrolled(s_1)$ $(3 + 7)$
$13. Takes(Alice, FAI)$ $(1 + 12)$
$14. \emptyset$ $(11 + 13)$

Who hated Caesar? (c) E. Delisle, UoT

1. Marcus was a man. 2. Marcus was a Roman. 3. All men are people. 4. Caesar was a ruler. 5. All Romans were either loyal to Caesar or hated him (or both). 6. Everyone is loyal to someone. 7. People try to assassinate rulers only if they are not loyal to them. 8. Marcus tries to assassinate Caesar.
9. Someone hated Caesar?

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1. $Man(Marcus)$

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1. $Man(Marcus)$
2. $Roman(Marcus)$

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1. $Man(Marcus)$
2. $Roman(Marcus)$
3. $\forall x_1 \,.\, (Man(x_1) \rightarrow People(x_1))$

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1. $Man(Marcus)$
2. $Roman(Marcus)$
3. $\neg Man(x_1) \vee People(x_1)$

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1. $Man(Marcus)$
2. $Roman(Marcus)$
3. $\neg Man(x_1) \vee People(x_1)$
4. $Ruler(Caesar)$

Reasoning on FOL: resolution exercises

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9. Someone hated Caesar?

1. $Man(Marcus)$
2. $Roman(Marcus)$
3. $\neg Man(x_1) \lor People(x_1)$
4. $Ruler(Caesar)$
5. $\forall x_2 \, . \, [Roman(x_2) \rightarrow (Loyal(x_2, Caesar) \lor Hated(x_2, Caesar))]$

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2. $Roman(Marcus)$
3. $\neg Man(x_1) \vee People(x_1)$
4. $Ruler(Caesar)$
5. $\neg Roman(x_2) \vee Loyal(x_2, Caesar) \vee Hated(x_2, Caesar)$

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9. Someone hated Caesar?

1. $Man(Marcus)$
2. $Roman(Marcus)$
3. $\neg Man(x_1) \vee People(x_1)$
4. $Ruler(Caesar)$
5. $\neg Roman(x_2) \vee Loyal(x_2, Caesar) \vee Hated(x_2, Caesar)$
6. $\forall x_3 \,.\, \exists x_4 \,.\, Loyal(x_3, x_4)$

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2. $Roman(Marcus)$
3. $\neg Man(x_1) \vee People(x_1)$
4. $Ruler(Caesar)$
5. $\neg Roman(x_2) \vee Loyal(x_2, Caesar) \vee Hated(x_2, Caesar)$
6. $Loyal(x_3, F_1(x_3))$

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2. $Roman(Marcus)$
3. $\neg Man(x_1) \lor People(x_1)$
4. $Ruler(Caesar)$
5. $\neg Roman(x_2) \lor Loyal(x_2, Caesar) \lor Hated(x_2, Caesar)$
6. $Loyal(x_3, F_1(x_3))$
7. $\forall x_5 \,.\, \forall x_6 \,.\, [(People(x_5) \land MightKill(x_5, x_6)) \rightarrow \neg Loyal(x_5, x_6)]$

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2. $Roman(Marcus)$
3. $\neg Man(x_1) \vee People(x_1)$
4. $Ruler(Caesar)$
5. $\neg Roman(x_2) \vee Loyal(x_2, Caesar) \vee Hated(x_2, Caesar)$
6. $Loyal(x_3, F_1(x_3))$
7. $\neg People(x_5) \vee \neg MightKill(x_5, x_6) \vee \neg Loyal(x_5, x_6)$

Reasoning on FOL: resolution exercises

Who hated Caesar? (c) E. Delisle, UoT

1. Marcus was a man. 2. Marcus was a Roman. 3. All men are people. 4. Caesar was a ruler. 5. All Romans were either loyal to Caesar or hated him (or both). 6. Everyone is loyal to someone. 7. People try to assassinate rulers only if they are not loyal to them. 8. Marcus tries to assassinate Caesar.
9. Someone hated Caesar?

1. $Man(Marcus)$
2. $Roman(Marcus)$
3. $\neg Man(x_1) \vee People(x_1)$
4. $Ruler(Caesar)$
5. $\neg Roman(x_2) \vee Loyal(x_2, Caesar) \vee Hated(x_2, Caesar)$
6. $Loyal(x_3, F_1(x_3))$
7. $\neg People(x_5) \vee \neg MightKill(x_5, x_6) \vee \neg Loyal(x_5, x_6)$
8. $MightKill(Marcus, Caesar)$

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9. Someone hated Caesar?

1. $Man(Marcus)$
2. $Roman(Marcus)$
3. $\neg Man(x_1) \vee People(x_1)$
4. $Ruler(Caesar)$
5. $\neg Roman(x_2) \vee Loyal(x_2, Caesar) \vee Hated(x_2, Caesar)$
6. $Loyal(x_3, F_1(x_3))$
7. $\neg People(x_5) \vee \neg MightKill(x_5, x_6) \vee \neg Loyal(x_5, x_6)$
8. $MightKill(Marcus, Caesar)$
9. $\neg \exists x_7 \,.\, Hated(x_7, Caesar)$

Who hated Caesar? (c) E. Delisle, UoT

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9. Someone hated Caesar?

1. $Man(Marcus)$
2. $Roman(Marcus)$
3. $\neg Man(x_1) \vee People(x_1)$
4. $Ruler(Caesar)$
5. $\neg Roman(x_2) \vee Loyal(x_2, Caesar) \vee Hated(x_2, Caesar)$
6. $Loyal(x_3, F_1(x_3))$
7. $\neg People(x_5) \vee \neg MightKill(x_5, x_6) \vee \neg Loyal(x_5, x_6)$
8. $MightKill(Marcus, Caesar)$
9. $\neg Hated(x_7, Caesar)$

Reasoning on FOL: resolution exercises

- $(5+9) \quad \Theta = \{x_7/x_2\}$
  $\Rightarrow$ 10. $\neg Roman(x_2) \vee Loyal(x_2, Caesar)$
- $(2+10) \quad \Theta = \{x_2/Marcus\}$
  $\Rightarrow$ 11. $Loyal(Marcus, Caesar)$
- $(7+11) \quad \Theta = \{x_5/Marcus, x_6/Caesar\}$
  $\Rightarrow$ 12. $\neg People(Marcus) \vee \neg MightKill(Marcus, Caesar)$
- $(8+12) \quad \Theta = \{\}$
  $\Rightarrow$ 13. $\neg People(Marcus)$
- $(3+13) \quad \Theta = \{x_1/Marcus\}$
  $\Rightarrow$ 14. $\neg Man(Marcus)$
- $(1+14) \quad \Theta = \{\}$
  $\Rightarrow$ Empty clause $\Rightarrow$ **UNSAT**

Reasoning on FOL: resolution exercises

Best of luck with your logic exam (c) H.Zhang, UoI

1. Anyone passing his/her logic exams and winning the lottery is happy. 2. Anyone who studies or is lucky can pass all exams. 3. John does not study but he is lucky. 4. Anyone who is lucky wins the lottery.
5. Is John happy?

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1. $\forall x \,.\, [(Passes(x, Logic) \wedge WinsLottery(x)) \rightarrow Happy(x)]$

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1. $\neg Passes(x_1, Logic) \vee \neg WinsLottery(x_1)) \vee Happy(x_1)$

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5. Is John happy?

1. $\neg Passes(x_1, Logic) \vee \neg WinsLottery(x_1)) \vee Happy(x_1)$

2. $\forall x \,.\, \forall y \,.\, [(Studies(x) \vee Lucky(x)) \rightarrow Passes(x, y)]$

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5. Is John happy?

1. $\neg Passes(x_1, Logic) \vee \neg WinsLottery(x_1)) \vee Happy(x_1)$

2a. $\neg Studies(x_2) \vee Passes(x_2, y_2)$ 2.b $\neg Lucky(x_3)) \vee Passes(x_3, y_3)$

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1. Anyone passing his/her logic exams and winning the lottery is happy. 2. Anyone who studies or is lucky can pass all exams. 3. John does not study but he is lucky. 4. Anyone who is lucky wins the lottery.
5. Is John happy?

1. $\neg Passes(x_1, Logic) \vee \neg WinsLottery(x_1)) \vee Happy(x_1)$

2a. $\neg Studies(x_2) \vee Passes(x_2, y_2)$ 2.b $\neg Lucky(x_3)) \vee Passes(x_3, y_3)$

3a. $\neg Studies(John)$ 3b. $Lucky(John)$

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5. Is John happy?

1. $\neg Passes(x_1, Logic) \vee \neg WinsLottery(x_1)) \vee Happy(x_1)$

2a. $\neg Studies(x_2) \vee Passes(x_2, y_2)$ 2.b $\neg Lucky(x_3)) \vee Passes(x_3, y_3)$

3a. $\neg Studies(John)$ 3b. $Lucky(John)$

4. $\forall x\, .\, [Lucky(x) \rightarrow WinsLottery(x)]$

Best of luck with your logic exam (c) H.Zhang, UoI

1. Anyone passing his/her logic exams and winning the lottery is happy. 2. Anyone who studies or is lucky can pass all exams. 3. John does not study but he is lucky. 4. Anyone who is lucky wins the lottery.
5. Is John happy?

1. $\neg Passes(x_1, Logic) \vee \neg WinsLottery(x_1)) \vee Happy(x_1)$

2a. $\neg Studies(x_2) \vee Passes(x_2, y_2)$ 2.b $\neg Lucky(x_3)) \vee Passes(x_3, y_3)$

3a. $\neg Studies(John)$ 3b. $Lucky(John)$

4. $\neg Lucky(x_4) \vee WinsLottery(x_4)$

Best of luck with your logic exam (c) H.Zhang, UoI

1. Anyone passing his/her logic exams and winning the lottery is happy. 2. Anyone who studies or is lucky can pass all exams. 3. John does not study but he is lucky. 4. Anyone who is lucky wins the lottery.
5. Is John happy?

1. $\neg Passes(x_1, Logic) \vee \neg WinsLottery(x_1)) \vee Happy(x_1)$

2a. $\neg Studies(x_2) \vee Passes(x_2, y_2)$ 2.b $\neg Lucky(x_3)) \vee Passes(x_3, y_3)$

3a. $\neg Studies(John)$ 3b. $Lucky(John)$

4. $\neg Lucky(x_4) \vee WinsLottery(x_4)$

5. $\neg Happy(John)$

Reasoning on FOL: resolution exercises

- $(1+5) \quad \Theta = \{x_1/John\}$
  $\Rightarrow$ 6. $\neg Passes(John, Logic) \vee \neg WinsLottery(John)$
- $(4+6) \quad \Theta = \{x_4/John\}$
  $\Rightarrow$ 7. $\neg Lucky(John) \vee \neg Passes(John, Logic)$
- $(3b+7) \quad \Theta = \{\}$
  $\Rightarrow$ 8. $\neg Passes(John, Logic)$
- $(2b+8) \quad \Theta = \{x_3/John, y_3/Logic\}$
  $\Rightarrow$ 9. $\neg Lucky(John)$
- $(3b+9) \quad \Theta = \{\}$
  $\Rightarrow$ Empty clause $\Rightarrow$ **UNSAT**

Hoofers Club (c) H.Zhang, UoI

1. Tony, Tom and Liz belong to the Hoofers Club. 2. Every member of the Hoofers Club is either a skier or a mountain climber or both. 3. No mountain climber likes rain, and all skiers like snow. 4. Liz dislikes whatever Tony likes and likes whatever Tony dislikes. 5. Tony likes rain and snow. 6. Is there a member of the Hoofers Club who is a mountain climber but not a skier?

Reasoning on FOL: hyper-resolution

Try solving the previous resolution exercises with hyper-resolution.