

**1 Euclid's Algorithm****20 pts****1.1 Sum decrease****5 pts**

<b>0</b>	Correct
<b>-5</b>	Blank
<b>(4 pts)</b>	<b>Analysis of <math>A' + B'</math></b>
<b>-0</b>	Correct
<b>-3</b>	Just restates proof from class
<b>-2</b>	Arrives at a correct intermediate bound and then skips directly to $3/2(A' + B')$ without justification
<b>-3.5</b>	Writes out an expression for $A', B'$ in terms of quotients/remainders and hand-waves the claimed bound
<b>(1 pts)</b>	<b>Conclusion on number steps</b>
<b>-0</b>	Correct given the analysis of $A' + B'$
<b>-1</b>	Incorrect or missing

**1.2 Expression for  $L, L'$** **3 pts**

<b>-0</b>	Correct
<b>-1</b>	Expression for $L'$ incorrect or missing
<b>-1</b>	Expression for $L$ incorrect or missing
<b>-1</b>	No justification provided
<b>-3</b>	Blank

**1.3 Define ratio  $\beta$** **5 pts**

<b>-0</b>	Correct
<b>-1</b>	$L, L'$ correct in previous part. Derives correct equation for $\beta$ but then solves it incorrectly

<b>-2</b>	$L, L'$ correct in previous part. An equation for $\beta$ is provided but the equation is incorrect
<b>-3</b>	Correctly solves for $\beta$ based on incorrect values of $L, L'$
<b>-5</b>	Blank or incorrect

#### 1.4 Improved upper bound

5 pts

<b>-0</b>	Correct
<b>-1</b>	Correct analysis of a suboptimal bound (possibly due to errors in previous parts) but better than the bound from Part (a)
<b>-3</b>	Fails to properly use the analysis of $g(A, B)$ and thus (implicitly) gets a bound based on $A + B$
<b>-5</b>	Correct bound that does not improve on Part (a)
<b>-5</b>	Gives a bound that uses $O(\cdot)$ notation
<b>-5</b>	Blank

#### 1.5 Tightness of bound

2 pts

<b>-0</b>	Correct
<b>-1</b>	Gives right sequence of numbers but no justification
<b>-1.5</b>	Gives wrong sequence of inputs but provides reasonable intuition
<b>-2</b>	Blank

## 2 Fibonacci Programming

34 pts

### 2.1 Estimate running time

15 pts

<b>-0</b>	Correct
<b>-3</b>	Does not have a reasonably useful way of describing how fast each algorithm is.

-4	Incorrect times: some method(s) should be significantly slower than the other(s) . This might be due to coding errors or an inadequate approach to thinking about how to compare or measure the runtimes of the algorithms .
-4	Incorrect qualitative comparison of runtimes for the methods not covered above.
-4	No precise timings provided (e.g. merely says, "matrix is slower than recursive").

## 2.2 First overflow

4 pts

-0	Correct
-1	Index off by 1
-2	Index off by 2
-4	Incorrect (e.g. claims to have calculated the desired Fibonacci number, but value is incorrect)

## 2.3 Modular Fibonacci

15 pts

-0	Correct
-3	Incorrect recursive value: should be approx $y$ . We'll accept between $x$ and $z$ —reasonable answers, depending on your system and implementation.
-4	Incorrect iterative value: should be approx $y$ ; we'll accept between $x$ and $z$ .
-3	Incorrect matrix value: should be approx $y$ ; we'll accept $x$ to $z$
-15	Blank

## 3 Big-Oh Notation

15 pts

### 3.1 Compare the given functions

10 pts

0	All of the statements are given.
-1	Missing statements when $i = j$ .

<b>-3</b>	Missing some true statements of the requested type that don't affect the overall ordering of the functions.
<b>-1</b>	Statements that are made are inconsistent with each other. (Further rubric items assume the most similar consistent ordering.)
<b>-2</b>	Statements that are made correspond to one transposition from the correct asymptotic order of the functions
<b>-5</b>	Statements that are made correspond to two transpositions from the correct asymptotic order of the functions
<b>-8</b>	Statements that are made correspond to more than two transposition from the correct asymptotic order of the functions
<b>-1</b>	Small mistake (described in a comment)
<b>-10</b>	Blank or no correct statement.

### 3.2 Counterexample

5 pts

<b>-0</b>	Function satisfies the conditions given in the problem.
<b>-1</b>	Function is asymptotically incomparable with some reasonable function, but not the given ones.
<b>-2</b>	Function is outside a certain common set of functions in which it'd be harder to find a correct one, but still incorrect.
<b>-5</b>	incorrect/missing
<b>-1</b>	Function takes values outside $\mathbb{R}^+$ .

## 4 Recurrences

20 pts

### 4.1 Part a.i.

5 pts

<b>-0</b>	Correct
<b>-4</b>	Gives asymptotic solution using master theorem
<b>-2</b>	Approach correct modulo simple algebraic error
<b>-1</b>	Forgets to verify that proposed solution is consistent with base case
<b>-5</b>	blank/no proof

**4.2 Part a.ii.****5 pts**

<b>-0</b>	Correct
<b>-4</b>	Gives asymptotic solution using master theorem
<b>-2</b>	Approach correct modulo simple algebraic error
<b>-1</b>	Forgets to verify that proposed solution is consistent with base case
<b>-5</b>	blank/no proof

**4.3 Part b.i.****3 pts**

<b>-0</b>	Correct
<b>-3</b>	Uses master theorem but arrives at wrong conclusion
<b>-4</b>	Tries to unroll recurrence “by hand” and handwaves the asymptotic behavior
<b>-5</b>	blank/no proof

**4.4 Part b.ii.****7 pts**

<b>-0</b>	Correct
<b>-3</b>	Applies the correct change of variable but final answer is an expression in terms of the new variable
<b>-5</b>	Applies the correct change of variable but incorrectly applies master theorem
<b>-2</b>	Doesn't address the floor function by arguing that $T$ is increasing and thus suffices to consider tower exponentials
<b>-7</b>	blank/no proof

**5 TwistedBubbleSort****25 pts****5.1 Different from bubble sort****2 pts**

<b>-0</b>	Correctly mentions one difference.
<b>-2</b>	Blank or no correct difference.

**5.2  $i$ -th iteration****5 pts**

<b>-0</b>	Completely correct
<b>-2</b>	Neglects base case
<b>-2</b>	Makes correct partial observations but does not give full proof
<b>-5</b>	blank/no proof

**5.3 Correctness****10 pts**

<b>-0</b>	Completely correct
<b>-1</b>	Neglects base case
<b>-4</b>	Makes correct partial observations but does not give full proof
<b>-4</b>	Makes erroneous claims about intermediate state of array
<b>-10</b>	blank/no useable observations in proof

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**6 InsertionSort****0 pts**

<b>-0</b>	Correct
<b>-0</b>	Incorrect