





Dynamax: A Python package for probabilistic state space modeling with JAX

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Summary

State space models (SSMs) are fundamental tools for modeling sequential data. They are broadly used across engineering disciplines like signal processing and control theory, as well as scientific domains like neuroscience (Vyas et al., 2020), genetics (Durbin et al., 1998), ecology (Patterson et al., 2008), computational ethology (Weinreb et al., 2024), economics (Jacquier et al., 2002), and climate science (Ott et al., 2004). Fast and robust tools for state space modeling are crucial to researchers in all of these application areas.

State space models specify a probability distribution over a sequence of observations, y_1, \dots, y_T , where y_t denotes the observation at time t . The key assumption of an SSM is that the observations arise from a sequence of *latent states*, z_1, \dots, z_T , which evolve according to a *dynamics model* (aka transition model). An SSM may also use inputs (aka controls or covariates), u_1, \dots, u_T , to steer the latent state dynamics and influence the observations. For example, in a neuroscience application from Vyas et al. (2020), y_t represents a vector of spike counts from ~ 1000 measured neurons, and z_t is a lower dimensional latent state that changes slowly over time and captures correlations among the measured neurons. If sensory inputs to the neural circuit are known, they can be encoded in u_t . In the computational ethology application of Weinreb et al. (2024), y_t represents a vector of 3D locations for several key points on an animal's body, and z_t is a discrete behavioral state that specifies how the animal's posture changes over time. In both examples, there are two main objectives: First, we aim to infer the latent states z_t that best explain the observed data; formally, this is called *state inference*. Second, we need to estimate the dynamics that govern how latent states evolve; formally, this is part of the *parameter estimation* process. Dynamax provides algorithms for state inference and parameter estimation in a variety of SSMs.

There are a few key design choices to make when constructing an SSM:

- What is the type of latent state? E.g., is z_t a continuous or discrete random variable?
- How do the latent states evolve over time? E.g., are the dynamics linear or nonlinear?
- How are the observations distributed? E.g., are they Gaussian, Poisson, etc.?

Some design choices are so common they have their own names. Hidden Markov models (HMM) are SSMs with discrete latent states, and linear dynamical systems (LDS) are SSMs with continuous latent states, linear dynamics, and additive Gaussian noise. Dynamax supports canonical SSMs and allows the user to construct bespoke models as needed, simply by inheriting from a base class and specifying a few model-specific functions.

Finally, even for canonical models, there are several algorithms for state inference and parameter

estimation. Dynamax provides robust implementations of several low-level inference algorithms to suit a variety of applications, allowing users to choose among a host of models and algorithms for their application. More information about state space models and algorithms for state inference and parameter estimation can be found in the textbooks by Murphy (2023) and Särkkä & Svensson (2023).

Statement of need

Dynamax is an open-source Python package for state space modeling. Since it is built with JAX (Bradbury et al., 2018), it supports just-in-time (JIT) compilation for hardware acceleration on CPU, GPU, and TPU machines. It also supports automatic differentiation for gradient-based model learning. While other libraries exist for state space modeling in Python (Corenflos & Särkkä, 2021; Johnson, 2020; Linderman et al., 2020; Seabold & Perktold, 2010; Weiss et al., 2024), Dynamax provides a combination of low-level inference algorithms and high-level modeling objects that can support a wide range of research applications in JAX.

The API for Dynamax is divided into two parts: a set of core, functionally pure, low-level inference algorithms, and a high-level, object oriented module for constructing and fitting probabilistic SSMs. The low-level inference API provides message passing algorithms for several common types of SSMs. For example, Dynamax provides JAX implementations for:

- Forward-Backward algorithms for discrete-state hidden Markov models (HMMs),
- Kalman filtering and smoothing algorithms for linear Gaussian SSMs,
- Extended and unscented generalized Kalman filtering and smoothing for nonlinear and/or non-Gaussian SSMs, and
- Parallel message passing routines that leverage GPU or TPU acceleration to perform message passing in $O(\log T)$ time on a parallel machine (Hassan et al., 2021; Särkkä & García-Fernández, 2020; Stone, 1975). Note that these routines are not simply parallelizing over batches of time series, but rather using a parallel algorithm with sublinear depth or span.

The high-level model API makes it easy to construct, fit, and inspect HMMs and linear Gaussian SSMs. Finally, the online Dynamax documentation and tutorials provide a wealth of resources for state space modeling experts and newcomers alike.

Dynamax has supported several publications. The low-level API has been used in machine learning research (Chang et al., 2023; Lee et al., 2023; Zhao & Linderman, 2023). Special purpose libraries have been built on top of Dynamax, like the Keypoint-MoSeq library for modeling animal behavior (Weinreb et al., 2024) and the Structural Time Series in JAX library, sts-jax (Li & Murphy, 2022). Finally, the Dynamax tutorials are used as reference examples in a major machine learning textbook (Murphy, 2023).

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References

- Bradbury, J., Frostig, R., Hawkins, P., Johnson, M. J., Leary, C., Maclaurin, D., Necula, G., Paszke, A., VanderPlas, J., Wanderman-Milne, S., & Zhang, Q. (2018). *JAX: Composable transformations of Python+NumPy programs* (Version 0.3.13). <http://github.com/google/jax>

- 86 Chang, P. G., Durán-Martín, G., Shestopaloff, A., Jones, M., & Murphy, K. P. (2023). Low-
87 rank extended Kalman filtering for online learning of neural networks from streaming
88 data. In S. Chandar, R. Pascanu, H. Sedghi, & D. Precup (Eds.), *Proceedings of the*
89 *2nd conference on lifelong learning agents* (Vol. 232, pp. 1025–1071). PMLR. <https://doi.org/10.48550/arXiv.2305.19535>
90
- 91 Corenflos, A., & Särkkä, S. (2021). *Code companion for Bayesian Filtering and Smoothing*
92 (Version 1.0). <https://github.com/EEA-sensors/Bayesian-Filtering-and-Smoothing>
- 93 Durbin, R., Eddy, S. R., Krogh, A., & Mitchison, G. (1998). *Biological sequence analysis: Prob-*
94 *abilistic models of proteins and nucleic acids*. <https://doi.org/10.1017/cbo9780511790492>
- 95 Hassan, S. S., Särkkä, S., & García-Fernández, Á. F. (2021). Temporal parallelization of
96 inference in hidden Markov models. *IEEE Transactions on Signal Processing*, 69, 4875–4887.
97 <https://doi.org/10.1109/TSP.2021.3103338>
- 98 Jacquier, E., Polson, N. G., & Rossi, P. E. (2002). Bayesian analysis of stochastic volatility
99 models. *Journal of Business & Economic Statistics*, 20(1), 69–87. <https://doi.org/10.1198/073500102753410408>
100
- 101 Johnson, M. J. (2020). *PyHSMM: Bayesian inference in HSMMs and HMMs* (Version 0.0.0).
102 <https://github.com/mattjj/pyhsmm>
- 103 Lee, H. D., Warrington, A., Glaser, J., & Linderman, S. (2023). Switching autoregressive low-
104 rank tensor models. *Advances in Neural Information Processing Systems*, 36, 57976–58010.
105 <https://doi.org/10.48550/arXiv.2306.03291>
- 106 Li, X., & Murphy, K. (2022). *Structural time series (STS) in JAX*. <https://github.com/probml/sts-jax>
107
- 108 Linderman, S., Antin, B., Zoltowski, D., & Glaser, J. (2020). *SSM: Bayesian Learning and*
109 *Inference for State Space Models* (Version 0.0.1). <https://github.com/lindermanlab/ssm>
- 110 Murphy, K. P. (2023). *Probabilistic machine learning: Advanced topics*. MIT Press. <http://probml.github.io/book2>
111
- 112 Ott, E., Hunt, B. R., Szunyogh, I., Zimin, A. V., Kostelich, E. J., Corazza, M., Kalnay,
113 E., Patil, D., & Yorke, J. A. (2004). A local ensemble Kalman filter for atmospheric
114 data assimilation. *Tellus A: Dynamic Meteorology and Oceanography*, 56(5), 415–428.
115 <https://doi.org/10.3402/tellusa.v56i5.14462>
- 116 Patterson, T. A., Thomas, L., Wilcox, C., Ovaskainen, O., & Matthiopoulos, J. (2008).
117 State-space models of individual animal movement. *Trends in Ecology & Evolution*, 23(2),
118 87–94. <https://doi.org/10.1016/j.tree.2007.10.009>
- 119 Särkkä, S., & García-Fernández, Á. F. (2020). Temporal parallelization of Bayesian smoothers.
120 *IEEE Transactions on Automatic Control*, 66(1), 299–306. <https://doi.org/10.1109/TAC.2020.2976316>
121
- 122 Särkkä, S., & Svensson, L. (2023). *Bayesian filtering and smoothing* (Vol. 17). Cambridge
123 University Press. <https://doi.org/10.1017/CBO9781139344203>
- 124 Seabold, S., & Perktold, J. (2010). Statsmodels: Econometric and statistical mod-
125 eling with python. *9th Python in Science Conference*. <https://doi.org/10.25080/majora-92bf1922-011>
126
- 127 Stone, H. S. (1975). Parallel tridiagonal equation solvers. *ACM Transactions on Mathematical*
128 *Software (TOMS)*, 1(4), 289–307. <https://doi.org/10.1145/355656.355657>
- 129 Vyas, S., Golub, M. D., Sussillo, D., & Shenoy, K. V. (2020). Computation through neural
130 population dynamics. *Annual Review of Neuroscience*, 43(1), 249–275. <https://doi.org/10.1146/annurev-neuro-092619-094115>
131

- 132 Weinreb, C., Pearl, J. E., Lin, S., Osman, M. A. M., Zhang, L., Annapragada, S., Conlin,
133 E., Hoffmann, R., Makowska, S., Gillis, W. F., Jay, M., Ye, S., Mathis, A., Mathis, M.
134 W., Pereira, T., Linderman, S. W., & Datta, S. R. (2024). Keypoint-MoSeq: Parsing
135 behavior by linking point tracking to pose dynamics. *Nature Methods*, 21(7), 1329–1339.
136 <https://doi.org/10.1038/s41592-024-02318-2>
- 137 Weiss, R., Du, S., Grobler, J., Courneau, D., Pedregosa, F., Varoquaux, G., Mueller, A.,
138 Thirion, B., Nouri, D., Louppe, G., Vanderplas, J., Benediktsson, J., Buitinck, L., Korobov,
139 M., McGibbon, R., Lattarini, S., Niculae, V., Gramfort, A., Lebedev, S., ... Rockhill, A.
140 (2024). *Hmmlearn* (Version 0.3.2). <https://github.com/hmmlearn/hmmlearn>
- 141 Zhao, Y., & Linderman, S. (2023). Revisiting structured variational autoencoders. *International*
142 *Conference on Machine Learning*, 42046–42057. [https://doi.org/10.48550/arXiv.2305.](https://doi.org/10.48550/arXiv.2305.16543)
143 [16543](https://doi.org/10.48550/arXiv.2305.16543)

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