

# Dynamax: A Python package for probabilistic state space modeling with JAX

Scott W. Linderman<sup>1¶</sup>, Peter Chang<sup>3</sup>, Giles Harper-Donnelly<sup>4</sup>, Aleyna Kara<sup>5</sup>, Xinglong Li<sup>6</sup>, and Kevin Murphy<sup>2¶</sup>

<sup>1</sup> Department of Statistics and Wu Tsai Neurosciences Institute, Stanford University, USA <sup>2</sup> Google DeepMind, USA <sup>3</sup> CSAIL, Massachusetts Institute of Technology, USA <sup>4</sup> Cambridge University, UK <sup>5</sup> Computer Science Department, Technical University of Munich Garching, Germany <sup>6</sup> Statistics Department, University of British Columbia, Canada ¶ Corresponding author

DOI: [10.xxxxxx/draft](https://doi.org/10.xxxxxx/draft)

## Software

- [Review](#) ↗
- [Repository](#) ↗
- [Archive](#) ↗

Editor: [Open Journals](#) ↗

Reviewers:

- [@openjournals](#)

Submitted: 01 January 1970

Published: unpublished

## License

Authors of papers retain copyright and release the work under a Creative Commons Attribution 4.0 International License ([CC BY 4.0](#)).

## Summary

State space models (SSMs) are fundamental tools for modeling sequential data. They are broadly used across engineering disciplines like signal processing and control theory, as well as scientific domains like neuroscience (Vyas et al., 2020), genetics (Durbin et al., 1998), ecology (Patterson et al., 2008), computational ethology (Weinreb et al., 2024), economics (Jacquier et al., 2002), and climate science (Ott et al., 2004). Fast and robust tools for state space modeling are crucial to researchers in all of these application areas.

State space models specify a probability distribution over a sequence of observations,  $y_1, \dots, y_T$ , where  $y_t$  denotes the observation at time  $t$ . The key assumption of an SSM is that the observations arise from a sequence of *latent states*,  $z_1, \dots, z_T$ , which evolve according to a *dynamics model* (aka transition model). An SSM may also use inputs (aka controls or covariates),  $u_1, \dots, u_T$ , to steer the latent state dynamics and influence the observations. For example, in a neuroscience application from Vyas et al. (2020),  $y_t$  represents a vector of spike counts from  $\sim 1000$  measured neurons, and  $z_t$  is a lower dimensional latent state that changes slowly over time and captures correlations among the measured neurons. If sensory inputs to the neural circuit are known, they can be encoded in  $u_t$ . In the computational ethology application of Weinreb et al. (2024),  $y_t$  represents a vector of 3D locations for several key points on an animal's body, and  $z_t$  is a discrete behavioral state that specifies how the animal's posture changes over time. In both examples, there are two main objectives: First, we aim to infer the latent states  $z_t$  that best explain the observed data; formally, this is called *state inference*. Second, we need to estimate the dynamics that govern how latent states evolve; formally, this is part of the *parameter estimation* process. Dynamax provides algorithms for state inference and parameter estimation in a variety of SSMs.

There are a few key design choices to make when constructing an SSM:

- What is the type of latent state? E.g., is  $z_t$  a continuous or discrete random variable?
- How do the latent states evolve over time? E.g., are the dynamics linear or nonlinear?
- What is the conditional distribution of the observations? E.g., are they Gaussian, Poisson, etc.?

Some design choices are so common that they have their own names. For example, hidden Markov models (HMM) are SSMs with discrete latent states, and linear dynamical systems (LDS) are SSMs with continuous latent states, linear dynamics, and additive Gaussian noise. Dynamax supports these canonical SSMs and allows you to construct more complex models if needed.

Finally, even for canonical models, there are several algorithms for state inference and parameter

estimation. Dynamax provides robust implementations of several low-level inference algorithms to suit a variety of applications, allowing users to choose among a host of models and algorithms for their application. More information about state space models and algorithms for state inference and parameter estimation can be found in the textbooks by Murphy (2023) and Särkkä & Svensson (2023).

## Statement of need

Dynamax is an open-source Python package for state space modeling. Since it is built with JAX (Bradbury et al., 2018), it supports just-in-time (JIT) compilation for hardware acceleration on CPU, GPU, and TPU machines. It also supports automatic differentiation for gradient-based model learning. While other libraries exist for state space modeling in Python (Corenflos & Särkkä, 2021; Johnson, 2020; Linderman et al., 2020; Seabold & Perktold, 2010; Weiss et al., 2024), some using JAX (Duran-Martin et al., 2022), Dynamax provides a unique combination of low-level inference algorithms and high-level modeling objects that can support a wide range of research applications.

The API for Dynamax is divided into two parts: a set of core, functionally pure, low-level inference algorithms, and a high-level, object oriented module for constructing and fitting probabilistic SSMs. The low-level inference API provides message passing algorithms for several common types of SSMs. For example, Dynamax provides JAX implementations for:

- Forward-Backward algorithms for discrete-state hidden Markov models (HMMs),
- Kalman filtering and smoothing algorithms for linear Gaussian SSMs,
- Extended and unscented generalized Kalman filtering and smoothing for nonlinear and/or non-Gaussian SSMs, and
- Parallel message passing routines that leverage GPU or TPU acceleration to perform message passing in sublinear time (Hassan et al., 2021; Särkkä & García-Fernández, 2020; Stone, 1975).

The high-level model API makes it easy to construct, fit, and inspect HMMs and linear Gaussian SSMs. Finally, the online Dynamax documentation and tutorials provide a wealth of resources for state space modeling experts and newcomers alike.

Dynamax has supported several publications. The low-level API has been used in machine learning research (Chang et al., 2023; Lee et al., 2023; Zhao & Linderman, 2023). More sophisticated, special purpose models on top of Dynamax, like the Keypoint-MoSeq library for modeling postural dynamics of animals (Weinreb et al., 2024). Finally, the Dynamax tutorials are used as reference examples in a major machine learning textbook (Murphy, 2023).

## Acknowledgements

A significant portion of this library was developed while S.W.L. was a Visiting Faculty Researcher at Google and P.C., G.H.D., A.K., and X.L. were Google Summer of Code participants.

## References

- Bradbury, J., Frostig, R., Hawkins, P., Johnson, M. J., Leary, C., Maclaurin, D., Necula, G., Paszke, A., VanderPlas, J., Wanderman-Milne, S., & Zhang, Q. (2018). *JAX: Composable transformations of Python+NumPy programs* (Version 0.3.13). <http://github.com/google/jax>
- Chang, P. G., Durán-Martín, G., Shestopaloff, A., Jones, M., & Murphy, K. P. (2023). Low-rank extended Kalman filtering for online learning of neural networks from streaming data. In S.

- 86 Chandar, R. Pascanu, H. Sedghi, & D. Precup (Eds.), *Proceedings of the 2nd conference*  
87 *on lifelong learning agents* (Vol. 232, pp. 1025–1071). PMLR.
- 88 Corenflos, A., & Särkkä, S. (2021). *Code companion for Bayesian Filtering and Smoothing*  
89 (Version 1.0). <https://github.com/EEA-sensors/Bayesian-Filtering-and-Smoothing>
- 90 Duran-Martin, G., Murphy, K., & Kara, A. (2022). *JSL: JAX State-Space models (SSM)*  
91 *Library*. <https://github.com/probml/JSL>
- 92 Durbin, R., Eddy, S. R., Krogh, A., & Mitchison, G. (1998). *Biological sequence analysis:*  
93 *Probabilistic models of proteins and nucleic acids*.
- 94 Hassan, S. S., Särkkä, S., & García-Fernández, Á. F. (2021). Temporal parallelization of  
95 inference in hidden Markov models. *IEEE Transactions on Signal Processing*, 69, 4875–4887.
- 96 Jacquier, E., Polson, N. G., & Rossi, P. E. (2002). Bayesian analysis of stochastic volatility  
97 models. *Journal of Business & Economic Statistics*, 20(1), 69–87.
- 98 Johnson, M. J. (2020). *PyHSMM: Bayesian inference in HSMMs and HMMs* (Version 0.0.0).  
99 <https://github.com/mattjj/pyhsmm>
- 100 Lee, H. D., Warrington, A., Glaser, J., & Linderman, S. (2023). Switching autoregressive low-  
101 rank tensor models. *Advances in Neural Information Processing Systems*, 36, 57976–58010.
- 102 Linderman, S., Antin, B., Zoltowski, D., & Glaser, J. (2020). *SSM: Bayesian Learning and*  
103 *Inference for State Space Models* (Version 0.0.1). <https://github.com/lindermanlab/ssm>
- 104 Murphy, K. P. (2023). *Probabilistic machine learning: Advanced topics*. MIT Press. [http://](http://probml.github.io/book2)  
105 [probml.github.io/book2](http://probml.github.io/book2)
- 106 Ott, E., Hunt, B. R., Szunyogh, I., Zimin, A. V., Kostelich, E. J., Corazza, M., Kalnay, E.,  
107 Patil, D., & Yorke, J. A. (2004). A local ensemble Kalman filter for atmospheric data  
108 assimilation. *Tellus A: Dynamic Meteorology and Oceanography*, 56(5), 415–428.
- 109 Patterson, T. A., Thomas, L., Wilcox, C., Ovaskainen, O., & Matthiopoulos, J. (2008).  
110 State-space models of individual animal movement. *Trends in Ecology & Evolution*, 23(2),  
111 87–94.
- 112 Särkkä, S., & García-Fernández, Á. F. (2020). Temporal parallelization of Bayesian smoothers.  
113 *IEEE Transactions on Automatic Control*, 66(1), 299–306.
- 114 Särkkä, S., & Svensson, L. (2023). *Bayesian filtering and smoothing* (Vol. 17). Cambridge  
115 University Press.
- 116 Seabold, S., & Perktold, J. (2010). Statsmodels: Econometric and statistical modeling with  
117 python. *9th Python in Science Conference*.
- 118 Stone, H. S. (1975). Parallel tridiagonal equation solvers. *ACM Transactions on Mathematical*  
119 *Software (TOMS)*, 1(4), 289–307.
- 120 Vyas, S., Golub, M. D., Sussillo, D., & Shenoy, K. V. (2020). Computation through neural  
121 population dynamics. *Annual Review of Neuroscience*, 43(1), 249–275.
- 122 Weinreb, C., Pearl, J. E., Lin, S., Osman, M. A. M., Zhang, L., Annapragada, S., Conlin,  
123 E., Hoffmann, R., Makowska, S., Gillis, W. F., Jay, M., Ye, S., Mathis, A., Mathis, M.  
124 W., Pereira, T., Linderman, S. W., & Datta, S. R. (2024). Keypoint-MoSeq: Parsing  
125 behavior by linking point tracking to pose dynamics. *Nature Methods*, 21(7), 1329–1339.  
126 ISBN: 1548-7105
- 127 Weiss, R., Du, S., Grobler, J., Cournapeau, D., Pedregosa, F., Varoquaux, G., Mueller, A.,  
128 Thirion, B., Nouri, D., Louppe, G., Vanderplas, J., Benediktsson, J., Buitinck, L., Korobov,  
129 M., McGibbon, R., Lattarini, S., Niculae, V., Gramfort, A., Lebedev, S., ... Rockhill, A.  
130 (2024). *Hmmlearn* (Version 0.3.2). <https://github.com/hmmlearn/hmmlearn>

- <sup>131</sup> Zhao, Y., & Linderman, S. (2023). Revisiting structured variational autoencoders. *International*  
<sup>132</sup> *Conference on Machine Learning*, 42046–42057.

DRAFT