




# Dynamax: A Python package for probabilistic state space modeling with JAX

Scott W. Linderman<sup>1,2¶</sup>, Peter Chang<sup>3</sup>, Giles Harper-Donnelly<sup>4</sup>, Aleyna Kara<sup>5</sup>, Xinglong Li<sup>6</sup>, and Kevin Murphy<sup>2¶</sup>

<sup>1</sup> Department of Statistics and Wu Tsai Neurosciences Institute, Stanford University, USA <sup>2</sup> Google Research, USA <sup>3</sup> CSAIL, Massachusetts Institute of Technology, USA <sup>4</sup> Cambridge University, UK <sup>5</sup> Boğaziçi University, Turkey <sup>6</sup> University of British Columbia, Canada ¶ Corresponding author

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## Summary

State space models (SSMs) are fundamental tools for modeling sequential data. They are broadly used across engineering disciplines like signal processing and control theory, as well as scientific domains like neuroscience, genetics, ecology, and climate science. Fast and robust tools for state space modeling are crucial to researchers in all of these application areas.

State space models specify a probability distribution over a sequence of observations,  $y_1, \dots, y_T$ , where  $y_t$  denotes the observation at time  $t$ . The key assumption of an SSM is that the observations arise from a sequence of *latent states*,  $z_1, \dots, z_T$ , which evolve according to a *dynamics model* (aka transition model). An SSM may also use inputs (aka controls or covariates),  $u_1, \dots, u_T$ , to steer the latent state dynamics and influence the observations.

For example, SSMs are often used in neuroscience to model the dynamics of neural spike train recordings (Vyas et al., 2020). Here,  $y_t$  is a vector of spike counts from each of, say, 100 measured neurons. The activity of nearby neurons is often correlated, and SSMs can capture that correlation through a lower dimensional latent state,  $z_t$ , which may change slowly over time. If we know that certain sensory inputs may drive the neural activity, we can encode them in  $u_t$ . A common goal in neuroscience is to infer the latent states  $z_t$  that best explain the observed neural spike train; formally, this is called *state inference*. Another goal is to estimate the dynamics that govern how latent states evolve; formally, this is part of the *parameter estimation* process. Dynamax provides algorithms for state inference and parameter estimation in a variety of SSMs.

The key design choices when constructing an SSM include the type of latent state (is  $z_t$  a continuous or discrete random variable?), the dynamics that govern how latent states evolve over time (are they linear or nonlinear?), and the conditional distribution of the observations (are they Gaussian, Poisson, etc.?). Canonical examples of SSMs include hidden Markov models (HMM), which have discrete latent states, and linear dynamical systems (LDS), which have continuous latent states with linear dynamics and additive Gaussian noise. Dynamax supports these canonical examples as well as more complex models.

More information about state space models and algorithms for state inference and parameter estimation can be found in the textbooks by Murphy (2023) and Särkkä & Svensson (2023).

## Statement of need

Dynamax is an open-source Python package for state space modeling. Since it is built with JAX (Bradbury et al., 2018), it supports just-in-time (JIT) compilation for hardware acceleration on CPU, GPU, and TPU machines. It also supports automatic differentiation for gradient-based

41 model learning. While other libraries exist for state space modeling in Python (Corenflos &  
42 Särkkä, 2021; Johnson, 2020; Linderman et al., 2020; Seabold & Perktold, 2010; Weiss et al.,  
43 2024), some using JAX (Duran-Martin et al., 2022), Dynamax provides a unique combination of  
44 low-level inference algorithms and high-level modeling objects that can support a wide range  
45 of research applications.

46 The API for Dynamax is divided into two parts: a set of core, functionally pure, low-level  
47 inference algorithms, and a high-level, object oriented module for constructing and fitting  
48 probabilistic SSMs. The low-level inference API provides message passing algorithms for several  
49 common types of SSMs. For example, Dynamax provides JAX implementations for:

- 50 ■ Forward-Backward algorithms for discrete-state hidden Markov models (HMMs),
- 51 ■ Kalman filtering and smoothing algorithms for linear Gaussian SSMs,
- 52 ■ Extended and unscented Kalman filtering and smoothing for nonlinear Gaussian SSMs,
- 53 ■ Conditional moment filtering and smoothing algorithms for models with non-Gaussian  
54 emissions, and
- 55 ■ Parallel message passing routines that leverage GPU or TPU acceleration to perform  
56 message passing in sublinear time.

57 The high-level model API makes it easy to construct, fit, and inspect HMMs and linear Gaussian  
58 SSMs. Finally, the online Dynamax documentation and tutorials provide a wealth of resources  
59 for state space modeling experts and newcomers alike.

60 Dynamax has supported several publications. The low-level API has been used in machine  
61 learning research (Chang et al., 2023; Lee et al., 2023; Zhao & Linderman, 2023). More  
62 sophisticated, special purpose models on top of Dynamax, like the Keypoint-MoSeq library for  
63 modeling postural dynamics of animals (Weinreb et al., 2024). Finally, the Dynamax tutorials  
64 are used as reference examples in a major machine learning textbook (Murphy, 2023).

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