




Dynamax: A Python package for probabilistic state space modeling with JAX

Scott W. Linderman¹[✉], Peter Chang², Giles Harper-Donnelly³, Aleya Kara⁴, Xinglong Li⁵, Gerardo Duran-Martin⁶, and Kevin Murphy⁷[✉]

¹ Department of Statistics and Wu Tsai Neurosciences Institute, Stanford University, USA ² CSAIL, Massachusetts Institute of Technology, USA ³ Cambridge University, England, UK ⁴ Computer Science Department, Technical University of Munich Garching, Germany ⁵ Statistics Department, University of British Columbia, Canada ⁶ Queen Mary University of London, England, UK ⁷ Google DeepMind, USA
[✉] Corresponding author

DOI: [10.xxxxxx/draft](https://doi.org/10.xxxxxx/draft)

Software

- [Review](#) 
- [Repository](#) 
- [Archive](#) 

Editor: [Open Journals](#) 

Reviewers:

- [@openjournals](#)

Submitted: 01 January 1970

Published: unpublished

License

Authors of papers retain copyright and release the work under a Creative Commons Attribution 4.0 International License ([CC BY 4.0](#))

Summary

State space models (SSMs) are fundamental tools for modeling sequential data. They are broadly used across engineering disciplines like signal processing and control theory, as well as scientific domains like neuroscience (Vyas et al., 2020), genetics (Durbin et al., 1998), ecology (Patterson et al., 2008), computational ethology (Weinreb et al., 2024), economics (Jacquier et al., 2002), and climate science (Ott et al., 2004). Fast and robust tools for state space modeling are crucial to researchers in all of these application areas.

State space models specify a probability distribution over a sequence of observations, y_1, \dots, y_T , where y_t denotes the observation at time t . The key assumption of an SSM is that the observations arise from a sequence of *latent states*, z_1, \dots, z_T , which evolve according to a *dynamics model* (aka transition model). An SSM may also use inputs (aka controls or covariates), u_1, \dots, u_T , to steer the latent state dynamics and influence the observations. For example, in a neuroscience application from Vyas et al. (2020), y_t represents a vector of spike counts from ~ 1000 measured neurons, and z_t is a lower dimensional latent state that changes slowly over time and captures correlations among the measured neurons. If sensory inputs to the neural circuit are known, they can be encoded in u_t . In the computational ethology application of Weinreb et al. (2024), y_t represents a vector of 3D locations for several key points on an animal's body, and z_t is a discrete behavioral state that specifies how the animal's posture changes over time. In both examples, there are two main objectives: First, we aim to infer the latent states z_t that best explain the observed data; formally, this is called *state inference*. Second, we need to estimate the dynamics that govern how latent states evolve; formally, this is part of the *parameter estimation* process. Dynamax provides algorithms for state inference and parameter estimation in a variety of SSMs.

There are a few key design choices to make when constructing an SSM:

- What is the type of latent state? E.g., is z_t a continuous or discrete random variable?
- How do the latent states evolve over time? E.g., are the dynamics linear or nonlinear?
- How are the observations distributed? E.g., are they Gaussian, Poisson, etc.?

Some design choices are so common they have their own names. Hidden Markov models (HMM) are SSMs with discrete latent states, and linear dynamical systems (LDS) are SSMs with continuous latent states, linear dynamics, and additive Gaussian noise. Dynamax supports canonical SSMs and allows the user to construct bespoke models as needed, simply by inheriting from a base class and specifying a few model-specific functions. For example, see the *Creating Custom HMMs* tutorial in the Dynamax documentation.

43 Finally, even for canonical models, there are several algorithms for state inference and parameter
44 estimation. Dynamax provides robust implementations of several low-level inference algorithms
45 to suit a variety of applications, allowing users to choose among a host of models and algorithms
46 for their application. More information about state space models and algorithms for state
47 inference and parameter estimation can be found in the textbooks by Murphy (2023) and
48 Särkkä & Svensson (2023).

49 Statement of need

50 Dynamax is an open-source Python package for state space modeling. Since it is built with JAX
51 (Bradbury et al., 2018), it supports just-in-time (JIT) compilation for hardware acceleration on
52 CPU, GPU, and TPU machines. It also supports automatic differentiation for gradient-based
53 model learning. While other libraries exist for state space modeling in Python (Corenflos
54 & Särkkä, 2021; Johnson, 2020; Linderman et al., 2020; Seabold & Perktold, 2010; Weiss
55 et al., 2024) and Julia (Dalle, 2024), Dynamax provides a diverse combination of low-level
56 inference algorithms and high-level modeling objects that can support a wide range of research
57 applications in JAX. Additionally, Dynamax implements parallel message passing algorithms
58 that leverage the associative scan (a.k.a., parallel scan) primitive in JAX to take full advantage
59 of modern hardware accelerators. Currently, these primitives are not natively supported in
60 other frameworks like PyTorch. While various subsets of these models and algorithms may be
61 found in other libraries, Dynamax is a “one stop shop” for state space modeling in JAX.

62 The API for Dynamax is divided into two parts: a set of core, functionally pure, low-level
63 inference algorithms, and a high-level, object oriented module for constructing and fitting
64 probabilistic SSMs. The low-level inference API provides message passing algorithms for several
65 common types of SSMs. For example, Dynamax provides JAX implementations for:

- 66 ■ Forward-Backward algorithms for discrete-state hidden Markov models (HMMs),
- 67 ■ Kalman filtering and smoothing algorithms for linear Gaussian SSMs,
- 68 ■ Extended and unscented generalized Kalman filtering and smoothing for nonlinear and/or
- 69 non-Gaussian SSMs, and
- 70 ■ Parallel message passing routines that leverage GPU or TPU acceleration to perform
- 71 message passing in $O(\log T)$ time on a parallel machine (Hassan et al., 2021; Särkkä
- 72 & García-Fernández, 2020; Stone, 1975). Note that these routines are not simply
- 73 parallelizing over batches of time series, but rather using a parallel algorithm with
- 74 sublinear depth or span.

75 The high-level model API makes it easy to construct, fit, and inspect HMMs and linear Gaussian
76 SSMs. Finally, the online Dynamax documentation and tutorials provide a wealth of resources
77 for state space modeling experts and newcomers alike.

78 Dynamax has supported several publications. The low-level API has been used in machine
79 learning research (Chang et al., 2023; Lee et al., 2023; Zhao & Linderman, 2023). Special
80 purpose libraries have been built on top of Dynamax, like the Keypoint-MoSeq library for
81 modeling animal behavior (Weinreb et al., 2024) and the Structural Time Series in JAX library,
82 sts-jax (Li & Murphy, 2022). Finally, the Dynamax tutorials are used as reference examples in
83 a major machine learning textbook (Murphy, 2023).

84 Acknowledgements

85 A significant portion of this library was developed while S.W.L. was a Visiting Faculty Researcher
86 at Google and P.C., G.H.D., A.K., and X.L. were Google Summer of Code participants.

References

- Bradbury, J., Frostig, R., Hawkins, P., Johnson, M. J., Leary, C., Maclaurin, D., Necula, G., Paszke, A., VanderPlas, J., Wanderman-Milne, S., & Zhang, Q. (2018). *JAX: Composable transformations of Python+NumPy programs* (Version 0.3.13). <http://github.com/google/jax>
- Chang, P. G., Durán-Martín, G., Shestopaloff, A., Jones, M., & Murphy, K. P. (2023). Low-rank extended Kalman filtering for online learning of neural networks from streaming data. In S. Chandar, R. Pascanu, H. Sedghi, & D. Precup (Eds.), *Proceedings of the 2nd conference on lifelong learning agents* (Vol. 232, pp. 1025–1071). PMLR. <https://doi.org/10.48550/arXiv.2305.19535>
- Corenflos, A., & Särkkä, S. (2021). *Code companion for Bayesian Filtering and Smoothing* (Version 1.0). <https://github.com/EEA-sensors/Bayesian-Filtering-and-Smoothing>
- Dalle, G. (2024). HiddenMarkovModels.jl: Generic, fast and reliable state space modeling. *Journal of Open Source Software*, 9(96), 6436.
- Durbin, R., Eddy, S. R., Krogh, A., & Mitchison, G. (1998). *Biological sequence analysis: Probabilistic models of proteins and nucleic acids*. <https://doi.org/10.1017/cbo9780511790492>
- Hassan, S. S., Särkkä, S., & García-Fernández, Á. F. (2021). Temporal parallelization of inference in hidden Markov models. *IEEE Transactions on Signal Processing*, 69, 4875–4887. <https://doi.org/10.1109/TSP.2021.3103338>
- Jacquier, E., Polson, N. G., & Rossi, P. E. (2002). Bayesian analysis of stochastic volatility models. *Journal of Business & Economic Statistics*, 20(1), 69–87. <https://doi.org/10.1198/073500102753410408>
- Johnson, M. J. (2020). *PyHSMM: Bayesian inference in HSMMs and HMMs* (Version 0.0.0). <https://github.com/mattjj/pyhsmm>
- Lee, H. D., Warrington, A., Glaser, J., & Linderman, S. (2023). Switching autoregressive low-rank tensor models. *Advances in Neural Information Processing Systems*, 36, 57976–58010. <https://doi.org/10.48550/arXiv.2306.03291>
- Li, X., & Murphy, K. (2022). *Structural time series (STS) in JAX*. <https://github.com/probml/sts-jax>
- Linderman, S., Antin, B., Zoltowski, D., & Glaser, J. (2020). *SSM: Bayesian Learning and Inference for State Space Models* (Version 0.0.1). <https://github.com/lindermanlab/ssm>
- Murphy, K. P. (2023). *Probabilistic machine learning: Advanced topics*. MIT Press. <http://probml.github.io/book2>
- Ott, E., Hunt, B. R., Szunyogh, I., Zimin, A. V., Kostelich, E. J., Corazza, M., Kalnay, E., Patil, D., & Yorke, J. A. (2004). A local ensemble Kalman filter for atmospheric data assimilation. *Tellus A: Dynamic Meteorology and Oceanography*, 56(5), 415–428. <https://doi.org/10.3402/tellusa.v56i5.14462>
- Patterson, T. A., Thomas, L., Wilcox, C., Ovaskainen, O., & Matthiopoulos, J. (2008). State-space models of individual animal movement. *Trends in Ecology & Evolution*, 23(2), 87–94. <https://doi.org/10.1016/j.tree.2007.10.009>
- Särkkä, S., & García-Fernández, Á. F. (2020). Temporal parallelization of Bayesian smoothers. *IEEE Transactions on Automatic Control*, 66(1), 299–306. <https://doi.org/10.1109/TAC.2020.2976316>
- Särkkä, S., & Svensson, L. (2023). *Bayesian filtering and smoothing* (Vol. 17). Cambridge University Press. <https://doi.org/10.1017/CBO9781139344203>
- Seabold, S., & Perktold, J. (2010). Statsmodels: Econometric and statistical mod-

- 133 eling with python. *9th Python in Science Conference*. [https://doi.org/10.25080/](https://doi.org/10.25080/majora-92bf1922-011)
134 [majora-92bf1922-011](https://doi.org/10.25080/majora-92bf1922-011)
- 135 Stone, H. S. (1975). Parallel tridiagonal equation solvers. *ACM Transactions on Mathematical*
136 *Software (TOMS)*, 1(4), 289–307. <https://doi.org/10.1145/355656.355657>
- 137 Vyas, S., Golub, M. D., Sussillo, D., & Shenoy, K. V. (2020). Computation through neural
138 population dynamics. *Annual Review of Neuroscience*, 43(1), 249–275. [https://doi.org/10.](https://doi.org/10.1146/annurev-neuro-092619-094115)
139 [1146/annurev-neuro-092619-094115](https://doi.org/10.1146/annurev-neuro-092619-094115)
- 140 Weinreb, C., Pearl, J. E., Lin, S., Osman, M. A. M., Zhang, L., Annapragada, S., Conlin,
141 E., Hoffmann, R., Makowska, S., Gillis, W. F., Jay, M., Ye, S., Mathis, A., Mathis, M.
142 W., Pereira, T., Linderman, S. W., & Datta, S. R. (2024). Keypoint-MoSeq: Parsing
143 behavior by linking point tracking to pose dynamics. *Nature Methods*, 21(7), 1329–1339.
144 <https://doi.org/10.1038/s41592-024-02318-2>
- 145 Weiss, R., Du, S., Grobler, J., Cournapeau, D., Pedregosa, F., Varoquaux, G., Mueller, A.,
146 Thirion, B., Nouri, D., Louppe, G., Vanderplas, J., Benediktsson, J., Buitinck, L., Korobov,
147 M., McGibbon, R., Lattarini, S., Niculae, V., Gramfort, A., Lebedev, S., ... Rockhill, A.
148 (2024). *Hmmlearn* (Version 0.3.2). <https://github.com/hmmlearn/hmmlearn>
- 149 Zhao, Y., & Linderman, S. (2023). Revisiting structured variational autoencoders. *International*
150 *Conference on Machine Learning*, 42046–42057. [https://doi.org/10.48550/arXiv.2305.](https://doi.org/10.48550/arXiv.2305.16543)
151 [16543](https://doi.org/10.48550/arXiv.2305.16543)

DRAFT